2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

A Method for Model-Reduction of Nonlinear Building Thermal Dynamics

Siddharth Goyal and Prabir Barooah

***Abstract*— We propose a method for model-reduction of a class of non-linear models that are relevant to modeling thermal dynamics of multi-zone buildings. These models can have large state-space dimension even for a moderate number of zones. Reduced order models of building thermal dynamics can be useful to model-based control for improving energy efficiency, especially to computationally intensive ones such as Model Predictive Control (MPC). Although there are a number of well-developed techniques for model reduction of LTI systems, the same cannot be said about non-linear systems. The method we propose exploits the linear portion of the model to compute a transformation (by using balanced realization) and a specific sparsity pattern of the non-linear portion to obtain the reduced order model. Simulations are presented with a four-zone building model, which show that the prediction of the zone temperatures and humidity ratios by the reduced model is quite close to that from the full-scale model, even when substantial reduction of model order is specified.**

I. INTRODUCTION

Buildings are one of the primary consumers of energy in worldwide, and particularly in the United States. Inefficiencies in the building technologies, particularly in operating the HVAC (heating, ventilation and air conditioning) systems cause a significant fraction of energy consumed by building to be wasted. Part of the reason is that HVAC systems are operated on a pre-designed schedule of zone-wise temperature set points that local PID controllers in every zone try to maintain. There is growing interest in developing techniques that seek to compute the optimal building control signals to minimize building-wide energy consumption, such as MPC (model predictive control) [18].

Control techniques that seek to determine the optimal control signals to minimize energy consumption require a model of the building's thermal dynamics, i.e., a model that relate the control signals to the space temperatures (average temperatures of the zones of the building). A first-principles based model of the thermal dynamics of a building can be constructed from energy and mass balance equations by assuming well-mixed air in the zones, so that each zone is characterized by a single space temperature value. The air in the zone is modeled as a thermal capacitor and each solid surface separating two zones (walls, windows, etc.) is modeled as a thermal resistor and capacitor. Phenomena such as non-uniform mixing of air inside a room, heat transfer due to convection, buoyancy driven flows, etc. are neglected. This results in a lumped parameter model that in the form of a system of coupled non-linear ODEs. Still, such a model suffers from large state space dimension, which increases quite steeply as the the number of zones increases. For instance, a 4-zone building model has a state dimension of about 40. Depending on the number of floors, zones, and their layout, the model of a building with 100 zones can have a state dimension that exceeds 1000. Thus, control signal computations with model-based control techniques, especially ones such as MPC that requires on-line optimization, becomes challenging with such a model.

In this paper, we propose a method for reduction of the order of such models of building thermal dynamics. A secondary contribution of the paper is the first-principles based model of building thermal dynamics, which we call the "full-scale model". This full-scale model is a combination of a linear-time-invariant system and a non-linear component. The linear component comes from the lumped RC network models of solid surfaces of the building (walls, windows, floors and ceilings), while the non-linear part comes from the non-linear dependencies of enthalpy on mass flow rate of air, humidity, and temperature. The outputs of the reduced-order model are the space temperatures and humidities of the zones, and the inputs are kept same as those of the full-scale model. Since the number of outputs is $2N$ for a N-zone building (temperatures and humidities of the N zones), the state dimension of the reduced model, though user-specified, has a minimum possible value of $2N$.

Although there are a number of well-developed techniques for model reduction of linear systems, model reduction of non-linear systems is much less developed. A few notable work in this area are [16], [17], [15], with the stronger results obtained for k-power bilinear systems [15]. Our method avoids the difficulties in computing the energy function that is required for the method of [16], and does not require simulation data as is needed by the method of [17]. The proposed method leverages existing methods for model reduction of LTI system. A coordinate transformation is first carried out by using only the linear portion of the thermal model by applying standard balanced realization technique. The specific sparsity structure of the non-linear portion is then exploited to truncate the state of the full-scale model in the transformed coordinates. Although we use balanced realization to compute the transformation, other methods of linear model reduction that lead to a state transformation of the LTI part, such as that in [11], may be potentially used as well. Simulations are provided to show the predictive power of the proposed method.

S. Goyal and Prabir Barooah are with the Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, Florida, USA, email:{siddgoya,pbarooah}@ufl.edu. This work has been supported by the National Science Foundation by Grants CNS-0931885 and ECCS-0955023.

The rest of the paper is organized as follows. Section II describes briefly the full-scale model of building thermal dynamics. The proposed method for order reduction of this model is described in Section III. Results from simulations are presented in Section IV.

II. Full-scale Model of Building Thermal Dynamics

A common configuration of HVAC systems used in modern buildings is the so-called variable-air-volume (VAV) system, where a building is divided into a number of "zones". The schematic of a building with a VAV system with four zones is shown in Figure 1. One or more air handling units (AHUs) condition a mixture of return air (RA) and outside air (OA) by passing it across a cooling coil where the temperature and humidity of the air is brought into desired values. The flow rate of conditioned air supplied into each of the zones is controlled through dampers in the "VAV boxes" of the respective zones. The dampers in a zone are commanded by a local controller that computes its control command based on the space temperature of that zone and the desired temperature value.

In practice, the control inputs to the system are the fan speeds, the damper positions of the VAV boxes, and the flow rate of chilled water through the cooling coil in the AHU, which determine the flow rates and temperature of the supply air into the zones. In this paper, we ignore the "upstream" side of the dynamics and concentrate on modeling the "downstream" side (see Figure 1). The downstream part of the building's thermal dynamics is affected by the following externally specified variables, which are the inputs to the model: (i) characteristics of the supply air (flow rate, temperature and humidity), (ii) thermal loads due to occupants, equipments and lights, (iii) thermal loads due to solar radiation, (iv) outside temperatures and humidity.

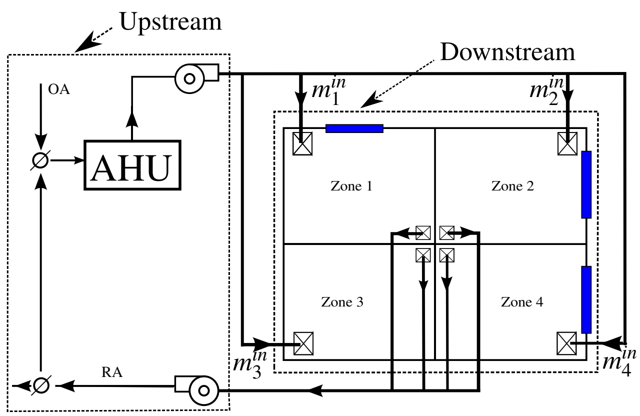


Fig. 1. A schematic of a 4-zone building HVAC system.

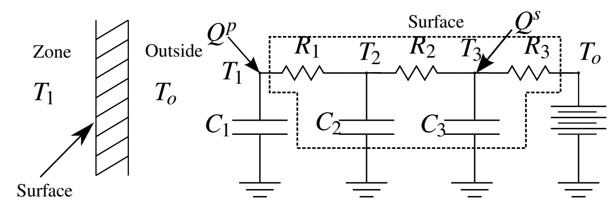
The reason for ignoring the upstream side, which includes the AHU dynamics, is twofold. First, the size of the downstream model increases very fast with the number of zones, but the size of the upstream model increases only with the number of AHUs which is typically small for a large building. Thus, the downstream model requires model reduction techniques much more than the upstream model. Second, the AHU has the fastest dynamics in the HVAC system, with a time constant of about a minute [12], whereas the thermal dynamics of the zones are far slower with time constants in the order tens of minutes [13] to hours [14]. As a result, it may be possible to replace the dynamics of the AHUs and ducts by static gains without significant loss of accuracy, as long as the system does not operate in the unstable parts of the fan and AHU characteristics.

The main variables of interest that the model is required to predict are $T_1,\ldots T_N, W_1,\ldots W_N$, where T_i and W_i are the temperature and humidity ratio in the i^{th} zone respectively. The vector v of input signals to the building thermal dynamics is defined below ($i = 1,\ldots,N$).

$$v = [m_1^{in},\ldots,m_N^{in}, W_1^{in},\ldots,W_N^{in}, T_1^{in},\ldots T_N^{in}, Q_1^p,\ldots Q_N^p, \\ Q_1^s,\ldots Q_N^s, T_0, W_{OA}]^T, \tag{1}$$

where W_i^{in} is the humidity ratio of conditioned air entering into the i^{th} zone, T_i^{in} is the temperature of conditioned air entering into the i^{th} zone, Q_i^p is the rate of heat generated by people in the i^{th} zone, Q_i^s is the solar radiation entering in the i^{th} zone, T_0 and W_{OA} are the temperature and humidity ratio of outside air respectively.

As discussed in Section I, only conductive heat transfer in considered. A model of a building's thermal dynamics can be constructed by combining elemental models of conductive interaction between two zones separated by a solid surface such as a wall. The most extensively used version of lumped parameter models for conduction is called the 3R2C model, in which a solid surface separating two volumes of air is modeled by a network of three resistors and two capacitors, as shown in Figure 2. Such RC network models are well established and experimentally validated [1], [14]. The temperature of each zone is also assigned a node and capacitance.



(a) surface element (b) RC-network model

Fig. 2. A lumped RC-network model for conductive interaction between the outside and an internal air space separated by a single surface.

For a building consisting of a number of *surface elements* (wall, window, ceiling and floor), the lumped RC model for each surface element can be concatenated to produce a RC-network of the entire building that models the conduction among the zones. We now briefly review the model structure; the interested reader is referred to [10] for details. It is assumed that air density remains constant and uniform in the zone, so that the thermal capacitance of a zone does not change with time. The number of *nodes* in the RC network model of a multi-zone building is denoted by n. The model for the dynamics of the space temperatures in a building, when considering only conduction across surface elements, is now a LTI system:

$$C\dot{T} = A^a T + B^a U, \tag{2}$$

where $T = [T_1, \ldots, T_n]^T$ is the state vector that contains the temperatures of not only the zones but also of the internal nodes of the surface elements, and the input vector $U = [T_0, Q_1^p, \ldots, Q_N^p, Q_1^s, \ldots, Q_N^s]^T$ contains the outside temperature as well as the heat gains from solar radiation and occupants of all the zones, some of which can be 0. These nodes are indexed in such a way that first N components of T correspond to the space temperature of N zones, and remaining $n-N$ states correspond to the internal node temperatures of the surface elements. The elements of the matrices $C, A^a \in \mathbb{R}^{n\times n}$ and $B^a \in \mathbb{R}^{n\times(2N+1)}$ are determined by the capacitances and resistances of the zones as well as that of the internal nodes of all the surface elements.

The model above does not take into account the energy exchange between the zones and the outside due to the supplied conditioned air and extraction of the return air. To accommodate these effects, extra term that accounts for the enthalpy of the air is needed. The overall dynamics of T_i, the temperature of zone i, can be expressed as

$$C_i\dot{T}_i = A^a_{i*}T + B^a_{i*}U + m_i^{in} h_i^{in}(T_i^{in}, W_i^{in}) - m_i^{out} h_i^{out}(T_i, W_i) \\ i = 1, 2, \ldots N \tag{3}$$

where C_i is the thermal capacity of the i^{th} zone, A^a_{i*} and B^a_{i*} denote the i^{th} row of matrices A^a and B^a, respectively, $m_i^{out}(t)$ is the flow rate of air leaving zone i, $h^{in}(\cdot)$ is the enthalpy of the incoming (supply) air, and $h^{out}(\cdot)$ is the enthalpy of the outgoing air. It is assumed that air leaving a zone through ventilation ducts has the same temperature and humidity ratio as air present in the zone. The enthalpies for incoming and outgoing air in (3) can be computed from psychometric equations [4] as

$$h_i^{in} = C_{pa}T_i^{in} + W_i^{in}(h_{we} + C_{pw}T_i^{in}) \tag{4}$$

$$h_i^{out} = C_{pa}T_i + W_i(h_{we} + C_{pw}T_i) \tag{5}$$

where C_{pa} is specific heat capacity of air at constant pressure, h_{we} is the evaporation heat of water at 0°C, C_{pw} is specific heat capacity of water vapor at constant pressure. The humidity ratio W_i is directly tied to the occupancy of zone i due to perspiration. Humidity dynamics can be derived from mass balance and gas laws as

$$\frac{dW_i}{dt} = \frac{R_g T_i n_i^p \omega_{H_2O}}{V_i P_i} + \frac{R_g T_i (W_i^{in} - W_i) m_i^{in}}{V_i P_i (1 + W_i^{in})} \tag{6}$$

where R_g is ideal gas constant and P_i, V_i are the pressure and volume of the i^{th} zone, respectively [10]. Eq. (6) can be compactly written as $\dot{W} = g(T_z, W)$, where $T_z = [T_1 \;\; T_2 \;\ldots\; T_N]^T$ and $W = [W_1 \;\; W_2 \;\ldots\; W_N]^T$. Combining (3) - (6), the full-scale model of thermal dynamics in a building is obtained:

$$\dot{T} = AT + BU + f(T, W, v) \tag{7}$$

$$\dot{W} = g(T_z, W, v), \tag{8}$$

where A and B are matrices of dimension $n\times n$ and $n\times(2N+1)$ respectively, $f(T, W, v)$ is the nonlinear part in (3) that captures the enthalpy difference of supplied conditioned and outgoing return air. Note that U is a sub-vector of v, which is the vector of all inputs specified in (1). It is also important to note that f has a special structure; only its first N entries are potentially non-zero, which correspond to thermal loads of the N zones. The remaining entries of f are zeros. This fact will be useful in the proposed model reduction method.

III. PROPOSED MODEL REDUCTION METHOD

We start with a brief review of the classical balanced truncation method for LTI systems that is used in the proposed method.

A. *Review of balanced truncation method for LTI system*

Consider a linear time invariant system with a $p\times m$ transfer function $G(s)$ with a minimal realization

$$\dot{x} = Ax + Bu, \;\; y = Cx + Du \tag{9}$$

where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}, C \in \mathbb{R}^{p\times n}$ and $D \in \mathbb{R}^{p\times m}$. Consider a transformation $x_b = Rx$ which gives us the transformed realization

$$\dot{x_b} = A_b x_b + B_b u, \;\; y_b = y = C_b x_b + Du, \\ A_b = RAR^{-1}, \;\; B_b = RB, \;\; C_b = CR^{-1}. \tag{10}$$

This is called balanced realization if R is chosen in a way that controllability and observability Gramians are both equal and diagonal [8]. Suppose we want to reduce the full-scale n^{th} order system (10) to a r^{th} order system. Decompose A_b, B_b, C_b as

$$A_b = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}, \; B_b = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}, \; C_b = \begin{bmatrix} C_1^T \\ C_2^T \end{bmatrix}^T \tag{11}$$

where $A_{11} \in \mathbb{R}^{r\times r}, A_{12} \in \mathbb{R}^{(n-r)\times r}, A_{21} \in \mathbb{R}^{(n-r)\times r}, A_{22} \in \mathbb{R}^{(n-r)\times(n-r)}, B_1 \in \mathbb{R}^{r\times m}, B_2 \in \mathbb{R}^{(n-r)\times m}, C_1 \in \mathbb{R}^{p\times r}$ and $C_2 \in \mathbb{R}^{p\times(n-r)}$. The system

$$\dot{x_r} = A_{11}x_r + B_1 u, \;\; y_r = C_1 x_r + Du \tag{12}$$

is a reduced order model of (9), where states corresponding to the $n-r$ smallest eigenvalues of the controllability and observability gramians are ignored [9], [8].

B. *Application of balanced truncation to nonlinear building thermal model*

Recall that the temperature dynamics of the building thermal model is

$$\dot{T} = AT + BU + f(T, W, v) \tag{13}$$

where $T \in \mathbb{R}^n$ contains the n temperature states. Since the vector T contains the temperatures of the internal nodes of the surface elements, $n > N$, where N is the number of zones. We re-index the entries of T into "zone temperatures" and "internal node temperatures" to obtain

$$T = \begin{bmatrix} T_z^T, & T_n^T \end{bmatrix}^T \tag{14}$$

where $T_n \in \mathbb{R}^{(n-N)}$ is the vector of temperature of the internal nodes of surface elements. Due to the special structure of f, (the first N entries being non-zero and remaining entries being zero) f can be re-indexed as

$$f(T,W,v) = [f_a^T(T,W,v)\ 0^T_{(n-N)\times 1}]^T, \text{ where } f_a \in \mathbb{R}^N \quad (15)$$

We now introduce a fictitious output of the following form:

$$Y = CT, \quad C \in \mathbb{R}^{p\times n}(p \geq N) \quad (16)$$

with the constraint that Y contains T_z as a sub-vector. With appropriate indexing, C can be decomposed as

$$C = \begin{bmatrix} I_{N\times N} & 0_{N\times(n-N)} \\ C_z & C_n \end{bmatrix} \quad (17)$$

where I is an identity matrix, $C_z \in \mathbb{R}^{(p-N)\times N}$ and $C_n \in \mathbb{R}^{(p-N)\times(n-N)}$. Using (14) and (17), Y can be expressed as

$$Y = \begin{bmatrix} T_z \\ C_zT_z + C_nT_n \end{bmatrix} := \begin{bmatrix} Y_z \\ Y_n \end{bmatrix} \quad (18)$$

where $Y_z = T_z \in \mathbb{R}^N$ and $Y_n \in \mathbb{R}^{(n-N)}$. Combining (13) -(18), overall dynamics can be rewritten as

$$\begin{bmatrix} \dot{T} \\ \dot{W} \end{bmatrix} = \begin{bmatrix} AT + BU + [f_a^T(Y_z,W,v)\ \ 0^T_{(n-N)\times 1}]^T \\ g(Y_z,W,v) \end{bmatrix} \quad (19)$$
$$Y = CT$$

Note that the nonlinear function $f(T,W,v)$ is transformed into another nonlinear function $[f_a^T(Y_z,W,v)\ \ 0^T_{(n-N)\times 1}]^T$. The new nonlinear function $f_a^T(Y_z,W,v)$ uses output Y_z instead of the temperature state T. This nonlinear function transformation is possible only if the output vector Y contains T_z.

Let $T_b := RT$, where $R \in \mathbb{R}^{n\times n}$ the co-ordinate transformation that leads to a balanced realization of the system $\dot{T} = AT + BU$, where A, B are the corresponding matrices from (7). Eq. (7)-(8) can now be expressed as

$$\begin{bmatrix} \dot{T}_b \\ \dot{W} \end{bmatrix} = \begin{bmatrix} A_bT_b + B_bU + R[f_a^T(Y_z,W,v)\ \ 0^T_{(n-N)\times 1}]^T \\ g(Y_z,W,v) \end{bmatrix} \quad (20)$$
$$Y = C_bT_b$$
$$\text{where } A_b = RAR^{-1},\ B_b = RB,\ C_b = CR^{-1}$$

Note that the computation of R is solely based on the LTI part of (7). Decomposing $A_b, B_b, C_b, T, f(Y,W,v)$ gives us the following matrices.

$$A_b = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix},\ B_b = \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}$$
$$C_b = \begin{bmatrix} C_1 & C_2 \end{bmatrix},\ R = \begin{bmatrix} R_{11} & R_{12} \\ R_{21} & R_{22} \end{bmatrix}$$

where matrices $A_{11} \in \mathbb{R}^{r\times r}, A_{12}, A_{21} \in \mathbb{R}^{(n-r)\times r}, A_{22} \in \mathbb{R}^{(n-r)\times(n-r)}, B_1 \in \mathbb{R}^{r\times m}, B_2 \in \mathbb{R}^{(n-r)\times m}, C_1 \in \mathbb{R}^{p\times r}, C_2 \in \mathbb{R}^{p\times(n-r)}, R_{11} \in \mathbb{R}^{r\times r}, R_{12}, R_{21} \in \mathbb{R}^{(n-r)\times r}, R_{22} \in \mathbb{R}^{(n-r)\times(n-r)}$. We now truncate the last $n-r$ states of T_b, which leads to the following $(r+N)^{th}$ order reduced system

$$\begin{bmatrix} \dot{T}_k \\ \dot{W}_k \end{bmatrix} = \begin{bmatrix} A_{11}T_k + B_1U + R_{11}h(Y_k,W_k,u) \\ g(Y_k,W_k) \end{bmatrix} \quad (21)$$
$$Y_k = C_1T_k$$

where $h(Y_k,W_k,v) = [f_a^T(Y_z,W_k,v)\ 0]^T \in \mathbb{R}^{r\times 1}$. The notation $f_a^T(Y_k,W_k,v)$ denotes a function that is obtained by replacing the elements of Y_z (W) that appear in the function $f_a(Y_z,W,v)$ by the the corresponding elements of Y_k (W_k). The implicit assumption here is that the effect of the truncated $n-r$ states is not significant in the nonlinear term. Simulation results in next section suggest that this assumption holds well up to a particular order (r) of reduced model. Eq. (21) is the reduced order model of the full-scale system model (7)-(8). In the reduced model, W_k is the vector of zone humidity ratios and the first N entries of Y_k are the space temperatures for the N zones.

Using the transformation $x_b = Rx$, given the initial temperature $T(0)$ and humidity ratio $W(0)$ of the full-scale model, initial value of the state ($[T_k^T(0),\ W_k^T(0)]^T$) can be calculated as

$$\begin{bmatrix} T_k(0) \\ W_k(0) \end{bmatrix} = \begin{bmatrix} [R_{11}\ R_{12}]T(0) \\ W(0) \end{bmatrix}. \quad (22)$$

C. Non-Dimensionalization

Before applying the technique developed in the previous section to the model (7), (8) directly, the states and inputs need to be non-dimensionalized by appropriate scaling in order to achieve numerical robustness. To see the need for this, notice that the input vector U in (7) contains variables such as outside temperature and heat gains from solar radiation and occupants, which differ significantly in magnitudes depending on the units of measurement used. For an LTI model $\dot{x} = Ax + Bu$, if two input signals have equal effect on the state but one has a much higher typical magnitude than the other, the entry(-ies) of the B matrix corresponding to the larger input is likely to be smaller than those that correspond to the smaller input. In such a situation balanced truncation may incorrectly determine certain inputs to have little effect on the output. Effect of inputs on outputs should not depend on the choice of units of measurements, and non-dimensionalizing the equations of the model before model reduction ameliorates such numerical issues.

Therefore, we scale the variables T, T_o, Q^s and Q^p as

$$\bar{T} = \frac{T}{T(0)},\quad \bar{T}_o = \frac{T_o}{T_{o_{avg}}},\quad \bar{Q}^s = \frac{Q^s}{Q^s_{avg}},\quad \bar{Q}^p = \frac{Q^p}{Q^p_{avg}}, \quad (23)$$

where $T_{o_{avg}}$ is the average of maximum and minimum of the outside temperatures range expected, Q^s_{avg} is the average of maximum and minimum heat gain of a zone from solar radiation, and Q^p_{avg} is the average of maximum and minimum heat generated by people in a zone. Eq. (7) can now be re-expressed in terms of the non-dimensional variables defined above, which is denoted as

$$\dot{\bar{T}} = A_s\bar{T} + B_s\bar{U} + f_s(\bar{T},W,\bar{v}) \quad (24)$$

where $\bar{U} = [\bar{T}_o\ \ \bar{Q}^{s^T}\ \ \bar{Q}^{p^T}]^T$, and $\bar{v}$ is the scaled counterpart of v. Instead of applying balanced transformation to the LTI part of (7), it is applied to the LTI part of (24) and the transformation matrix R described in Section III-B is obtained. This new R matrix so obtained is used in the full-scale model defined in (7) and rest of procedure is same as described in Section III-B.

IV. Simulation Results

Simulations are carried out for a four-zone building that is shown schematically in Figure 1. All four zones have an equal floor area of $25m^2$, each wall is 5 meters wide by 3 meters tall. This provides a volumetric area of $75m^3$ for each zone. Zone 1 has a small window ($5m^2$) on the north facing wall, whereas zones 2 and 4 have a larger window ($7m^2$ each) on the east facing wall. Zone 3 does not have a window. Wall thermal resistances and capacitances are obtained from Carrier's Hourly Analysis Program (HAP)[6]. The HVAC system used for both the buildings is designed to supply maximum flow rate of 0.25 kg/s per zone at the temperature of $12.78°C$. These design choices were made after consulting with a HVAC expert. The number of people in a zone is chosen as a random integer that is uniformly distributed between 0 and 4. Outside temperature, outside humidity ratio and solar radiation data is obtained for a summer day (05/24/1996) of Gainesville, Fl [5].

Numerical results presented here are obtained from simulations conducted in MATLAB©using ode45. The inputs in the vector U are kept constant for every 10 minute intervals. A PI controller for each zone is used in the full-scale model to determine the flow rates of conditioned air to track the desired zone temperatures, which are set as $19°C$ for all the zones. The mass flow rates computed by the PI controllers are used as inputs to the reduced order model. All temperatures and humidity ratios are initialized at $24°C$ and 0.01 respectively. In figures and figure captions, superscript r represents the results obtained from reduced order model and legends 1, 2, 3 and 4 represent the results for the 1^{st}, 2^{nd}, 3^{rd} and 4^{th} zone, respectively. Inputs such as outside temperature, outside humidity ratio, mass flow rates (obtained from the PI controller) and total internal loads are shown in Figure 3.

The full-scale model for the four-zone building has 40 states. We applied the proposed method to construct two reduced order models for this system: (i) one with 14 states and (ii) one with 8 states. For a four zone building, 8 is the minimum possible order using the proposed method. Figures 4 and 5 show the space temperatures and humidity ratios for the 14^{th}-order reduced model. It is clear from Figure 4 that the temperature and humidity ratio predictions by the reduced model are close to the predictions by the full-scale model for all the zones. Predictions by the 8^{th} order reduced model are shown in Figure 6 and Figure 7. Temperature predictions by the 8^{th} order reduced model show larger error in both transient and steady state behavior. However, humidity ratio predictions are close to those by the full-scale model, as seen from Figure 7. It seems to suggest that the effect of temperature variation on humidity ratio is not large. Comparing the error in zone temperature predictions by the 14^{th} and 8^{th} order reduced models illustrates the compromise between prediction accuracy and model order.

V. Conclusion

This paper presents a method for model reduction of a class of non-linear systems that model conductive thermal

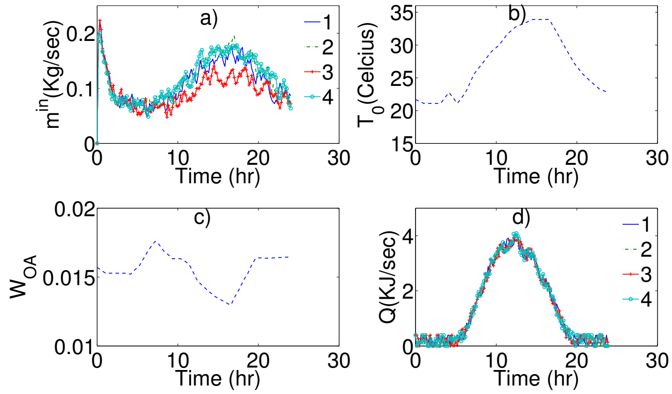


Fig. 3. Inputs to the four-zone building model: mass flow rates (m_i^{in}), outside temperature (T_O), outside humidity ratio (W_{OA}) and total internal load (Q) for the 4 zones.

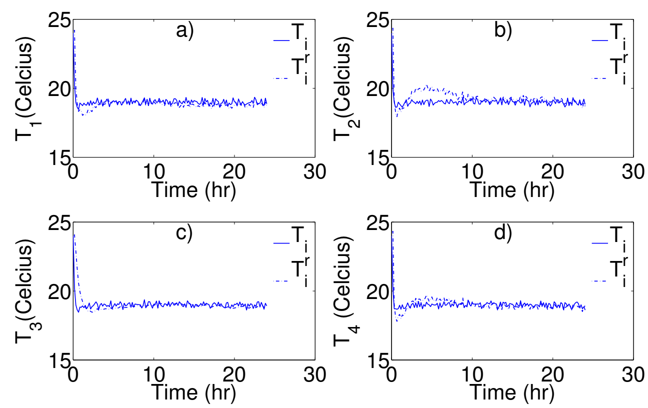


Fig. 4. Space temperatures for full-scale 40^{th} order (T_i) and reduced 14^{th} order (T_i^r) model for a four-zone building, i=1,2,...4.

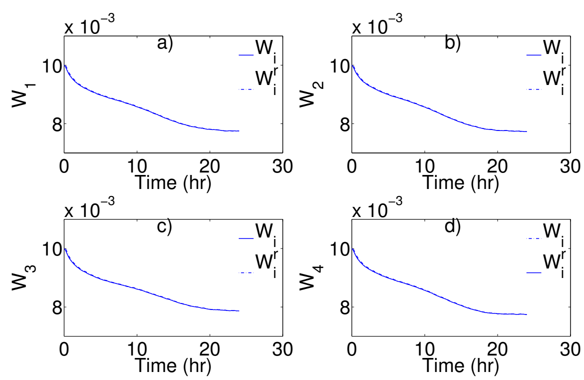


Fig. 5. Zone humidity ratios for full-scale 40^{th} order (W_i) and reduced 14^{th} order (W_i^r) model for a four-zone building, i=1,2,...4.

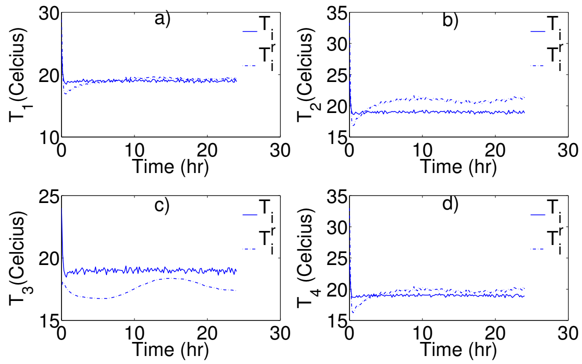


Fig. 6. Space temperatures for full-scale 40^{th} order (T_i) and reduced 8^{th} order (T_i^r) model for a four-zone building, i=1,2,...4.

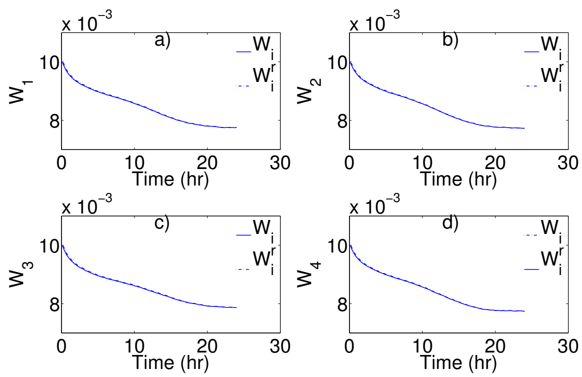


Fig. 7. Zone humidity ratios for full-scale 40^{th} order (W_i) and reduced 8^{th} order (W_i^r) model for a four-zone building, i=1,2,...4.

interactions in a multi-zone building. The full-scale model of the building thermal dynamics, which is itself a lumped parameter model, has a larger number of states even for a moderate number of zones. The proposed model reduction technique is seen to work exceedingly well in simulations - the prediction of the zone temperatures and humidities are quite close to the predictions of the full-scale model even with substantial order reduction. It is observed that appropriate scaling of the states of the full-scale model, before applying the reduction method, is crucial for the reduced model to have accurate predictive power. Although we did not report it here due to lack of space, without such scaling the reduced model's predictions are considerably poorer. Since the number of outputs of the model is twice the number of zones, the minimum order of the reduced model achievable by this method is also twice the number of zones. Further work is needed on the method if further order reduction is desired. Another avenue for future research is to provide theoretical guarantees on the difference between the input-output map of the reduced model and the full-scale model.

We finally note that althogh the full-scale model here ignores convection, if convection between pairs of zones can be modeled as RC-networks, the proposed method is applicable for order reduction of a model that contains both convection and conduction effects. Preliminary work on modeling convection with RC-networks is under way [19].

VI. Acknowledgement

The authors gratefully acknowledge Prof. H. H. Ingley's help in determining the specifications of the HVAC system used in the simulations of the four-zone building, and thank Prof. Prashant Mehta for helpful discussions.

References

[1] M.M. Gouda and S. Danaher and C.P. Underwood, Building thermal model reduction using nonlinear constrained optimization, *Building and Environment*, 2001.

[2] M.M. Gouda and C.P. Underwood and S. Danaher, Modelling the robustness properties of HVAC plant under feedback control, *Building Services Engineering Research and Technology*, Vol. 24, Issue 4, pp. 271-280, Nov. 2003.

[3] M. G. Safonov and R. Y. Chiang. A Schur method for balanced-truncation model reduction. *IEEE Trans. Autom. Control,* 34:729-733, 1989.

[4] *ASHRAE Handbook : Fundamentals (SI Edition)*, Chapter 30: Nonresidential Cooling and Heating Load Calculation Procedures, ASHRAE, 2005.

[5] National Solar Radiation Data Base (NSRDB), [Online] `http://rredc.nrel.gov/solar/old_data/nsrdb/1991-2005/tmy3/`

[6] Carrier, *Hourly Analysis Program 4.20a*, 4.20a, 2004.

[7] B. C. Moore. Principal component analysis in linear system: controllability, observability and model reduction.*IEEE Trans. Autom. Control*, AC-26:17-32, 1981.

[8] Kemin Zhou and John Doyle. *Balanced Model Reduction, Essentials of Robust Control*, Prentice Hall, New Jersey, 1998, Chapter 7.

[9] A. Megretski, J. White, L. Daniel, K. Wilcox, *Lecture notes on Model Reduction*, MIT Chapter 5, Course 6.242, Fall 2004.

[10] Siddharth Goyal, Prabir Barooah, Modeling Thermal Dynamics in Multi-Zone Buildings, *Technical Report*, University of Florida, Gainesville, Aug, 2010, [Online]`http://plaza.ufl.edu/siddgoya/Homepage/Publications.html`

[11] Kun Deng, Prabir Barooah, Prashant Mehta and Sean Meyn, Building Thermal Model Reduction via Aggregation of States, *In Proceedings of the American Control Conference*, June 30 - July 2, 2010, pp. 5118-5123,Baltimore, MD.

[12] Bourdouxhe, J.-P., M. Grodent, and J. Lebrun. Reference Guide for Dynamic Models of HVAC Equipment. *American Society of Heating, Refrigerating, and Air-Conditioning Engineers, Inc*, 1998. Atlanta.

[13] Bourhan Tashtoush and M. Molhim and M. Al-Rousan, Dynamic model of an HVAC system for control analysis, *Energy*, 30 (2005), pp. 1729-1745.

[14] M.M. Gouda and S. Danaher and C.P. Underwood, Low-order model for the simulation of a building and its heating system, *Building services energy research technology*, 2000, 21(3) 199-208.

[15] Al-Baiyat SA, Bettayeb M, Al-Saggaf UM. New model reduction scheme for bilinear systems, *International Journal of Systems Science*, 1994; 25:1631-1642.

[16] Scherpen, J.M.A., Balancing for nonlinear systems. *Systems and Control Letters*, 1993, 21(2):143-153.

[17] S. Lall, J. E. Marsden, and S. Glavaski, A subspace approach to balanced truncation for model reduction of nonlinear control systems, *Int. J. Robust Nonlinear Control*, 12 (2002), pp. 519-535.

[18] Oldewurtel, F. , Parisio, A., Jones, C.N., Morari, M., Gyalistras, D., Gwerder, M., Stauch, V., Lehmann, B., Wirth, K., Energy Efficient Building Climate Control using Stochastic Model Predictive Control and Weather Predictions, *In Proceedings of the American Control Conference*, Baltimore, MD, 2010.

[19] Siddharth Goyal, Chenda Liao, and Prabir Barooah, Identification of multi-zone building thermal interaction model from data, *submitted for conference publication*. [Online] `http://plaza.ufl.edu/siddgoya/Homepage/Publications.html`