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Multi-Mode Adaptive Positive Position Feedback: An Experimental Study

Ryan Orszulik and Jinjun Shan

***Abstract*— A vibration suppression strategy is developed for a flexible manipulator with a collocated piezoelectric sensor/actuator pair. A control law is developed based upon positive position feedback and is augmented with an adaptive parameter estimator based on the recursive least squares method to update the first two natural frequencies of the structure online. For the positive position feedback control law, accurate targeting of the modes is critical for vibration control. Experiments are then conducted to show that the controller can be used to suppress the vibrations of a structure with unknown natural frequencies.**

I. INTRODUCTION

Flexible manipulator systems exhibit many advantages over their rigid counterparts. They possess a higher load ratio, and a large increase in the speed of the links is possible. They require less power to produce the same acceleration as the rigid links which have the same load carrying capacity, hence inexpensive and smaller actuators are sufficient. Because of the high performance requirements, consideration of structural flexibility in robots arms is a real challenge. Unfortunately, taking into account the flexibility of the arm leads to the appearance of oscillations at the tips of the links during the motion. These oscillations make the control problems of such systems very difficult. There has been extensive research on active vibration control of flexible systems, see for example [1]. Many control strategies have been used in the control of lightweight flexible structures. These control strategies include, but are not limited to: adaptive control [2], fuzzy logic control [3], H_∞ control [4], and time-optimal control [5].

With the developments in sensor/actuator technologies, many researchers have concentrated on vibration control using smart materials such as shape memory alloys (SMA) and piezoelectric transducers among others. Piezoelectric materials have been applied in structural vibration control as well as in structural acoustics because of their advantages of fast response, large force output and the fact that they generate no magnetic field in the conversion of electrical energy into mechanical motion. A significant number of papers have been written on positive position feedback (PPF) [6], [7], [8], [9]. It has been shown to be a solid vibration control strategy for flexible systems with smart materials, particularly with the PZT (lead zirconium titanate) type of piezoelectric material. However, the effectiveness of PPF deteriorates when the natural frequencies of the structure are poorly known or changing due to, for example, the presence of a tip mass. There has been significantly less work done on adaptive PPF than there has been on just PPF. Kwak *et al*. [10] presented a method for adaptively tuning the controller frequency of the PPF controller for a single mode of a cantilever beam using a gradient update law, and also illustrate their implementation of a real-time controller. Rew *et al*. [11] proposed an adaptive PPF controller for a plate in a fixed-free configuration in which estimated natural frequencies are adjusted at every time step. Baz and Hong [12] presented an adaptive modal PPF controller where the AMPPF controller parameters are adjusted in an adaptive manner in order to follow the performance of an optimal reference model (for a cantilever beam). In previous work by the authors, simulations were conducted for two mode APPF [13] and in [14] experiments were carried out for a single mode APPF controller.

The first two sections provide general derivations of the positive position feedback controller and the adaptive estimator for any number of modes. PPF control is then designed to damp the first two modes of the constrained beam with frequency estimates provided by the adaptive online parameter estimator based upon the recursive least squares method with forgetting factor. Next, the experimental setup is briefly outlined and the experimental results of the controller are shown. Finally, a brief discussion is given and conclusions are given.

II. CONTROLLER DEVELOPMENT

A. Positive Position Feedback

Positive Position Feedback (PPF) was devised by Goh and Caughey in 1985 and is an appropriate control method for an active structure equipped with strain actuators and sensors such as PZT or PVDF. PPF is essentially a second order filter that is used to apply high frequency gain stabilization by improving the frequency roll-off of the system [15]. Alternatively, PPF works by using a second-order system which is forced by the position response of the structure. This response is then fed back to give the force input to the structure.

Considering the scalar case first, PPF can be described by the two coupled differential equations where the first equation describes the structure, and the second describes the compensator [6] as

$$\begin{aligned} \ddot{\xi} + 2\zeta\omega\dot{\xi} + \omega^2\xi &= g\omega^2\eta \\ \ddot{\eta} + 2\zeta_f\omega_f\dot{\eta} + \omega_f^2\eta &= \omega_f^2\xi \end{aligned} \tag{1}$$

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R. Orszulik is a Ph.D. candidate at the Department of Earth and Space Science and Engineering, York University, Toronto, Canada orszulik@yorku.ca

J. Shan is with the Department of Earth and Space Science and Engineering, York University, Toronto, Canada jjshan@yorku.ca

where ξ is the modal coordinate, η is the filter coordinate, ζ and ζ_f are the structural damping and filter damping ratios, ω and ω_f are the structural natural frequency and filter frequency, and g is the scalar gain. It is shown in [6] that the necessary and sufficient condition for stability is

$$0 < g < 1 \tag{2}$$

The next major advantage of PPF is that the transfer function of the controller rolls off quickly (as can be seen from its Bode plot). This is good because it makes the PPF controller well-suited for control of low-frequency modes of a structure with well-separated modes. This is also a major advantage due to the fact that the system will not be influenced by unmodeled high frequency dynamics.

There are three possible output conditions for a PPF controller based upon the choice of controller frequency ω_c [7]:

(1) if $\omega \ll \omega_f$ this is termed active flexibility;
(2) if $\omega \approx \omega_f$ this is termed active damping;
(3) if $\omega \gg \omega_f$ this is termed active stiffness

In order to effectively damp out a structural mode, obviously the case of active damping is required. Thus the controller frequency should be selected to be close to the modal frequency.

More than one mode of vibration in a beam can be damped at a time. In order to damp the first two modes of a vibrating cantilever beam, two PPF controllers are required in parallel where each is tuned to the natural frequency of the mode it is to damp. This means that the frequency of η_1 is chosen to be close to that of ξ_1 while the frequency of η_2 is chosen to be close to that of ξ_2.

For the general multivariable case, the system equations become

$$\begin{aligned} \ddot{\xi} + D\dot{\xi} + \Omega\xi &= C^T G\eta \\ \ddot{\eta} + D_f\dot{\eta} + \Omega_f\eta &= \Omega_f C\xi \end{aligned} \tag{3}$$

where G is the diagonal gain matrix, C is the participation matrix, Ω and Ω_f are the diagonal modal and filter frequency matrices, and D and D_f are the diagonal modal and filter damping matrices. In this case, stability can be guaranteed [6] if and only if

$$\Omega - C^T GC > 0 \tag{4}$$

where greater than zero means positive definite. Another important property of PPF, is that all spillover into uncontrolled or unmodelled modes is stabilizing [6].

A positive position feedback controller is developed in this section for the beam using a single collocated PZT sensor/actuator pair. The dynamic equation of the structure in modal coordinates is

$$\ddot{x} + Z_s\dot{x} + \Omega_s x = S_m^T hu \tag{5}$$

where x is the vector of modal coordinates, Z_s is the damping matrix, Ω_s is the frequency matrix, S_m^T is the matrix of mass normalized eigenvectors of the system, h is the actuator influence matrix, and u is the input to the actuator (voltage in this case). The sensor (or output) equation can be seen as

$$y = p^T S_m x \tag{6}$$

where p is the sensor influence matrix.

The equation describing the controller is given as

$$\ddot{\eta} + Z_f\dot{\eta} + \Omega_f\eta = \Omega_f E y \tag{7}$$

where η is the vector of controller coordinates, Z_f is the controller damping matrix, Ω_f is the controller frequency matrix, and E is the modal participation factor matrix, which will be defined shortly. The actuator input equation is given as

$$u = E^T G\eta \tag{8}$$

where G is the gain matrix.

Since S_m^T is the matrix of mass normalized eigenvectors, the modal participation factor matrix can be defined as

$$E = S_m^T M r \tag{9}$$

where M is the global mass matrix of the system, and r is a matrix of ones with the same number of rows as M, and the number of columns equal to the number of collocated sensor/actuator pairs.

The four equations describing the system, Eqs. (5-8) can be combined into two second order differential equations as

$$\ddot{x} + Z_s\dot{x} + \Omega_s x = S_m^T h E^T G\eta \tag{10}$$

$$\ddot{\eta} + Z_f\dot{\eta} + \Omega_f\eta = \Omega_f E p^T S_m x \tag{11}$$

Now the structure and controller equations will be placed into state space (or first order form) for ease of analysis. The structural equations become

$$\dot{\hat{x}} = A\hat{x} + Bu \tag{12}$$

$$y = C\hat{x} \tag{13}$$

where

$$\begin{aligned} A &= \begin{bmatrix} 0 & I \\ -\Omega_s & -Z_s \end{bmatrix} \\ B &= \begin{bmatrix} 0 \\ S_m^T h \end{bmatrix} \\ C &= \begin{bmatrix} p^T S_m & 0 \end{bmatrix} \end{aligned}$$

and the controller equations become

$$\dot{\hat{\eta}} = \hat{A}\hat{\eta} + \hat{B}y$$

$$u = \hat{C}\hat{\eta}$$

where

$$\begin{aligned} \hat{A} &= \begin{bmatrix} 0 & I \\ -\Omega_f & -Z_f \end{bmatrix} \\ \hat{B} &= \begin{bmatrix} 0 \\ \Omega_f E \end{bmatrix} \\ \hat{C} &= \begin{bmatrix} E^T G & 0 \end{bmatrix} \end{aligned}$$

B. Adaptive Parameter Estimation

Since the structural transfer function is SISO, it can be put in transfer function form through

$$G(s) = \frac{Z(s)}{R(s)} = C(sI - A)B \tag{14}$$

where

$$R(s) = s^n + a_{n-1}s^{n-1} + \ldots + a_1 s + a_0 \tag{15}$$

$$Z(s) = b_m s^m + \ldots + b_1 s + b_0 \tag{16}$$

which allows the adaptive law to be developed generically. Now the system is of the form

$$y = G(s)u = \frac{Z(s)}{R(s)}u \tag{17}$$

where y, u are the output and input of the plant which can also be expressed as [16]

$$y^{(n)} + a_{n-1}y^{(n-1)} + \ldots + a_1\dot{y} + a_0 y = b_m u^{(m)} + \ldots + b_1\dot{u} + b_0 u \tag{18}$$

Lumping all of the unknown parameters into the vector

$$\theta^* = \begin{bmatrix} b_m & \ldots & b_0 & a_{n-1} & \ldots & a_0 \end{bmatrix}^T \tag{19}$$

and filtering both sides of Eq. (18) with a monic Hurwitz polynomial [16] given by

$$\frac{1}{\Lambda(s)} = \frac{1}{s^n + \lambda_{n-1}s^{n-1} + \ldots + \lambda_1 s + \lambda_0} \tag{20}$$

the static parametric model can be obtained as [16]

$$z = \theta^{*T}\phi \tag{21}$$

where

$$z = \frac{s^n}{\Lambda(s)}y \tag{22}$$

$$\theta^* = \begin{bmatrix} b_m & \ldots & b_0 & a_{n-1} & \ldots & a_0 \end{bmatrix}^T \tag{23}$$

$$\phi = \begin{bmatrix} \frac{s^m}{\Lambda(s)}u & \ldots & \frac{1}{\Lambda(s)}u & -\frac{s^{n-1}}{\Lambda(s)}y & \ldots & -\frac{1}{\Lambda(s)}y \end{bmatrix}^T \tag{24}$$

The estimation model can now be defined as

$$\hat{z} = \theta^T\phi \tag{25}$$

where $\hat{z}$ and θ is the estimate of z and θ^* at each time t. The estimation error can then be defined as

$$\varepsilon = \frac{z - \hat{z}}{m_s^2} = \frac{z - \theta^T\phi}{m_s^2} \tag{26}$$

where m_s^2 is referred to as the normalizing signal and is designed to bound ϕ from above [16]. A typical choice for the normalizing signal is

$$m_s^2 = 1 + \alpha\phi^T\phi \tag{27}$$

where $\alpha > 0$.

The cost function is a convex function of θ with a global minimum and is given by

$$\begin{aligned} J(\theta) = \; & \tfrac{1}{2}\int_0^t e^{-\beta(t-\tau)}\frac{[z(\tau) - \theta^T(t)\phi(\tau)]^2}{m_s^2(\tau)}d\tau \\ & + \tfrac{1}{2}e^{-\beta t}(\theta - \theta_0)^T Q_0(\theta - \theta_0) \end{aligned} \tag{28}$$

where $Q_0 = Q_0^T > 0$, $\beta > 0$ are design constants, and $\theta_0 = \theta(0)$ is the initial parameter estimates of the unknowns. This cost function serves to deweight previous data and includes a penalty on the error in the initial guess.

Following the derivation presented by Ioannou *et.al.* [16], the recursive least squares algorithm with forgetting factor is obtained as

$$\dot{\theta} = P\varepsilon\phi \tag{29}$$

$$\dot{P} = \begin{cases} \beta P - P\frac{\phi\phi^T}{m_s^2}P & ||P|| \leq R_0 \\ 0 & \text{otherwise} \end{cases} \tag{30}$$

where $P(0) = P_0 = Q_0^{-1}$. Here, R_0 is a scalar that serves as an upper bound for $||P||$, since in this case, with $\beta > 0$, $P(t)$ may grow without bound.

III. EXPERIMENTAL RESULTS

The experimental setup is shown in Fig. 1 where the flexible manipulator has a collocated piezoelectric sensor/actuator pair. The two piezoelectrics are MIDE QP10W that have been bonded to the beam. One PZT patch can be seen as the sensor, and will output a voltage when the beam undergoes deformation, the other patch will act as the actuator, and will strain based upon the voltage supplied to it by the control system. The sensor data can be acquired from the PZT without any additional circuitry, however, the actuator is driven by the QPA200 high voltage amplifier. This amplifier accepts signals between +/- 10 V and can amplify them to an output range of +/- 200 V. A sample rate is chosen as 1 kHz and the entire process is controlled from Simulink through Quanser's Q8 data acquisition board and QuaRC interface which allows for real-time control.

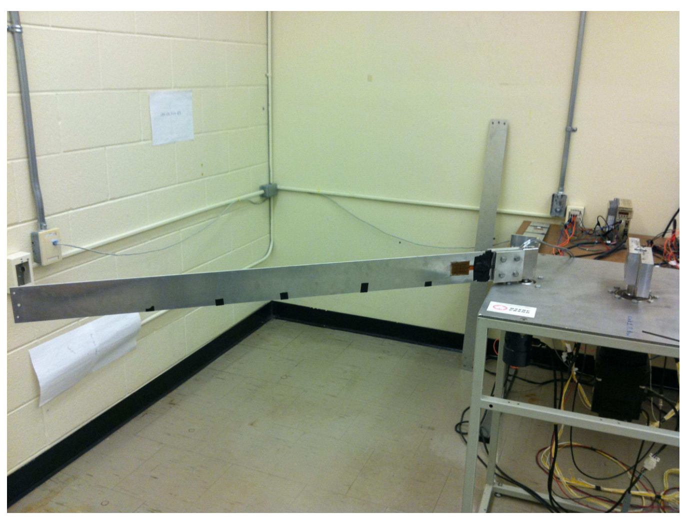

Fig. 1: Single-link flexible manipulator at SDCNLab of York University

From the FFT of the open loop response, the actual natural frequencies of the first two modes can be found as 10.0091 and 60.4882 rad/s respectively, although they are assumed unknown. To excite vibrations in a repeatable manner, the piezoelectric actuator is used to excite vibration using the addition of two sinusoidal signals at the beam's first two natural frequencies along with a noise component for 10 seconds (not shown in the plots). The three parameters for the estimation algorithm are chosen to be: $\beta = 5$, $Q_0 = 10^9\,\mathrm{I}$, and the initial paramter estimate is taken as a vector of ones implying no prior knowledge of the system. During the control run, the estimator finds the first two natural frequencies to be 10.0246 and 60.0935 rad/s respectively, which are very close to the true values. It takes the estimator approximately three seconds to converge on the correct frequency values as can be seen in Figs. 4 and 5.

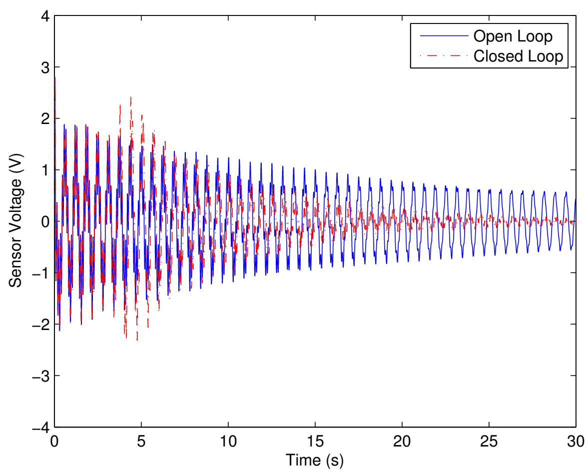

Fig. 2: Piezoelectric sensor voltage due to vibration

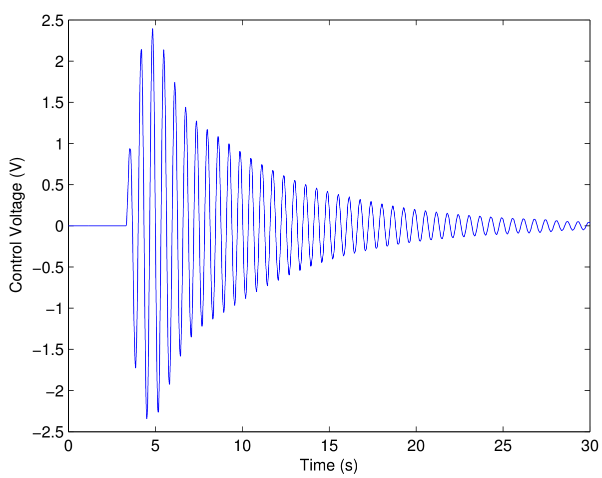

Fig. 3: Voltage supplied by control to amplifier

The PPF control takes approximately 25 seconds to damp out the vibrations as can be seen in Fig. 2, with the corresponding control voltage seen in Fig. 3. This is relatively quick due to the beam's extremely small intrinsic damping and given that free vibration would continue in time into the minute range. One important effect should be discussed in Fig. 2: When the control comes online at approximately

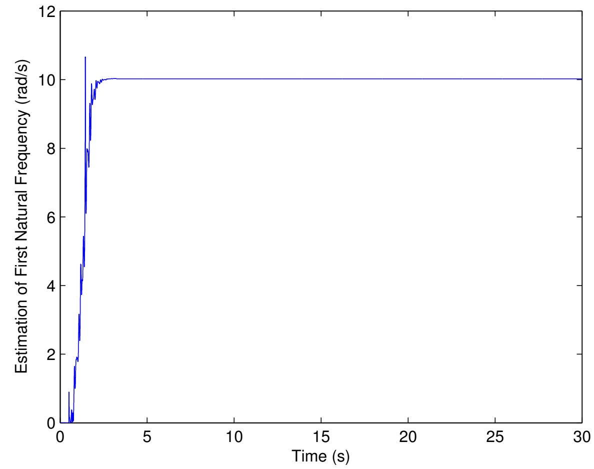

Fig. 4: Estimation of the first natural frequency in rad/s

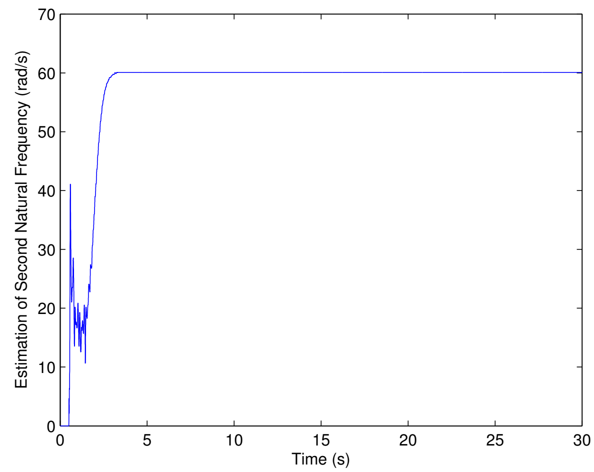

Fig. 5: Estimation of the second natural frequency in rad/s

3 seconds it can be seen that the voltage detected by the piezoelectric sensor rises before being damped out by the PPF control. This is due to the effect of feedthrough, which collocated piezoelectric sensor/actuator pairs are particularly susceptible to [15]. From a mathematical point of view, for the first three seconds, when the control is off, the tranfer function is simply that of a monic vibrating system subjected to an arbitrary impulse. When the control comes on, the transfer function that describes the input-output relationship between the sensor and actuator takes over, however, its denominator remains the same, while its numerator will be altered [17], [18]. Physically, this is due to the fact that for the collocated sensor/actuator pair there will be some direct energy transmission from the actuator to the sensor. Thus the sensor voltage is made up of two components, that of the beam response due to the actuator, and some direct transmission of the force due to the collocation of the actuator [18]. However, this is not a problem for the PPF control law, as it was proven to be stable even in the presence

of feedthrough in [19].

IV. CONCLUSIONS

A positive position feedback controller with an adaptive parameter estimator is developed to estimate the first two natural frequencies of a beam with a collocated piezoelectric sensor/actuator pair. With the proposed adaptive PPF controller, control of the vibrations can be achieved without accurate knowledge of the structure's natural frequencies beforehand. An experimental study of the controller is conducted and the results verified the effectiveness of the proposed approach.

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