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Robust Composite Adaptive Fuzzy Identification and Control for a Class of MIMO Nonlinear Systems

D. Kim, H. Chung, S. Bhasin, and W. E. Dixon

Abstract— **A robust adaptive fuzzy identification and trajectory tracking control approach is developed for a class of multi-input-multi-output (MIMO) nonlinear systems in the presense of unmodeled uncertainties, parametric uncertainties, and external disturbances. A sliding mode-based fuzzy model identification/observer method is used to provide additional feedback of the unknown system dynamics. The model identification error is used along with the tracking error as a composite adaptive update law for a fuzzy logic based feeedforward term. The fuzzy logic feedforward term is used in conjunction with a recently developed robust integral of the sign of the error (RISE) feedback method to yield a continuous controller that achieves semi-global asymptotic trajectory tracking.**

I. INTRODUCTION[1]

Fuzzy logic systems (FLSs) can be applied to develop controllers for complex and practical problems where accurate mathematical models may not be available. Specifically, fuzzy systems can be used to approximate any nonlinear system within an arbitrarily small residual error if a sufficient number of rules are used. A salient motivating property of FLSs is the approximation capability through the use of linguistic information from human experts [1]–[3]. The approximation capabilities of FLS are improved by incorporating an adaptive control scheme so that the fuzzy rules model the uncertain structure and the adaptive control strategy adjusts the fuzzy parameters through on-line fuzzy update laws [4]–[6]. Over the past decade, researchers (cf. [7]–[9]) have included a sliding mode control feedback term (resulting in a adaptive fuzzy sliding mode control (AFSMC)) as a means to adaptively compensate for the uncertainties, while robustly compensating for the redidual approximation error and other bounded disturbances. However, sliding mode control generally creates notable problems in practical applications such as the chattering phenomena which is a byproduct caused by the use of the discontinuous feedback control (rather than high-gain) switched at an infinite frequency with a finite amplitude to offset uncertainties and disturbances [10], [11]. To reduce the effects of discontinuous feedback, techniques such as sliding surface strategies and chattering free problems have been proposed [12]–[14].

Recently, a new continuous high gain feedback control method was developed coined the robust integral of the sign of the error (RISE) in [15]. The RISE technique can be applied to yield an asymptotic tracking result despite the presence of sufficiently smooth bounded exogeneous disturbances [15], [16]. In [15], the technique was used to obtain an asymptotic tracking result in the presence of the mixed structured and unstructured uncertainties.

For an affine nonlinear control system with additive disturbances, an integrated identification/observer and tracking strategy is developed using an adaptive FLS approach combined with the RISE method to eliminate the residual approximation error and other added unmodeled disturbances. The contribution of this result includes new control development and analysis for a novel integrated identification and tracking algorithm for dynamical systems, where human experience can be included in the development of the fuzzy rule set [17], [18]. For the tracking control development, a fuzzy logic feedforward term is augmented by a RISE feedback term to compensate for additive disturbances and the residual fuzzy approximation error. The fuzzy rule set is updated using adaptation laws that are a composite of the tracking error and the model identification error. A Lyapunov-based stability analysis is used to conclude that the robust adaptive fuzzy identification and tracking approach with the composite adaptation law yields semi-global asymptotic tracking in the presence of unmodeled dynamics, parametric uncertainty, and exogeneous disturbances. This paper is organized as follows. In Section II, the brief description for FLSs is intoduced. Section III describes the control development with the RISE feedback term. Section IV shows robust identification using the sliding mode term. In Section V, a Lyapunov stability analysis for the RISE fuzzy techique is presented. Conclusions are provided in Section VI.

II. DESCRIPTION FOR FUZZY LOGIC SYSTEMS (FLSS)

A fuzzy logic system maps from an input vector $x = [x_1, x_2, \cdots, x_n]^T \in U \subseteq \mathbb{R}^n$ to an output vector $y(x) \in \mathbb{R}$ where $U = U_1 \times \cdots \times U_n$ and $U_i \in \mathbb{R}$. A MIMO fuzzy system consists of three main components denoted as fuzzy rule bases, fuzzification, and defuzzification operators. The fuzzy rule base is composed of a collection of fuzzy If-Then rules in the following:

$$R^l \; : \quad \text{If } x_1 \text{ is } F_1^l \text{ and } x_2 \text{ is } F_2^l \text{ and } \cdots \text{ and } x_n \text{ is } F_n^l \\ \text{Then } y_1 \text{ is } G_1^l \text{ and } y_2 \text{ is } G_2^l \cdots \text{ and } y_n \text{ is } G_n^l$$

where F_i^l and G_i^l are fuzzy sets in the rule R, $l = 1, 2, ..., M;\ i = 1, 2, ..., n$. The number M denotes a total number of fuzzy If-Then rules in the rule base. By using the singleton fuzzifier, product inference engine, and center-average defuzzifier, the final output of the fuzzy logic system

[1]Dohee Kim, Shubhendu Bhasin, and Warren E. Dixon are with the Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville FL 32611-6250, USA Email: {ndsatellite, sbhasin, wdixon}@ufl.edu. Heetae Chung are with the Division of Digital Media, Pusan University of Foreign Studies, Busan, Korea Email: htchung@pufs.ac.kr

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can be written as [19]

$$y(x)=\frac{\sum_{l=1}^{M}\bar{y}^{l}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)} \tag{1}$$

where $\bar{y}^l$ is the point at which the μ_{G^l} has a maximum value under the assumption that $\mu_{G^l}\left(\bar{y}^l\right)=1$ and the membership function $\mu_F(x)$ represents the grade of membership of x in a fuzzy set F and has the interval $[0,1]$. In (1), provided that the membership function $\mu_F(x)$ is fixed and $\bar{y}^l$ is regarded as adjustable parameters, (1) can be rewritten as

$$y_i=\theta_i^T\xi(x) \qquad i=1,2,...,n \tag{2}$$

where $\theta_i=[\theta_{i,1},\theta_{i,2},...,\theta_{i,M}]^T\in\mathbb{R}^M$ is a vector of adjustable parameters, and $\xi(x)=[\xi_1,...,\xi_M]^T\in\mathbb{R}^M$ denotes the fuzzy basis function and each element ξ_l is defined as

$$\xi_l=\frac{\prod_{i=1}^{n}\mu_{F_i^l}(x_i)}{\sum_{l=1}^{M}\left(\prod_{i=1}^{n}\mu_{F_i^l}(x_i)\right)}.$$

For MIMO systems, (2) can be extended as

$$y=\Theta^T\xi(x) \tag{3}$$

where $\Theta=[\theta_1,\theta_2,...,\theta_n]\in\mathbb{R}^{M\times n}$.

III. Robust Trajectory Tracking using Fuzzy Logic Systems

Consider an MIMO nonlinear dynamical system written as

$$\dot{x}=f(x)+g(x)u(t)+d(t) \tag{4}$$

where $x(t)\triangleq[x_1,x_2,\cdots,x_n]^T\in\mathbb{R}^n$ is the system state, $g(x)\in\mathbb{R}^{n\times m}$ is a continuous function, $u(t)\in\mathbb{R}^m$ is the control input, $f(x)\in\mathbb{R}^n$ is an unknown continuous function, locally Lipschitz in $x(t)$, and $d(t)\in\mathbb{R}^n$ is an exogeneous disturbance. For the nonlinear plant provided in (4), the tracking control objective is for the system state $x(t)$ to track a given desired reference trajectory $x_d(t)$ in the presence of uncertainties and disturbances.

Assumption 1: The system state $x(t)\in\mathbb{R}^n$ is measurable.

Assumption 2: The funtion $g(x)$ is known and invertible if the rows of $g(x)$ are linearly independent and if $m=n$. The matrix inverse $g^+(x)$ is bounded.

Assumption 3: The desired trajectory $x_d(t)\in\mathbb{R}^n$ is designed such that $x_d^{(i)}(t)\in\mathcal{L}_\infty,\ i=0,1,...,n$.

Assumption 4: The disturbance term and its first and second time derivatives are bounded i.e. $d(t),\dot{d}(t),\ddot{d}(t)\in\mathcal{L}_\infty$. In (4), since $f(x)$ is unknown, the controller $u(t)$ is designed as a self-tuning adaptive controller constructed in terms of the estimate function $\hat{f}(x\mid\Theta)$ resulting from a fuzzy logic system (FLS) where the fuzzy parameters are updated by an online adaptation law. Using FLS, the nonlinear system of (4) can be equivalently represented as

$$\dot{x}=\hat{f}(x\mid\Theta)+g(x)u(t)+\varepsilon(x,\Theta)+d(t) \tag{5}$$

where the fuzzy approximation error $\varepsilon(x,\Theta)\in\mathbb{R}^n$ is defined as

$$\varepsilon\triangleq\left[f(x)-\hat{f}(x\mid\Theta)\right]. \tag{6}$$

To quantify the tracking objective, a tracking error $e(t)\in\mathbb{R}^n$ is defined as

$$e(t)=x(t)-x_d(t) \tag{7}$$

where the desired state trajectory $x_d(t)$ is defined so that the system output $x(t)$ tracks a smooth reference trajectory $x_d(t)$. To facilitate the subsequent stability analysis, a filtered tracking error $r(t)\in\mathbb{R}^n$ is defined as

$$r=\dot{e}+\alpha e \tag{8}$$

where $\alpha\in\mathbb{R}$ denotes a positive constant. The system equations in (5) and the tracking error in (7) can be used to rewrite the filtered tracking error as

$$r=\hat{f}(x\mid\Theta)+g(x)u+\varepsilon+d-\dot{x}_d+\alpha e \tag{9}$$

where the fuzzy parameter Θ of the estimate term $\hat{f}(\hat{x}\mid\Theta)$ is defined in (3). Let $\Theta^*\in\mathbb{R}^{M\times n}$ denote the optimal estimation parameters defined as [3]

$$\Theta^*\triangleq\arg\min_{\Theta\in\mathbb{U}_f}\left[\sup_{x\in\mathbb{U}_x}\left\|f(x)-\hat{f}(x\mid\Theta)\right\|\right]. \tag{10}$$

In (10), $\mathbb{U}_f$ and $\mathbb{U}_x$ are compact sets for the fuzzy parameter Θ and the system state $x(t)$, respectively, defined as

$$\begin{aligned}\mathbb{U}_f &\triangleq \left\{\Theta\in\mathbb{R}^{M\times n}:\|\Theta\|\le M_f\right\} \\ \mathbb{U}_x &\triangleq \left\{x\in\mathbb{R}^n:\|x\|\le M_x\right\},\end{aligned} \tag{11}$$

where M_f and M_x are designed parameters, and $\|\cdot\|$ represents a Euclidean 2-norm. Using the definition of the optimal parameter matrix in (10), the minimum fuzzy approximation error $w(x)\in\mathbb{R}^n$ is defined as

$$w\triangleq\left[f(x)-\hat{f}(x\mid\Theta^*)\right]. \tag{12}$$

In (9), adding and subtracting $\hat{f}(x\mid\Theta^*)$ and using the minimum fuzzy approximation error $w(x,\Theta^*)$, the filtered tracking error in (9) can be written as

$$\begin{aligned}r \;=\;& w+\left[\hat{f}(x\mid\Theta^*)-\hat{f}(x\mid\Theta)\right]+\hat{f}(x\mid\Theta) \\ &-\dot{x}_d+g(x)u+d+\alpha e.\end{aligned} \tag{13}$$

Remark 1: A FLS can approximate a smooth function within an arbitrary small residual error. The minimum approximation error $w(x,\Theta^*)$ can be bounded on a compact set assuming that $\|w(x,\Theta^*)\|<w_1$, $\|w'(x,\Theta^*)\|<w_2$, and $\|w''(x,\Theta^*)\|<w_3$ with a known bound w_1, w_2, and w_3, respectively where $w'=\frac{\partial w}{\partial x}$ and $w''=\frac{\partial^2 w}{\partial x^2}$ [20], [21].

Based on (13), the controller $u(t)\in\mathbb{R}^m$ is designed as

$$u(t)=g^+(x)\left[-\hat{f}(x\mid\Theta)+\dot{x}_d-\mu\right] \tag{14}$$

where $\mu(t)$ is a RISE feedback term that is included to ensure asymptotic tracking in the presence of the FLS reconstruction uncertainty and the additive disturbances. Since $g(x) \in \mathbb{R}^{n\times m}$ is non-square, the pseudo-inverse $g^{+}(x) = g^{T}(x)\left\{g(x)g^{T}(x)\right\}^{-1} \in \mathbb{R}^{m\times n}$ is used, where $g(x)g^{+}(x) = I_{n\times n}$ provided $g(x)g^{T}(x)$ is non-singular and $n \leq m$. The RISE feedback term $\mu(t) \in \mathbb{R}^{n}$ in (14) is defined as [15], [16]

$$\mu(t) \triangleq (k_r + k)e(t) - (k_r + k)e(0) + v(t). \tag{15}$$

In (15), $v(t) \in \mathbb{R}^{n}$ is the generalized solution to

$$\dot{v}(t) = (k_r + k)\alpha e(t) + \beta_1 sgn(e(t)),\ v(0) = 0, \tag{16}$$

where k_r, k, $\beta_1 \in \mathbb{R}$ denote positive constant control gains, and $sgn(\cdot)$ is a vector signum function defined as

$$sgn(e) \triangleq \left[\begin{array}{cccc} sgn(e_1) & sgn(e_2) & ... & sgn(e_n) \end{array}\right].$$

Substituting the controller $u(t)$ designed in (14) into (13) yields

$$r = w + \left[\hat{f}(x \mid \Theta^{*}) - \hat{f}(x \mid \Theta)\right] + \alpha e + d - \mu. \tag{17}$$

The singleton fuzzifier defined in (3) can be replaced for the estimate $\hat{f}$ as

$$\hat{f}(x \mid \Theta) \triangleq \Theta^{T}\xi(x), \qquad \hat{f}(x \mid \Theta^{*}) \triangleq \Theta^{*T}\xi(x) \tag{18}$$

where $\xi(x) = [\xi_1, ..., \xi_M]^{T} \in \mathbb{R}^{M}$ is a vector of fuzzy basis functions. The mismatch parameter matrix $\tilde{\Theta}(t) \in \mathbb{R}^{M\times n}$ is also defined as

$$\tilde{\Theta} \triangleq \Theta^{*} - \Theta. \tag{19}$$

After substituting (18) and (19), the filtered tracking error in (17) can be written as

$$r = w + \tilde{\Theta}^{T}\xi(x) + \alpha e + d - \mu. \tag{20}$$

IV. Robust Identification using Fuzzy Logic Systems

The identification model plus sliding mode term for identifying the nonlinear plant in (5) can be represented as

$$\dot{\hat{x}} = \hat{f}(x \mid \Theta) + g(x)u + \beta_2 sgn(\tilde{x}), \tag{21}$$

where $\hat{x}(t) \triangleq [\hat{x}_1, \hat{x}_2, \cdots, \hat{x}_n]^{T} \in \mathbb{R}^{n}$ is the system state of the identification model, $\tilde{x}(t) \in \mathbb{R}^{n}$ denotes a measurable identification error and is defined as

$$\tilde{x}(t) \triangleq x(t) - \hat{x}(t), \tag{22}$$

and $\beta_2 \in \mathbb{R}$ denotes a positive constant. The fuzzy parameter Θ in (21) is identical to the parameter used in the tracking controller (14) as seen in Fig. 1 so that the parameter can approach one common optimal point. The identification control objective is to drive the estimated state $\hat{x}(t)$ to the system state $x(t)$.

Remark 2: The sliding mode term helps the model to identify the nonlinear system with additive disturbance and provides robust properties against exogeneous disturbance and unmodeled dynamics. The measurable identification error $\tilde{x}(t)$ in (22) is used to update the parameter of the fuzzy system.

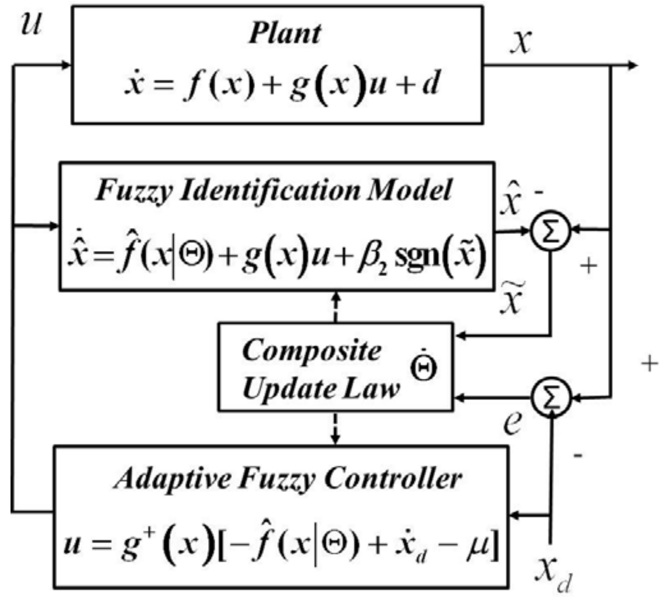

Fig. 1. Overall scheme of robust identification and tracking control using composite upadate law

After subtracting (21) from (5), the time derivative of the identification error equation is defined as

$$\dot{\tilde{x}} = \left[f(x) - \hat{f}(x \mid \Theta)\right] + \varepsilon + d - \beta_2 sgn(\tilde{x}). \tag{23}$$

Based on (10), (11), and (12), (23) can be written as

$$\dot{\tilde{x}} = w + \left[\hat{f}(x \mid \Theta^{*}) - \hat{f}(x \mid \Theta)\right] + d - \beta_2 sgn(\tilde{x}). \tag{24}$$

Using (18) and (19), the identification error dynamics in (24) can be written as

$$\dot{\tilde{x}} = h + \tilde{\Theta}^{T}\xi(x) - \beta_2 sgn(\tilde{x}) \tag{25}$$

where $h(t) \in \mathbb{R}^{n}$ is a disturbance term consisting of the fuzzy approximation error $w(t)$ and the exogeneous disturbance $d(t)$ as

$$h \triangleq w + d, \tag{26}$$

and in terms of Assumption 4 and Remark 1, the disturbance term $h(t)$ can be bounded as

$$\|h\| \leq \bar{h} \tag{27}$$

where $\bar{h}$ is a bounding positive constant.

To develop the integrated stability analysis for both robust tracking and robust identification using the FLS, the derived equations are rearranged and several useful properties are defined. To facilitate the subsequent stability analysis, the time derivative of (20) is given as

$$\begin{aligned}\dot{r} \;\; = \;\; & \alpha\dot{e} + w'(x, \Theta^{*})\dot{x} + \tilde{\Theta}^{T}\xi'(x)\dot{x} + \dot{\tilde{\Theta}}^{T}\xi(x) \\ & + \dot{d} - \dot{\mu}.\end{aligned} \tag{28}$$

In (28), $\dot{x}(t)$ is replaced by using (7) and (8) and after differentiating the RISE term of (15), the following expression is obtained as

$$\begin{aligned}\dot{r} &= \alpha(r-\alpha e)+w'(x,\Theta^*)(r-\alpha e+\dot{x}_d) \\ &\quad +\tilde{\Theta}^T\xi'(x)(r-\alpha e+\dot{x}_d)-\dot{\Theta}^T\xi(x)+\dot{d} \\ &\quad -(k_d+k)r-\beta_1 sgn(e).\end{aligned} \quad (29)$$

Rearranging the terms of (29), the closed-loop filtered error can be expressed as

$$\dot{r}=\tilde{N}+N-e-(k_d+k)r-\beta_1 sgn(e) \quad (30)$$

where the auxiliary function $\tilde{N}(t)\in\mathbb{R}^n$ is denoted as

$$\begin{aligned}\tilde{N} &\triangleq \alpha(r-\alpha e)+w'(x,\Theta^*)(r-\alpha e) \\ &\quad -\dot{\Theta}^T\xi(x)+\tilde{\Theta}^T\xi'(x)(r-\alpha e)+e,\end{aligned} \quad (31)$$

and N is defined as

$$N\triangleq\dot{d}+N_{B1}+N_{B2}. \quad (32)$$

In (32), $N_{B1}(t)\in\mathbb{R}^n$ is defined as

$$N_{B1}=w'(x,\Theta^*)\dot{x}_d, \quad (33)$$

and N_{B2} is defined as

$$N_{B2}=\tilde{\Theta}^T\xi'(x)\dot{x}_d. \quad (34)$$

The composite adaptive parameter update law for the FLSs is designed based on the subsequent stability analysis as

$$\dot{\Theta}=\Gamma proj\left\{\alpha\xi'(x)\dot{x}_d e^T+\xi(x)\tilde{x}^T\right\} \quad (35)$$

where $\Gamma\in\mathbb{R}^{Mn\times Mn}$ denotes a constant, positive-definite, diagonal adaptation gain matrix, and $proj(\cdot)$ denotes a projection algorithm utilized to guarantee that the i^{th} element of Θ can be bounded as

$$\underline{\Theta}_i\leq\Theta_i\leq\overline{\Theta}_i \quad (36)$$

where $\underline{\Theta}_i$, $\overline{\Theta}_i\in\mathbb{R}$ denotes a known, constant lower and upper bound for each element of $\hat{\Theta}(t)$. The selection of $\hat{f}(x\mid\Theta)$ in the FLS involves a human expert's knowledge in choosing the initial value $\Theta(0)$ of the parameter or the randomly chosen values. In any case, the parmater $\Theta(t)$ is properly adjusted by the adaptive update law [19], [22]. The composite parameter update law can enhance the parameter update performance in that the update law utilizes two sources of information to adjust one fuzzy parameter Θ. Substituting the update laws of (35) for the function of (31) and using the mean value theorem, the function $\tilde{N}(t)$ can be upper bounded as

$$\left\|\tilde{N}\right\|\leq\rho(\|z\|)\|z\| \quad (37)$$

where $z(t)\in\mathbb{R}^{3n}$ is defined as

$$z(t)\triangleq\left[\begin{array}{ccc} e^T & r^T & \tilde{x}^T\end{array}\right]^T, \quad (38)$$

and the bounding function $\rho(\cdot)\in\mathbb{R}$ is a positive, globally invertible, non-decreasing function. On the basis of Assumptions 4, the bounding properties for $\dot{d}(t)$ and $\ddot{d}(t)$ can be developed as

$$\left\|\dot{d}\right\|\leq\rho_1,\quad\left\|\ddot{d}\right\|\leq\rho_2. \quad (39)$$

Considering Assumption 3 and Remark 1, the bounds for the $N_{B1}(t)$ are obtained as

$$\|N_{B1}\|\leq\rho_3,\quad\left\|\dot{N}_{B1}\right\|\leq\rho_4+\rho_5\|z\|, \quad (40)$$

and using Assumption 3 and (36), the bounds for the $N_{B2}(t)$ are as follows:

$$\|N_{B2}\|\leq\rho_6,\quad\left\|\dot{N}_{B2}\right\|\leq\rho_7+\rho_8\|z\|, \quad (41)$$

where $\rho_i\in\mathbb{R}$, $(i=1,2,...,8)$ denote computable positive bounding constants. To facilitate the subsequent stability analysis, let $D\subset\mathbb{R}^{3n+2}$ be a domain containing $y(t)=0$, where $y(t)\in\mathbb{R}^{3n+2}$ is defined as

$$y(t)\triangleq\left[\begin{array}{ccc} z^T(t) & \sqrt{P(t)} & \sqrt{Q(t)}\end{array}\right]^T. \quad (42)$$

The auxiliary function $P(t)\in\mathbb{R}$ in (42) is the generalized solution to the differential equation

$$\begin{aligned}\dot{P} &= -L(t), \\ P(0) &= \beta_1\sum_{j=1}^{n}|e_j(0)|+e^T(0)(N(0))\end{aligned} \quad (43)$$

where the subscript j denotes j^{th} element of $e(0)$ and the auxiliary function $L(t)\in\mathbb{R}$ is defined as

$$L\triangleq r^T\left(\dot{d}+N_{B1}-\beta_1 sgn(e)\right)+\dot{e}^T N_{B2}-\beta_3\|z\|^2. \quad (44)$$

In (44), $\beta_1,\beta_3\in\mathbb{R}$ are positive constants selected by the following sufficient conditions:

$$\begin{aligned}\beta_1 &> \max\left(\rho_1+\rho_3+\rho_6,\ \rho_1+\rho_3+\frac{\rho_2}{\alpha}+\frac{\rho_4}{\alpha}+\frac{\rho_7}{\alpha}\right) \\ \beta_3 &> \rho_5+\rho_8.\end{aligned} \quad (45)$$

Provided that the sufficient conditions introduced in (45) are satisfied, then $P(t)\geq 0$. The auxiliary function $Q(t)\in\mathbb{R}$ included in (42) is defined as

$$Q(t)\triangleq\frac{1}{2}tr\left(\tilde{\Theta}^T\Gamma^{-1}\tilde{\Theta}\right), \quad (46)$$

where the function $Q(t)\geq 0$ since Γ is positive-definite.

V. Stability Analysis

Theorem: The robust adaptive fuzzy logic controller (14) constructed from fuzzy modeling rules to identify and control a class of uncertain nonlinear dynamic systems, which is along with the composite adaptive fuzzy parameter update laws given in (35), ensures that all systems signals are bounded and that the tracking error $e(t)$ and identification error $\tilde{x}(t)$ are regulated such that

$$\|e(t)\|\to 0,\qquad\|\tilde{x}(t)\|\to 0\quad\text{as}\quad t\to\infty \quad (47)$$

provided that the control gain k in (15) is chosen so that it has a sufficiently large value based on the initial conditions of the states, the gain conditions given in (45) are satisfied, and the following sufficient gain conditions are satisfied as

$$\beta_2>\bar{h}\qquad\lambda>\beta_3 \quad (48)$$

where β_2, $\bar{h}$, β_3, and λ are introduced in (21), (27), (45), and (57), respectively.

Proof: To prove the integrated system identification and tracking control result, consider a positive definite function $V\left(e, r, \tilde{x}, \tilde{\Theta}\right)$ defined as

$$V \triangleq \frac{1}{2}e^T e + \frac{1}{2}r^T r + \frac{1}{2}\tilde{x}^T \tilde{x} + P + Q. \tag{49}$$

The defined Lyapunov function candidate satisfies the following inequalities:

$$U_1(y) \leq V(y, t) \leq U_2(y) \tag{50}$$

where the continuous positive definite fuctions $U_1(y)$, $U_2(y) \in \mathbb{R}$ are defined as

$$U_1(y) \triangleq \frac{1}{2}\|y\|^2, \qquad U_2(y) \triangleq \|y\|^2$$

where $U_1(y)$, $U_2(y) \in \mathbb{R}$ are continuous positive definite functions. Using (8), (24), (30), (43), the differential equations of the closed-loop system are continuous except in the set $\{y|\tilde{x} = 0 \ or \ e = 0\}$. Using Filippov's differential inclusion [23]–[26], the existence of solutions can be established for $\dot{y} = f(y)$, where $f(y) \in \mathbb{R}^{3n+2}$ denotes the right-hand side of the the closed-loop error signals. Under Filippov's framework, a generalized Lyapunov stability theory can be used (see [26]–[29] for further details). The generalized time derivative of (49) exists almost everywhere (a.e.), and $\dot{V}(y) \in^{a.e.} \dot{\tilde{V}}(y)$ where

$$\dot{\tilde{V}} = \bigcap_{\zeta \in \partial V(y)} \zeta^T K \begin{bmatrix} \dot{z}^T & \frac{1}{2}P^{-\frac{1}{2}}\dot{P} & \frac{1}{2}Q^{-\frac{1}{2}}\dot{Q} \end{bmatrix}^T$$

where ∂V is the generalized gradient of V [27], and $K[\cdot]$ is defined as [28], [29]

$$K[f](y) \triangleq \bigcap_{\delta > 0} \bigcap_{\mu N = 0} \overline{co} f(B(x, \delta) - N),$$

where $\bigcap_{\mu N=0}$ denotes the intersection of all sets N of Lebesgue measure zero, $\overline{co}$ denotes convex closure, and $B(x, \delta)$ represents a ball of radius δ around x. Since $V(y)$ is a Lipschitz continuous regular function,

$$\begin{aligned} \dot{\tilde{V}} &= \nabla V^T K \begin{bmatrix} \dot{z}^T & \frac{1}{2}P^{-\frac{1}{2}}\dot{P} & \frac{1}{2}Q^{-\frac{1}{2}}\dot{Q} \end{bmatrix}^T \\ &\subset \begin{bmatrix} z^T & 2P^{\frac{1}{2}} & 2Q^{\frac{1}{2}} \end{bmatrix} \\ &\quad K \begin{bmatrix} \dot{z}^T & \frac{1}{2}P^{-\frac{1}{2}}\dot{P} & \frac{1}{2}Q^{-\frac{1}{2}}\dot{Q} \end{bmatrix}^T. \end{aligned}$$

From (8), (25), (30), (34), and (43), $\dot{\tilde{V}}(t)$ can be expressed as

$$\begin{aligned} \dot{\tilde{V}} \subset\ & r^T \left(\tilde{N} + N_{B1} + \dot{d} - (k_r + k) r - \beta_1 sgn(e) - e\right) \\ & + (\dot{e} + \alpha e)^T \tilde{\Theta}^T \xi'(x) \dot{x}_d + e^T (r - \alpha e) \\ & + \tilde{x}^T \left(h + \tilde{\Theta}^T \xi(x) - \beta_2 sgn(\tilde{x})\right) \\ & - r^T \left(\dot{d} + N_{B1} - \beta_1 sgn(e)\right) \\ & - \dot{e}^T N_{B2} + \beta_3 \|z\|^2 - tr\left(\tilde{\Theta}^T \Gamma_1^{-1} \dot{\Theta}\right). \end{aligned} \tag{51}$$

where [29]

$$K[sgn(e)] = SGN(e)$$

such that

$$SGN(e) = \begin{cases} 1 & e > 0 \\ [-1, 1] & e = 0 \\ -1 & e < 0 \end{cases}.$$

By arranging the terms and using the composite fuzzy parameter update laws given in (35), the expression in (51) can be simplified as

$$\begin{aligned} \dot{\tilde{V}} \subset\ & r^T \tilde{N} - r^T (k_r + k) r - \alpha e^T e \\ & + \tilde{x}^T h + \beta_2 \tilde{x}^T sgn(\tilde{x}) + \beta_3 \|z\|^2. \end{aligned} \tag{52}$$

The expression in (52) can be upper bounded as

$$\begin{aligned} \dot{\tilde{V}} \subset\ & -k_r \|r\|^2 - \alpha \|e\|^2 - k \|r\|^2 + \bar{h} \|\tilde{x}\| \\ & - \beta_2 \sum_{j=1}^{n} |\tilde{x}_j| + \left\|\tilde{N}\right\| \|r\| + \beta_3 \|z\|^2. \end{aligned} \tag{53}$$

The bounding condition (37) for $\left\|\tilde{N}\right\|$ and the fact that

$$\|\tilde{x}\| \leq \sum_{j=1}^{n} |\tilde{x}_j| \tag{54}$$

yields

$$\begin{aligned} \dot{\tilde{V}} \subset\ & -k_r \|r\|^2 - \alpha \|e\|^2 - \left\{k \|r\|^2 - \rho(\|z\|) \|z\| \|r\|\right\} \\ & - \left(\beta_2 - \bar{h}\right) \|\tilde{x}\| + \beta_3 \|z\|^2. \end{aligned} \tag{55}$$

Choosing β_2 according to the gain condition in (48) and completing the squares with respect to $\|r\|$, (55) can be upper bounded as

$$\dot{\tilde{V}} \subset -\left(\lambda - \beta_3 - \frac{\rho^2(\|z\|)}{4k}\right) \|z\|^2 \tag{56}$$

where

$$\lambda \triangleq \min\{\alpha, k_r\}. \tag{57}$$

The bounding result given in (56) achieves the following equivalent inequality:

$$\dot{\tilde{V}} \leq -U(y) \tag{58}$$

where $U(y) = c\|z\|^2$ is a continuous positive semi-definite function for a positive constant $c \in \mathbb{R}$. The function $y(t) \in \mathbb{R}^{3n+2}$ is defined on the domain

$$D \triangleq \left\{y(t) \in \mathbb{R}^{3n+2} \mid \|y\| \leq \rho^{-1}\left(2\sqrt{k(\lambda - \beta_3)}\right)\right\}.$$

The inequality conditions in (50) and (58) show that $V \in \mathcal{L}_\infty$ exists in the domain D. Using (49), $r(t)$, $e(t)$, $\tilde{x}(t)$, $P(t)$, $Q(t) \in \mathcal{L}_\infty$ in the domain D. From $r(t)$, $e(t) \in \mathcal{L}_\infty$ and (8), $\dot{e}(t) \in \mathcal{L}_\infty$ in D. Further, (7), $e(t)$, $\dot{e}(t) \in \mathcal{L}_\infty$, and $x_d(t)$, $\dot{x}_d(t) \in \mathcal{L}_\infty$ by Assumption 3 can be used to show that $x(t)$, $\dot{x}(t) \in \mathcal{L}_\infty$. In addition to $x(t)$ and $\dot{x}(t) \in \mathcal{L}_\infty$ from (4) and (5), $f(x)$, $g(x)$, $u(t) \in \mathcal{L}_\infty$ in D. Since $u(t) \in \mathcal{L}_\infty$, the estimates $\hat{f}(x \mid \Theta) \in \mathcal{L}_\infty$ in D and the RISE term

$\mu(t) \in \mathcal{L}_\infty$ in D from (14). From (36) and $\hat{f}(x \mid \Theta) \in \mathcal{L}_\infty$ in D, it is said that the fuzzy parameters are bounded like following: $\Theta(t)$, $\xi(x) \in \mathcal{L}_\infty$. Moreover, using (22) and $x(t)$, $\tilde{x}(t) \in \mathcal{L}_\infty$ shows $\hat{x}(t) \in \mathcal{L}_\infty$ in D. The fact that the estimates $\hat{f}(x \mid \Theta) \in \mathcal{L}_\infty$ and the controller $u(t) \in \mathcal{L}_\infty$ in D shows $\dot{\hat{x}} \in \mathcal{L}_\infty$ in D from (21) and then $\dot{\tilde{x}} \in \mathcal{L}_\infty$ in D from $\dot{x}(t) \in \mathcal{L}_\infty$, $\dot{\hat{x}} \in \mathcal{L}_\infty$ in D, and (22). Based on the bounded property of the fuzzy basis function $\xi(\cdot)$, the sign function $sgn(\cdot)$, Assumption 3, 4, and $\dot{\Theta}(t) \in \mathcal{L}_\infty$ along the parameter update laws, we can prove that $\dot{r}(t) \in \mathcal{L}_\infty$ in D from (30) and then $\dot{z}(t) \triangleq \begin{bmatrix} \dot{e}^T & \dot{r}^T & \dot{\tilde{x}}^T \end{bmatrix}^T \in \mathcal{L}_\infty$ in D. Therefore, $U(y)$ is uniformly continuous in D. Now, consider the set S satisfying $S \subset D$ denoted as

$$S \triangleq \left\{ y(t) \in D \mid U_2(y) \leq \left\{ \rho^{-1}\left(2\sqrt{k(\lambda - \beta_3)} \right) \right\}^2 \right\}. \tag{59}$$

The region of attraction in (59) is arbitrarily large and can include any initial condition by increasing the control gain k (i.e., a semi-global result), and hence

$$c\|z\|^2 \to 0 \quad as \quad t \to \infty \;\; \forall y(0) \in S.$$

The above result shows that both the identification error $\tilde{x}(t)$ and the tracking error $e(t)$ go to zero as time goes to infinity under all initial condition $y(0)$ in the set S (i.e., $\tilde{x}(t)$, $e(t)$ $\to 0$ as $t \to \infty$, $\forall y(0) \in S$). ■

VI. Conclusion

A robust adaptive fuzzy identification and tracking approach is developed for an affine disturbed nonlinear dynamic system. A novel fuzzy identification method is used to generate additional approximate model knowledge that is combined with the tracking error in a composite adaptive update law. The developed controller uses a fuzzy logic based feedforward controller combined with a continuous RISE feedback term to compensate for parametric uncertainties, uncertain exogenous disturbances, and the residual fuzzy approximation error. A Lyapunov-based stability analysis indicates that the developed controller achieves semi-global asymptotic tracking under the sufficient gain conditions. Future work includes simulation and experimental results to illustrate the added performance value that is provided by the fuzzy identifier.

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