2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

# Multiple Model Adaptive Estimation of Satellite Attitude using MEMS Gyros

Hoday Stearns, Masayoshi Tomizuka
Department of Mechanical Engineering
University of California, Berkeley

*Abstract*— **Recently, MEMS gyroscopes are receiving attention for future use attitude determination systems in microsatellites, because of advantages of light weight, low cost, and low power consumption. However, the high noise levels and bias drifts of these sensors currently limits use for high-precision applications. A multiple model adaptive estimation algorithm is employed to estimate the magnitudes of noise variances in a gyroscope model. The estimated values are used in an extended Kalman filter for attitude estimation. It is demonstrated through simulations and experiments that the adaptive estimation of noise parameters improves attitude estimation performance.**

## I. Introduction

Modern satellites depend on accurate knowledge of attitude for navigation and pointing of antennae, solar panels, and scientific equipment. *Attitude* is the rotational orientation of an object in three-dimensional space; it may be thought of as the rotational coordinate transformation needed to transform reference frame axes to the body-fixed axes. Popular representations of attitude include Euler angles, rotation matrices, or quaternions. Satellites are equipped with an Attitude Determination and Control System (ADCS) to estimate and control the attitude of the spacecraft. A typical ADCS system consists of a collection of different types of sensors for estimating attitude (relative attitude sensors, absolute attitude sensors), actuators and algorithms. Sensors on board an ADCS include rate sensors (gyroscopes), star trackers, sun sensors, earth horizon sensors, and magnetometers.

Recently, due to the improving performance of MEMS sensors, there has been increased interest in using MEMS gyroscopes in ADCS systems as replacements for fiber-optics gyros, or as complementary sensors [1]. MEMS sensors have advantages of being low-cost, light weight, and low power consumption. In the case of employing MEMS gyroscopes in ADCS, several challenges arise. The noise levels of these sensors are high, and gyroscope bias drifts over time [2] [3] [4] [5]. The gyroscope parameters are also sensitive to conditions such as temperature and operating voltage.

In ADS, gyroscope measurements are combined with readings from absolute attitude sensors to compute optimal attitude estimates. A survey of several attitude estimation filters is presented in [6]. One of the most common is an extended Kalman filter (EKF) filtering algorithm [7] [8] [9] [10]. The EKF is based on a kinematic model of attitude dynamics based on gyroscope measurements, with corrections coming from the external attitude sensor.

The accuracy of the EKF is affected by the values of parameters chosen for the noise model, but often, these noise statistics are not known accurately beforehand. Especially for MEMS gyroscopes, the noise variances are higher than those for traditional fiber-optics gyroscopes and may vary with operating conditions. Adaptive estimation methods are being developed and employed to estimate the noise statistics online [11]. One of these is the multiple model adaptive estimation (MMAE) algorithm. MMAE is an adaptive algorithm for estimating unknown model parameters in real time from Kalman filter estimation residuals. Previously, MMAE has been applied for identifying sensor and actuator hardware failures [12] [13] [14], in intertial navigation systems filters [15], and for estimating gyroscope noise statistics [16] [17] [18] [19]b[20].

This paper proposes a 2-stage attitude estimation filter integrating an MMAE filter for estimating gyroscope noise statistics with an attitude estimation EKF. It is demonstrated through simulations that the MMAE filter successfully identifies gyroscope biases and noise statistics parameters. In addition, experiments are performed with a 3-axis gyroscope and demonstrate that MMAE estimation of noise statistics improves attitude quaternion estimation in actual use. The contribution of this paper is the evaluation of MMAE noise parameter estimation on the attitude filter performance, and experimental verification of the performance improvement due to the MMAE algorithm.

The organization of this paper is as follows: In section 2, a gyroscope model is presented, followed by presentation of MMAE algorithm and attitude EKF equations. In section 3, MMAE performance in gyroscope model parameters estimation is demonstrated through simulation results. In section 4, MMAE performance in attitude estimation is investigated through both simulation and experimental results. Finally, conclusions are drawn in section 5.

## II. Background

### A. Gyro Model

In this research, a three-axis gyroscope is considered. Each gyro axis has its own bias. The model is given by

$$\begin{aligned}\mathbf{b}[k+1] &= \mathbf{b}[k]+\boldsymbol{\eta}_1[k]\\ \tilde{\boldsymbol{\omega}}[k] &= \boldsymbol{\omega}[k]+\mathbf{b}[k]+\boldsymbol{\eta}_2[k].\end{aligned} \tag{1}$$

where $\tilde{\boldsymbol{\omega}} = [\ \tilde{\omega}_1,\ \ \tilde{\omega}_2,\ \ \tilde{\omega}_3\ ]^T$, are the measurements of the x,y, and z axes gyros; $\boldsymbol{\omega} = [\ \omega_1,\ \ \omega_2,\ \ \omega_3\ ]^T$ is the array of true angular rates around body-fixed x, y, and z axes; $\mathbf{b} = [\ b_1,\ \ b_2,\ \ b_3\ ]^T$ is the array of biases of x, y, and z axes gyros, and $\boldsymbol{\eta}_1$, $\boldsymbol{\eta}_2$ are noises with $\boldsymbol{\eta}_1 \sim N(\mathbf{0}, Q\mathrm{I}_3)$, $\boldsymbol{\eta}_2 \sim N(\mathbf{0}, R\mathrm{I}_3)$, and $Q, R > 0$. It is assumed that the noises of the separate axes are independent of each other, and have the same covariance.

The discretized model is used to construct a Kalman filter for estimating bias **b**. However, it is often the case that there is uncertainty in the model used for constructing the filter. In the discrete gyro model Eq. 1, the noise parameters Q and R are not accurately known, and can also change over time depending on operating conditions. In the next section, we describe an adaptive algorithm, known as Multiple Model Adaptive Estimation, and apply it to estimate noise parameters $Q$ and $R$ in real time.

### B. Multiple Model Adaptive Estimation

Kalman filters are widely used to estimate the states of linear systems. However, if inaccurate model parameters are used to construct the filter, the state estimate accuracy will degrade, and may even diverge. Multiple model adaptive estimation (MMAE) is a technique for estimating unknown model parameters within a Kalman filter while simultaneously estimating state.

A block diagram of the MMAE scheme is shown in Fig. 1. The input and output data of a plant with unknown parameters is collected and passed to a bank of Kalman filters. The filter bank contains many parallel filters, called *hypothesis* filters, each constructed using a model based on a different guess for the values of the unknown parameters. Each filter in the bank represents a hypothesis about the true parameter value. The output of each filter is compared with the true system output measurement, and the filters with the lowest estimation residual are expected to represent the most accurate models. The MMAE estimate of system state is computed to be the weighted average of the elemental filter state estimates; similarly, the value of the unknown parameter is also computed to be the weighted average of the parameter guesses in each of the elemental filters. The weights are given by the a-posteriori probability of obtaining the residual, conditioned on all past input/output data history.

Suppose that there is a discrete linear system given by the following

$$\begin{aligned}x(t+1) &= Fx(t)+Gw(t)\\ z(t) &= Hx(t)+v(t)\end{aligned}$$

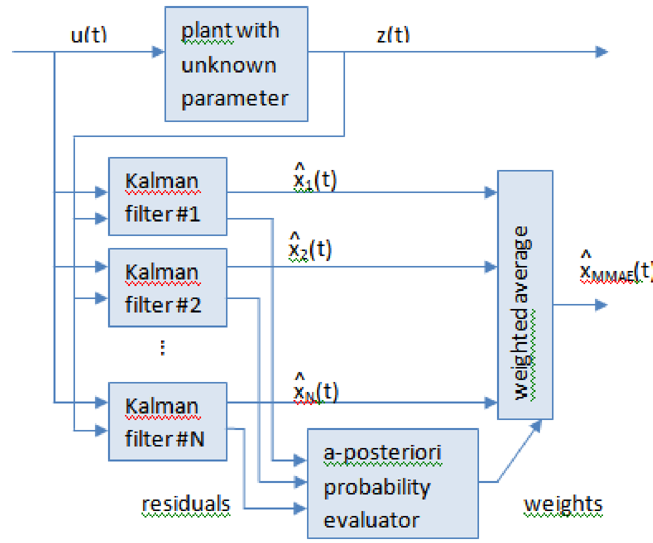


Fig. 1. Block diagram of MMAE scheme

The system is affected by process and measurement noises, both assumed to be zero-mean, white, independent Gaussian noises with covariances $Q$ and $R$. It is well known that the optimal estimate for the state may be found from the Kalman filter state estimate, whose equations are summarized here:

Predict:

$$\begin{aligned}\hat{x}^-(t+1) &= F\hat{x}^+(t)\\ P^-(t+1) &= FP^+(t)F^T+GQG^T\end{aligned}$$

Correct:

$$\begin{aligned}K(t) &= P^-(t)H^T(HP^-(t)H^T+R)^{-1}\\ \hat{x}^+(t) &= \hat{x}^-(t)+K(t)(z(t)-H\hat{x}^-(t))\\ P^+(t) &= (I-K(t)H)P^-(t)\end{aligned}$$

In the above equations, $\hat{x}^-(t)$ represents the a-priori expected value of the state given the history of all past measurements, and $\hat{x}^+(t)$ represents the a-posteriori expected value of the state given the history of all past measurements and present measurement, i.e. $\hat{x}^+(t) = E[x(t)|Z_t]$, where $Z_t := [z(0), \ldots z(t)]^T$. It is implicitly assumed that the model is known precisely.

However, suppose that there exist unknown model parameters. Let the unknown parts be parameterized by a parameter vector **a**. The value of **a** is unknown, but is known to take on a value from within a known set of values $\mathbf{a}_1, \mathbf{a}_2, \ldots \mathbf{a}_N$ (assume discrete and finite). Consider the probability that **a** takes a certain value $\mathbf{a}_i$, given the history $Z_t$ of all present and previous system outputs (this is known as the a-posteriori conditional probability),

$$p_i(t) := \Pr\left[\mathbf{a} = \mathbf{a}_i | Z(t) = Z_t\right] \tag{2}$$

It can be shown that the a-posteriori conditional probability at time $t$ may be computed recursively [11],

$$p_i(t) = \frac{f_{\hat{z}_i^-(t)|\mathbf{a}_i,Z(t-1)}\left(z(t)|\mathbf{a}_i, Z_{t-1}\right)p_i(t-1)}{\sum_{n=1}^{N} f_{\hat{z}_n^-(t)|\mathbf{a}_n,Z(t-1)}\left(z(t)|\mathbf{a}_n, Z_{t-1}\right)p_n(t-1)}. \quad (3)$$

where

$$f_{\hat{z}_i^-(t)|\mathbf{a}_i,Z(t-1)}\left(z(t)|\mathbf{a}_i, Z_{t-1}\right) = \frac{1}{(2\pi)^{m/2}\left(det(S_i(t))\right)^{1/2}} e^{-\frac{1}{2}r_i^T(t)S_i^{-1}(t)R_i(t)} \quad (4)$$

and $r_i(t) = z(t) - \hat{z}_i^-(t)$, $\hat{z}_i^-(t) = H\hat{x}_i^-(t)$, $S_i(t) = HP_i^-(t)H^T + R$, and $m$ is the number of outputs. Then, the optimal parameter value estimate, defined as the expected value of the parameter, is

$$\mathbf{a}_{MMAE}(t) = E\left[\mathbf{a}|Z_t\right] = \sum_{n=1}^{N} p_n(t)\mathbf{a}_n. \quad (5)$$

An MMAE algorithm is applied to the discrete gyro model in Eqs. **??** to estimate noise covariances $Q$ and $R$, with the goal of ultimately improving the state estimation accuracy of the attitude estimation filter. Based on the discrete gyro model, several Kalman filters for estimating gyro bias are created with several guesses for the values of noise parameters $Q$ and $R$. Finally, an MMAE filter compares the filter residuals and identifies the expected value of bias and also noise parameters. The identified noise parameter values are next used in an attitude estimation filter, for optimally weighting gyro and attitude sensor measurements, to be described in the next section.

### *C. Attitude Estimation Filter*

In this section, the quaternion representation for attitude will be introduced, followed by presentation of an attitude estimation extended Kalman filter.

Attitude refers to the rotational orientation of an object in three dimensional space. A quaternion $\mathbf{q} = \begin{bmatrix} q_1, & q_2, & q_3, & q_4 \end{bmatrix}^T$is a 4-element array which describes a rotation as the following:

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_{13} \\ q_4 \end{bmatrix} = \begin{bmatrix} \mathbf{v}\sin(\theta/2) \\ \cos(\theta/2) \end{bmatrix}$$

where $\mathbf{q}_{13} = \begin{bmatrix} q_1, & q_2, & q_3 \end{bmatrix}^T$, $\theta$ is the angle of rotation, and $\mathbf{v} = \begin{bmatrix} v_1, & v_2, & v_3 \end{bmatrix}^T$ is a unit vector denoting the direction of the axis rotated around.

Quaternion kinematics follow the relation

$$\dot{\mathbf{q}} = \frac{1}{2}\Omega(\boldsymbol{\omega})\mathbf{q} \quad (6)$$

where $\boldsymbol{\omega} = \begin{bmatrix} \omega_1 & \omega_2 & \omega_3 \end{bmatrix}^T$ is the angular rates around the three body-fixed axes, and

$$\Omega(\boldsymbol{\omega}) = \begin{bmatrix} 0 & \omega_3 & -\omega_2 & \omega_1 \\ -\omega_3 & 0 & \omega_1 & \omega_2 \\ \omega_2 & -\omega_1 & 0 & \omega_3 \\ -\omega_1 & -\omega_2 & -\omega_3 & 0 \end{bmatrix}.$$

The above equation may be discretized to obtain

$$\mathbf{q}(t+1) = e^{\frac{1}{2}\Omega(\boldsymbol{\omega})\Delta t}\mathbf{q}(t).$$

The satellite's attitude quaternion can be measured by sensors such as star trackers etc. The sensor model including noise is

$$\mathbf{z} = \mathbf{q} + \frac{1}{2}\Xi(\mathbf{q})\mathbf{v} \quad (7)$$

where $\mathbf{z} \in \Re^4$ is sensor output,

$$\Xi(\mathbf{q}) = \begin{bmatrix} q_4 & -q_3 & q_2 \\ q_3 & q_4 & -q_1 \\ -q_2 & q_1 & q_4 \\ -q_1 & -q_2 & -q_3 \end{bmatrix}$$

and $\mathbf{v} \sim N(\mathbf{0}, R_s)$ where $R_s \in \Re^{3\times 3}$.

Gyroscope measurements are used to propagate attitude estimates continuously (via Eq. 6) between slower star tracker update instances; star tracker measurements (Eq. 7) are used to correct the quaternion estimate and to update the estimate of gyroscope biases.

The quaternion kinematics equation (Eq. 6) is nonlinear in the variables we wish to estimate, namely the attitude quaternion and gyroscope bias. Methods for estimating the state of nonlinear systems include extended Kalman filters, particle filters, and sigma-point filters. Extended Kalman filters (EKF) are estimators constructed by linearizing the state and output equations of a nonlinear system around the a-posteriori state estimate trajectory. For a derivation of the attitude EKF, refer to references [7] [11].

The attitude estimation EKF equations are summarized below:

Propagation

$$\begin{aligned} \hat{\mathbf{q}}^-(t+1) &= e^{\frac{1}{2}\Omega\left(\tilde{\boldsymbol{\omega}}(t)-\hat{\mathbf{b}}^-(t)\right)\Delta t}\hat{\mathbf{q}}^-(t) \\ \hat{\mathbf{b}}^-(t+1) &= \hat{\mathbf{b}}^-(t) \\ \dot{P} &= F(t)P(t) + P(t)F(t)^T \\ &\quad + G(t)Q_{\text{EKF}}G(t)^T \end{aligned}$$

Gain

$$K(t) = P(t)H(t)^T(H(t)P(t)H(t)^T + R_s)^{-1}$$

Correction

$$\begin{aligned}
\mathbf{x}(t) &= \begin{bmatrix} \delta\boldsymbol{\alpha}(t) \\ \Delta\hat{\mathbf{b}}(t) \end{bmatrix} = 2K\Xi(\hat{\mathbf{q}}^-(t))^T\mathbf{z}(t) \\
\hat{\mathbf{q}}^+(t) &= \hat{\mathbf{q}}^-(t) + \frac{1}{2}\Xi(\hat{\mathbf{q}}^-(t))\delta\boldsymbol{\alpha}(t)^T \\
\hat{\mathbf{b}}^+(t) &= \hat{\mathbf{b}}^-(t) + \Delta\hat{\mathbf{b}}(t) \\
P^+(t) &= (I_6 - K(t)H(t))P^-(t)
\end{aligned}$$

$$F(t) = \begin{bmatrix} -[(\tilde{\boldsymbol{\omega}} - \hat{\mathbf{b}})\times] & -\frac{1}{2}I_3 \\ 0_{3\times3} & 0_{3\times3} \end{bmatrix}$$

$$G(t) = \begin{bmatrix} -\frac{1}{2}I_3 & 0_{3\times3} \\ 0_{3\times3} & I_3 \end{bmatrix}$$

$$H(t) = \begin{bmatrix} I_3 & 0_{3\times3} \end{bmatrix}$$

$$Q_{\text{EKF}} = \begin{bmatrix} RI_3 & 0_{3\times3} \\ 0_{3\times3} & QI_3 \end{bmatrix}$$

and the meaning of the notation $[(\cdot)\times]$ is given by

$$[\boldsymbol{\omega}\times] = \begin{bmatrix} 0 & -\omega_3 & \omega_2 \\ \omega_3 & 0 & -\omega_1 \\ -\omega_2 & \omega_1 & 0 \end{bmatrix}.$$

## III. MMAE FOR NOISE STATISTICS ESTIMATION

The MMAE algorithm for identifying gyro noise parameters is demonstrated in simulation. It is shown that the MMAE algorithm identified unknown parameter values with reasonable accuracy. First, a Kalman filter was created for estimating a gyro's bias based on the discrete gyro model in Eqs. **??**. However, accurate values for the variances $Q$ and $R$ of process and measurement noises are unavailable. Reasonable ranges for the possible value of $Q$ and $R$ were chosen, and 7 guesses were picked for each of $Q$ and $R$ from within this range. A bank of 49 hypothesis filters in total, covering each combination of parameter values, was created and simulated. Then, an MMAE filter (Eqs. 2 through **??**) is used to compute the conditional probability of each one, which yields a conditional probability estimate of the parameter values, and also the bias. The results are shown in Figs. 2 through 6.

Fig. 2 shows a time plot of the estimates for bias of each of the hypothesis filters, together with the true bias. It is seen that the individual Kalman filter bias estimates are weighted averages of the system output and zero, and that the weighting depends on the ratio of the $Q$ and $R$ values used. In Fig. 3, the a-priori conditional probability of Eq. 2 for each Kalman filter in the filter bank is plotted. Recall that this is the probability a certain Kalman filter is using accurate model values. By the end of the simulated time period, the filter with $Q =$ and $R =$ received the highest weight. In Figs. 4 and 5, the MMAE estimates for $Q$ and $R$ are plotted. It is seen that the MMAE identified the correct

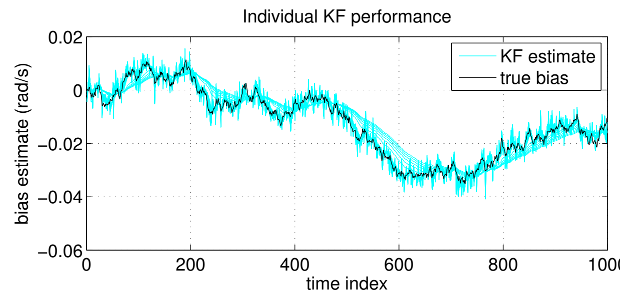


Fig. 2. Bias estimates of individual Kalman filters in filter bank, true bias

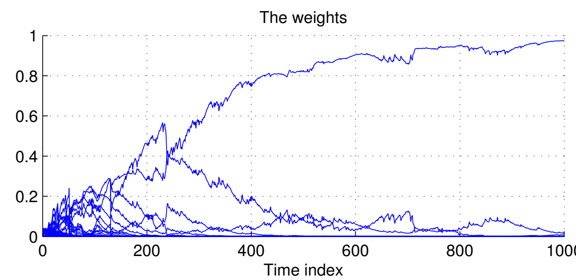


Fig. 3. Weighting of each filter in filter bank. The bold line denotes the weight for the filter with correct values for the parameters being estimated.

values well. Finally, Fig. 6 shows the MMAE estimate for bias plotted along with the individual filter estimates and the true bias. The MMAE estimated bias approximated the true bias well.

## IV. MMAE FOR ATTITUDE ESTIMATION IMPROVEMENT

Unlike in simulation, where the values of noise variances are known, in experiment, these values are not known accurately in general. Consequently, the estimation accuracy of MMAE for identifying these parameters cannot be evaluated directly. Another method for verifying the MMAE performance must be devised. Perhaps one of the most natural ways of evaluating MMAE for the attitude estimation filter is to compare the estimation accuracy of the attitude quaternion in the cases of using MMAE-estimated noise statistics values and without using the MMAE. For instance, the benchmark

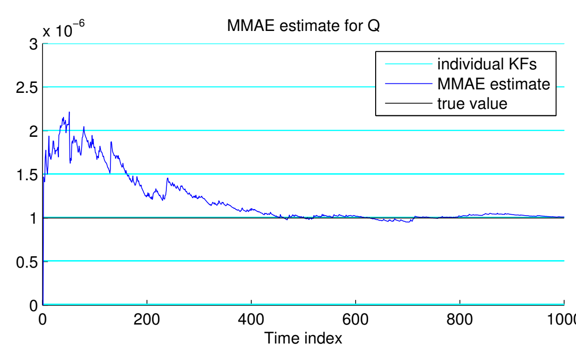


Fig. 4. MMAE estimate for $Q$, true value

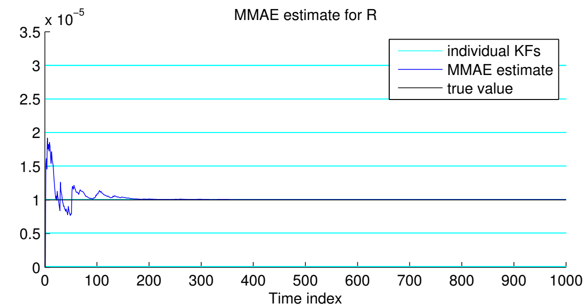


Fig. 5. MMAE estimate for $R$, true value

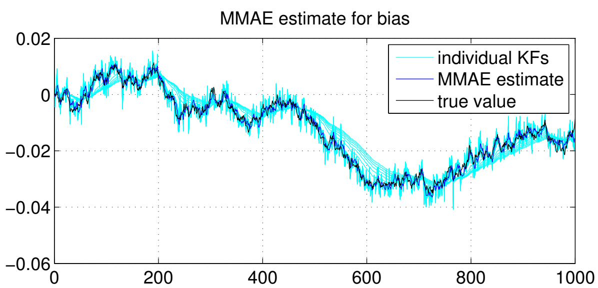


Fig. 6. MMAE bias estimate, bias estimates of individual Kalman filters in filter bank, true bias

case may consist of an EKF using noise statistics taken from the gyroscope datasheets.

### A. Simulation

First, simulation results of the attitude estimation filter are presented. The performance of two attitude filters are compared: one using MMAE estimated values for noise statistics, and the other using incorrect values. The attitude quaternion estimation error is shown in Fig. 7. The attitude filter 1, which uses correct $Q$ and $R$ values, gives better accuracy than filter 2; the RMS error of the quaternion estimation (components 1-3) was reduced by 11.3 %.

The bias estimation errors for each of the three axes are shown in Fig. 8. The peak error is less for EKF with MMAE than for without MMAE.

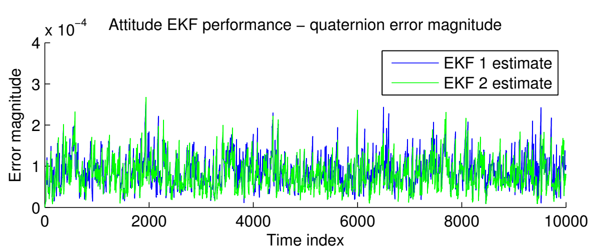


Fig. 7. Performance of attitude estimation filter– Quaternion estimation error (magnitude of components 1-3) for EKF 1 (with MMAE) and EKF 2 (without MMAE)

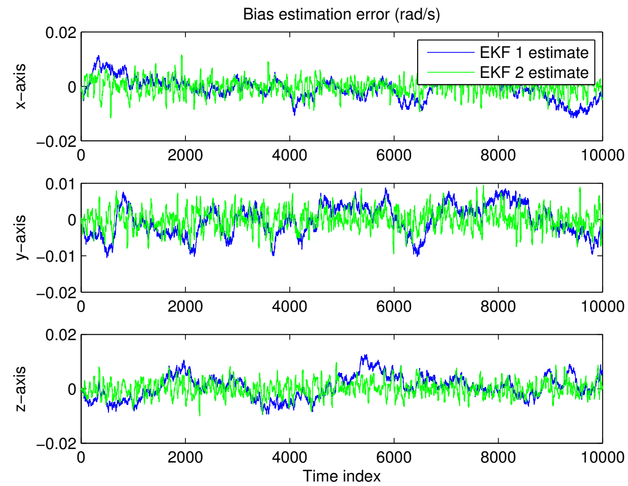


Fig. 8. Performance of attitude estimation filter– Bias estimation error for EKF 1 (with MMAE) and EKF 2 (without MMAE)

### B. Experiment

Next, MMAE is applied to experimental data. A 6-degree-of-freedom robot arm is used as a testbed. A 3-axis gyro ADIS16400 is mounted on the end effector. True attitude measurements are calculated from the joint encoders. The star camera readings are simulated. The robot arm was moved through a trajectory that creates the end-effector Euler angles plotted in Fig. 9.

Performances of two attitude filters are compared: the first filter uses $Q$ and $R$ values estimated by MMAE, and the second filter uses values from the datasheet. In Fig. 10, the quaternion estimation error of the attitude estimation EKF is plotted for both of these filters. It is seen that the filter with MMAE values estimated end-effector attitude more accurately. The average value of the norm of the first three components of multiplicative quaternion estimation error decreased from $2.73 \times e^{-4}$ to $2.38 \times e^{-4}$, and the variance of the quaternion estimation error decreased from $1.76 \times e^{-8}$ to $1.32 \times e^{-8}$. This result suggests that employing extra computation in MMAE yields slight performance improvement in attitude estimation accuracy, rather than using pre-identified nominal values.

## V. Conclusion

MMAE was applied to adaptively estimate unknown noise parameters for a gyroscope sensor in real-time operation. Accurate parameter estimation improved gyroscope bias estimates as well as attitude quaternion estimates. Simulation results were presented to examine the performance of MMAE algorithm for gyro noise parameter and bias estimation, and experimental results were presented to verify the performance. It was found that MMAE estimated noise variances reasonably.

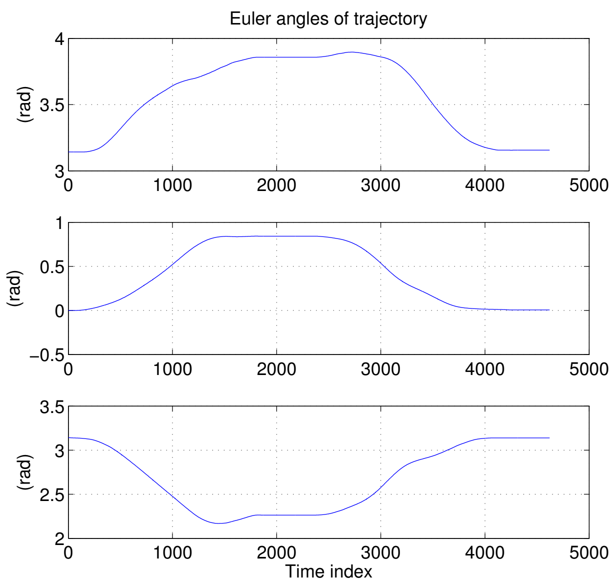


Fig. 9. Euler angles of test trajectory

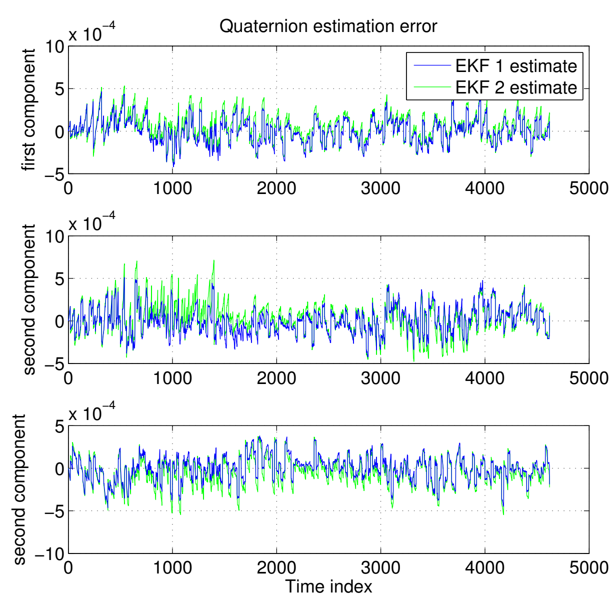


Fig. 10. Performance of attitude estimation filter– Quaternion estimation error for EKF 1 (with MMAE) and EKF 2 (without MMAE)

## VI. Acknowledgements

The authors gratefully acknowledge the contribution of Professor Mohammed Almajed and Dr. Badr Alsuwaidan at King Abdulaziz City for Science and Technology.

## References

[1] M. Al-Majed and B. Alsuwaidan, "A new testing platform for attitude determination and control subsystems: Design and applications," in *Advanced Intelligent Mechatronics, 2009. AIM 2009. IEEE/ASME International Conference on*, Singapore, 2009.

[2] P. Aggarawal, Z. Syed, X. Niu, and N. El-Sheimy, "Cost-effective testing and calibration of low cost mems sensors for integrated positioning, navigation and mapping systems," in *XXIII FIG Congress*, 2006.

[3] Q. Lam, N. Stamatakos, C. Woodruff, and S. Ashton, "Gyro modeling and estimation of its random noise sources," in *Guidance, Navigationa nd Control Conference and Exhibit*, Austin, Texas, 2003.

[4] H. Chang, L. Xue, W. Qin, G. Yuan, and W. Yuan, "An integrated mems gyroscope array with higher accuracy output," *Sensors*, vol. 8, pp. 2886–2899, 2008.

[5] N. El-Sheimy, H. Hou, and X. Niu, "Analysis and modeling of inertial sensors using allan variance," *IEEE Transactions on Instrumentation and Measurement*, vol. 57, no. 1, 2008.

[6] J. Crassidis, F. L. Markley, and Y. Cheng, "Survey of nonlinear attitude estimation methods," *Journal of Guidance, Control, and Dynamics*, vol. 30, no. 1, 2007.

[7] E. J. Lefferts, F. L. Markley, and M. D. Shuster, "Kalman filtering for spacecraft attitude estimation," *Journal of Guidance, Control, and Dynamics*, vol. 5, no. 5, pp. 417–429, 1982.

[8] F. L. Markley, "Attitude estimation or quaternion estimation?" *Advances in Astronautical Sciences*, vol. 115, pp. 113–127, 2003.

[9] M. D. Shuster, "The quaternion in kalman filtering," in *AAS/AIAA Astrodynamics Conference*, vol. 85, Victoria, British Columbia, 1993, pp. 25–37.

[10] J. Bijker and W. Steyn, "Kalman filter configurations for a low-cost loosely integrated inertia navigation system on an airship," *Control Engineering Practice*, vol. 16, pp. 1509–1518, 2008.

[11] P. Maybeck, *Stochastic Models, Estimation and Control*, ser. Mathematics in Science and Engineering. Elsevier, 1979, vol. 141.

[12] K. A. Fisher and P. S. Maybeck, "Multiple model adaptive estimation with filter spawning," in *Proceedings of the American Control Conference*.

[13] P. Maybeck, "Multiple model adaptive algorithms for detecting and compensating sensor and actuator/surface failures in aircraft flight control systems," *International Journal of Robust and Nonlinear Control*, vol. 9, no. 14, pp. 1051–1070.

[14] C. Hajiyev and H. E. Soken, "Adaptive kalman filter with the filter gain correction applied to uav flight dynamics," in *17th Mediterranean Conference on Control and Automation*.

[15] C. Hide, T. Moore, and M. Smith, "Adaptive kalman filtering for low-cost ins/gps," *The Journal of Navigation*, vol. 56, no. 01, pp. 143–152, 2003. [Online]. Available: http://dx.doi.org/10.1017/S0373463302002151

[16] Y. Sun, "Robust spacecraft attitude estimation using multiple-model approaches," Ph.D. dissertation, State University of New York at Buffalo, 2007.

[17] J. Crassidis, "Multiple gyro study with noise model parameter estimation," University at Buffalo, State University of New York, Tech. Rep., 2009.

[18] Q. Lam and J. Crassidis, "Precision attitude determination using a multiple model adaptive estimation scheme," in *Aerospace Conference, 2007 IEEE*, 2007, pp. 1 –20.

[19] ——, "Evaluation of a multiple model adaptive estimation scheme for space vehicle's enhanced navigation solution," in *AIAA Guidance, Navigation and Control Conference and Exhibit*, Hilton Head, South Carolina.

[20] A. Mohamed and K. Schwarz, "Adaptive kalman filtering for ins/gps," *Journal of Geodesy*, vol. 73, no. 4, pp. 193–203.