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Mean-Square H_∞ Filter Design: Application to a 2DOF Helicopter

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***Abstract*— This paper designs the central finite-dimensional H_∞ filter for linear stochastic systems with integral-quadratically bounded deterministic disturbances, that is suboptimal for a given threshold γ with respect to a modified Bolza-Meyer quadratic criterion including the attenuation control term with the opposite sign. The original H_∞ filtering problem for a linear stochastic system is reduced to the corresponding mean-square H_2 filtering problem, using the technique proposed in [1]. In the example, the designed filter is applied to estimation of the pitch and yaw angles of a two degrees of freedom (2DOF) helicopter.**

I. INTRODUCTION

Over the past two decades, considerable attention has been paid to the H_∞ estimation problem for deterministic and stochastic systems. The seminal papers on H_∞ control ([1]) and estimation ([2], [3], [4]) established a background for consistent treatment of controller/filtering problems in the H_∞ framework. The H_∞ filter design implies that the resulting closed-loop filtering system is robustly stable and achieves a prescribed level of attenuation from the disturbance input to the output estimation error in L_2/l_2-norm. A large number of results on this subject have been reported for systems in the general situation (see, for example, [5]–[23] and references therein). Sufficient conditions for existence of an H_∞ filter, where the filter gain matrices satisfy Riccati equations, were obtained for linear deterministic systems in [4] and linear systems with state delay in [24] or with measurement delay in [25]. However, the criteria of existence and suboptimality of solution for the central H_∞ filtering problems based on the reduction of the original H_∞ problem to the induced H_2 one, similar to those obtained in [1], [4] for linear systems, remain yet undeveloped for linear stochastic systems with integral-quadratically bounded deterministic disturbances.

This paper presents the central (see [1] for definition) finite-dimensional mean-square H_∞ filter for linear stochastic systems, that is suboptimal for a given threshold γ with respect to a modified Bolza-Meyer quadratic criterion including the attenuation control term with the opposite sign. In contrast to results previously obtained for linear systems [4], [24], [25], this paper reduces the original H_∞ filtering problem to the corresponding mean-square H_2 filtering problem, using the technique proposed in [1].

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Designing the central suboptimal mean-square H_∞ filter for linear stochastic systems presents a significant advantage in filtering theory and practice, since (1) it enables one to address filtering problems for linear stochastic time-varying systems, where the linear matrix inequality technique is hardly applicable and the Hamilton-Jacobi-Bellman equation-based methods fail to provide a closed-form solution, (2) the obtained mean-square H_∞ filter is suboptimal, that is, optimal for any fixed γ with respect to the H_∞ noise attenuation criterion, and (3) the obtained mean-square H_∞ filter is finite-dimensional.

It should be commented that the proposed design of the central suboptimal mean-square H_∞ filters for linear stochastic systems with integral-quadratically bounded disturbances naturally carries over from the design of the optimal mean-square H_2 filters for linear stochastic systems with unbounded disturbances (white noises). The entire design approach creates a complete filtering algorithm for handling the linear stochastic time-varying systems with unbounded or integral-quadratically bounded disturbances optimally for all thresholds γ uniformly or for any fixed γ separately. A similar algorithm for linear deterministic systems was developed in [4].

The designed filter is applied to estimation of the pitch and yaw angles of a two degrees of freedom (2DOF) helicopter. The simulation results show a reliable performance of the filter, in particular, the obtained attenuation level is five times less than a given threshold.

The paper is organized as follows. Section 2 presents the mean-square H_∞ filter problem statement for linear stochastic time-varying systems. The central suboptimal mean-square H_∞ filter is designed in Section 3. In Section 4, the designed filter is applied to estimation of the pitch and yaw angles of a two degrees of freedom (2DOF) helicopter. Conclusions are given in Section 5.

II. MEAN-SQUARE H_∞ FILTERING PROBLEM STATEMENT

Let (Ω, F, P) be a complete probability space with an increasing right-continuous family of σ-algebras $F_t, t \geq t_0$, and let $(W_1(t), F_t, t \geq t_0)$ and $(W_2(t), F_t, t \geq t_0)$ be independent Wiener processes. Consider the following linear stochastic time-varying system $\mathscr{S}_1$:

$$\begin{aligned}
dx(t) &= (A(t)x(t) + B(t)u(t) + G(t)\omega(t))dt + b(t)dW_1(t), \\
x(t_0) &= x_0, && (1) \\
dy_1(t) &= C_1(t)x(t)dt + h(t)dW_2(t), && (2) \\
y_2(t) &= C_2(t)x(t) + H(t)\omega(t), && (3) \\
z(t) &= L(t)x(t), && (4)
\end{aligned}$$

where $x(t) \in \mathbb{R}^n$ is the unmeasured state, $u(t) \in \mathbb{R}^l$ is is a known input signal,$y_1(t) \in \mathbb{R}^{m_1}$ and $y_2(t) \in \mathbb{R}^{m_2}$ are the measured observations, $z(t) \in \mathbb{R}^q$ is the output to be estimated, $\omega(t) \in \mathscr{L}_2^s[0,\infty)$ is the deterministic disturbance input, $A(t)$, $B(t)$,$G(t)$, $b(t)$, $C_1(t)$, $h(t)$, $C_2(t)$, $H(t)$, and $L(t)$ are known deterministic continuous time-varying functions of appropriate dimension. The initial condition $x_0 \in R^n$ is a Gaussian random variable such that x_0, $W_1(t) \in \mathbb{R}^{p_1}$, and $W_2(t) \in \mathbb{R}^{p_2}$ are independent. It is assumed that $h(t)h^T(t)$ is a positive definite matrix.

For the system given by (1)–(4), the following assumptions are made over the time interval $[t_0,t_1]$:

- $(A(t),b(t))$ is stabilizable and $(C_1(t),A(t))$ is detectable; $(\mathscr{C}_1)$
- $(A(t),G(t))$ is stabilizable and $(C_2(t),A(t))$ is detectable; $(\mathscr{C}_2)$
- $(A(t),B(t))$ is stabilizable and $(L(t),A(t))$ is detectable, and $(\mathscr{C}_3)$
- $H(t)G^T(t) = 0$ and $H(t)H^T(t)$ is a positive definite matrix. $(\mathscr{C}_4)$

As usual, the first two assumptions ensure that the estimation error, provided by the designed filter, converge to zero [26]. The noise orthogonality condition $H(t)G^T(t) = 0$ is technical and represents the independence between the state and measurement deterministic disturbances. Extensive comments on the assumption $(\mathscr{C}_4)$ can be found in [1].

The filtering problem to be addressed is as follows: develop a central suboptimal mean-square $\mathscr{H}_\infty$ filter for the linear stochastic system $(\mathscr{S}_1)$ as a linear filter based on the observations $\{y_1(s),t_0 \le s \le t\}$ and $\{y_2(s),t_0 \le s \le t\}$ such that the following three requirements are satisfied.

1) The resulting dynamics of the estimation error $E(x(t)) - m(t)$, where $x(t)$ is the state of $(\mathscr{S}_1)$ and $m(t)$ is the mean-square H_∞ estimate produced by the designed filter, is asymptotically stable in the absence of disturbances, $\omega(t) \equiv 0$. Here, $E(x(t))$ denotes the expectation of stochastic process $x(t)$.
2) The variance of the mean-square H_∞ estimate $m(t)$ of the system state $x(t)$, based on the observation process $Y(t) = \{y_1(s), 0 \le s \le t\}$, is equal to the minimum estimation error variance ([27])
$$E[(x(t)-E(x(t)\mid F_t^Y))(x(t)-E(x(t)\mid F_t^Y))^T \mid F_t^Y] \quad (5)$$
at every time moment t. Here, $E[\xi(t) \mid F_t^Y]$ means the conditional expectation of a stochastic matrix process $\xi(t) = (x(t)-E(x(t)\mid F_t^Y))(x(t)-E(x(t)\mid F_t^Y))^T$ with respect to the σ - algebra F_t^Y generated by the observation process $Y(t)$ in the interval $[t_0,t]$.
3) Given a noise attenuation level γ, the $\mathscr{H}_\infty$ noise attenuation condition (6) is ensured. More specifically, for any nonzero disturbance input $\omega(t) \in \mathscr{L}_2^s[0,\infty)$, the inequality
$$\|z(t)-L(t)m(t)\|_2^2 < \gamma^2 \left\{ \|\omega(t)\|_2^2 + E(x^T(t_0))RE(x(t_0)) \right\} \quad (6)$$
holds, where $\|f(t)\|_2^2 := \int_{t_0}^{t_1} f^T(t)f(t)dt$, t_1 is the selected filter horizon, R is a symmetric positive definite matrix, and γ is a given real positive scalar.

III. Central Suboptimal Mean-Square H_∞ Filter Design

The proposed design of the suboptimal mean-square H_∞ filter for linear stochastic systems is based on the general result (see Theorem 3 in [1]) reducing the H_∞ controller problem to the corresponding optimal H_2 controller problem. In this paper, only the filtering part of this result, valid for the entire controller problem, is used. Then, the optimal mean-square Kalman-Bucy filter for linear stochastic systems [28] and the H_∞ filter for linear systems (Theorem 4 in [4]) are employed to obtain the desired result, which is given by the following theorem.

Theorem 1. *The central suboptimal mean-square H_∞ filter for the linear stochastic system (1)–(4), ensuring the minimum of the mean-square criterion (5) and the H_∞ noise attenuation condition (6), is given by the equation for the mean-square H_∞ estimate $m(t)$*
$$dm(t) = (A(t)m(t)+B(t)u(t))dt + P(t)C_1^T(t)(h(t)h^T(t))^{-1}[dy_1(t)-C_1(t)m(t)dt] + S(t)C_2^T(t)(H(t)H^T(t))^{-1}[y_2(t)-C_2(t)m(t)]dt, \quad (7)$$
with initial condition $m(t_0) = E(x(t_0) \mid F_{t_0}^Y)$, where the matrix function $P(t)$ (minimum estimation error variance) is the solution of the differential Riccati equation
$$\dot{P}(t) = A(t)P(t)+P(t)A^T(t) + b(t)b^T(t) - P(t)C_1^T(t)(h(t)h^T(t))^{-1}C_1(t)P(t), \quad (8)$$
with initial condition $P(t_0) = E[(x(t_0)-m(t_0))(x(t_0)-m(t_0))^T \mid F_{t_0}^Y]$, and the symmetric matrix function $S(t)$ is the solution of the differential Riccati equation
$$\dot{S}(t) = A(t)S(t)+S(t)A^T(t)+G(t)G^T(t) - S(t)[C_2^T(t)(H(t)H^T(t))^{-1}C_2(t)-\gamma^{-2}L^T(t)L(t)]S(t), \quad (9)$$
with initial condition $S(t_0) = R^{-1}$.

Proof. First, let us design the estimate $\bar{x}(t)$ satisfying the minimum variance condition (5) of Section 2. As known [27], this mean-square estimate is given by the conditional expectation $\bar{x}(t) = E(x(t) \mid F_t^Y)$ of the system state $x(t)$ with respect to the σ - algebra F_t^Y, generated by the observations (2) in the interval $[t_0,t]$, and is produced by the Kalman-Bucy filter [28] applied to the linear stochastic system (1) over the linear observations (2) in the presence of Gaussian disturbances (Wiener processes) $W_1(t)$ and $W_2(t)$. The corresponding filtering equations for the estimate $\bar{x}(t)$ and the estimation error variance $\bar{P}(t)$ take the form
$$d\bar{x}(t) = (A(t)\bar{x}(t)+B(t)u(t)+G(t)\omega(t))dt + \bar{P}(t)C_1^T(t)(h(t)h^T(t))^{-1}[dy_1(t)-C_1(t)\bar{x}(t)dt], \quad (10)$$
with the initial condition $\bar{x}(t_0) = E(x(t_0) \mid F_{t_0}^Y)$, and
$$\dot{\bar{P}}(t) = A(t)\bar{P}(t)+\bar{P}(t)A^T(t)+b(t)b^T(t)- \quad (11)$$

$$\bar{P}(t)C_1^T(t)(h(t)h^T(t))^{-1}C_1(t)\bar{P}(t),$$

with the initial condition

$$\bar{P}(t_0) = E((x(t_0) - \bar{x}(t_0))(x(t_0) - \bar{x}(t_0))^T \mid F_{t_0}^Y).$$

Note that the latter equation coincides with (8). Now, applying the central suboptimal H_∞ filter for linear systems [4] to the estimate $\bar{x}(t)$ governed by equations (10), (11) yields the central suboptimal mean-square H_∞ estimate equation (7), where the matrix function $\bar{P}(t)$ satisfies equation (8), and the matrix $S(t)$ in (7) satisfies equation (9).

Note that filter (7)–(9) yields, in view of Theorems 3 and 4 in [1], the asymptotic stability of the mean value $E(\bar{x}(t))$ of the estimate (10) in the absence of disturbances and the prescribed attenuation level γ for this variable: $\|E(\bar{x}(t))\|_2^2 < \gamma^2\|\omega(t)\|_2^2 + E(x^T(t_0))RE(x(t_0))$. Since $E(\bar{x}(t)) = E(x(t))$ and the variances of the estimated errors produced by the estimates $\bar{x}(t)$ and $m(t)$ are equal, the conditions 1–3 of Section 2 hold. The theorem is proved. ■

Remark 1. The convergence of the designed mean-square H_∞ state estimate $m(t)$ to the real state value $x(t)$ is assured by the conditions $(\mathscr{C}_1)$ and $(\mathscr{C}_2)$ in view of the results of Theorem 7.4 and Section 7.7 in [26]. Note that boundedness of the noise-output H_∞ norm for the system $(\mathscr{S}_1)$, controlled by filter (7)–(9), i.e., admissibility of the mean-square H_∞ filter (7)–(9), is determined by the conditions I–III of Theorem 3 in [1].

Remark 2. According to the comments in Subsection V.G in [1], the obtained central mean-square H_∞ filter (7)–(9) presents a natural choice for H_∞ filter design among all admissible H_∞ filters satisfying the inequality (6) for a given threshold γ, since it does not involve any additional actuator loop (i.e., any additional external state variable) in constructing the filter gain matrix. Moreover, the obtained central mean-square H_∞ filter has the suboptimality property, i.e., it minimizes the criterion

$$J = \|z(t) - L(t)m(t)\|_2^2 - \gamma^2\left(\|\omega(t)\|_2^2 + E(x_0^T)RE(x_0)\right)$$

Remark 3. Following the discussion in Subsection V.G in [1], note that the complementarity condition always holds for the obtained filter (7)–(9), since the positive definiteness of the initial condition matrix R implies the positive definiteness of the filter gain matrix $S(t)$ as the solution of (9).

IV. Example

This section presents the design of the central suboptimal mean-square H_∞ filter to estimate the pitch and yaw angles for a 2DOF helicopter, ensuring the minimum of the mean-square criterion (5) and the H_∞ noise attenuation condition (6) holds for $\gamma = 1.1$.

Let the 2DOF helicopter system with the state space representation

$$\dot{x}(t) = Ax(t) + B(t)u(t) + G\omega(t) + b\psi_1(t), x(t_0) = x_0 \quad (12)$$

$$y(t) = C_1x(t) + h\psi_2(t) \quad (13)$$

$$y_2(t) = C_2x(t) + H\omega(t) \quad (14)$$

$$z(t) = Lx(t), \quad (15)$$

where the state vector is: $x = [\Theta, \Psi, \dot{\Theta}, \dot{\Psi}]^T$, in which Θ and Ψ are pitch and yaw angles respectively, $\dot{\Theta}$ and $\dot{\Psi}$ are pitch and yaw rates respectively. The matrices are:

$$A = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -9.2751 & 0 \\ 0 & 0 & 0 & -3.4955 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 2.3667 & 0.0790 \\ 0.2410 & 0.7913 \end{bmatrix} \quad b = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0.9024 & 0.0876 \\ 0.0919 & 0.8772 \end{bmatrix}$$

$$G = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0.9024 & 0.0876 & 0 & 0 \\ 0.0919 & 0.8772 & 0 & 0 \end{bmatrix}$$

$$C_1 = C_2 = L = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} \quad h = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$H = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

Here, $u(t)$ is the motor voltage input, $\omega(t)$ is an L_2^2 disturbance input, $\psi_1(t)$ and $\psi_2(t)$ are Gaussian white noises, which are the weak mean square derivatives of standard Wiener processes $W_1(t)$ and $W_2(t)$ (see [27]), respectively. The Wiener processes are considered independent of each other and of a Gaussian random variable x_0 serving as the initial condition in (12). Equations (12) and (13) present the conventional form for equations (1) and (2), which is actually used in practice [29]. It can be easily verified that the noise orthogonality condition holds for the system (12)–(15).

The filtering problem to be addressed is the same as described at Section 2. The filtering horizon is set from $t_0 = 0$ to $t_1 = 80$ s.

The central suboptimal mean-square H_∞ filter takes the following form for the system (12)–(15)

$$\dot{m}(t) = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -9.2751 & 0 \\ 0 & 0 & 0 & -3.4955 \end{bmatrix} m(t) \quad (16)$$

$$+P(t)\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\left(y(t) - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} m(t)\right)$$

$$+S(t)\begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}\left(y_2(t) - \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{bmatrix} m(t)\right),$$

with $m(0)=m_0=E(x_0 \mid F_0^Y)$, where $S(t)$ and $P(t)$ are the solutions to the differential Riccati equations

$$\dot{S}(t)=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -9.2751 & 0 \\ 0 & 0 & 0 & -3.4955 \end{bmatrix} S(t) \tag{17}$$

$$+S(t)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -9.2751 & 0 \\ 0 & 1 & 0 & -3.4955 \end{bmatrix}$$

$$-S(t)\begin{bmatrix} 0.1736 & 0 & 0 & 0 \\ 0 & 0.1736 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} S(t)$$

$$+\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.8220 & 0.1597 \\ 0 & 0 & 0.1597 & 0.7779 \end{bmatrix}, \quad S(0)=S_0=R^{-1},$$

and

$$\dot{P}(t)=\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -9.2751 & 0 \\ 0 & 0 & 0 & -3.4955 \end{bmatrix} P(t) \tag{18}$$

$$+P(t)\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 1 & 0 & -9.2751 & 0 \\ 0 & 1 & 0 & -3.4955 \end{bmatrix}$$

$$-P(t)\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} P(t)+\begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0.8220 & 0.1597 \\ 0 & 0 & 0.1597 & 0.7779 \end{bmatrix},$$

$$P(0)=E[(x_0-m_0)(x_0-m_0)^T \mid F_0^Y]=P_0,$$

respectively.

Numerical simulations results are obtained solving the system (12)–(15), and the filtering equations (16)-(18), with the following initial values: $x_0=[-0.7069,0,0,0]^T$, $P_0=diag(10,10,15,5)$, $R=[0.8,0.5,0.2,0.1;0.5,0.8,0.7,0.2;0.2,0.7,0.9,0.4;0.1,0.2,0.4,0.9]$, and $m_0=[0.1745,-0.5236,-0.15,-0.4]^T$. The motor voltage $u(t)=[12.495,-3.835]^T$, the L_2 disturbance $\omega(t)=[\omega_1(t),\omega_2(t),\omega_3(t),\omega_4(t)]^T$ is realized as $\omega_1(t)=1/(1+t)$, $\omega_2(t)=0.1(1-e^{-0.3t})$, $\omega_3(t)=0.1745/(1+t)$, and $\omega_4(t)=0.0873(1-e^{-0.3t})$. The attenuation level value is set to $\gamma=1.1$. The disturbances $\psi_1(t)$ and $\psi_2(t)$ in (12),(13) are realized using the built-in MatLab white noise function.

As a result of the numerical simulation, the following graphs are presented: graph of the noise-output H_∞ norm (Figure 1); graphs of the pitch angle and the corresponding estimation error (Figure 2); graph of the yaw angle and the corresponding estimation error (Figure 3).

Note that the maximum value of the noise-output H_∞ norm $T=\|z(t)-L(t)m(t)\|/(\|\omega(t)\|_2^2+E(x_0)RE(x_0))^{1/2}$ is 0.208 in the considered simulation interval, which is five times less than the given H_∞ attenuation level, $\gamma=1.1$. In addition, the estimation errors converge to zero.

V. Conclusions

This paper designs the central finite-dimensional H_∞ filter for linear stochastic systems with integral-quadratically bounded deterministic disturbances, that is suboptimal for a given threshold γ with respect to a modified Bolza-Meyer quadratic criterion including the attenuation control term with opposite sign. The designed filter is applied to estimation of the pitch and yaw angles of a two degrees of freedom (2DOF) helicopter. The simulation results show a reliable performance of the filter, in particular, the obtained attenuation level is five times less than a given threshold. This significant improvement is obtained due to the more reasonable selection of the filter gain matrix in the designed filter. Although this conclusion follows from the developed theory, the numerical simulation serves as a convincing illustration. The presented approach would be applied in the future to obtain the central suboptimal mean-square H_∞ filters for nonlinear polynomial stochastic systems.

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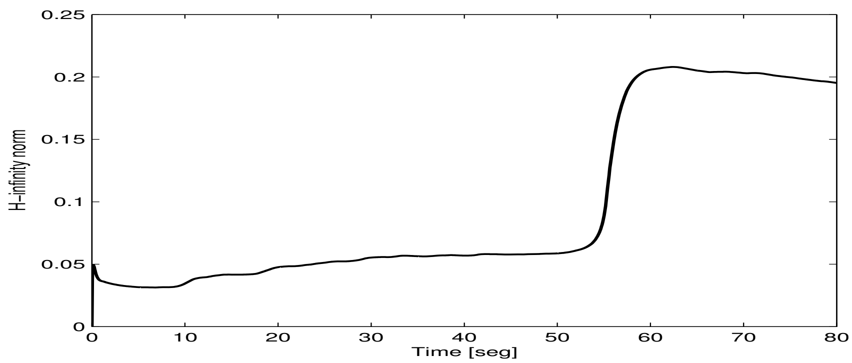

Fig. 1. Noise-output H_∞ norm T.

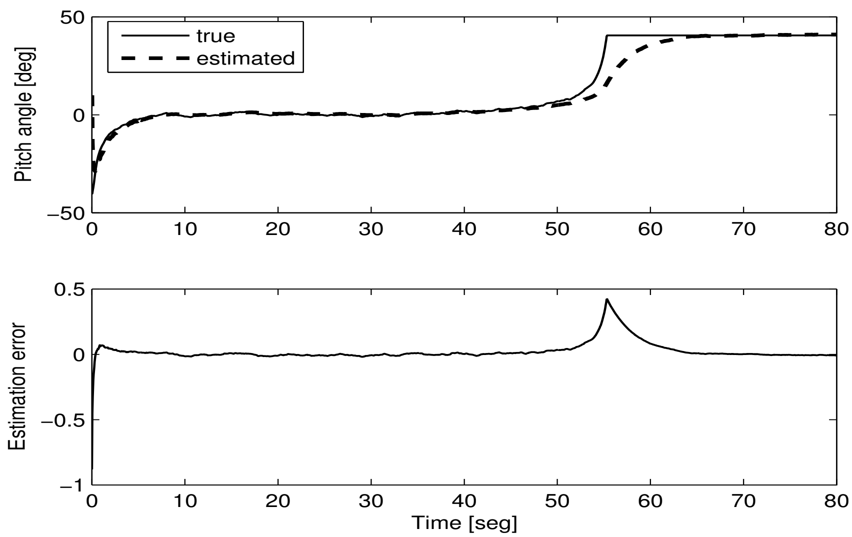

Fig. 2. **Above.** Pitch angle. **Below.** Estimation error.

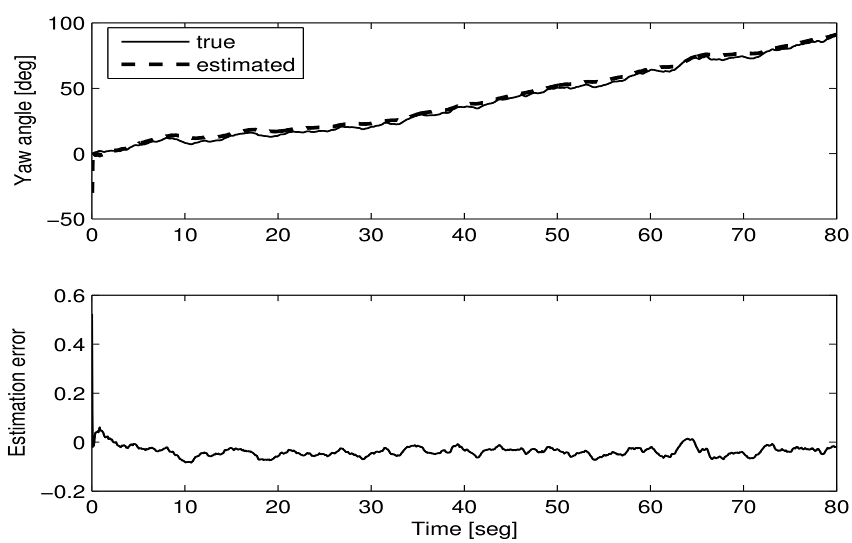

Fig. 3. **Above.** Yaw angle. **Below.** Estimation error.