

Observer scheme for linear recycling systems with time delays

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Abstract—In this paper a new control methodology is proposed for unstable linear time-delay systems with recycle. For this kind of systems time delays are present in the forward and backward paths increasing control difficulty. The strategy is based on the observation that if some internal system signals were known then it would be possible to remove the backward dynamics. In this way, a controller feedback could be designed by considering only the dynamics of the forward loop. To carry out this strategy an asymptotic observer is proposed to estimate the internal signals needed. Necessary and sufficient conditions to assure convergence of the proposed observer-predictor are given. An overall procedure for the proposed methodology is provided and numerical simulations to illustrate its performance are presented.

I. INTRODUCTION

Recycling processes are commonly found in chemical industry. Recycling systems enable the matter and energy to be recovered in an industrial process. For instance, a typical plant configuration is formed by a reactor/separator process, where reactants are recycled back to the reactor [1], [2], [3].

The so called snowball effect is observed in the operation of many chemical plants with recycle streams. Snowball means that a small change in a load variable causes a very large change in the flow rates around the recycle loop. Although snowballing is a steady state phenomenon and has nothing to do with dynamics, it depends on the control structure [4]. Disadvantages of snowball effect has drawn the attention of some researchers. Luyben [5], [6], [7], studied the effects of recycle loops on process dynamics and their implications to plant-wide control. Taiwo [8], discussed the robust control for recycling plants and proposed the concept of recycle compensation to recuperate inherent process dynamics, i.e. dynamics without recycle. Scali and Ferrari [9], analyzed the problem under same idea. Similar approaches were extended by Lakshminarayanan and Takada [10], and Kwok et. al [11].

Due to the snowball effect, control of systems with recycle loops are somewhat difficult and interesting in their own. Sometimes, transport delays that can not be neglected, are present in recycled systems significantly increasing its control difficulty. It is known that when recycle loops and time delays occur, exponential terms appear in forward and backwards paths. In state space representation recycled system

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with delay correspond to systems with delays on the input and the state. Control problem of recycled system become even more difficult when the forward path is unstable.

Model approximation has been proposed to remove the exponential terms from the transfer function denominator of a delayed system, such as the method of moments [12], and Pade-Taylor approximations [13], [14], [15]. Other techniques, such as the seasonal time-series model [11], have been proposed to obtain an approximate model to represent recycle systems. Del Muro et. al. [16] proposed an approximate model to represent recycle systems by using discrete-time approach. In turn, such approximate models can be used for stability analysis or control design [17], [8], [9], [18], [19], [20], [21], [22], [23].

In this work the problem of recycled system composed of an unstable first order plant in the direct path and a stable system of order n in the recycle loop is addressed. The work is organized as follows: Section II presents the problem formulation and the class of the systems considered in this work. The general idea of the solutions is also outlined in this Section. Here the need of an observer-predictor arises. Section III presents the control proposed. This Section is divided in three parts. Firstly a preliminary result concerning the stability of a class of input delay systems is presented. Then a scheme to estimate some internal signal of the system is proposed. Based on the estimation of the necessary internal variables the overall control scheme is presented in the last part of Section III. Some simulations results are described in Section IV. Such results illustrated the performance of the control here proposed. Finally Section V presents some conclusions.

II. PROBLEM FORMULATION

Let us consider the class of recycling system shown in Figure 1, which can be described as,

$$Y(s) = \begin{bmatrix} G_d & G_d G_r \end{bmatrix} \begin{bmatrix} U(s) \\ Y(s) \end{bmatrix} \quad (1)$$

with,

$$G_d = G_1(s)e^{-\tau_1 s} = \frac{b}{s-a}e^{-\tau_1 s} \quad (2a)$$

$$G_r = G_2(s)e^{-\tau_2 s} = \frac{N(s)}{D(s)}e^{-\tau_2 s} \quad (2b)$$

where $G_d(s)$, and $G_r(s)$ are transfer functions of the forward (or direct) and backward (or recycle) paths, respectively; $\tau_1, \tau_2 \geq 0$ are the time delays associated to $G_d(s)$, and $G_r(s)$. $a, b \in \mathbb{R}$, with $a > 0$, that is G_d is unstable; $N(s)$

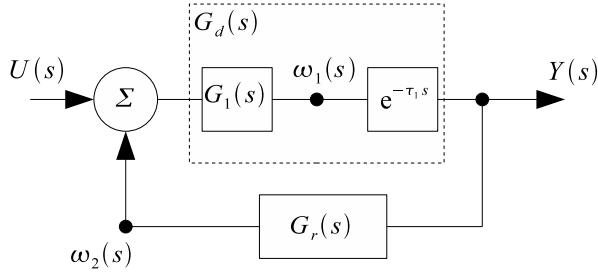


Fig. 1. A process with recycle

and $D(s)$ are polynomials on the complex variable s . $U(s)$ is the process input and $Y(s)$ is the process output.

The closed-loop transfer function of system (1) is given by

$$G_t(s) = \frac{D(s)be^{-\tau_1 s}}{(s-a)D(s) - bN(s)e^{-(\tau_1+\tau_2)s}} \quad (3)$$

Note that exponential terms appear explicitly in numerator and denominator of $G_t(s)$. Stability of (3) is determined by the roots of its characteristic equation

$$Q(s) = (s-a)D(s) - bN(s)e^{-(\tau_1+\tau_2)s} = 0 \quad (4)$$

More precisely, the overall path $U(s) \rightarrow Y(s)$ is stable if and only if all the roots of $Q(s)$ are contained in the open left-half complex plane. It is well known that the transcendental term in $Q(s)$ induces an infinite number of roots preventing the use of classical control design techniques and stability analysis methods.

Let us to describe some ideas behind the methodology proposed. With reference to Figure 1, if signal ω_2 were known, then we could set

$$U(s) = R_1(s) - \omega_2(s) \quad (5)$$

obtaining the system shown in Figure 2. Then it would be possible to design $R_1(s)$ as $R_1(s) = (R(s) - \omega_1(s))J(s)$ like in Figure 3. Since ω_1 and ω_2 are internal system signals an observer-predictor scheme to estimate these variables is developed in the following section.

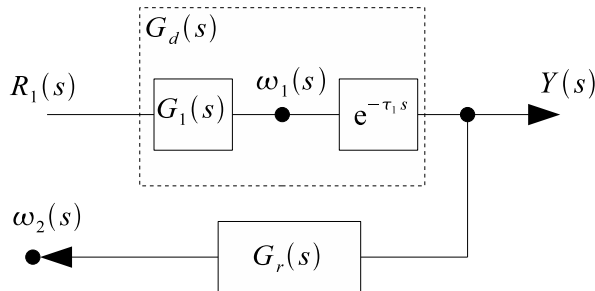


Fig. 2. System of Fig. 1 after applying $U(s)$ given by (5)

III. OBSERVER-PREDICTOR BASED CONTROL

A. A preliminary stability result

In this section a preliminary result on the stability of an unstable first order system plus time delay is presented. This

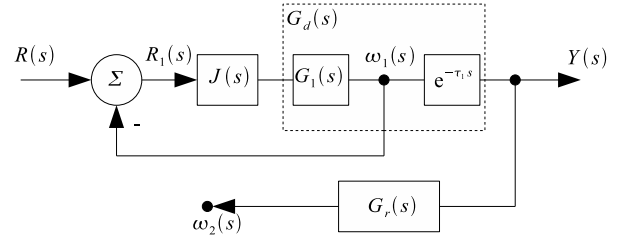


Fig. 3. A control structure for the system of Fig. 2

result will be used later in the proof of the observer-predictor convergence.

Lemma 1: Consider the unstable input-output delay system

$$\frac{Y(s)}{U(s)} = G(s)e^{-\tau s} = \frac{b}{s-a}e^{-\tau s}, \quad a > 0 \quad (6)$$

with a proportional output feedback

$$U(s) = R(s) - kY(s) \quad (7)$$

There exist a proportional gain k such that the closed loop system

$$\frac{Y(s)}{R(s)} = \frac{be^{-\tau s}}{s-a+kb e^{-\tau s}} \quad (8)$$

is stable if and only if $\tau < \frac{1}{a}$.

Proof: The proof use the well known fact that a discrete time model derived from a continuous time system is equal to its continuous counterpart if the sampling period $T \rightarrow 0$. It is carried out by discretizing the system and then showing that all the poles remain inside the unitary circle when the sampling period tends to zero iff $\tau < \frac{1}{a}$.

Discretizing model (6) using a zero order hold and a sampling period $T = \frac{\tau}{n}$ with $n \in \mathbb{N}$, it is obtained,

$$G(z) = \frac{b}{a} \frac{(e^{aT} - 1)}{z^n(z - e^{aT})} \quad (9)$$

Model (9) in closed loop with the (discretized) output feedback (7) produces the characteristic polynomial,

$$p(z) = z^n(z - e^{aT}) + k \frac{b}{a} (e^{aT} - 1) \quad (10)$$

Let us to analyze the root locus of (10). Open loop system has n poles at the origin and one at $z = e^{aT}$, since there are not finite zeros there exist $n+1$ branches to infinity, $n-1$ starting at the origin and two starting at a point located over the real axis between the origin and $z = e^{aT}$. This point can be found by considering,

$$\frac{dk}{dz} = \frac{d}{dz} \left[-\frac{z^n(z - e^{aT})}{\frac{b}{a}(1 - e^{aT})} \right] = 0,$$

producing the equation,

$$(n+1)z^n - nz^{n-1}e^{aT} = 0,$$

that has $n-1$ roots at the origin and one at $z = \frac{n}{n+1}e^{a\frac{\tau}{n}}$. If the starting point over the real axis is located inside the unit circle, the closed loop system could have a region of

stability, otherwise the system is unstable for any k .

The corresponding property of the associated continuous system (6)-(7) is obtained by considering the limit as $n \rightarrow \infty$, this is,

$$\lim_{n \rightarrow \infty} z = \lim_{n \rightarrow \infty} \frac{n}{n+1} e^{a\frac{\tau}{n}} = 1. \quad (11)$$

Since this limit point is located on the stability boundary, it is not difficult to see that if $a\tau < 1$ (i.e., $\tau < 1/a$) the limit tends to one from the left and then, there exist k that places the breaking point inside the unitary circle. In the case that $a\tau \geq 1$ it is not possible to stabilize the system by static output feedback (i.e., the limit tends to one from the right and a couple of poles are outside the unitary circle).

For the remaining $n - 1$ poles, from the corresponding characteristic equation for the continuous case obtained by considering $n \rightarrow \infty$, it is obtained,

$$\begin{aligned} \lim_{n \rightarrow \infty} p(z) &= \lim_{n \rightarrow \infty} \left[z^n(z - e^{aT}) + k\frac{b}{a}(e^{aT} - 1) \right] \\ &= \lim_{n \rightarrow \infty} \left[z^n(z - e^{a\frac{\tau}{n}}) + k\frac{b}{a}(e^{a\frac{\tau}{n}} - 1) \right] \\ &= (z - 1) \lim_{n \rightarrow \infty} z^n \end{aligned} \quad (12)$$

Therefore, in this case, it is confirmed that one pole is located at $z = 1$ and we can note that the rest of them are at the origin. From the above developments, it is clear that when one pole is located in a neighborhood of the point $z = 1$, all the other poles are in a neighborhood of the origin. Then, we can finally state that the system can be stabilized iff $a\tau < 1$. ■

Stability of (8) has been previously studied in the literature. Lemma 1 can also be proved using classical frequency domain, D-decomposition or even by the classical Pontryagin Method [24], [25], [26], [27], [28]. The proof presented here is a simple one and its main idea (discretizing) can be applied to other kind linear time delay systems. Furthermore it allows to easily establish the following result.

Corollary 2: Consider system (6)-(7) with $\tau < 1/a$. Then, a k that stabilizes the closed loop system, satisfies $a/b < k < a/b + \sigma$, for some $\sigma > 0$.

Proof: Analyzing the root locus associated to the discrete system, it is possible to see that the open loop system has n poles at the origin and one at $z = e^{aT}$ without finite zeros. Then, there are $n - 1$ branches going to infinity and a pair converging to a point on the real axis located between the origin and $z = 1$ (stability region). Note that if $k = 0$ the system is unstable. The gain k that takes the systems to the border of the stability region ($z = 1$) is obtained by evaluating k for $z = 1$, this is,

$$k = - \left. \frac{z^n(z - e^{aT})}{\frac{b}{a}(1 - e^{aT})} \right|_{z=1} = \frac{a}{b} \quad (13)$$

Then by Lemma 1 the proof is concluded. ■

B. Prediction Strategy

To estimate ω_1 and ω_2 in Figure 1 the observer-predictor depicted in Figure 4 is proposed. Its convergence is established in the following result.

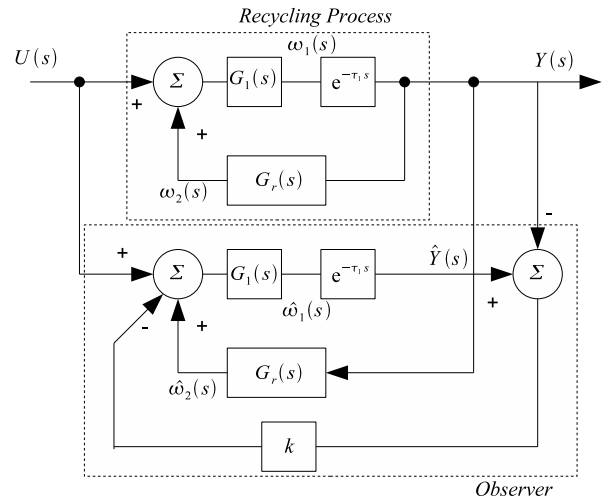


Fig. 4. Observer predictor proposed

Theorem 3: Consider the observer-predictor scheme shown in Figure 4, with G_r a stable transfer function. There exists constant k such that

$$\lim_{t \rightarrow \infty} [\omega_i - \hat{\omega}_i] = 0, \text{ for } i = 1, 2, \quad (14)$$

if and only if $\tau_1 < 1/a$.

Proof: A state space representation of the observer-predictor scheme shown in Figure 4 is

$$\begin{aligned} \dot{\mathbf{x}}(t) &= \mathbf{A}\mathbf{x}(t) + \mathbf{A}_1\mathbf{x}(t - \tau_1) \\ &\quad + \mathbf{A}_2\mathbf{x}(t - \tau_2) + \mathbf{B}u(t) \end{aligned} \quad (15a)$$

$$\mathbf{y}(t) = \mathbf{C}\mathbf{x}(t - \tau_1) \quad (15b)$$

with,

$$\mathbf{x}(t) = [x_d(t) \quad x_r(t) \quad \hat{x}_d(t) \quad \hat{x}_r(t)]^T$$

$$\mathbf{y}(t) = [y(t) \quad \hat{y}(t)]^T$$

$$\mathbf{B} = [\mathbf{B}_d \quad 0 \quad \mathbf{B}_d \quad 0]^T$$

$$\mathbf{A} = \begin{bmatrix} A_d & 0 & 0 & 0 \\ 0 & A_r & 0 & 0 \\ 0 & 0 & A_d & 0 \\ 0 & 0 & 0 & A_r \end{bmatrix}$$

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 0 & 0 & 0 \\ B_r C_d & 0 & 0 & 0 \\ B_r k C_d & 0 & -B_d k C_d & 0 \\ B_r C_d & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} 0 & B_d C_r & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & B_d C_r \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$\mathbf{C} = \begin{bmatrix} C_d & 0 & 0 & 0 \\ 0 & 0 & C_d & 0 \end{bmatrix}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state vector, $u \in \mathbb{R}$ is the input, $\mathbf{y} \in \mathbb{R}^2$ is the output, $\tau_1 \geq 0$ and $\tau_2 \geq 0$ are the time delays present in the system. $A_d \in \mathbb{R}^{n \times n}$, $B_d \in \mathbb{R}^{n \times 1}$, and $C_d \in$

$\mathbb{R}^{1 \times n}$ are matrices and vectors parameters that corresponds to the forward loop in the process, and $A_r \in \mathbb{R}^{n \times n}$, $B_r \in \mathbb{R}^{n \times 1}$, and $C_r \in \mathbb{R}^{1 \times n}$ are matrices and vectors parameters that corresponds to backward path in the process, $\hat{x}(t)$ is the estimation of $x(t)$.

Defining the state prediction errors $e_{x_d}(t) = \hat{x}_d(t) - x_d(t)$, $e_{x_r}(t) = \hat{x}_r(t) - x_r(t)$, and the output estimation $e_y(t) = \hat{y}(t) - y(t)$, it is possible to describe the behavior of the error signals as,

$$\begin{bmatrix} \dot{e}_{x_d}(t) \\ \dot{e}_{x_r}(t) \\ e_y(t + \tau_1) \\ e_{\omega_2}(t + \tau_2) \end{bmatrix} = \begin{bmatrix} A_d & 0 & -B_d k & B_d \\ 0 & A_r & 0 & 0 \\ C_d & 0 & 0 & 0 \\ 0 & C_r & 0 & 0 \end{bmatrix} \begin{bmatrix} e_{x_d}(t) \\ e_{x_r}(t) \\ e_y(t) \\ e_{\omega_2}(t) \end{bmatrix} \quad (16)$$

Note that,

$$e_y(t) = C_d e_{x_d}(t - \tau_1) \quad (17)$$

$$e_{\omega_2}(t) = C_r e_{x_r}(t - \tau_2) \quad (18)$$

System (16) can be rewritten as

$$\dot{e}_{x_d}(t) = A_d e_{x_d}(t) - B_d k C_d e_{x_d}(t - \tau_1) + B_d C_r e_{x_r}(t - \tau_2) \quad (19a)$$

$$\dot{e}_{x_r}(t) = A_r e_{x_r}(t) \quad (19b)$$

Since A_r is a Hurwitz matrix, the stability of system (19) can be analyzed by considering the partial dynamics

$$\dot{e}_{x_d}(t) = A_d e_{x_d}(t) - B_d k C_d e_{x_d}(t - \tau_1) \quad (20)$$

or equivalently,

$$\begin{bmatrix} \dot{e}_{x_d}(t) \\ e_y(t + \tau_1) \end{bmatrix} = \begin{bmatrix} A_d & -B_d k \\ C_d & 0 \end{bmatrix} \begin{bmatrix} e_{x_d}(t) \\ e_y(t) \end{bmatrix} \quad (21)$$

Consider now a state space realization of system (8). It is easy to see that this dynamics can be written in state space form as,

$$\begin{bmatrix} \dot{x}(t) \\ y(t + \tau_1) \end{bmatrix} = \begin{bmatrix} A_d & -B_d k \\ C_d & 0 \end{bmatrix} \begin{bmatrix} x_d(t) \\ y(t) \end{bmatrix} + \begin{bmatrix} B_d \\ 0 \end{bmatrix} u(t) \quad (22)$$

Comparing (22) and (21) it is clear that Lemma 1 can be applied to system (21). Hence the result of the theorem follows. ■

C. Proposed control scheme

Based on the estimation of internal signals $\hat{\omega}_1$, $\hat{\omega}_2$ we proceed to implement the ideas exposed in Section II using $\hat{\omega}_1$ and $\hat{\omega}_2$. The complete control scheme is proposed in Figure 5. The proposed methodology can be summarized as follows:

- 1) Make sure that the conditions of Theorem 3 are satisfied, that is, $G_r(s)$ a stable transfer function and $\tau_1 < 1/a$ for the unstable first order delayed plant.
- 2) Tune the parameter k using Corollary 2.
- 3) Design of a controller $J(s)$ based on the free delay model of the forward path $G_1(s)$. A PI or PID control based strategy can be considered.
- 4) Finally, implement the general control structure as it is shown in Figure 5.

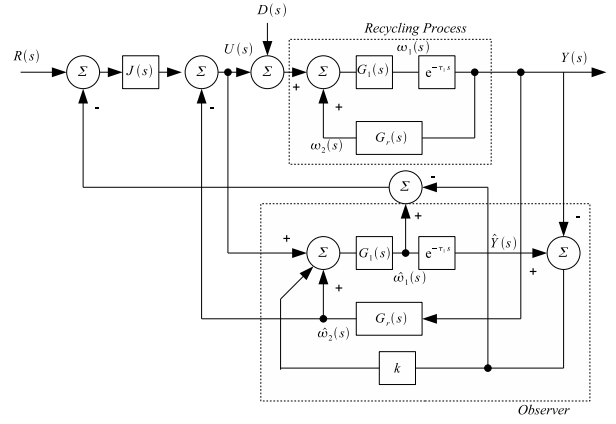


Fig. 5. Proposed control schema

IV. SIMULATION RESULTS

In this section, some academic examples show the performance of observer based control strategy previously proposed.

Example 4: Consider the recycled time delay system of the form (1) with,

$$G_d = \frac{5.3}{s-1} e^{-0.8s}, \quad G_r = \frac{1}{s+2} e^{-2s}. \quad (23)$$

Following the procedure above described, it is obtained a proportional gain $k = 0.21$. Instead of a single controller $J(s)$, the free delay direct path can be stabilized by a two degree of freedom PI [29], obtaining a general feedback of the form,

$$U(s) = R(s)G_{ff}(s) - G_c(s)(\hat{\omega}_1(s) - e_y(t)) - \hat{\omega}_2(s) \quad (24)$$

with

$$G_{ff}(s) = 1.2 \left(0.3 + \frac{1}{s} \right) \quad \text{and} \quad G_c(s) = 1.2 \left(1 + \frac{1}{s} \right). \quad (25)$$

To evaluate the output signal evolution, it was considered a positive unit step input and a step disturbance $D(s)$ acting at $t = 30$ sec. In Figure 6, the continuous line shows the output response when the exact knowledge of the model parameters is assumed; the dashed line presents the output signal when the time delay in the forward path is increased by 6%. Figure 7, shows an experiment where $D(s) = 0$ and the error output initial condition $e_y(0)$ is 0.1.

From figures 6 and 7 it can be seen the observer predictor convergence and the well behaved of the control based on estimated signals.

Example 5: Consider now the recycled system (1) with,

$$G_d = \frac{1}{s-0.25} e^{-2s}, \quad G_r = \frac{10}{(s+1)(s+2)} e^{-2s}. \quad (26)$$

In this case, the proportional feedback (7) is implemented by considering $k = 0.3$. The general control feedback (24) is obtained by considering,

$$G_{ff}(s) = 1.5 \left(0.4 + \frac{1}{4s} \right) \quad \text{and} \quad G_c(s) = 1.5 \left(1 + \frac{1}{4s} \right) \quad (27)$$

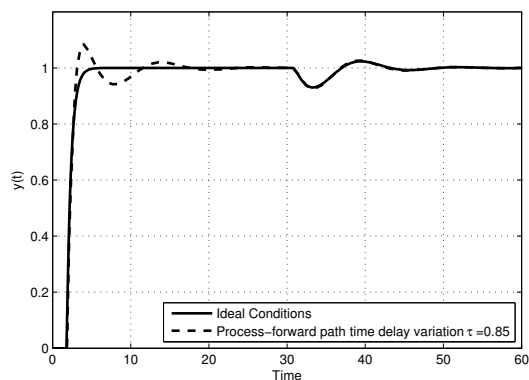


Fig. 6. Output signal in example 4

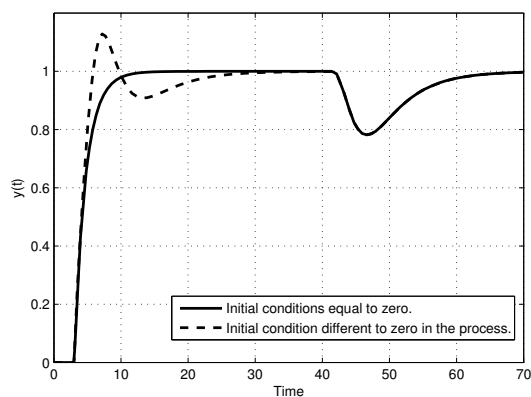


Fig. 8. Output signal in example 5

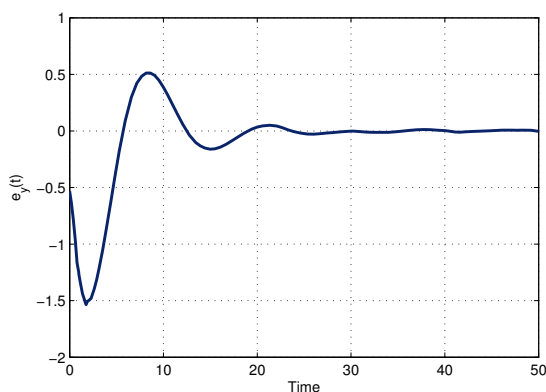


Fig. 7. Estimation error $e_y(t)$, example 4

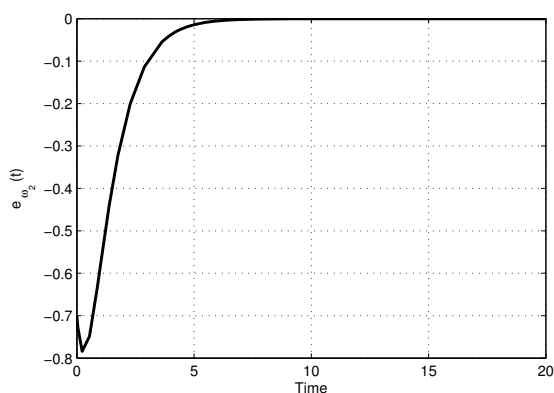


Fig. 9. Estimation error $e_{\omega}(t)$, example 5

Like in the previous example, a step disturbance $D(s) = -0.05$ is introduced at $t = 40$ sec. Figure 8 shows the evolution of the output signal with zero initial conditions (continuous line) and the case when the initial condition in the recycle path is set at 0.01 . Figure 9 shows the output error signal $e_{\omega_2}(t)$ when $D(s) = 0$ and a small initial condition of magnitude 0.07 is presents in the backward path. Again, figures 8 and 9 the observer predictor convergence and good control performance is obtained.

V. CONCLUSIONS

Unstable processes with significant time delay and recycle loop is a challenging control problem. This work presents explicit conditions for the construction of an stabilizing observer based controller scheme for such class of systems. The observer-prediction strategy allow to estimate some internal variables of the process that are used to: i) remove the dynamics of backward loop in the recycling process and ii) design a stabilizing control law for the free delay model of the forward path. The methodology is evaluated by numerical simulations. The overall controller which includes a PI with two-degree of freedom is capable to reduce the output overshooting as it is shown in simulations.

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