

# Control of Dual Loop EGR Air-Path Systems for Advanced Combustion Diesel Engines by a Singular Perturbation Methodology

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**Abstract**—This paper presents a method to control dual loop EGR air-path systems for Diesel engines running advanced combustion modes. Considering the different time scales (fast and slow) dynamics of high pressure loop EGR (HPL-EGR) and low pressure loop EGR (LPL-EGR), a decomposition control method for a singularly perturbed system was utilized to achieve systematic air-path control, including the control of temperature, pressure, and oxygen fraction in intake manifold. Variable geometry turbocharger (VGT) was used to control the pressure before a high-pressure throttle (HP-Throttle) valve to accommodate the constraints of other actuators, such as dual-loop EGR and HP-Throttle. Effectiveness of such a control methodology was shown by simulation results based on a high-fidelity GT-Power Diesel engine model.

## NOMENCLATURE

1	Intake manifold
2	Section before high-pressure throttle
3	Exhaust manifold
4	Section before low-pressure compressor
$v$	Valve
$a$	Ambient
$c$	compressor
$t$	turbo
$hegr$	High pressure loop EGR valve
$legr$	Low pressure loop EGR valve
$hthr$	High pressure throttle
$W_x$	Gas flow rate through $x$
$W_{in}$	Engine intake gas flow rate
$p_x$	Pressure in $x$
$m_x$	Mass in $x$
$V_x$	Volume of $x$
$P_x$	Power of $x$
$F_x$	Oxygen fraction in $x$
$T_x$	Temperature at $x$
$\tau_x$	Time constant of $x$
$R$	Air constant
$N_e$	Engine speed (rpm)
$\gamma$	Specific heat ratio
$V_d$	Engine displacement volume

$c_p$  Specific heat of air at constant pressure  
(kJ/kg/K)

## I. INTRODUCTION

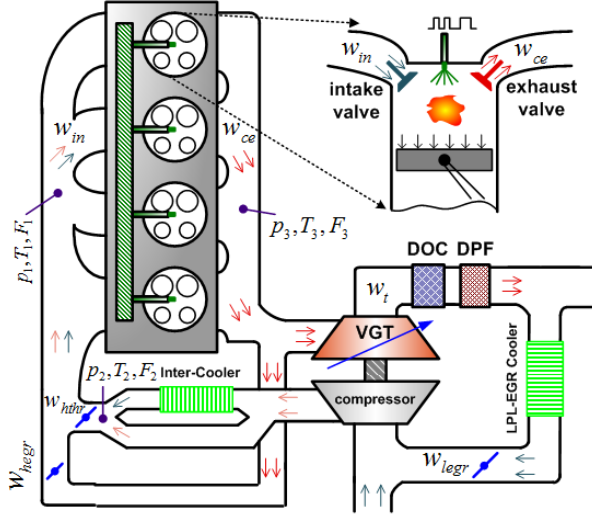
ADVANCED combustion modes, including homogenous charge compression ignition (HCCI), low temperature diffusion combustion (LTDC), and premixed charge compression ignition (PCCI), possess promising combustion properties in improving fuel efficiency and reducing engine-out emissions [13][5][12]. However, due to their stringent requirements on in-cylinder conditions [15][17][19][20] and the complexity of combustion processes, sophisticated air-path control, which can control the combustion boundary conditions (e.g. in-cylinder temperature, pressure, and oxygen fraction at intake valve closing), become crucial in realizing widespread applications of advanced combustion modes in production Diesel engines. Fortunately, a variety of actuators, such as variable geometry turbocharger (VGT), two-stage turbo-charging system, dual-loop exhaust gas recirculation (EGR), and variable valve actuation (VVA), have been recently developed for providing the authorities of controlling the advanced combustion modes in both steady-state and transient operations [19].

Different air-path system control methods for VGT, two-stage turbocharger systems, EGR, and VVA have been proposed in the literature [1][12][14]. However, few studies were conducted for control and cooperation of air-path systems having dual-loop EGR systems, which can significantly enhance the control authorities on both air-path variables and in-cylinder thermal dynamic boundary conditions [17][19][4].

In [1][12][14], coordinated control of EGR and VGT was investigated. In [1], the authors developed a coordinated EGR-VGT controller for passenger car Diesel engines. In [12], a switching logic and a set of linear controllers were designed to achieve EGR and VGT coordinated control. A sliding mode controller in [14] and a model predictive control (MPC) algorithm in [10] were applied for Diesel engines. Comparisons of different Diesel engine control algorithms were presented in [8]. In [16], a hybrid robust air-path system control approach was developed for transient operations of Diesel engines running LTC and conventional combustion modes. However, most of these control methods are implemented on conventional EGR and VGT systems. In [9], a LQG control algorithm was designed for a two-stage turbo system with a HPL-EGR. The mean-value models and a control method for Diesel engines with a dual-loop EGR and a single-stage turbocharger were introduced in [17][18]. In

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[4], the authors proposed a dual-loop EGR control method, which deals with HPL-EGR and LPL-EGR separately by motion planning. Cooperative control methods of dual-loop EGR systems are still absent.



**Figure 1. A schematic diagram for a Diesel engine capable of advanced multiple combustion modes (air-path actuators in blue).**

As Figure 1 illustrates, in a dual-loop EGR system, the LPL-EGR loop has a substantially larger volume and consequently a much slower dynamics than those of the HPL-EGR. Such a significant difference in time scale of the dynamics makes the coordination of the dual-loop EGR system challenging. For such systems with different time scale dynamics, the singular perturbation control methods described in [1][11][3] may offer some effective means. In the singular perturbation control methods, the original system can be decomposed into two separated subsystems, with lower dimensions, which can describe the different time scale dynamics separately. Respective control laws can be designed for the different dynamic scale subsystems, and the final control law can be therefore derived by the so-called composite feedback control law, which combines the control signals from subsystems together [1][11][3]. The stability analysis of the singular perturbation method was addressed in [11].

In this paper, a systematic air-path control method that cooperates dual-loop EGR, HP-Throttle, and VGT, for simultaneously controlling the intake manifold pressure, temperature, and oxygen fraction in a seamless way is derived. Such a new air-path control method expands the conventional Diesel engine control scope by adding the temperature dimension, which is critical for control of advanced combustion modes.

The arrangement of the rest of this paper is as follows. In section II, the control-oriented model of the air-path system is introduced. Section III describes the decomposition control method for air-path dual-loop EGR control (viewed as a singularly perturbed system) and a Lyapunov function based VGT control. In section IV, the validation of such a control methodology is shown by applying it to a high-fidelity GT-Power engine model and by comparisons to a

conventional PID control method. Conclusive remarks are presented in the end.

## II. AIR-PATH MODELING

In this section, control-oriented models for a Diesel engine air-path system that consists of a dual-loop EGR, a high-pressure throttle, and a VGT are introduced.

### A. Intake Manifold Model

The dynamic models for the intake manifold are derived based on the mass and energy conservations as well as the ideal gas law [17][7].

The intake/exhaust manifold pressure models can be described as:

$$\dot{p}_1 = \frac{RT_1}{V_1} (W_{hegr} + W_{hthr} - W_{in}), \quad (1)$$

$$\dot{T}_1 = \frac{RT_1}{p_1 V_1} (W_{hegr} (\gamma T_{hegr} - T_1) + W_{hthr} (\gamma T_2 - T_1) + W_{in} (T_1 - \gamma T_e)), \quad (2)$$

$$\dot{F}_1 = \frac{RT_1}{p_1 V_1} (W_{hegr} (F_3 - F_1) + W_{hthr} (F_2 - F_1)). \quad (3)$$

Here, the engine intake gas flow rate  $W_{in}$  can be estimated by the speed-density model as

$$W_{in} = \frac{\eta_v p_1 N_e V_d}{120 RT_1}. \quad (4)$$

### B. Control Volume before HP-Throttle

The gas dynamics in the control volume before HP-Throttle can be described as follows:

$$\dot{T}_2 = \frac{RT_2}{p_2 V_2} (W_{legr} (\gamma T_{legr} - T_2) + W_a (\gamma T_a - T_2) + W_{hthr} (T_2 - \gamma T_1)), \quad (5)$$

$$\dot{F}_2 = \frac{RT_2}{p_2 V_2} (W_{legr} (F_3 (t - d) - F_2) + W_a (F_a - F_2)). \quad (6)$$

Eq.(5) holds with the assumption that the inter-cooler after compressor can compensate the temperature increasing effect caused by the compressor. Since the temperatures of LPL-EGR, ambience, and inter-cooler are in the same level, which is significantly lower than that of HPL-EGR gas, the assumption here will not influence the nature of the dynamics described in (5).

### C. Resultant Control-Oriented Model of Dual-Loop EGR System

In a typical dual-loop EGR air-path system, LPL-EGR has a large volume (from LPL-EGR valve to HP throttle), i.e.  $V_2 \gg V_1$ . Denote  $\alpha_1 = \frac{RT_1}{p_1 V_2}$ ,  $\alpha_2 = \frac{RT_2}{p_2 V_2}$ ,  $\epsilon = \frac{V_1}{V_2}$ .  $x_1 = T_2$ ,  $x_2 = F_2$ ,  $z_1 = p_1$ ,  $z_2 = T_1$ ,  $z_3 = F_1$ ,  $u_1 = W_{hegr}$ ,  $u_2 = W_{ht}$ ,  $u_3 = W_{legr}$ . Then according to (1)(2)(3)(5)(6), the resultant models can be generated as:

$$\dot{x}_1 = \alpha_2 (\gamma T_{legr} - x_1) u_3 + \alpha_2 (\gamma T_a - x_1) W_a + \alpha_2 (x_1 - \gamma z_2) u_2 = f_1(x, z, u), \quad (7)$$

$$\dot{x}_2 = \alpha_2 (F_3 (t - d) - x_2) u_3 + \alpha_2 (F_a - x_2) W_a = f_2(x, z, u), \quad (8)$$

$$\epsilon \dot{z}_1 = \alpha_1 z_1 u_1 + \alpha_1 z_1 u_2 - \alpha_1 z_1 W_{in} = g_1(x, z, u), \quad (9)$$

$$\epsilon \dot{z}_2 = \alpha_1 (\gamma T_{hegr} - z_2) u_1 + \alpha_1 (\gamma x_1 - z_2) u_2 + \alpha_1 (z_2 - \gamma T_e) W_{in} = g_2(x, z, u), \quad (10)$$

$$\epsilon \dot{z}_3 = \alpha_1 (F_3 - z_3) u_1 + \alpha_1 (x_2 - z_3) u_2 = g_3(x, z, u). \quad (11)$$

For simplicity, vector fields  $[f_1(\cdot), f_2(\cdot)]^T$  and  $[g_1(\cdot), g_2(\cdot), g_3(\cdot)]^T$  are denoted as  $f(\cdot)$  and  $g(\cdot)$ , respectively, in the following. With the feedback control law,  $f(x, z, u)$  and  $g(x, z, u)$  can be written as  $f(x, z)$  and  $g(x, z)$ .

While the ultimate goal is to control the in-cylinder conditions of advanced combustion mode engines by the complex air-path system, in this paper, the control objective is to regulate the intake manifold pressure  $p_1$ , temperature  $T_1$ , and oxygen fraction  $F_1$  simultaneously.

#### D. Variable Geometry Turbocharger

A turbo-charging system can increase the intake manifold pressure by utilizing the energy from the high-pressure high-temperature exhaust gas. The powers of the turbine and compressor are shown as:

$$P_t = W_t C_p T_3 \eta_t \left( 1 - \left( \frac{p_{DPF}}{p_{exh}} \right)^{\frac{\gamma-1}{\gamma}} \right), \quad (12)$$

$$P_c = W_c C_p T_4 \frac{1}{\eta_c} \left( \left( \frac{p_d}{p_a} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right), \quad (13)$$

where the  $p_{DPF}$  is the pressure upstream of a Diesel particulate filter. It is normally higher than the ambient pressure and can be described as  $p_{DPF} = p_a + \Delta p_{ex}$ .  $\Delta p_{ex}$  is approximated as a constant pressure difference.

The time derivative of the power of the compressor can be described as [5][14]:

$$\dot{P}_c = \frac{1}{\tau_c} (P_t - P_c). \quad (14)$$

#### E. Control Actuators

To realize the aforementioned air-path control objective, four actuators can be adjusted, including HPL-EGR valve, LPL-EGR valve, VGT, and HP-Throttle. These actuators can be used to control the pressure, temperature, oxygen fraction in intake manifold, and the pressure before HP-Throttle. To reduce the complexity of the model, in this control algorithm design, the nonlinear effects of the actuators were bypassed by assuming the gas flow rates through these valves can be well controlled by the orifice equation, the gas flow rate through these valves can be estimated as:

$$W_v = \frac{C_D A_v p_u}{\sqrt{RT_d}} \left( \frac{p_d}{p_u} \right)^{1/\gamma} \left\{ \frac{2\gamma}{\gamma-1} \left[ 1 - \left( \frac{p_d}{p_u} \right)^{(\gamma-1)/\gamma} \right] \right\}, \quad (15)$$

for subsonic flow  $\{p_d/p_u > [2/(\gamma+1)]^{\gamma/(\gamma-1)}\}$ , and

$$W_v = \frac{C_D A_v p_u}{\sqrt{RT_d}} \sqrt{\frac{2}{\gamma+1}} \left[ \frac{2}{\gamma+1} \right]^{(\gamma+1)/2(\gamma-1)}, \quad (16)$$

for choked flow  $\{p_d/p_u \leq [2/(\gamma+1)]^{\gamma/(\gamma-1)}\}$ , where  $A_v$  is the effective open area for the valve,  $p_u$  is the upstream stagnation pressure,  $T_d$  is the downstream stagnation temperature, and  $p_d$  is the downstream stagnation pressure.

The effective areas in these equations can be controlled by adjusting the valve openings. In this sense, if the gas flow rates can be determined as inputs by the control algorithm, then each required effective area can be calculated by the pressure difference across the valve and the downstream gas temperature.

### III. SINGULAR PERTURBATION CONTROL METHODOLOGY

For systems with fast and slow dynamics, a composite control based on the singular perturbation control method [11]

is introduced here. Unlike traditional singular perturbation problem, there are two direct actuators (HP-Throttle and HPL-EGR) and three control variables (pressure, temperature, and oxygen fraction) in the intake manifold, as can be seen from Figure 1. Therefore, additional authorities, including the desired states in the control volume before HP-Throttle, need to be utilized, which will be further addressed later.

The references of  $x_1, x_2, z_1, z_2, z_3$  are denoted as  $x_{1,d}, x_{2,d}, z_{1,d}, z_{2,d}, z_{3,d}$ , respectively.  $z_{1,d}, z_{2,d}, z_{3,d}$  are based on the control objective and  $x_{1,d}, x_{2,d}$  are designed to facilitate the control algorithm.

#### A. Control Algorithm Design for Decomposed Subsystems

The control law can be designed with respect to fast and slow manifolds separately, by decomposing the system above into reduced and boundary-layer subsystem. Then the stability of overall system will be analyzed thereafter.

A reduced system can be derived by letting  $\epsilon = 0$ , i.e., neglecting the fast dynamics of  $[z_1, z_2, z_3]^T$ . In this reduced system  $z_{1,s}, z_{2,s}, z_{3,s}, u_{1,s}, u_{2,s}$ , and  $u_{3,s}$  are used instead of  $z_1, z_2, z_3, u_1, u_2$ , and  $u_3$  in the original system. Thus, we have  $z_1 = z_{1,s} + z_{1,f}$ ,  $z_2 = z_{2,s} + z_{2,f}$ ,  $z_3 = z_{3,s} + z_{3,f}$ ,  $u_1 = u_{1,s} + u_{1,f}$ ,  $u_2 = u_{2,s} + u_{2,f}$ ,  $u_3 = u_{3,s} + u_{3,f}$ .  $z_{1,f}, z_{2,f}, z_{3,f}, u_{1,f}, u_{2,f}$  and  $u_{3,f}$  indicate the counterparts of fast dynamics. When  $\epsilon = 0$ , by (9)-(11), we have

$$\alpha_1 z_{1,s} u_{3,s} + \alpha_1 z_{1,s} u_{2,s} - \alpha_1 z_{1,s} W_{in} = 0, \quad (17)$$

$$\alpha_1 (\gamma T_{hegr} - z_{2,s}) u_{3,s} + \alpha_1 (\gamma x_1 - z_{2,s}) u_{2,s} + \alpha_1 (z_{2,s} - \gamma T_e) W_{in} = 0, \quad (18)$$

$$\alpha_1 (F_3 - z_{3,s}) u_{3,s} + \alpha_1 (x_2 - z_{3,s}) u_{2,s} = 0. \quad (19)$$

i.e.,

$$u_{3,s} + u_{2,s} - W_{in} = 0, \quad (20)$$

$$T_{hegr} u_{3,s} + x_1 u_{2,s} - T_e W_{in} = 0, \quad (21)$$

$$z_{3,s} = \frac{F_3 u_{3,s} + x_2 u_{2,s}}{u_{3,s} + u_{2,s}}. \quad (22)$$

To be noted, without influence the above conditions,  $z_{1,s}$  and  $z_{2,s}$  can be chosen as any values, which are other authorities in the control law design.

By (20) and (21),

$$u_{2,s} = \frac{T_{hegr} - T_e}{T_{hegr} - x_1} W_{in}, \quad (23)$$

$$u_{3,s} = \frac{T_e - x_1}{T_{hegr} - x_1} W_{in}. \quad (24)$$

The reduced dynamic system, therefore, can be derived as:

$$\dot{x}_1 = \alpha_2 (\gamma T_{legr} - x_1) u_{1,s} + \alpha_2 (\gamma T_a - x_1) W_a + \alpha_2 (x_1 - \gamma z_{2,s}) u_{2,s} = f_1(x, z_s), \quad (25)$$

$$\dot{x}_2 = \alpha_2 (F_3(t - \tau) - x_2) u_{1,s} + \alpha_2 W_a (F_a - x_2) = f_2(x, z_s), \quad (26)$$

With respect to (25)(26), choose Lyapunov Function Candidate (LFC) as:

$$V = \frac{1}{2} \theta_1 (x_1 - x_{1,d})^2 + \frac{1}{2} \theta_1 (x_2 - x_{2,d})^2. \quad (27)$$

Then,

$$\nabla_{(x-x_d)} V \cdot (f(x, z_s) - \dot{x}_{1,d}) = \theta_1 (x_1 - x_{1,d}) (\dot{x}_1 - \dot{x}_{1,d}) + \theta_1 (x_2 - x_{2,d}) (\dot{x}_2 - \dot{x}_{2,d}). \quad (28)$$

To assure (28) negative definite, let

$$\dot{x}_1 - \dot{x}_{1,d} = -(x_1 - x_{1,d}), \quad (29)$$

$$\dot{x}_{2,s} - \dot{x}_{2,d} = -(x_2 - x_{2,d}). \quad (30)$$

Then, we have:

$$u_{1,s} = \frac{-(x_2 - x_{2,d}) + \dot{x}_{2,d} - \alpha_2 W_a (F_a - x_2)}{\alpha_2 (F_3 (t - \tau) - x_2)}, \quad (31)$$

$$\dot{x}_{1,d} = (x_1 - x_{1,d}) + \alpha_2 (\gamma T_{legr} - x_1) u_{1,s} + \alpha_2 (\gamma T_a - x_1) W_a + \alpha_2 (x_1 - \gamma z_{2,s}) u_{2,s}. \quad (32)$$

Let  $\tau = \frac{t}{\epsilon}$ , then a boundary-layer subsystem can be defined as:

$$\frac{dz_{1,f}}{d\tau} = \alpha_1 z_{1,f} u_{3,f} + \alpha_1 z_{1,f} u_{2,f} - \alpha_1 z_{1,f} W_{in} = g_1(x, z_f), \quad (33)$$

$$\frac{dz_{2,f}}{d\tau} = \alpha_1 (\gamma T_{hegr} - z_{2,f}) u_{3,f} + \alpha_1 (\gamma x_1 - z_{2,f}) u_{2,f} + \alpha_1 (z_{2,f} - \gamma T_e) W_{in} = g_2(x, z_f), \quad (34)$$

$$\frac{dz_{3,f}}{d\tau} = \alpha_1 (F_3 - z_{3,f}) u_{3,f} + \alpha_1 (\gamma x_2 - z_{2,f}) u_{2,f} = g_3(x, z_f). \quad (35)$$

Regarding the fast dynamics  $z_1, z_2, z_3$ , choose LFC as:

$$W = \frac{1}{2} \theta_2 \|z - z_d\|^2. \quad (36)$$

Then, the partial derivative of  $W$  with respect to  $(z - z_d)$  along  $g(x, z_f) - \epsilon \dot{z}_d$  can be written as:

$$\begin{aligned} \nabla_{(z-z_d)} W \cdot (g(x, z_f) - \epsilon \dot{z}_d) = & \theta_2 (z_1 - z_{1,d}) \left( \frac{dz_{1,f}}{d\tau} - \epsilon \dot{z}_{1,d} \right) + \\ & \theta_2 (z_2 - z_{2,d}) \left( \frac{dz_{2,f}}{d\tau} - \epsilon \dot{z}_{2,d} \right) + \\ & \theta_2 (z_3 - z_{3,d}) \left( \frac{dz_{3,f}}{d\tau} - \epsilon \dot{z}_{3,d} \right). \end{aligned} \quad (37)$$

To guarantee (37) negative definite, let

$$\frac{dz_{1,f}}{d\tau} - \epsilon \dot{z}_{1,d} = -(z_1 - z_{1,d}), \quad (38)$$

$$\frac{dz_{2,f}}{d\tau} - \epsilon \dot{z}_{2,d} = -(z_2 - z_{2,d}), \quad (39)$$

$$\frac{dz_{3,f}}{d\tau} - \epsilon \dot{z}_{3,d} = -(z_3 - z_{3,d}). \quad (40)$$

Then, the following control law can be derived by solving (38)-(40) as:

$$u_{2,f} = A - u_{3,f}, \quad (41)$$

$$u_{3,f} = \frac{-(z_1 - z_{1,d}) + \epsilon \dot{z}_{2,d} - \alpha_1 (z_{2,f} - \gamma T_e) W_{in} - \alpha_1 (z_{2,f} - \gamma T_e) A}{\alpha_1 \gamma T_{hegr} + \alpha_1 \gamma x_1}, \quad (42)$$

$$x_2 = \frac{\alpha_1 (F_3 - z_{3,f}) u_{3,f} + \alpha_1 z_{3,f} (A - u_{3,f}) - \epsilon \dot{z}_{3,d} + (z_3 - z_{3,d})}{\alpha_1 (u_{3,f} - A)}, \quad (43)$$

$$\text{where } A = \frac{-(z_1 - z_{1,d}) + \epsilon \dot{z}_{1,d} + \alpha_1 z_{1,f} W_{in}}{\alpha_1 z_{1,f}}.$$

Since  $x_2$  is a state in the system, it cannot be pre-designed to satisfy (43). Therefore, in the control law,  $x_{2,d}$  is used instead of  $x_{2,s}$ . Then (43) becomes:

$$x_{2,d} = \frac{\alpha_1 (F_3 - z_{3,f}) u_{3,f} + \alpha_1 z_{3,f} (A - u_{3,f}) - \epsilon \dot{z}_{3,d} + (z_3 - z_{3,d})}{\alpha_1 (u_{3,f} - A)}. \quad (44)$$

Consequently, by this modification, according to (35), (40) becomes:

$$\frac{dz_{3,f}}{d\tau} - \epsilon \dot{z}_{3,d} = -(z_3 - z_{3,d}) + \alpha_1 \gamma (x_2 - x_{2,d}) u_{2,f}. \quad (45)$$

Thus, (28) and (37) can be rewritten as:

$$\nabla_{(x-x_d)} V \cdot (f(x, z_s) - \dot{x}_{1,d}) = -\theta_1 \|x - x_d\|^2, \quad (46)$$

$$(\nabla_{(z-z_d)} W) (g(x, z_f) - \epsilon \dot{z}_d) = -\theta_2 \|z - z_d\|^2 + \quad (47)$$

$$\theta_2 (z_3 - z_{3,d}) \alpha_1 \gamma (x_2 - x_d) u_{2,f} \leq -\theta_2 \|z - z_d\|^2 +$$

$$\beta \|z - z_d\| \|x - x_d\|,$$

where  $\beta = \alpha_1 \max(u_{2,f})$ , and  $\|\cdot\|$  represents the 2-norm.

The composite control law, therefore, can be generated as:

$$u_1 = u_{1,s}, \quad (48)$$

$$u_2 = u_{2,s} + u_{2,f}, \quad (49)$$

$$u_3 = u_{3,s} + u_{3,f}. \quad (50)$$

$x_{1,d}$  and  $x_{2,d}$  are designed to satisfy (32) and (44).  $z_{2,s}$  is chosen as:

$$z_{2,s} = \frac{(\gamma z_{2,d} - x_1) u_2 + x_1 u_{2,s}}{\gamma u_{2,s}}. \quad (51)$$

## B. Stability Analysis for Overall System

**Theorem:** The vector field  $f$  fulfills the following condition, i.e., a positive constant  $m$  exists, such that:

$$\|f(x, z) - f(x, z_s)\| \leq m \|z - z_d\|. \quad (52)$$

**Proof:** By (47)(51)(7) and (25), we have:

$$f(x, z) - f(x, z_s) = \alpha_2 (x_1 - \gamma z_2) u_2 - \alpha_2 (x_1 - \gamma z_{2,s}) u_{2,s} = \quad (53)$$

$$\alpha_2 u_2 \gamma (z_2 - z_{2,d}) \leq m |z_2 - z_{2,d}|,$$

where  $m = \max(\alpha_2, u_2, \gamma)$ .

Considering (8)(26), we have

$$f_2(x, z) - f_2(x, z_s) = 0. \quad (54)$$

With (53) and (54), conclusion (52) can be easily achieved.

Choose an overall LFC as:  $M = (1 - \lambda)V(x) + \lambda W(z)$ , where  $\lambda$  is a constant, such that  $0 < \lambda < 1$ . With  $\epsilon \frac{dz_s}{d\tau} = 0$ , according to (52)(46)(47),

$$\begin{aligned} \dot{M} = (1 - \lambda) \dot{V} + \lambda \dot{W} & = (1 - \lambda) \nabla_{(x-x_d)} V \cdot (f(x, z) - \dot{x}_{1,d}) + \\ & \nabla_{(z-z_d)} W \cdot \left( \epsilon \frac{dz_s}{d\tau} + \epsilon \frac{dz_f}{d\tau} - \dot{z}_d \right) \\ & = (1 - \lambda) \nabla_{(x-x_d)} V \cdot (f(x, z_s) - \dot{x}_{1,d}) + \\ & (1 - \lambda) \nabla_{(x-x_d)} V \cdot (f(x, z) - f(x, z_s)) + \\ & \frac{\lambda}{\epsilon} (\nabla_{z-z_d} W) \cdot (g(x, z_f) - \epsilon \dot{z}_d) \end{aligned} \quad (55)$$

$$= -\theta_1 (1 - \lambda) \|z_1 - z_{1,d}\|^2 +$$

$$(1 - \lambda) m \|x - x_d\| \|z - z_d\| -$$

$$\theta_2 \frac{\lambda}{\epsilon} \|z - z_d\|^2 + \frac{\beta}{\epsilon} \|z - z_d\| \|x - x_d\|$$

$$= - \left( \frac{\|x - x_d\|}{\|z - z_d\|} \right)^T T \left( \frac{\|x - x_d\|}{\|z - z_d\|} \right).$$

Here,  $T = \begin{pmatrix} \theta_1 (1 - \lambda) & -(1 - \lambda) m \\ -\frac{\beta}{\epsilon} & \theta_2 \frac{\lambda}{\epsilon} \end{pmatrix}$ . If the control law can

guarantee that  $T$  is positive definite, then the asymptotic stability can be therefore ensured. As  $\theta_1 (1 - \lambda) > 0$ , this condition is equivalent to that the determinant of  $T$  is positive, i.e.,

$$|T| = \theta_1 \theta_2 \lambda (1 - \lambda) - \beta (1 - \lambda) m > 0. \quad (56)$$

As can be realized, for given  $\lambda, m, \epsilon$  and  $\beta$ , by choosing  $\theta_1$  and  $\theta_2$  properly, the condition (56) can be satisfied.

## C. VGT Control

Considering the constraints of the actuators (including LPL-EGR valve, HPL-EGR valve and HP-Throttle), the bounds of  $\alpha_2$  can be determined according to (48)-(50), i.e., constant values  $\underline{\alpha}_2$  and  $\bar{\alpha}_2$  can be found, such that:

$$\underline{\alpha}_2 \leq \alpha_2 \leq \bar{\alpha}_2. \quad (57)$$

As  $\alpha_2 = \frac{RT_2}{P_2 V_2}$ , the desired bounds of  $P_2$  can be therefore determined:

$$\underline{P}_2 \leq P_2 \leq \bar{P}_2. \quad (58)$$

Here we set the objective of  $P_2$  in the center the bounds, i.e.,

$$P_{2,d} = \frac{\bar{P}_2 - \underline{P}_2}{2}. \quad (59)$$

Thus, the VGT can be used to adjust the exhaust pressure towards the objective value in (59).

For turbocharger control, a quadratic LFC is chosen as:

$$V_t = \frac{1}{2}(p_2 - p_{2,d})^2 + \frac{1}{2}(\mathbf{P}_c - \mathbf{P}_{c,d})^2, \quad (60)$$

where  $\mathbf{P}_{c,d}$  is the desired power of compressor as:

$$\mathbf{P}_{c,d} = W_{hthr} C_p T_4 \frac{1}{\eta_c} \left( \left( \frac{p_{2,d}}{p_a} \right)^{\frac{\gamma-1}{\gamma}} - 1 \right). \quad (61)$$

Take the derivative of (60), then,

$$\dot{V}_t = (p_2 - p_{2,d})(\dot{p}_2 - \dot{p}_{2,d}) + (\mathbf{P}_c - \mathbf{P}_{c,d})(\dot{\mathbf{P}}_c - \dot{\mathbf{P}}_{c,d}), \quad (62)$$

Let  $\dot{p}_2 - \dot{p}_{2,d} = -l_1(p_2 - p_{2,d})$  and  $\dot{\mathbf{P}}_c - \dot{\mathbf{P}}_{c,d} = -l_2(\mathbf{P}_c - \mathbf{P}_{c,d})$ . According to (14), the control law of turbocharger can be then easily derived as:

$$W_t = \frac{1}{k_t} (\dot{\phi} + \phi - ((p_2 - p_{2,d}) \frac{RT_2}{V_2 k_c})), \quad (63)$$

where

$$\phi = k_c \left( -\frac{l_1 V_2}{RT_2} (p_2 - p_{2,d}) + W_{hthr} \right). \quad (64)$$

#### IV. SIMULATION STUDIES

##### A. Simulation Setup

The foregoing control methodology was evaluated through co-simulations between Matlab/SIMULINK and a GT-Power (an industry standard software package), 1D computational engine model. The engine model is for a four-cylinder Diesel engine with a dual-loop EGR, a HP-Throttle, and a VGT. The control signals include mass flow rates through the HPL-EGR valve, LPL-EGR valve, HP-throttle valve, and VGT. The outputs here are the intake manifold pressure  $p_1$ , temperature  $T_1$  and oxygen fraction  $F_1$ . Engine runs at a constant speed, i.e. 1200 rpm, and the fuel injection amount is constantly 20mg per cylinder per cycle.

To better show the effectiveness of the proposed air-path control law, comparisons with a carefully-tuned conventional PID control method were conducted. Without changing the control law of VGT, in the PID control, the other actuators were decoupled by their physical properties: the HPL-EGR mass flow rate, which will bring high temperature gas from exhaust manifold, was used to control the temperature in intake manifold; the LPL-EGR mass flow rate was utilized to control the oxygen fraction in intake manifold; the HP-Throttle was used to control the intake manifold pressure.

##### B. Simulation Results

In advanced combustion mode control, the profiles of intake conditions (pressure, temperature, and oxygen fraction) vary according to the specific requirements of the combustion modes and transient process. Without loss of generality, here the references are assumed as instant jumps to show the effectiveness and performance of the designed control method.

As can be seen from the simulation results presented in Figure 2 and the actuator signals shown in Figure 3, the

singularly perturbed decomposition control method shows good performance, particularly during the transient. As conventional control methods, such as PID control, lack the appropriate mechanisms in handling the interconnections between different time scale dynamics, the transient performance will be influenced by such a system attribute. In the PID control method, even after carefully choosing the control parameters and adding feedforward control contributions, the coupling effects of the control variables still exist as shown in Figure 4 and Figure 5. For example, as indicated in the second and third subplots in Figure 4, there was an overshoot on the intake manifold temperature during the transient, and the transient of oxygen fraction was influenced when HPL-EGR tried to tune the temperature back to the desired value. In addition, in real practice, PID control may require extensive parameter calibrations regarding to different operating conditions, which however is not necessary for the method developed in this paper.

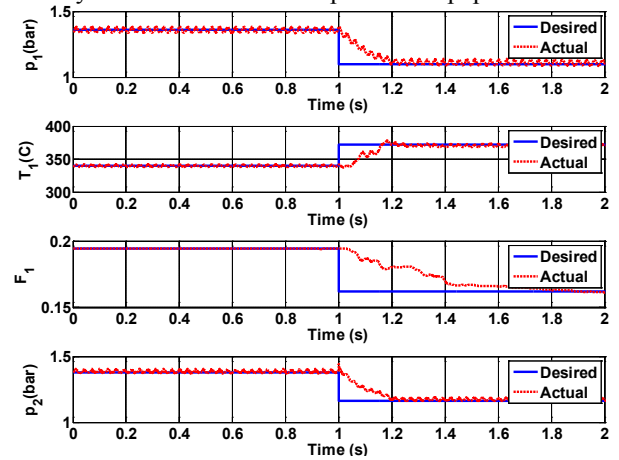


Figure 2. Outputs of singularly perturbed design method.

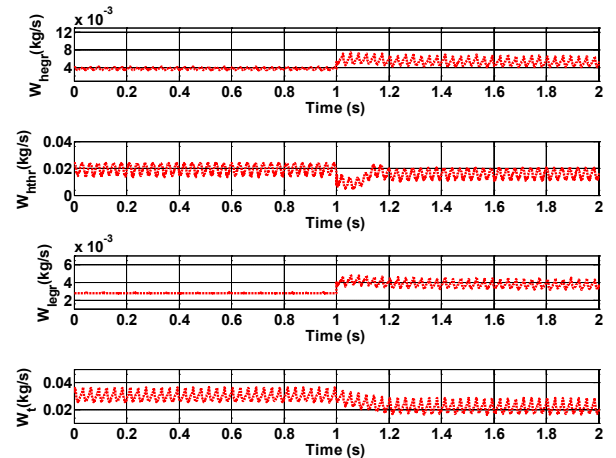


Figure 3. Input signals for singularly perturbed method.

#### V. CONCLUSIONS AND FUTURE WORK

In this paper, by using a singularly perturbed method, the two time-scale dynamics in dual-loop EGR air-path systems were investigated. By decomposing the original system into two lower dimension systems and designing the control law separately, a composite feedback control law was derived. The effectiveness of such a control method for complex Diesel engine air-path system was demonstrated by simulations using a GT-Power, high-fidelity, 1D

computational, engine model with comparisons to a conventional PID control algorithm.

The future work will primarily include the experimental investigations of the control method as well as the combinations of the precise fuel injection control approach [21] with air-path system control for advanced combustion mode engine control.

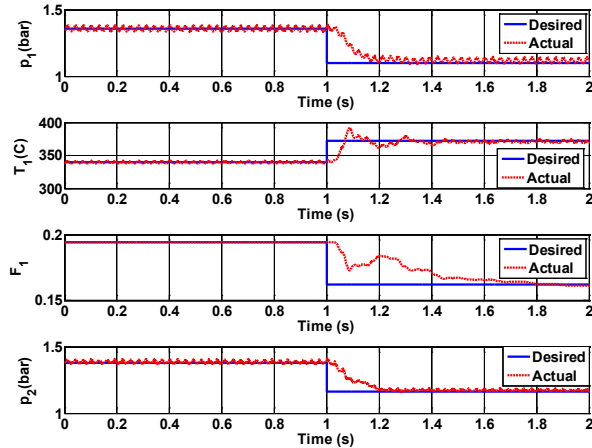


Figure 4. Output signals for PID control method.

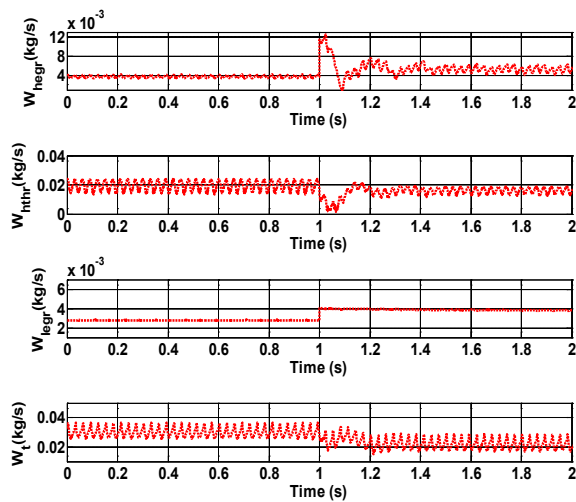


Figure 5. Input signals for PID control method.

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