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Persistent Excitation by Deterministic Signals in Subspace MISO Hammerstein System Identification

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Abstract— **The persistent excitation issue is considered for MISO Hammerstein system identification using the subspace approach. A signal generator that provides deterministic persistently exciting (PE) sequences is developed. These are obtained by operating amplitude weighting and phase shifting on a discrete Dirac Comb signal of finite length. The proposed signal generation procedure presents quite interesting features e.g. the signals data samples are deterministic, weakly constrained, and their size do not need to be too large. Theoretically, the signal generator design relies on a technical lemma establishing the PE property for the considered identified systems. The effectiveness of the proposed class of exciting sequences is confirmed by simulation.**

I. INTRODUCTION

BLOCK oriented nonlinear models has been proven to be a quite interesting tool in capturing many biologic, chemical and electrical nonlinear systems behavior. Among these, the most popular models are those composed of a linear dynamical subsystem and a static nonlinear element connected in series namely Hammerstein and Wiener models. Several identification schemes have been developed for this class of systems following various approaches including e.g. frequency, stochastic, blind, iterative optimization and others (see e.g. Giri and Bai [1] and references therein). Presently, the focus is made on the State Space Subspace Identification approach which captured a particular attention in control systems community. This class of algorithms had been introduced and developed for MIMO linear systems during the nineties by P. Van Overschee and B. De Moor [2] and references therein, and extended a decade after to both SISO and MIMO Hammerstein and Wiener nonlinear systems by Lovera *et al*. [3], Gomez and Baeyens [4] and Naitali *et al*. [5].

From a theoretical point of view, there is no denying that subspace algorithms are accurate and robust. However, to guarantee the consistency of the system parameter estimates, the input sequence applied while the identification experiment must be so that the virtual input sequence of the equivalent multivariable linear system is Persistently Exciting (PE) of appropriate order.

In the present work, parametric identification of multi-input single-output (MISO) Hammerstein systems is addressed based on the subspace approach. The focus is specifically made on the achievement of the PE property that guarantees the consistency of the subspace algorithm results, by using deterministic input sequences. To this end, a technical lemma is established showing that a class of PE input sequences for MISO Hammerstein systems can be generated by operating phase-shifting and amplitude-weighting on a specific mother mono variable signal. The latter has been previously constructed by the authors for polynomial basis function (PBF) based subspace identification of SISO Hammerstein system [5]. Compared to white Gaussian noise (WGN) and multi-level pseudo random sequence (MLPRS), the proposed vector input sequence procedure presents quite interesting features: (i) there is no (theoretical) need to large set of input and output data samples; (ii) the design procedure provides the user with some freedom, accordingly, the sequence wave form can be shaped to meet some desirable requirements e.g. uniform excitation of the input variation range, keeping the system around an operation point.

The paper is organized as follows: the identification problem is formulated in section II. In section III, the estimation procedure is described. In section IV, the main theorem, that describes key design elements for generating PE sequences, is established. In section V The effectiveness of the proposed sequence design procedure is illustrated by numerical simulation.

II. IDENTIFICATION PROBLEM FORMULATION

A. Class of systems

We are considering MISO nonlinear systems that can be well represented by the Hammerstein block-oriented multi channel model of Fig. 1. The model consists of m channels each one being composed of a SISO Hammerstein system.

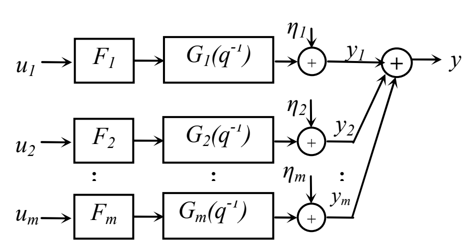

Fig. 1. Multi-branch block-oriented nonlinear system

In discrete-time, the system model output $y(t,\Theta)$ is related to the m scalar inputs, denoted $u_1(t),..,u_m(t)$, through the nonlinear dynamical equation:

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$$y(t,\Theta)=\sum_{r=1}^{m}G_r(q^{-1})F_r(u_r(t))+e(t) \quad (1)$$

$$G_r(q^{-1})=\frac{B_r(q^{-1})}{A_r(q^{-1})}=\frac{b_{r,1}q^{-1}+..+b_{r,n_r}q^{-n_r}}{1+a_{r,1}q^{-1}+..+a_{r,n_r}q^{-n_r}} \quad (2)$$

$$F_r(u_r(t))=F_{r,d_r}\cdot\Psi_{r,d_r}(u_r(t))+\varepsilon_{r,d_r}(t) \quad (3)$$

$$\Theta=\left(A_r,B_r,F_{r,d_r}\right)_{r=1..m}\in\Omega\subset\Re^n \quad (4)$$

$$n=m+\sum_{r=1}^{m}(2n_r+d_r) \quad (5)$$

where $A_r(q^{-1})$, $B_r(q^{-1})$ designate the denominator and numerator of the transfer function $G_r(q^{-1})$ that characterizes the dynamics of the linear subsystem of the r^{th} channel; $F_r(\cdot)$ is a function describing its static nonlinear element; d_r is the dimension of the multivariable nonlinear function basis of the system inputs $\Psi_{r,d_r}(\cdot)\overset{\Delta}{=}\left(\psi_{r,k}(\cdot)\right)_{k=1,..,d_r}$ on which the line vector F_{r,d_r} is developed; $e(t)$ denotes the equation error resulting from the measurement noises η_i $(i=1\ldots m)$, and $\varepsilon_{r,d_r}(t)$ being a function standing for the nonlinear element modeling error, and which tends to zero when d_r tends to infinity. Obviously, none of $B_r(q^{-1})$ and $F_r(.)$ is identically zero. Furthermore the system subject to the following constraints: (i) the integers n_r and d_r are bounded from above by known integers n_r^* and d_r^*; and (ii) the considered linear subsystems are BIBO stable.

B. Identification Problem Statement

The identification problem at hand is to determine accurate parameter estimates of the linear and nonlinear elements of all channels, based on a given set of consistent input-output data of the system (1-5). Specifically, we seek the determination of the vector $\hat{\Theta}$ that minimizes the output estimation error criterion i.e.

$$\hat{\Theta}=\arg\min_{\Theta\in\Omega}V^2(\Theta) \quad (6)$$

$$V^2(\Theta)=\sum_{t=0}^{Ns-1}\left(y(t)-y(t,\Theta)\right)^2 \quad (7)$$

where $\Omega\subset\Re^n$ is the related search space and where N_s designates the number of measured data samples. The main difficulties in such minimization problem are three folds: (i) the multivariable feature and its high dimensionality; (ii) the strong nonlinearity of the output error with respect to the unknown parameters vector Θ; (iii) the design of an input sequence ensuring the necessary PE requirement.

III. Subspace Identification Of Hammerstein Block Oriented Multi Input Nonlinear Systems

In this section, the subspace identification approach is extended to Hammerstein multi-channel systems (Fig. 1). This is performed in four steps:

(i) The system model is reformulated as a series combination of a known MIMO static nonlinear subsystem and an unknown MISO linear subsystem whose inputs and output sequences are absolutely known.

(ii) The state space matrices of the linear part are estimated using a subspace identification algorithm.

(iii) Estimates of the state space matrices of all channels are recovered from those of the diagonal blocks of MISO linear subsystem matrices given by their canonical realization in the modal form.

(iv) Finally, estimates of all channel nonlinearity vectors and input matrices are obtained the linear MISO subsystem input matrices estimates through singular values decompositions (SVD).

A. Multivariable State Space Model

The input-output model (1) can be given a state space representation of the form:

$$\begin{cases} x_r(t+1)=A_r x_r(t)+B_r\cdot F_r(u_r(t))+\varepsilon_r'(t) \\ y(t)=\sum_{r=1}^{m}\left(C_r x_r(t)+\eta_r(t)\right) \end{cases} \quad (8)$$

where $x_r(t)$ denote the r^{th} channel state vector, $A_r\in\Re^{n_r\times n_r}$, $B_r\in\Re^{n_r\times 1}$ and $C_r\in\Re^{1\times n_r}$ stand for the state, input and output matrices of the linear dynamic subsystem of the r^{th} channel; $\varepsilon_r(t)\in\Re^{n_r}$ and $\eta_r(t)\in\Re$ being real noise sequences standing for the state and output errors.

Consequently, considering the column vector $\Psi_r(u_r(t))$ as an intermediate input, the discrete state space equation of each channel can be expressed by:

$$x_r(t+1)=A_r x_r(t)+B_{r,d_r}\cdot\Psi_{r,d_r}(u_r(t))+\varepsilon_r''(t) \quad (9)$$

where B_{r,d_r} is a one-rank input matrix defined by :

$$B_{r,d_r}=B_r\cdot F_{r,d_r}\in\Re^{n_r\times d_r} \quad (10)$$

with $\varepsilon_r''(t)\in\Re^{n_r\times 1}$ is a zero-mean and bounded noise.

$$\varepsilon_r''(t)=\varepsilon_r(t)+\varepsilon_r'(u_r(t)) \quad (11)$$

Now, let $U(t)$ be the vector input obtained by the concatenation of all function basis of all channel inputs i.e.

$$U(t)=\Psi(u(t))=\left(\Psi_{1,d_r}(u_1(t))^T,..,\Psi_{m,d_r}(u_m(t))\right)^T\in\Re^{M\times 1} \quad (12)$$

with:

$$u(t)=[u_1(t),u_2(t),..,u_m(t)]^T \in \Re^m \text{ and } M=\sum_{r=1}^{m} d_r \quad (13\text{-}14)$$

and let $X(t)$ be the global state vector by:

$$X(t)=\left(x_1^T(t),x_2^T(t),..,x_m^T(t)\right)^T \in \Re^{N\times 1} \text{ with } N=\sum_{r=1}^{m} n_r \quad (15\text{-}16)$$

Then, it follows from (9), (12) and (13) that the initial system input-output model (1-5) can be compacted in the following state space form:

$$\begin{cases} X(t+1)=\mathrm{A}X+\mathrm{B}U(t)+\mathscr{E}(t) \\ y(t)=\Gamma X+\mathscr{V}(t) \end{cases} \quad (17)$$

where:

$\mathrm{A}\in\Re^{N\times N}$, $\mathrm{B}\in\Re^{N\times M}$ and $\Gamma\in\Re^{1\times N}$ designate the global state, input and output block-diagonal matrices of the linear part in (16), they are given by:

$$A=diag(A_1,..,A_m); \quad (18)$$

$$B=diag(B_{1,d_1},..B_{m,d_m}); \quad (19)$$

$$\Gamma=[C_1,..,C_m] \quad (20)$$

$\mathscr{E}\in\Re^{N\times 1}$ and $\mathscr{V}\in\Re$ are bounded and zero-mean equation errors. The new system model (17) shows that the initial multi-channel Hammerstein system (Fig. 1) includes two subsystems connected in series: (i) a fully known. static nonlinear operator $\Psi(\cdot)$ and (ii) an unknown linear subsystem represented by the triplet $(\mathrm{A},\mathrm{B},\Gamma)$, with input $U(t)$ and output y; and which can be estimated by using the state space subspace identification Algorithm

B. Computing the Channels Matrices Estimates

Let $\hat{\mathrm{A}}\in\Re^{N\times N}$, $\hat{\mathrm{B}}\in\Re^{N\times M}$, and $\hat{\Gamma}\in\Re^{1\times N}$ be the system matrices estimates provided by the robust version of the combined deterministic-stochastic subspace identification algorithm namely *subid* [2] of the linear multivariable system whose multivariable input is U and whose scalar output is y defined by (12) and (17) respectively, and let $\mathrm{P}_c\in\Re^{N\times N}$ be a similarity matrix which transforms the system matrices estimates to the canonical state space realization given in the modal form (21-23).

$$\hat{\mathrm{A}}_c=\mathrm{P}_c^{-1}\hat{\mathrm{A}}\mathrm{P}_c; \quad \hat{\mathrm{B}}_c=\mathrm{P}_c^{-1}\hat{\mathrm{B}}; \quad \hat{\Gamma}_c=\hat{\Gamma}\mathrm{P}_c \quad (21\text{-}23)$$

Then, the state space matrices estimates of the r^{th} channel can be retrieved from the multivariable state space matrices estimates as follows:

- The state and input matrices estimates $\hat{A}_r$ and $\hat{B}_{r,d_r}$ are forwardly obtained by the r^{th} diagonal blocks of size $n_r\times n_r$ and $n_r\times d_r$ of the canonical state and input matrices $\hat{\mathrm{A}}_c$ and $\hat{\mathrm{B}}_c$ respectively (24-25).

$$\hat{A}_r=\hat{\mathrm{A}}_c(p_r+1:p_r+n_r,p_r+1:p_r+n_r) \quad (24)$$

$$\hat{B}_{r,d_r}=\hat{\mathrm{B}}_c(p_r+1:p_r+n_r,q_r+1:q_r+d_r) \quad (25)$$

where $$p_r=\sum_{k=1}^{r-1} n_k \text{ and } q_r=\sum_{k=1}^{r-1} d_k \quad (26\text{-}27)$$

- The output matrix is given by the r^{th} row block of $\hat{\Gamma}_c$:

$$\hat{C}_r=\hat{\Gamma}_c(p_r+1:p_r+n_r) \quad (28)$$

- The polynomial vector F_{r,d_r} and the input matrix B_r are recovered up to an arbitrary multiplicative constant λ_r from the SVD of the input matrix estimate $\hat{B}_{r,d_r}$ (29-32) where σ_r is its unique nonzero singular value:

$$\hat{B}_{r,d_r}=U_rS_rV_r^T \quad (29)$$

$$\hat{B}_r=\frac{1}{\lambda_r}U_r[\sigma_r,0,..0]^T,\ \hat{F}_{r,d_r}=\lambda_r[1\ \ 0\ \ ..\ \ 0]\cdot V_r^T,\ \lambda_r\in\Re^* \quad (30\text{-}32)$$

Theoretically, these formulas show that all channel parameters can simultaneously be identified from input output data. However, to be really effective the multivariable input $u(t)$ must provide the required PE properties to the virtual input $U(t)$. This is the subject of the next section where design elements of a class of PE deterministic pulse sequences are capitalized as a technical lemma.

IV. VECTOR INPUT SEQUENCE DESIGN FOR PE PURPOSE

A. Persistent Excitation

To guarantee the consistency of the parameter estimator described in subsections III.B, the virtual multivariable input sequence $U(t)$ must meet the PE requirement stated in the following definition [2].

Definition 1. *Let* $\{U(t)\}$ *be any real multivariable sequence of finite dimension* M *and* $\mathbf{U}_{0|2i-1,j}$ *the block Hankel matrix of* $\{U(t)\}$ having $2iM$ rows and *j* columns. *Then, the input sequence* $\{U(t)\}$ *is persistently exciting of order* $2i$ *if the input covariance matrix* $R_{2i}^{U,U}$ *is of full rank, where*

$$R_{2i}^{U,U}\overset{\Delta}{=}\Phi\left[\mathbf{U}_{0|2i-1,j},\mathbf{U}_{0|2i-1,j}\right]=E_j\left(\mathbf{U}_{0|2i-1,j}\cdot\mathbf{U}_{0|2i-1,j}^T\right) \quad (33)$$

with $E_j(\cdot)$ *the expectation operator:*

$$E_j(\cdot)\overset{\Delta}{=}\lim_{j\to+\infty}\frac{1}{j}(\cdot) \quad (34)$$ □

The input design issue is to determine a real vector input sequences $u(t)$ that provide the virtual input $U(t)$ with the necessary PE property.

B. Identification Experiment Design

Lemma. *Let* $\{s_{1,k}\},\{s_{2,k}\},..,\{s_{m,k}\}$ *be any* m *real* *sequences of length d and let $\{\phi_r\}$ be any strictly increasing integer sequence of length m.*

Define m scalar signals $u_1(t),..,u_m(t)$ derived, for some integer number i^*, on the interval $t \in \lfloor 0, 2i^* d \lfloor$ as follows:

$$u_r(t) = \sum_{k=0}^{d-1} s_{r,k}\delta(t - \phi_r - 2ki^*), \forall r \in \{1,..,m\} \quad (35)$$

$$\text{where } \delta(t) = \begin{cases} 1 & \text{if } t = 0 \\ 0 & \text{otherwise} \end{cases} \quad (36)$$

Consider a md − variable signal $U(t)$ defined on the interval $t \in [0, 2i + j - 2]$, for some integers i and j, by:

$$U(t) = \left[\Psi_{r,d}^T\right]_{r=1,..m}^T \overset{\Delta}{=} \left[\psi_{r,k}(u_r(t))\right]_{k=1,..,d;r=1,..,m}^T \in \Re^{md} \quad (37)$$

where $(\psi_{r,k}(\cdot))_{k=1,..,d}$ is any zero-kernel functions basis i.e:

$$\psi_{r,k}(0) = 0, \ \forall (r,k) \in \{1,..,m\} \times \{1,..,d\} \quad (38)$$

and let $\mathbf{U}_{0|2i-1,0|j-1}$ be the Hankel matrix of $U(t)$ starting from t=0 and having $2i$ block-rows and j columns.

Then, there exist two integers i_0^*, j_0 such that, for any $i^* \geq i_0^*$, and any $j \geq j_0$, the covariance matrix $R_{2i}^{U,U}$ is of full rank, provided that:

$$\text{(i) } rank\left(R_{2i^*}^{\Psi_{r,d}(u_r),\Psi_{r,d}(u_r)}\right) = 2di^* \quad (39)$$

$$\text{(ii) } \phi_m - \phi_1 \leq 2(i^* - i) \quad (40)$$

$$\text{(iii) } \min_{r=2,..m,c=1..r-1} (\phi_r - \phi_c) \geq 2i \quad (41)$$

□

Proof: Let $\mathbf{U}_{0|2i-1,j}$ and ${}^r\mathbf{V}_{0|2i-1,j}$ be the block Hankel matrices, of $U(t)$ and of $\Psi_{r,d}(u_r(t))$ respectively, starting from $t_0 = 0$ with $2i$ row blocks and j columns:

$$\mathbf{U}_{0|2i-1,j} = \begin{pmatrix} U(0) & U(1) & .. & U(j-1) \\ U(1) & U(2) & .. & U(j) \\ .. & .. & .. & .. \\ U(2i-1) & U(2i) & .. & U(j+2i-2) \end{pmatrix} \in \Re^{2i \cdot d \times j} \quad (42)$$

$${}^r\mathbf{V}_{0|2i-1,j} = \begin{pmatrix} \Psi_{r,d}(u_r(0)) & \Psi_{r,d}(u_r(1)) & .. & \Psi_{r,d}(u_r(j-1)) \\ \Psi_{r,d}(u_r(1)) & \Psi_{r,d}(u_r(2)) & .. & \Psi_{r,d}(u_r(j)) \\ ... & ... & .. & ... \\ \Psi_{r,d}(u_r(2i-1)) & \Psi_{r,d}(u_r(2i)) & ... & \Psi_{r,d}(u_r(2i+j-2)) \end{pmatrix} \quad (43)$$

Consider the matrix $\mathbf{V}$ obtained by concatenating all the ${}^r\mathbf{V}_{0|2i-1,j}$'s block Hankel matrices, i.e.:

$$\mathbf{V} = \left({}^1\mathbf{V}_{0|2i-1,j}^T, {}^2\mathbf{V}_{0|2i-1,j}^T, .., {}^m\mathbf{V}_{0|2i-1,j}^T\right)^T \in \Re^{2im \cdot d \times j} \quad (44)$$

Then, since the matrices $\mathbf{V}$ and $\mathbf{U}_{0|2i-1,j}$ can be obtained from each other by rows permutations, the dimensions of the subspaces spanned by these matrices are equal, therefore if the covariance matrix of one of them is of full rank so will be the case for the other. Hence, the full rank property of the covariance matrix $R_{2i}^{U,U}$ can be guaranteed by ensuring the full rank property of the following covariance matrix:

$$R_{2i}^{\mathbf{VV}} = \lim_{j \to +\infty} \frac{1}{j} \mathbf{V}\mathbf{V}^T \quad (45)$$

$$\mathbf{V}\mathbf{V}^T = \begin{pmatrix} {}^1\mathbf{V}_{0|2i-1,j}\,{}^1\mathbf{V}_{0|2i-1,j}^T & {}^1\mathbf{V}_{0|2i-1,j}\,{}^2\mathbf{V}_{0|2i-1,j}^T & ... & {}^1\mathbf{V}_{0|2i-1,j}\,{}^m\mathbf{V}_{0|2i-1,j}^T \\ {}^2\mathbf{V}_{0|2i-1,j}\,{}^1\mathbf{V}_{0|2i-1,j}^T & {}^2\mathbf{V}_{0|2i-1,j}\,{}^2\mathbf{V}_{0|2i-1,j}^T & ... & {}^2\mathbf{V}_{0|2i-1,j}\,{}^m\mathbf{V}_{0|2i-1,j}^T \\ ... & ... & ... & ... \\ {}^m\mathbf{V}_{0|2i-1,j}\,{}^1\mathbf{V}_{0|2i-1,j}^T & {}^m\mathbf{V}_{0|2i-1,j}\,{}^2\mathbf{V}_{0|2i-1,j}^T & ... & {}^m\mathbf{V}_{0|2i-1,j}\,{}^m\mathbf{V}_{0|2i-1,j}^T \end{pmatrix} \quad (46)$$

Therefore, if for any r the covariance matrix of $\Psi_{r,d}(u_r(t))$ is of rank $2di^*$ (43), it is so for any $i \leq i^*$. Consequently, all diagonal blocks of $\mathbf{V}\,\mathbf{V}^T$ are of full rank. Hence, to guarantee the full rank property of $\mathbf{V}\,\mathbf{V}^T$ and thereby of $R_{2i}^{U,U}$, it suffices to find a weak condition on the sequences $u_r(t)$ that guarantees the nullity of all off-diagonal blocks in (46), i.e. one seeks the property:

$${}^r\mathbf{V}_{0|2i-1,j}\,{}^c.\mathbf{V}_{0|2i-1,j}^T = 0_{2di \times 2di}, \ \forall r \neq c, \quad (47)$$

It is checked that any non diagonal block ${}^r\mathbf{V}_{0|2i-1,j}\,{}^c\mathbf{V}_{0|2i-1,j}^T$ of the covariance matrix $\mathbf{V}_{0|2i-1,j}\mathbf{V}_{0|2i-1,j}^T$ (48), contains the element (49), at the intersection of the p^{th}-row and the q^{th}-column of the block located at the r^{th}-row block and the c^{th}-column block,

$${}^r\mathbf{V}_{0|2i-1,j}\,{}^c\mathbf{V}_{0|2i-1,j}^T = \left(\sum_{k=0}^{j-1} \Psi_{r,d}(u_r(p+k))\Psi_{c,d}(u_c(q+k))^T\right)_{\substack{p=0..2i-1,\\ q=0..2i-1}} \in \Re^{2id \times 2id} \quad (48)$$

$$\left({}^r\mathbf{V}_{0|2i-1,j}\,{}^c\mathbf{V}_{0|2i-1,j}^T\right)(p,q) = \sum_{k=0}^{j-1} \Psi_{r,m}(u_r(p+k))\Psi_{c,d}(u_c(q+k))^T \in \Re^{d \times d} \quad (49)$$

This can be further developed as follows:

$$\left({}^r\mathbf{V}_{0|2i-1,j}\,{}^c\mathbf{V}_{0|2i-1,j}^T\right)(p,q) = \sum_{k=0}^{j-1} \begin{bmatrix} \psi_{r,1}(u_r(p+k))\psi_{c,1}(u_c(q+k)) & ... & \psi_{r,1}(u_r(p+k))\psi_{c,d}(u_c(q+k)) \\ \psi_{r,2}(u_r(p+k))\psi_{c,1}(u_c(q+k)) & ... & \psi_{r,2}(u_r(p+k))\psi_{c,d}(u_c(q+k)) \\ ... & ... & ... \\ \psi_{r,d}(u_r(p+k))\psi_{c,1}(u_c(q+k)) & ... & \psi_{r,d}(u_r(p+k))\psi_{c,d}(u_c(q+k)) \end{bmatrix} \quad (50)$$

The component at the μ^{th}-row and ν^{th}-column is explicitly given by:

$$\left(\left({}^r\mathbf{V}_{0|2i-1,j}\,{}^c\mathbf{V}_{0|2i-1,j}^T\right)(p,q)\right)(\mu,\nu) = \sum_{k=0}^{j-1} \psi_{r,\mu}(u_r(p+k))\psi_{r,\nu}(u_c(q+k)) \quad (51)$$

and which can be forced to zero by making $u_c(q+k) = 0$ when $u_r(p+k) \neq 0$ and vice versa, since that

$\psi_{r,k}(0) = 0$ (38). Having (35), This amounts to let the integer i^* and the integer sequence $\{\phi_r\}$ be such that the two equalities (52) do not simultaneously hold where the notation $(\bmod 2i^*)$ stands for the congruence modulo $2i^*$.

$$\begin{cases} p + k - \phi_r \equiv 0 \pmod{2i^*} \\ q + k - \phi_c \equiv 0 \pmod{2i^*} \end{cases} \tag{52}$$

Bearing this in mind and knowing that both p and q vary strictly between 0 and 2i-1, if (40) and (41) hold, it comes that the following double inequality (53) is satisfied,

$$0 < \phi_r - \phi_c - (p - q) < 2i^* \tag{53}$$

Consequently (52) do not holds, therefore all the off diagonal blocks of $\mathbf{V}\,\mathbf{V}^T$ are zeros. Furthermore, to make possible the full rank property of $R_{2i}^{U,U}$ the number columns of the Hankel matrix $\mathbf{U}_{0|2i-1,j}$ must be enough large, i.e. $j \geq 2mdi$. Consequently there exist $j_0 = 2mdi$ and $i_0^* = i + (\phi_m - \phi_1)/2$ such that , for any $i^* \geq i_0^*$, and any $j \geq j_0$, the covariance matrix $R_{2i}^{U,U}$ is of full rank and which is equal to $2mdi$, which establishes the lemma ■

C. *Pulse Based Input PE Signals for Subspace Identification of MISO Hammerstein Systems*

Theorem. Consider the N^{th} – order linear input-output state space model of the MISO Hammerstein system (17) subject to the multivariable input signal $U(t)$ defined on the interval $t \in [0, 2i + j - 2]$, for some integers i and j, by:

$$U(t) = \left[\Psi_{r,d}(u_r(t))^T\right]_{r=1,..,m}^T \in \Re^{md\times 1} \tag{54}$$

where $$\Psi_{r,d}(u_r(t)) \overset{\Delta}{=} [\psi_1(u_r(t)) \quad \psi_2(u_r(t)) \quad .. \quad \psi_d(u_r(t))]^T \tag{55}$$

and where:

$$\forall x \in \Re, \psi_k(x) = x^k, \forall k \in \{1,..,d\} \tag{56}$$

the signal $u_r(t)$ *being defined, for some integer numbers* i^* *and* φ, *by:*

$$u_r(t) = \sum_{k=0}^{d-1} s_{r,k}\,\delta(t - \phi_r - 2ki^*), \forall r \in \{1,..,m\} \tag{57}$$

where $$\phi_r = (r-1)\varphi \tag{58}$$

and where the real sequences $\{s_{1,k}\},..,\{s_{m,k}\}$ *of length* d *are such that:*

$$s_{r,k} \neq s_{r,k'} \quad \forall k \neq k' \tag{59}$$

Then, the multivariable input signal $U(t)$ *(54) is PE for the system (17) provided that:*

(i) $$i \geq i_0 = N + 1 \tag{60}$$

(ii) $$\varphi \geq \varphi_0 = 2i \tag{61}$$

(iii) $$i^* \geq i_0^* = i + \left[(m-1)\frac{\varphi}{2}\right] \tag{62}$$

(iv) $$j \geq j_0 = \max\left(2i(d-1) + (m-1)\phi_0, 2i(md+1)\right) \tag{63}$$

□

Proof: It was proved in [5] that, if $s_{r,k} \neq s_{r,k'}$ for all $k \neq k'$, and any $r = 1,...,m$, then the covariance matrix of the vector sequence $\Psi_{r,d}(u_r(t))$ defined by (55-56) is of full rank, provided that j is enough large so that any signal $u_r(t)$ contains at least d pulses, i.e.:

$$rank\left(R_{2i^*}^{\Psi_{r,d}(u_r),\Psi_{r,d}(u_r)}\right) = 2di^* \tag{64}$$

$$2i + j - 1 \geq 2i^*(d-1) + (m-1)\varphi \tag{65}$$

Therefore since that $\Psi_{r,d}(0) = 0_{d\times 1}$, and that the arithmetic progression sequences ϕ_r (58) is strictly increasing, all the conditions (38-41) of the lemma are satisfied provided that $\varphi \geq 2i$. Consequently, and knowing that in the subspace identification it is assumed on one hand that the half number (i) of block rows of the input Hankel matrix must be strictly larger than the system order (N), and on the other hand that its number of columns (j) must be larger than the number of rows ($2i(md+1)$) of the matrix concatenating the input and output Hankel matrices [2], there exist $i_0 = N + 1$, $\varphi_0 = 2i$, $i_0^* = i + [(m-1)\varphi/2]$ and $j_0 = \max(2i(md+1), 2i^*(d-1) + (m-1)\varphi - 2i + 1)$, such that for any $i \geq i_0$, $\varphi \geq \varphi_0$, $i^* \geq i_0^*$ and $j \geq j_0$, the covariance matrix $R_{2i}^{U,U}$ is of full rank ($2mdi$) and thereby the input sequence $U(t)$ is PE of order $2i$, which establishes the theorem ■

V. Numerical Example

To confirm the usefulness of the above theorem we consider the following numerical example:

$$F_1(v) = \frac{1}{2\pi}\sin(2\pi v);\ G_1(q^{-1}) = \frac{0.1251\,q^{-1} + 0.1166\,q^{-2}}{1 - 1.569\,q^{-1} + 0.811\,q^{-2}} \tag{66}$$

$$F_2(v) = \frac{\pi}{8}\arctan\left(\frac{8v}{\pi}\right);\ G_2(q^{-1}) = \frac{0.07417q^{-1} + 0.07224q^{-2}}{1 - 1.778q^{-1} + 0.9245q^{-2}} \tag{67}$$

$$F_3(v) = v\exp(-2v^2)G_3(q^{-1}) = \frac{0.0723q^{-1} + 0.06859q^{-2}}{1 - 1.714q^{-1} + 0.8546q^{-2}} \tag{68}$$

A. *Input sequence design*

The system (66-68) is excited by a 24-order PE sequence generated according to the proposed theorem by letting $i = 12; i^* = 36; \varphi = 24$ for $m = 3$, $n^* = 2$ and $d^* = 7$, yielding $M = 21$, $j = 528$ and a total number of samples $N_s = 599$. As mentioned in the theorem, any real sequence composed of different samples can be used to weight the samples the input sequences. Presently, the following simple alternated saw-tooth weighting sequence is used:

$$s_{r,k} = \gamma_r(-1)^k((m-1)\varphi_0 + 2ki^*) \tag{69}$$

where γ_r is a real number which chosen in accordance with the input range of the r^{th} input channel of the system. The input sequence thus defined as well as the resulting output sequence, are plotted in figure 3.

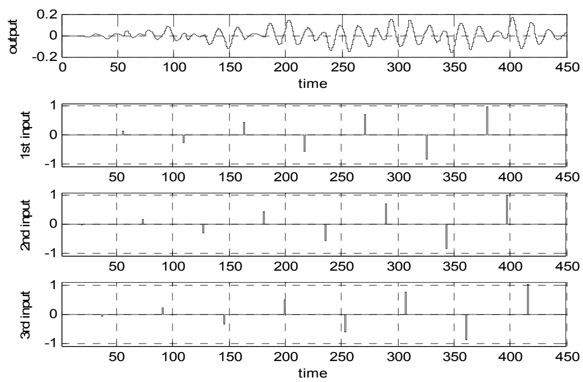

Figure 3 wave forms of the recorded input and output sequences

B. Identification results

The robust version of the Subspace Algorithm (subid) is applied using the previously generated input and output data by letting $i = 12$ and $j = 528$. The resulting estimates for the linear subsystems are the following:

$$\hat{A}_1 = \begin{bmatrix} 0.785 & 0.442 \\ -0.442 & 0.785 \end{bmatrix}, \hat{B}_1 = \begin{bmatrix} -1.063 \\ -6.245 \end{bmatrix}, \hat{C}_1 = \begin{bmatrix} -0.078 & -0.0066 \end{bmatrix} \quad (70)$$

$$\hat{A}_2 = \begin{bmatrix} 0.889 & 0.366 \\ -0.366 & 0.889 \end{bmatrix}, \hat{B}_1 = \begin{bmatrix} -3.01 \\ -2.67 \end{bmatrix}, \hat{C}_2 = \begin{bmatrix} -0.075 & 0.057 \end{bmatrix} \quad (71)$$

$$\hat{A}_3 = \begin{bmatrix} 0.857 & 0.347 \\ -0.347 & 0.857 \end{bmatrix}, \hat{B}_1 = \begin{bmatrix} 2.631 \\ 2.527 \end{bmatrix}, \hat{C}_3 = \begin{bmatrix} 0.0858 & -0.0607 \end{bmatrix} \quad (72)$$

The corresponding transfer functions are:

$$\hat{G}_1(q^{-1}) = \frac{0.1255\, q^{-1} + 0.1163\, q^{-2}}{1 - 1.569\, q^{-1} + 0.8111\, q^{-2}} \quad (73)$$

$$\hat{G}_2(q^{-1}) = \frac{0.07425\, q^{-1} + 0.07217\, q^{-2}}{1 - 1.778\, q^{-1} + 0.9245\, q^{-2}} \quad (74)$$

$$\hat{G}_3(q^{-1}) = \frac{0.07218\, q^{-1} + 0.0687\, q^{-2}}{1 - 1.714\, q^{-1} + 0.8547\, q^{-2}} \quad (75)$$

It is readily seen, comparing (73-75) and (66-68), that the transfer function coefficients are accurately estimated. The coefficient estimates of the 7^{th} degree-polynomial nonlinearities are:

$$\hat{F}_{1,7} = \begin{bmatrix} 1 & 0.041 & -6.255 & -0.316 & 10.213 & 0.431 & 5.122 \end{bmatrix} \quad (76)$$

$$\hat{F}_{2,7} = \begin{bmatrix} 1 & 0.03 & -1.596 & -0.188 & 2.207 & 0.221 & 1.197 \end{bmatrix} \quad (77)$$

$$\hat{F}_{3,7} = \begin{bmatrix} 1 & -0.002 & -1.99 & 0.02 & 1.83 & -0.036 & 0.77 \end{bmatrix} \quad (78)$$

Fig. 4 shows the graphical characteristics of the system polynomial static nonlinearities and their estimates. It is seen that the estimated curves are quite close to the true curves.

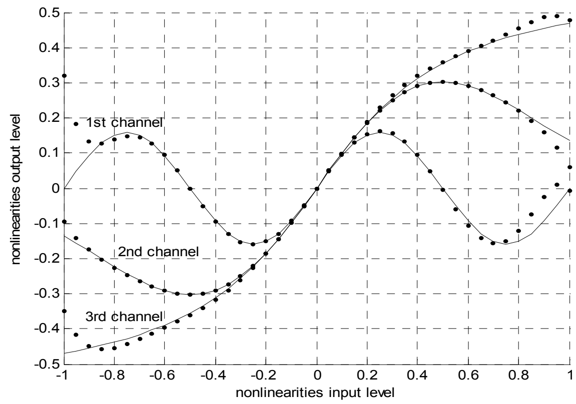

Fig. 4. Graphical representation of the system nonlinearities. Solid: true nonlinearities. Dotted: estimated nonlinearities.

Finally, Fig.5 shows the true system output and the estimated model output, when both are excited by the input sequence generated in Subsection A. It is readily seen the estimated model matches well the true system. The above observations confirm the high accuracy of the subspace identification when the system is excited by sequences generated according to the input design theorem of Section IV.C.

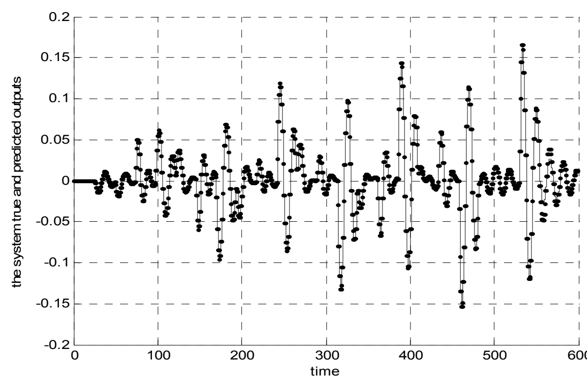

Fig. 5. True (-) and predicted (..) output sequences of the considered 3-channels Hammerstein system

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