The Role of Co-located Storage for Wind Power Producers in Conventional Electricity Markets

E. Bitar, R. Rajagopal, P. Khargonekar, K. Poolla

Abstract— In this paper we study the problem of optimizing contract offerings for an independent wind power producer (WPP) participating in conventional day-ahead forward electricity markets for energy. As wind power is an inherently variable source of energy and is difficult to predict, we explore the extent to which *co-located energy storage* can be used to improve expected profit and mitigate the the financial risk associated with shorting on the offered contracts. Using a simple stochastic model for wind power production and a model for the electricity market, we show that the problem of determining optimal contract offerings for a WPP with co-located energy storage can be solved using convex programming.

Index Terms— Renewable Energy, Smart Grid, Energy Storage, Electricity Markets

I. INTRODUCTION

Motivated by the dangers posed by global warming, there is great interest in renewable energy sources. Electric energy is the dominant form of energy consumption (accounting for more than 50 % in the US). Wind and solar energy are expected to become a much larger source of electric energy to meet the renewable energy production targets in many parts of the world [21]. These sources of electricity production are inherently uncertain, variable, and largely uncontrollable. Together, these characteristics constitute major challenges to the integration of these clean energy source into the electricity grid at deep penetration levels [17], [20], [18].

Electric energy storage represents one possible strategy to mitigate the impact of the inherent variability and uncertainty in wind and solar power. Indeed, hydro-power has traditionally been used for such purposes [11], [31]. In this paper, we focus on the scenario in which wind power producers (WPP) must sell their energy using contract mechanisms in *conventional* forward electricity markets. Our goal is to formulate and solve problems of optimal contract sizing for such wind power producers with dedicated co-located electric energy storage capacity. We explore the impact of optimal storage operation on contract sizing and profit. We start with a simple stochastic model for wind power production and a model for the electricity market.

Corresponding author: E. Bitar is with the Department of Mechanical Engineering, U.C. Berkeley *ebitar@berkeley.edu*

P. Khargonekar is with the Department of Electrical and Computer Engineering, University of Florida *ppk@ece.ufl.edu*

R. Rajagopal and K. Poolla are with the Department of Electrical Engineering and Computer Science, U.C. Berkeley {*ramr, poolla*}*@eecs.berkeley.edu*

We show that the problem of determining optimal contract offerings for WPP with co-located energy storage reduces to convex programming. We also show that the expected profit acquired by the wind power producer for optimal contract offerings is concave, non-decreasing in the parameter of energy storage capacity – revealing that greatest marginal benefit from energy storage is derived for a *small* amount of storage capacity.

Energy storage devices such as pumped-hydro, compressed air [24], sodium-sulfur batteries [32], etc. offer the possibility of adding storage to to "firm" wind generation power supply. Cavallo [8] wrote one of the earliest papers to make the case for joint operation of wind energy and storage (see [9] for a utility scale investigation of compressed air energy storage with wind). Greenblatt et al [23] compared gas turbines and compressed air energy storage in the context of wind as part of baseload electricity generation (see also [38] for a detailed report on CAES and wind energy). Denholm and Sioshansi [13] have studied energy storage and wind power in a transmission constrained system where they compare the economics of siting the storage system near the wind power generation site versus the load site. Electric and plug-in hybrid vehicles also represent potential distributed energy storage devices. Economic viability of compressed air energy storage (CAES) in a wind energy system in Denmark has recently been investigated in [27]. They also investigate the operation of this joint storage wind energy system in the Nordic spot and regulatory energy markets. They conclude that such a system is economically viable depending on the monthly payment from the regulating power market. A recent paper by DeCesaro and Porter [10] presents a summary of most wind integration studies to date.

Recently, Angarita et al [1] have investigated combined wind-hydro bids in an electricity pool market. They formulate a stochastic programming problem that accounts for uncertainty in the wind availability and prices of electricity in various markets. Using a "scenario based" approach to dealing with the uncertainty, they develop a linear programming solution which yields optimal offer curves and limits the risk of profit variability. Our work is related to this investigation, but employs analytical methods to yield computably optimal solutions.

As storage is currently quite expensive and the level of penetration of renewable energy is not very high, storage is not considered to be necessary for integrating wind into the electric grid for the 20% penetration levels ([17], p.229). A full analysis of the economics of storage in the context of renewable integration can be found in [12]. We note

that there are large investments in new technologies for energy storage research and development [16]. The Solar Energy Grid Integration Systems-Energy Storage (SEGIS-ES) project [40] represents a recent comprehensive effort along this direction in the US. A recent review of battery based storage technologies can be found in [14]. In their report, Denholm et al conclude that "It is clear that high penetration of variable generation (VG) increases the need for all flexibility options including storage, and it also creates market opportunities for these technologies. Evaluating the role of storage with VG sources requires continued analysis, improved data, and new techniques to evaluate the operation of a more dynamic and intelligent grid of the future." While we present our results in the context of joint optimization of wind and storage, we believe they can be generalized and extended to the situation of joint optimization of storage with the composite generation+load uncertainty in the grid.

The remainder of this paper is organized as follows. Section II provides a brief over overview of electricity markets. Section III delineates the mathematical models (wind power production, energy storage, market) employed in our analysis. Our main results are presented in section IV, followed with concluding remarks in section V.

II. ELECTRICITY MARKETS

We assume that the wind power producer is part of a power pool participating in electricity markets that are cleared by an external entity, such as an ISO or RTO. A common trading structure ([29], [30], [26]) consists of two successive *ex-ante* markets: a day-ahead (DA) forward market and a real-time (RT) spot market. The DA market permits participants to bid and schedule energy transactions for the following day. Depending on the region, the DA market closes for bids and schedules by 10 AM and clears by 1 PM on the day prior to the operating day. The schedules cleared in the DA market are financially binding and are subject to deviation penalties. As the schedules submitted to the DA market are cleared well in advance of the operating day, a RT spot market is employed to ensure the balance of supply and demand in real-time by allowing market participants to adjust their DA schedules based on more accurate wind and load forecasts. The RT market is cleared five to 15 minutes before the operating interval, which is on the order of five minutes.

For those market participants who deviate from their scheduled transactions agreed upon in the *ex-ante* markets, the ISO normally employs an *ex-post* deterministic settlement mechanism to compute asymmetric imbalance prices. This asymmetric pricing scheme for penalizing energy deviations reflects the energy imbalance of the control area as a whole and the *ex-ante* clearing prices. For example, if the overall system imbalance is negative, those power producers with a positive imbalance with respect to their particular schedules will receive a more favorable price than those producers who have negatively deviated from their schedules, and vice-versa.

For a more detailed analysis of electricity market systems in different regions, we refer the reader to [6], [7], [37].

III. MODELS: WIND, STORAGE, AND MARKETS

A. Wind Power Model

Wind power is modeled as a discrete-time random process $\{w_n \mid n \in \mathbb{N}\}$. For a fixed $n \in \mathbb{N}$, w_n is a continuous random variable whose cumulative distribution function (CDF) is assumed known and defined as $F(w, n) = \mathbb{P}\{w_n \leq w\}$. The random process $\{w_n\}$ takes on values in the unit interval $[0, 1]$, as wind power output is assumed to be normalized by the farm's nameplate capacity.

In the following results, we will be interested in *time-averaged* distributions defined on integer intervals of length $N \in \mathbb{N}$. For example, the time-averaged CDF on the integer interval $\{1, \cdots, N\}$ is defined as

$$F(w) = \frac{1}{N} \sum_{n=1}^{N} F(w, n) \tag{1}$$

Also, define $F^{-1} : [0, 1] \rightarrow [0, 1]$ as the *quantile function* corresponding to the CDF F. More precisely, for $\beta \in [0, 1]$, the β-*quantile* of F is given by

$$F^{-1}(\beta) = \inf \{x \in [0, 1] \; : \; \beta \leq F(x)\} \tag{2}$$

The quantile function corresponding to the time-averaged CDF will play a central role in our results.

B. Energy Storage Model

Consider the following linear difference equation as a dynamic model for a generic energy storage system [26].

$$e_{n+1} = (1 + \alpha h)e_n + h \left[\eta_{\text{inj}} P_{n,\text{inj}} - \frac{1}{\eta_{\text{ext}}} P_{n,\text{ext}} \right] \tag{3}$$

subject to the following constraints

$$0 \leq \quad e_n \quad \leq \overline{e} \tag{4}$$
$$0 \leq \quad P_{n,\text{inj}} \quad \leq \overline{P}_{\text{inj}} \tag{5}$$
$$0 \leq \quad P_{n,\text{ext}} \quad \leq \overline{P}_{\text{ext}} \tag{6}$$

The energy contained in the storage system at time n is denoted by e_n. The magnitude of the power extracted (injected) from (into) the storage system at time n is denoted by $P_{n,\text{ext}}$ ($P_{n,\text{inj}}$). The parameter $\alpha \leq 0$ is the dissipation coefficient on the stored energy, while $\eta_{\text{inj}}, \eta_{\text{ext}} \in [0, 1]$ model power injection and extraction efficiencies, respectively. The discretization step size is denoted by h.

C. Market Model

In our analysis, we take the perspective of a wind power producer (WPP) participating as a generator in an electricity market for energy. We employ a market model that consists of a *single* ex-ante DA forward market with an ex-post financial penalty for deviations from offered contracts. In the DA market, generators offer a portfolio of M time-ordered contracts for the delivery of power the following day. The *contract portfolio* $\mathbf{C} \in \mathbb{R}_+^M$ is structured as a sequence of

M power levels that are piecewise constant on intervals, typically, of length one-hour.

$$\mathbf{C} = \left[\, C^{(1)}, \cdots, C^{(M)} \, \right]$$

The time interval corresponding to contract $C^{(m)}$ is defined as the integer interval

$$\mathcal{N}_m = \{N(m-1)+1, \ \cdots, \ Nm\} \quad (7)$$

where $|\mathcal{N}_m| = N$. It follows naturally that the contract value C_n at time n is given by

$$C_n = \sum_{m=1}^{M} \mathbf{1}\{n \in \mathcal{N}_m\} C^{(m)} \quad (8)$$

where $\mathbf{1}\{\cdot\}$ is defined to be the indicator function. See Figure 1 for an example of a contract portfolio $\mathbf{C}$ (ex: $M = 24$) offered in a day-ahead forward market. As the power contracts are offered ex-ante, deviations naturally occur between the offered contracts and the realized wind power output.
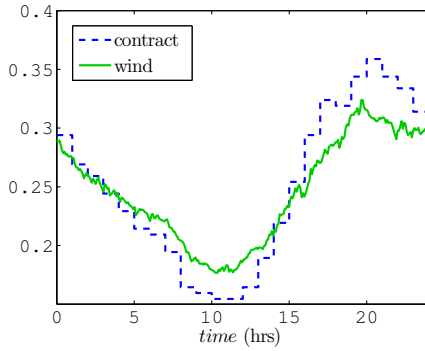

Fig. 1. Illustrative example of typical contract portfolio (dashed) offered ex-ante in day-ahead (DA) market. Contract intervals are of length one-hour. The wind power producer is subject to financial penalties for generation shortfalls realized ex-post – i.e. when the wind power (solid) dips below the offered contract (dashed).

The WPP receives a price p ($/MW-hour) – defined as the clearing price in the forward market – for each offered contract $C^{(m)}$. For uninstructed contract shortfalls (i.e. $w_n < C^{(m)}$ at time n) the generator pays at the imbalance penalty price q ($/MW-hour) for the associated shortfall. We make the following *assumptions* regarding prices and production costs:

A1 The WPP is assumed to be a *price taker* in the forward market, because the WPP capacity is assumed small relative to the whole market.

A2 In this formulation p and q are assumed to be *fixed* and *known*. However, this assumption can be relaxed to p and q random and time varying without affecting the tractability of the results as long as they are assumed to be independent of the wind process w_n.

A3 The WPP is assumed to have a *zero marginal cost of production*.

For a given contract portfolio $\mathbf{C}$, the *profit* acquired by the WPP on the interval $\{1, \cdots, NM\}$ is defined as

$$\Pi(\mathbf{C}, w) \ = \ h \sum_{m=1}^{M} \sum_{n \in \mathcal{N}_m} pC^{(m)} \ - \ q \left[C^{(m)} - w_n \right]^+ \quad (9)$$

where $x^+ := \max\{x, 0\}$ and h is the discretization time step. As wind power $\{w_n\}$ is modeled as a random process, we will be concerned with the *expected* profit $J(\mathbf{C})$:

$$J(\mathbf{C}) = \mathbb{E} \ \Pi(\mathbf{C}, w) \quad (10)$$

Here, the expectation is taken with respect to the random wind power process $w = \{w_n \mid n \in \mathbb{N}\}$.

Remark 3.1: (Storage) The introduction of energy storage will manifest in an augmented profit model in that the WPP will have recourse capability to cover contract shortfalls by drawing on stored energy. This scenario will analyzed in section IV-B. □

IV. MAIN RESULTS

A. Contract Sizing without Energy Storage

We begin by defining a profit maximizing portfolio $\mathbf{C}^*$ as

$$\mathbf{C}^* = \arg \max_{\mathbf{C} \in \mathbb{R}_+^M} J(\mathbf{C}). \quad (11)$$

In the absence of any energy storage capability, the opportunities for energy arbitrage between contract intervals evaporate and the decision of how much constant power to offer on interval i is independent of the decision on interval j for all $i \neq j$. Hence, the portfolio optimization (11) decouples into M independent optimization problems:

$$C^{(m)*} = \arg \max_{C \in \mathbb{R}_+} J(C) \qquad m = 1, \cdots, M \quad (12)$$

The formulation in (12) has be carefully studied [4] and is closely related to the *newsvendor* problem in operations research [33]. The main result is presented here.

Theorem 4.1: [4] Define the time-averaged distribution

$$F_m(w) = \frac{1}{N} \sum_{n \in \mathcal{N}_m} F(w, n)$$

An *optimal contract* $C^{(m)*}$ is given by

$$C^{(m)*} = F_m^{-1}(\gamma) \quad \text{where} \quad \gamma = \frac{p}{q}. \quad (13)$$

Remark 4.2: Properties of the optimal quantile rule (13), such as uniqueness, price elasticity of supply, and the effect penalty pricing are explored in detail in [4]. □

B. Energy Storage Formulation

As wind energy penetration levels increase, energy storage will play a more dominant role in facilitating the firming of wind power contracts in conventional electricity markets. We now consider the scenario in which the wind power producer (WPP) has a co-located energy storage device at it's disposal. As the capital cost of an energy storage system is quite prohibitive, a fundamental question in this context arises: what impact does energy storage capacity have on expected profit and what are optimal contract offerings in this context? We now formalize these questions as a constrained stochastic optimal control problem.

Recall section III-B and consider the linear difference equation (3) and constraints (4) - (6) as a model for the energy storage system. The energy storage system interfaces with the wind power producer through the power injection (extraction) variables $P_{n,\text{inj}}(P_{n,\text{ext}})$. Define the storage decision vector $u_n = [P_{n,\text{ext}},\ P_{n,\text{inj}}]^{\text{T}}$.

We assume that e_n and w_n are observed. For a particular time n, all of the information from the past relevant to the future is contained in the current storage state e_n and all past observed wind power realizations $w^n := \{w_i \mid i = 1, \cdots, n\}$. Hence, we consider storage operation policies of the form

$$u_n = g_n(e_n, w^n) = \begin{bmatrix} P_{n,\text{ext}} \\ P_{n,\text{inj}} \end{bmatrix}$$

where g_n is constrained to belong to the set of *feasible* operation policies guaranteeing that constraints (3) - (6) are satisfied. Let $g := \{g_n \mid n = 1, \cdots, NM\}$ and let $\mathcal{G}$ denote the set of all feasible operation policies g.

The expected profit corresponding to a particular operational policy and contract portfolio $(g, \mathbf{C})$ is defined as

$$J(g, \mathbf{C}) = \qquad\qquad\qquad (14)$$
$$\mathbb{E}\left[h \sum_{m=1}^{M} \sum_{n \in \mathcal{N}_m} pC^{(m)} - q\left[C^{(m)} - w_n + P_{n,\text{in}}^g - P_{n,\text{ext}}^g \right]^+ \right]$$

The superscript g is included to indicate the dependence on the control policy g. A profit maximizing storage operation policy and contract portfolio $(g^*, \mathbf{C}^*)$ are given by

$$(g^*, \mathbf{C}^*) = \underset{\substack{\mathbf{C} \geq 0 \\ g \in \mathcal{G}}}{\arg\max}\ J(g, \mathbf{C}) \qquad (15)$$
$$\text{subject to}\ \ (3) - (6)$$

In the proceeding sections (IV-C) - (IV-D), we explore various properties of this optimal contract sizing problem for a WPP with co-located storage.

Remark 4.3: (Optimal operational policy) For a given contract portfolio $\mathbf{C} \in \mathbb{R}_+^M$, it is straightforward to show that a *greedy* storage operational strategy belongs to the class of feasible optimal policies. The intuition is as follows. As there is no holding cost associated with stored energy, it is optimal to always inject the maximum allowed energy when

there is a surplus in generation (i.e. $w_k > C_n$) relative to the offered contract. Similarly, as there are *no price arbitrage* opportunities, because the penalty price q is time-invariant – when there is a shortfall in generation (i.e. $w_k \leq C_n$) relative to the offered contract, the optimal policy is to extract the maximum allowable energy from the storage needed to cover the shortfall. More formally:

If there is a *contract shortfall* (i.e., $\Delta_n = C_n - w_n > 0$),

$$g^*(e_n, w^n) = \begin{bmatrix} \min\left\{ \Delta_n,\ \frac{\eta_{\text{ext}}}{h}(e_n),\ \overline{P}_{\text{ext}} \right\} \\ 0 \end{bmatrix} \qquad (16)$$

If there is a *contract surplus* (i.e., $\Delta_n = C_n - w_n \leq 0$),

$$g^*(e_n, w^n) = \begin{bmatrix} 0 \\ \min\left\{ -\Delta_n,\ \frac{1}{h\eta_{\text{inj}}}(\overline{e} - e_n),\ \overline{P}_{\text{inj}} \right\} \end{bmatrix} \qquad (17)$$

Note that the optimal storage operational policy above is *stationary* and *causal*. $\qquad\qquad\qquad \square$

C. Contract Sizing with Energy Storage

In the day-ahead market, the WPP must offer a contract portfolio $\mathbf{C}$ for the delivery of power at some future time interval. We show that the problem of computing a profit maximizing portfolio $\mathbf{C}^*$ is a *convex optimization problem*.

Theorem 4.4: (Convexity Property) Let g^* be an optimal operational policy for a fixed $\mathbf{C} \in [0,1]^M$. Then $J(g^*, \mathbf{C})$ is *concave* in $\mathbf{C}$.

Proof: Without loss of generality, we prove the result for a single contract interval ($M = 1$, $\mathbf{C} = C \in [0,1]$). Define the random profit criterion

$$\Pi(g, C, w) = \sum_{n=1}^{N} phC - qh\left[C - w_n + P_{n,\text{in}}^g - P_{n,\text{ext}}^g \right]^+.$$

This next step does not hold in general. However in our case, the expectation and maximization operators commute, because – as indicated by equations (16) and (17) – our stationary policy is optimal for each realization of the wind process w.

$$J(g^*, C) = \max_{g \in \mathcal{G}}\ \mathbb{E}\ \Pi(g, C, w)$$
$$= \mathbb{E}\ \max_{g \in \mathcal{G}}\ \Pi(g, C, w)$$

Define $z(C, w) = \max_{g \in \mathcal{G}}\ \Pi(g, C, w)$. We first prove concavity of the optimal value $z(C, w)$ in the parameters (C, w). Consider the following linear programming (LP)

formulation

$$y(C, w) = \max_{s, e, P_{\text{inj}}, P_{\text{ext}}} \sum_{n=1}^{N} phC - qhs_n \qquad (18)$$

$$\text{subject to}$$

$$e_0 = 0$$

$$s_n \geq 0$$

$$s_n \geq C - w_n + P_{n,\text{inj}} - P_{n,\text{ext}}$$

$$(3) - (6) \ \forall \ n$$

where we have introduced a new slack decision variable s. It is straightforward to show that $z(C, w) = y(C, w)$ for all (C, w).

Hence, showing concavity of $z(C, w)$ in (C, w) is equivalent to showing concavity of $y(C, w)$. Let $\alpha \in [0, 1]$ and define

$$C^\alpha = \alpha C^1 + (1 - \alpha)C^2$$

$$w^\alpha = \alpha w^1 + (1 - \alpha)w^2$$

where $C^1, C^2 \in [0, 1]$ and $w^1, w^2 \in [0, 1]^N$. Moreover, let the set of vectors $\{s^*(C, w), \ P_{\text{inj}}^*(C, w), \ P_{\text{ext}}^*(C, w)\}$ be optimizers of (18) for particular C, w. Concavity of $y(C, w)$ in (C, w) is proven as follows.

$$y(C^\alpha, w^\alpha)$$

$$= phNC^\alpha - qh \sum_{n=1}^{N} s_n^*(C^\alpha, w^\alpha)$$

$$\geq phNC^\alpha - qh \sum_{n=1}^{N} \left[\alpha s_n^*(C^1, w^1) + (1 - \alpha)s_n^*(C^2, w^2) \right]$$

$$= \alpha y(C^1, w^1) + (1 - \alpha)y(C^2, w^2)$$

The inequality follows from the fact that

$$\alpha \begin{bmatrix} s^*(C^1, w^1) \\ P_{\text{inj}}^*(C^1, w^1) \\ P_{\text{ext}}^*(C^1, w^1) \end{bmatrix} + (1 - \alpha) \begin{bmatrix} s^*(C^2, w^2) \\ P_{\text{inj}}^*(C^2, w^2) \\ P_{\text{ext}}^*(C^2, w^2) \end{bmatrix}$$

is a feasible point for problem (18) with parameters (C^α, w^α). This feasibility holds because the parameters (C, w) enter linearly through the constraints in (18).

We, thus far, have shown that $z(C, w) = y(C, w)$ is concave in (C, w). Clearly then,

$$J(g^*, C) = \mathbb{E} \ z(C, w)$$

is concave in C, because a convex combination of concave functions is concave. ∎

D. Value of Energy Storage Capacity, $\overline{e}$

Storage capacity constitutes a large percentage of the capital cost associated with many energy storage modalities. Hence, in order to accurately amortize the the capital investment in storage capacity over a period of time, it is of vital importance to quantify the fiscal benefit to the WPP in terms of storage capacity (i.e. J^* as a function of $\overline{e}$). This

relation can then be used to optimally size the storage system so as to maximize return on investment. In Theorem 4.4, we proved that the problem of computing optimal contract offerings and the corresponding expected profit reduces to convex programming – a result that naturally lends itself to efficient computation of the return on investment curves.

In the following Theorem 4.5, we show that the optimal expected profit ($J^* := J(g^*, \mathbf{C}^*)$) derived by the WPP – with co-located storage of capacity $\overline{e}$ – is concave and non-decreasing in the capacity $\overline{e}$. This result reveals that the *greatest marginal benefit* is derived from a *small* amount of storage capacity. In fact, the marginal optimal expected profit with respect to the storage capacity $dJ^*/d\overline{e}$ can be analytically computed for $\overline{e}$ *small* [5].

Theorem 4.5: The optimal expected profit $J(g^*, \mathbf{C}^*)$ is concave and monotonically non-decreasing in the energy storage capacity $\overline{e}$.

Proof: Monotonicity is straightforward. Let $\epsilon > 0$. With a slight abuse of notation, let $J^*(\overline{e})$ denote the optimal expected profit corresponding to a system with storage capacity $\overline{e} \geq 0$. Clearly, $J^*(\overline{e} + \epsilon) \geq J^*(\overline{e})$, as the set of feasible solutions for problem (15) with capacity parameter $\overline{e}$ is a subset of the feasible set corresponding to a capacity parameter of $\overline{e} + \epsilon$. Concavity is proved analogously to theorem 4.4. ∎

For completeness, we present the result [5] that quantifies in closed-form the marginal expected optimal profit $dJ^*/d\overline{e}$ for *small* capacity $\overline{e}$.

Consider a contract portfolio $\mathbf{C} \in \mathbb{R}_+^M$ with individual contract duration of length N. Define the random variable $\xi(\mathbf{C})$ to be the number of times that the random process $\{w_n\}$ crosses the associated contract sequence $\{C_n\}$ from above. The random variable $\xi(\mathbf{C})$ can be interpreted as the number of *energy arbitrage opportunities* associated with the contract portfolio $\mathbf{C}$.

Theorem 4.6: [5] Let $\gamma := p/q$. Assume that (1) the energy storage system is non-dissipative (i.e. $\alpha = 0$), (2) no constraints on power extraction or injection, and (3) $e(0) = 0$.

Then the *marginal expected optimal profit* with respect to $\overline{e}$ at the origin is given by

$$\left. \frac{dJ^*}{d\overline{e}} \right|_{\overline{e}=0} = q \ \eta_{\text{inj}}\eta_{\text{ext}} \ \mathbb{E}\left[\ \xi(\mathbf{C}^*) \ \right] \qquad (19)$$

where $C^{(m)*} = F_m^{-1}(\gamma)$ for $m = 1, \cdots, M$.

Remark 4.7: (Intuition) The previous result has an intuitive interpretation in that the marginal value of storage capacity (for small amounts) is proportional to the expected number of *energy arbitrage opportunities*. Consider a system with small storage capacity $\epsilon > 0$. Each time the wind power process dips below the offered contract, the WPP has the opportunity extract ϵ energy from its storage device and

decrement it's financial penalty by $q(\eta_{\mathrm{inj}}\eta_{\mathrm{ext}})\epsilon$. Clearly then, the total expected financial benefit for small storage capacity is roughly $\approx q(\eta_{\mathrm{inj}}\eta_{\mathrm{ext}})\mathbb{E}[\xi]\epsilon$. $\qquad\square$

V. CONCLUSION

In this paper we have formulated and solved the problem of optimal contract sizing for a wind power producer with *co-located energy storage* participating in conventional electricity markets. We have shown that the problem of determining optimal contract offerings for a WPP with co-located energy storage can be solved using convex programming. Our results have the merit of providing key insights into the trade-offs between a variety of factors such as energy storage *capacity* and optimal expected profit. In the near term, we plan to identify efficient computational methodologies for solving the convex contract sizing problem outlined in this paper.

In our current and future work, we will investigate a number of intimately connected research directions: improved forecasting of wind power, optimization of reserve margins, firming of wind power, network aspects of renewable energy grid integration and storage optimization, and new market structures for facilitating integration of renewable sources. We are also studying the important case of markets with recourse where the producer has opportunities to adjust bids in successive stages. We are also developing large scale computational simulations which can be used to test the behavior of of simplified analytically tractable models and suggest new avenues for research applicable to real-world grid-scale problems.

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