2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Distributed Coordinated Tracking with Multiple Dynamic Leaders for Double-integrator Agents Using Only Position Measurements

Jianzhen Li, Wei Ren, and Shengyuan Xu

***Abstract*— This paper studies the distributed coordinated tracking problem for a group of autonomous vehicles modeled by double-integrator dynamics with multiple dynamic leaders. The objective is to drive the followers into the convex hull spanned by the dynamic leaders under the constraints that the velocities and the accelerations of both the leaders and the followers are not available, the leaders are neighbors of only a subset of the followers, and the followers have only local interaction. When the absolute position measurements of the vehicles are available, we propose a distributed finite-time coordinated tracking algorithm. Theoretical analysis shows that the followers will move into the convex hull spanned by the dynamic leaders in finite time if the network topology among the followers is undirected, for each follower there exists at least one leader that has a directed path to the follower, and the parameters in the algorithm are properly chosen. When the absolute position measurements are not available, we propose a distributed adaptive coordinated tracking algorithm using only the relative position measurements. Theoretical analysis shows that the followers will ultimately move into the convex hull spanned by the dynamic leaders under similar conditions to the case where the absolute position measurements are available.**

I. INTRODUCTION

The distributed multi-vehicle cooperative control has received increasing attention from researchers in different areas. This is due to its broad applications and its advantages such as low cost, high adaptivity, and easy maintenance, compared with its centralized counterpart. The study of distributed multi-vehicle cooperative control focuses on how to achieve collective objectives through local interaction. The leaderless consensus problem is a fundamental problem in distributed multi-vehicle cooperative control. The objective is to reach an agreement on certain quantities of interest among the vehicles through local interaction. Recently, significant progress has been made in the leaderless consensus problem [1]–[6].

A more challenging problem in distributed multi-vehicle cooperative control is the coordinated tracking problem, where there exists a single or multiple dynamic leaders. In the single-leader case, the objective is to drive the states of the followers to approach the state of the dynamic leader. This problem and its variants were investigated in [7]–[10]. A PD-like algorithm was proposed in [8] in a continuous-time setting and in [9] in a discrete-time setting for autonomous vehicles with single-integrator dynamics. In particular, [8] requires measurements of the leader's velocity, while [9] requires a small sampling period. The case where the autonomous vehicles are modeled by double-integrator dynamics was investigated in [7], [10] under a variable undirected network topology while in [11] under a strongly connected and balanced network topology in the presence of time delays. In particular, [7], [10], [11] require that the leader's acceleration be available to all the followers. Some variable structure-based algorithms have recently been reported in [12] for both single-integrator dynamics and double-integrator dynamics, which require less interaction than their counterparts in [7], [9]–[11]. However, all references mentioned above for double-integrator dynamics require that the velocity measurements be available.

In the multi-leader case, the objective is to drive the states of the followers into the convex hull spanned by those of the dynamic leaders, also called the containment control. Due to its widely application in practice, for example, a collection of autonomous vehicles secure and remove hazardous materials [14], the containment control problem has been investigated extensively. In [14], a stop-and-go strategy was proposed for vehicles modeled by single-integrator kinematics under a fixed undirected network topology. An extension to a switching directed network topology was given in [15] for single-integrator dynamics while in [16] for double-integrator dynamics. However, for autonomous vehicles with double-integrator dynamics, the algorithms proposed in [16] require the velocity measurements to be available.

In reality, it is more difficult to obtain velocity and acceleration measurements than position measurements. We are hence motivated to design distributed coordinated tracking algorithms for autonomous vehicles with double-integrator dynamics in the presence of multiple dynamic leaders using only position measurements. The case where there exists a single dynamic leader can be treated as a special case of multiple dynamic leaders. When the absolute position measurements of the vehicles are available, we propose a distributed finite-time coordinated tracking algorithm. In this algorithm, each follower has an observer. The followers only need to know the states (or the absolute positions of their neighbors, if their neighbors are leaders) of their neighbors's observers. We show that the followers are driven into the convex hull spanned by the dynamic leaders in finite time if the network topology among the followers is undirected, for each follower there exists at least one leader that has directed path to the follower, and the parameters in the algorithm are properly chosen. When the absolute position

J. Li and S. Xu are with the School of Automation at Nanjing University of Science and Technology, China.

W. Ren is with the Department of Electrical and Computer Engineering at Utah State University.

This work was supported by National Science Foundation under grant ECCS–1002393.

W. Ren is the corresponding author: wei.ren@usu.edu

measurements of the vehicles are not available, we propose a distributed adaptive coordinated tracking algorithm using the relative position measurements. We show that the followers will ultimately move into the convex hull spanned by the dynamic leaders under similar conditions to the case where the absolute position measurements are available.

The contributions of this paper are threefold. First, for vehicles with double-integrator dynamics, the velocity measurements are not required in the proposed algorithms. Therefore, these algorithms are easier to be implemented. Second, using the second algorithm in this paper, the bound on the accelerations of the leaders are not required to be known. Finally, when the accelerations of the leaders are not identical, the parameters in the second algorithm proposed in this paper are not required to satisfy any condition related to the network topology, while the parameters in the algorithm in [16] should satisfy a certain condition related to the network topology to guarantee that the followers will converge to the convex hull spanned by the dynamic leaders.

The rest of this paper is organized as follows. Section II introduces some preliminary knowledge used in this paper. Section III provides the main theoretical results. The paper is concluded in Section IV.

Notations: Define $\mathbf{1}_p \triangleq \begin{bmatrix} 1, & \cdots, & 1 \end{bmatrix}^T \in \mathbb{R}^p$. Given a vector $\nu = \begin{bmatrix} \nu_1, & \cdots, & \nu_p \end{bmatrix}^T \in \mathbb{R}^p$ and $\alpha \in \mathbb{R}$, define $\text{sig}(\nu)^\alpha \triangleq \begin{bmatrix} \text{sgn}(\nu_1) \mid \nu_1 \mid^\alpha, & \cdots, & \text{sgn}(\nu_p) \mid \nu_p \mid^\alpha \end{bmatrix}^T$ and $\text{sgn}(\nu) \triangleq \begin{bmatrix} \text{sgn}(\nu_1), & \cdots, & \text{sgn}(\nu_p) \end{bmatrix}^T$, where $\text{sgn}(\cdot)$ is the signum function.. We use $\text{diag}(\nu_1, \cdots, \nu_p)$ to denote the diagonal matrix of all ν_1, $\cdots$, ν_p, and I_p to denote the p by p identical matrix.

II. Background and Preliminaries

Consider a group of $n+s$ vehicles with double-integrator dynamics given by

$$\dot{x}_i(t) = v_i(t), \tag{1a}$$
$$\dot{v}_i(t) = u_i(t), \quad i = 1, \cdots, n+s, \tag{1b}$$

where $x_i(t)$, $v_i(t)$ and $u_i(t) \in \mathbb{R}^m$ are, respectively, the position, velocity and control input associated with the ith vehicle.

We use a graph $\mathcal{G} \triangleq (\mathcal{V}, \mathcal{E})$ to denote the network topology among vehicles 1 to $n+s$, where $\mathcal{V} \triangleq \{1, \cdots, n+s\}$ is the node set and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edge set. A directed edge $(j,i) \in \mathcal{E}$ if vehicle i can access information from vehicle j but not necessarily vice versa. An undirected edge $(j,i) \in \mathcal{E}$ if vehicle i and vehicle j can access information from each other. Here we assume that $(i,i) \notin \mathcal{E}$. The neighbor set $\mathcal{N}_i$ of vehicle i is defined as $\mathcal{N}_i \triangleq \{j \mid (j,i) \in \mathcal{E}\}$. Suppose that vehicles 1 to n have at least one neighbor and vehicles $n+1$ to $n+s$ have no neighbor. We call vehicles 1 to n the *followers* and vehicles $n+1$ to $n+s$ the *leaders*. A graph is undirected if $(i,j) \in \mathcal{E}$ implies that $(j,i) \in \mathcal{E}$. We assume that the graph associated with the followers is undirected and further assume that $a_{ij} = a_{ji}$, $i,j = 1, \cdots, n$. A directed path is a sequence of directed edges of the form (i_1, i_2), (i_2, i_3),..., where $i_j \in \mathcal{V}$. An undirected path is defined analogously. The adjacency matrix $\mathcal{A}_d \triangleq [a_{ij}] \in \mathbb{R}^{(n+s)\times(n+s)}$ is defined as $a_{ij} > 0$ if $(j,i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. It is easy to see that $a_{ij} = 0$, $i = n+1, \cdots, n+s$, $j = 1, \cdots, n+s$ because the leaders have no neighbors. The Laplacian matrix $\mathcal{L} \triangleq [l_{ij}] \in \mathbb{R}^{(n+s)\times(n+s)}$ is defined as $l_{ii} = \sum_{j=1, j\neq i}^{n+s} a_{ij}$ and $l_{ij} = -a_{ij}$, $i \neq j$. Note that $\mathcal{L}$ can be rewritten as

$$\mathcal{L} = \begin{bmatrix} L_1 & L_2 \\ 0_{s\times n} & 0_{s\times s} \end{bmatrix}, \tag{2}$$

where $L_1 \in \mathbb{R}^{n\times n}$ and $L_2 \in \mathbb{R}^{n\times s}$.

Definition 2.1: Let $\mathcal{C} \subseteq \mathbb{R}^p$. The set $\mathcal{C}$ is said to be convex if for any x and y in $\mathcal{C}$, the point $(1-\alpha)x + \alpha y$ is in $\mathcal{C}$ for any $\alpha \in [0,1]$. The convex hull of a set of points $X = \{x_1, \cdots, x_q\}$ is the minimal convex set containing all points in X. We use $\text{Co}(X)$ to denote the convex hull of X. Let $x_F(t) \triangleq \begin{bmatrix} x_1^T(t), & \cdots, & x_n^T(t) \end{bmatrix}^T$, $v_F(t) \triangleq \begin{bmatrix} v_1^T(t), & \cdots, & v_n^T(t) \end{bmatrix}^T$, $x_L(t) \triangleq \begin{bmatrix} x_{n+1}^T(t), & \cdots, & x_{n+s}^T(t) \end{bmatrix}^T$ and $v_L(t) \triangleq \begin{bmatrix} v_{n+1}^T(t), & \cdots, & v_{n+s}^T(t) \end{bmatrix}^T$. Also let $\Omega(t) \triangleq \text{Co}[x_L(t)]$ and $\Upsilon(t) \triangleq \text{Co}[v_L(t)]$.

The objective of the distributed coordinated tracking problem is to design $u_i(t)$ for all the followers such that the followers move into the convex hull spanned by the dynamic leaders, i.e., $\inf_{y(t)\in\Omega(t)} \|x_i(t) - y(t)\| \to 0$ and $\inf_{y(t)\in\Upsilon(t)} \|v_i(t) - y(t)\| \to 0$, $i = 1, \cdots, n$, as $t \to \infty$. We have the following assumption throughout the note.

Assumption 2.2: For each follower, there exists at least one leader that has a directed path to the follower.

III. Main Results

A. Coordinated Tracking Using Absolute Position Measurements

In this section, we assume that the absolute position measurements of the vehicles are available but the velocity measurements are not available. Let $u_i(t) = f_i(t)$, $i = n+1, \cdots, n+s$, where $f_i(t)$ is the acceleration input. We assume that $\|v_i(t)\|_\infty$ and $\|f_i(t)\|_\infty$, $i = n+1, \cdots, n+s$, are bounded. We propose the following coordinated tracking algorithm

$$u_i(t) = -\alpha\text{sgn}\{z_{1i}(t) + \beta\text{sig}[x_i(t) - \hat{x}_i(t)]^{\frac{1}{2}}\}, \tag{3a}$$
$$\dot{z}_{0i}(t) = z_{1i}(t) - k_1\text{sig}\left\{z_{0i}(t) - [x_i(t) - \hat{x}_i(t)]\right\}^{\frac{1}{2}}, \tag{3b}$$
$$\begin{aligned}\dot{z}_{1i}(t) = & -k_2\text{sgn}\left\{z_{0i}(t) - [x_i(t) - \hat{x}_i(t)]\right\} \\ & -\alpha\text{sgn}\{z_{1i}(t) + \beta\text{sig}[x_i(t) - \hat{x}_i(t)]^{\frac{1}{2}}\},\end{aligned} \tag{3c}$$
$$\begin{aligned}\dot{\hat{x}}_i(t) = & -k_3\text{sgn}\left\{\sum_{j=1}^{n} a_{ij}[\hat{x}_i(t) - \hat{x}_j(t)] \right. \\ & \left. + \sum_{j=n+1}^{n+s} a_{ij}[\hat{x}_i(t) - x_j(t)]\right\}, \\ & i = 1, \cdots, n,\end{aligned} \tag{3d}$$

where $\hat{x}_i(0) = 0$ for $i = 1, \cdots, n$, k_1, k_2, k_3, α and β are positive constant scalars, and a_{ij}, $i = 1, \cdots, n$, $j =$

$1, \cdots, n+s$, is the (i,j)th entry of the adjacency matrix $\mathcal{A}_d$.

Before moving on, we need the following lemma.

Lemma 3.1: Under Assumption 2.2, L_1 defined in (2) is symmetric positive definite. ■

Note from Lemma 3.1 that L_1 is invertible. Let $x_d(t) \triangleq [\, x_{d1}^T(t), \ \cdots, \ x_{dn}^T(t)\,]^T \triangleq -(L_1^{-1}L_2 \otimes I_m)x_L(t)$, where $x_{di}(t) \in \mathbb{R}^m$. Because $\|v_i(t)\|_\infty$ and $\|f_i(t)\|_\infty$, $i = n+1, \cdots, n+s$ are bounded, it follows that $\| \dot{x}_d(t) \|_\infty$ and $\| \ddot{x}_d(t) \|_\infty$ are also bounded. We hence assume that $\| \dot{x}_d(t) \|_\infty \leq \eta_a$ and $\| \ddot{x}_d(t) \|_\infty \leq \eta_b$.

Lemma 3.2: Under Assumption 2.2, $\inf_{y(t)\in\Omega(t)} \|x_{di}(t) - y(t)\| = 0$ and $\inf_{y(t)\in\Upsilon(t)} \|\dot{x}_{di}(t) - y(t)\| = 0$, $i = 1, \cdots, n$, for all t. Using (3d), $||\hat{x}_i(t) - x_{di}(t)|| \to 0$, $i = 1, \cdots, n$, in finite time if $k_3 > \eta_a$.

Proof: The first part of the lemma follows from Lemma 2.4 in [17]. Let $\hat{x}_i(t) = \hat{x}_{fi}(t)$, $i = 1, \cdots, n$ and $\hat{x}_i(t) = \dot{\sigma}_i(t)$, $i = n+1, \cdots, n+s$. Then, equation (3d) becomes equation (8) in [17], and $x_{di}(t)$ is equivalent to $\dot{\sigma}_{di}(t)$ in [17]. The second part of the lemma follows from the proof of Theorem 4.1 in [17]. ■

Lemma 3.3: [19] Consider the system

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) - k_1 \text{sig}[x_1(t)]^{\frac{1}{2}}, \\ \dot{x}_2(t) &= -k_2 \text{sgn}[x_1(t)] + \rho(t,x), \end{aligned}$$

where $x_1(t), x_2(t) \in \mathbb{R}$, k_1, k_2 are constant positive scalars and $\rho(t,x)$ is a bounded perturbation with $x \triangleq \begin{bmatrix} x_1(t) & x_2(t) \end{bmatrix}^T$. Suppose that there exists a symmetric positive-definite matrix P such that the linear matrix inequality

$$A^T P + PA + \varepsilon^2 C^T C + PBB^T P < 0 \tag{4}$$

is satisfied, where $A \triangleq \begin{bmatrix} -\frac{1}{2}k_1 & \frac{1}{2} \\ -k_2 & 0 \end{bmatrix}$, $B \triangleq \begin{bmatrix} 0 \\ 1 \end{bmatrix}$, $C \triangleq [\ 1 \quad 0\ ]$, and ε is a positive constant scalar. Then $x_1(t)$ and $x_2(t)$ will converge to zero in finite time for all bounded perturbations satisfying $| \rho(t,x) | \leq \varepsilon$. ■

Lemma 3.4: [20] Consider the system

$$\ddot{x}(t) = f(t,x) - \alpha K(t,x) \text{sgn}\{\dot{x}(t) + \beta \text{sig}[x(t)]^{\frac{1}{2}}\},$$

where $x(t) \in \mathbb{R}$, $| f(t,x) | \leq D$, $K_m \leq K(t,x) \leq K_M$, and α, β, D, K_m and K_M are constant positive scalars. Then, $x(t)$ and $\dot{x}(t)$ will converge to zero in finite time if $\alpha > \frac{1}{K_m}(D + \frac{\beta^2}{2})$. ■

Theorem 3.1: Under Assumption 2.2, using (3) for (1), $\inf_{y(t)\in\Omega(t)} \|x_i(t) - y(t)\| \to 0$ and $\inf_{y(t)\in\Upsilon(t)} \|v_i(t) - y(t)\| \to 0$ in finite time if $\alpha > \eta_b + \frac{\beta^2}{2}$, $k_3 > \eta_a$ and there exist $k_1 > 0$, $k_2 > 0$ and a symmetric positive-definite matrix P such that (4) is satisfied, where $\epsilon = \eta_b$. In particular, $||x_i(t) - x_{di}(t)|| \to 0$ and $||v_i(t) - \dot{x}_{di}(t)|| \to 0$, $i = 1, \cdots, n$, in finite time.

Proof: Note from Lemma 3.2 that there exists a $T_1 > 0$ such that $\hat{x}_i(t) = x_{di}(t)$, $i = 1, \cdots, n$, for all $t \geq T_1$. We next show that $x_i(t)$, $v_i(t)$, $\hat{x}_i(t)$, $z_{0i}(t)$ and $z_{1i}(t)$, $i = 1, \cdots, n$, will not diverge to infinity for all $t \in [0, T_1]$. Because from (3a) $\| u_i(t) \|_\infty \leq \alpha$, it is easy to see that $x_i(t)$ and $v_i(t)$ are bounded for all $t \in [0, T_1]$. Because from (3d) $||\dot{\hat{x}}_i(t)||_\infty \leq k_3$, it follows that $\hat{x}_i(t)$ is bounded for all $t \in [0, T_1]$, which implies that $x_i(t) - \hat{x}_i(t)$ is bounded for all $t \in [0, T_1]$. Because from (4) $\| \dot{z}_{1i}(t) \|_\infty \leq k_2 + \alpha$, it follows that $z_{1i}(t)$ is bounded for all $t \in [0, T_1]$. From (3b) we have that

$$\dot{\bar{z}}_{0i}(t) = z_{1i}(t) - [v_i(t) - \dot{\hat{x}}_i(t)] - k_1 \text{sig}[\bar{z}_{0i}(t)]^{\frac{1}{2}},$$

where $\bar{z}_{0i}(t) \triangleq z_{0i}(t) - [x_i(t) - \hat{x}_i(t)]$. Because $z_{1i}(t)$, $v_i(t)$ and $\dot{\hat{x}}_i(t)$ are bounded, we assume that $\| z_{1i}(t) - [v_i(t) - \dot{\hat{x}}_i(t)] \|_\infty < \gamma$ for all $t \in [0, T_1]$. Suppose that $| \bar{z}_{0il}(t_1) | > \frac{\gamma^2}{k_1^2}$ at a certain $t_1 \in [0, T_1]$, where $\bar{z}_{0il}(t)$ denotes the lth element of $\bar{z}_{0i}(t)$. If $\bar{z}_{0il}(t_1) > \frac{\gamma^2}{k_1^2}$, then it follows that

$$\begin{aligned} & \dot{\bar{z}}_{0il}(t_1) \\ = \ & z_{1il}(t_1) - [v_{il}(t_1) - \dot{\hat{x}}_{il}(t_1)] - k_1 \mid \bar{z}_{0il}(t_1) \mid^{\frac{1}{2}} \\ < \ & \gamma - k_1 \mid \bar{z}_{0il}(t_1) \mid^{\frac{1}{2}} < 0. \end{aligned}$$

If $\bar{z}_{0il}(t_1) < -\frac{\gamma^2}{k_1^2}$, then it follows that

$$\begin{aligned} & \dot{\bar{z}}_{0il}(t_1) \\ = \ & z_{1il}(t) - [v_{il}(t_1) - \dot{\hat{x}}_{il}(t_1)] + k_1 \mid \bar{z}_{0il}(t_1) \mid^{\frac{1}{2}} \\ > \ & -\gamma + k_1 \mid \bar{z}_{0il}(t_1) \mid^{\frac{1}{2}} > 0. \end{aligned}$$

Therefore, because $|\bar{z}_{0il}(0)|$ is bounded, $\bar{z}_{0il}(t)$ will not diverge to infinity for all $t \in [0, T_1]$, which implies that $z_{0il}(t)$ will not diverge to infinity for all $t \in [0, T_1]$. Thus $x_{di}(t)$ can be used to replace $\hat{x}_i(t)$ for $t \geq T_1$.

For $t \geq T_1$, because $\hat{x}_i(t) \equiv x_{di}(t)$, the closed-loop system of (1) using (3) becomes

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= -\alpha \text{sgn}\{z_{1i}(t) + \beta \text{sig}[\tilde{x}_i(t)]^{\frac{1}{2}}\}, \\ \dot{z}_{0i}(t) &= z_{1i}(t) - k_1 \text{sig}\left[z_{0i}(t) - \tilde{x}_i(t)\right]^{\frac{1}{2}}, \\ \dot{z}_{1i}(t) &= -k_2 \text{sgn}\left[z_{0i}(t) - \tilde{x}_i(t)\right] \\ &\quad -\alpha \text{sgn}\{z_{1i}(t) + \beta \text{sig}[\tilde{x}_i(t)]^{\frac{1}{2}}\}, \end{aligned}$$

where $\tilde{x}_i(t) \triangleq x_i(t) - x_{di}(t)$. It thus follows that for $t \geq T_1$

$$\begin{aligned} \dot{\tilde{z}}_{0i}(t) &= \tilde{z}_{1i}(t) - k_1 \text{sig}\left[\tilde{z}_{0i}(t)\right]^{\frac{1}{2}}, \\ \dot{\tilde{z}}_{1i}(t) &= -k_2 \text{sgn}\left[\tilde{z}_{0i}(t)\right] + \ddot{x}_{di}(t), \end{aligned}$$

where $\tilde{z}_{0i}(t) \triangleq z_{0i}(t) - \tilde{x}_i(t)$ and $\tilde{z}_{1i}(t) \triangleq z_{1i}(t) - \dot{\tilde{x}}_i(t)$. If there exists a symmetric positive definite matrix P such that (4) is satisfied, where $\epsilon = \eta_b$, it follows from Lemma 3.3 that there exists $T_2 > T_1$ such that $\tilde{z}_{0i}(t) = 0$ and $\tilde{z}_{1i}(t) = 0$ for all $t \geq T_2$, which implies that $z_{0i}(t) = \tilde{x}_i(t)$ and $z_{1i}(t) = \dot{\tilde{x}}_i(t)$ for all $t \geq T_2$. It follows from a similar statement as above that $x_i(t)$, $v_i(t)$, $z_{0i}(t)$, $z_{1i}(t)$ are all bounded for all $t \in [T_1, T_2]$. Thus $\dot{\tilde{x}}_i(t)$ can be used to replace $z_{1i}(t)$ for $t \geq T_2$.

For $t \geq T_2$, because $z_{1i}(t) \equiv \dot{\tilde{x}}_i(t)$, the closed-loop system of (1) using (3a) becomes

$$\ddot{\tilde{x}}_i(t) = -\alpha \text{sgn}\{\dot{\tilde{x}}_i(t) + \beta \text{sig}[\tilde{x}_i(t)]^{\frac{1}{2}}\} - \ddot{x}_{di}(t).$$

Because $\alpha > \eta_b + \frac{\beta^2}{2}$, it follows from Lemma 3.4 that there exists $T_3 > T_2$ such that $\tilde{x}_i(t) = 0$ and $\dot{\tilde{x}}_i(t) = 0$

for all $t \geq T_3$, which implies that $||x_i(t) - x_{di}(t)||$ and $||v_i(t) - \dot{x}_{di}(t)||$ will converge to zero in finite time. It follows from Lemma 3.2 that $\inf_{y(t)\in\Omega(t)} \|x_i(t) - y(t)\| \to 0$ and $\inf_{y(t)\in\Upsilon(t)} \|v_i(t) - y(t)\| \to 0$ in finite time. ■

Next we show how to choose the gains k_1 and k_2 in (3) such that there exists a symmetric positive-definite matrix P such that (4) is satisfied, where $\epsilon = \eta_b$.

Lemma 3.5: Given a constant $\varepsilon > 0$, there exists a symmetric-positive definite matrix P such that (4) is satisfied if $k_2 > \varepsilon$ and $k_2 + 2 - \sqrt{k_2^2 - \varepsilon^2} < k_1 < k_2 + 2 + \sqrt{k_2^2 - \varepsilon^2}$. ■

B. Coordinated Tracking Using Relative Position Measurements

In this section, we assume that the relative position measurements of the vehicles are available but the velocity measurements are not available. We also assume that $\|v_i(t)\|_\infty$, $\|f_i(t)\|_\infty$ and $\|\dot{f}_i(t)\|_\infty$, $i = n+1, \cdots, n+s$, are all bounded. We propose the following algorithm

$$\begin{aligned} u_i(t) &= -D_i(t)\text{sgn}\{\sum_{j=1}^{n+s} a_{ij}[x_i(t) - x_j(t)]\} \\ &\quad -k_1\{\sum_{j=1}^{n+s} a_{ij}[x_i(t) - x_j(t)]\} - k_2\hat{v}_i(t), \quad (5a) \\ \dot{\hat{v}}_i(t) &= -D_i(t)\text{sgn}\{\sum_{j=1}^{n+s} a_{ij}[x_i(t) - x_j(t)]\} \\ &\quad -k_1\{\sum_{j=1}^{n+s} a_{ij}[x_i(t) - x_j(t)]\} - k_2\hat{v}_i(t), \\ &\quad i = 1, \cdots, n, \quad (5b) \end{aligned}$$

where $\hat{v}_i(0) = 0$, $i = 1, \cdots, n$, $D_i(t) \triangleq \text{diag}[d_{i1}(t), \cdots, d_{im}(t)]$ with

$$\begin{aligned} d_{il}(t) &\triangleq |\sum_{j=1}^{n+s} a_{ij}[x_{il}(t) - x_{jl}(t)]| \\ &\quad + \int_0^t |\sum_{j=1}^{n+s} a_{ij}[x_{il}(\tau) - x_{jl}(\tau)]| \, d\tau, \\ &\quad l = 1, \cdots, m, \quad (6) \end{aligned}$$

$x_{il}(t)$ denotes the lth element of $x_i(t)$, and k_1 and k_2 are constant positive scalars.

Define $\psi_i(t) \triangleq \sum_{j=1}^{n+s} a_{ij}[x_i(t) - x_j(t)]$, $\phi_i(t) \triangleq \sum_{j=n+1}^{n+s} a_{ij}[k_2 v_j(t) + f_j(t)]$ and $\bar{v}_i(t) \triangleq \hat{v}_i(t) - v_i(t)$, $i = 1, \cdots, n$. Also define $\Psi(t) \triangleq [\ \psi_1^T(t), \ \cdots, \ \psi_n^T(t)\ ]^T$, $\Phi(t) \triangleq [\ \phi_1^T(t), \ \cdots, \ \phi_n^T(t)\ ]^T$ and $\bar{v}(t) \triangleq [\ \bar{v}_1^T(t), \ \cdots, \ \bar{v}_n^T(t)\ ]^T$. Because $\|v_i(t)\|_\infty$, $\|f_i(t)\|_\infty$ and $\|\dot{f}_i(t)\|_\infty$, $i = n+1, \cdots, n+s$, are all bounded, it is easy to see that $\Phi(t)$ and $\dot{\Phi}(t)$ are also bounded.

Lemma 3.6: Under Assumption 2.2, consider the function

$$\begin{aligned} V_1(t) &= V_2 + \int_0^t [\Psi(\tau) + \dot{\Psi}(\tau)]^T \\ &\quad \times \{k^*\text{sgn}[\Psi(\tau)] + (L_1^{-1} \otimes I_m)\Phi(\tau) + k_2\bar{v}(0)\}d\tau \end{aligned}$$

where k^* is a constant positive scalar and $V_2 \triangleq k^*\Psi^T(0)\text{sgn}[\Psi(0)] + \Psi^T(0)(L_1^{-1} \otimes I_m)\Phi(0) + k_2\Psi^T(0)\bar{v}(0)$. If

$$\begin{aligned} k^* &> \max\{\| (L_1^{-1} \otimes I_m)[\Phi(t) - \dot{\Phi}(t)] \|_\infty, \\ &\quad \| (L_1^{-1} \otimes I_m)\Phi(t) \|_\infty\} + k_2 \| \bar{v}(0) \|_\infty, \quad (7) \end{aligned}$$

then $V_1(t) \geq 0$.

Proof: See the appendix.

Theorem 3.2: Under Assumption 2.2, using (5) for (1), $\inf_{y(t)\in\Omega(t)} \|x_i(t) - y(t)\| \to 0$ and $\inf_{y(t)\in\Upsilon(t)} \|v_i(t) - y(t)\| \to 0$ as $t \to \infty$ if $k_1 > 0$ and $k_2 > 1$. In particular, $||x_i(t) - x_{di}(t)|| \to 0$ and $||v_i(t) - \dot{x}_{di}(t)|| \to 0$, $i = 1, \cdots, n$, as $t \to \infty$.

Proof: From (1b) and (5) we know that $\dot{v}_i(t) \equiv \dot{\hat{v}}_i(t)$, $i = 1, \cdots, n$, which implies that

$$\dot{\bar{v}}_i(t) \equiv 0, \quad i = 1, \cdots, n.$$

Therefore, we have that $\bar{v}_i(t) \equiv \bar{v}_i(0)$, $i = 1, \cdots, n$. Equation (5a) can be rewritten as

$$u_i(t) = -D_i(t)\text{sgn}[\psi_i(t)] - k_1\psi_i(t) - k_2\hat{v}_i(t), \quad i = 1, \cdots, n.$$

It follows that

$$\begin{aligned} &\ddot{\psi}_i(t) \\ &= \sum_{j=1}^{n+s} a_{ij}[\ddot{x}_i(t) - \ddot{x}_j(t)] \\ &= \sum_{j=1}^{n+s} a_{ij}u_i(t) - \sum_{j=1}^{n} a_{ij}u_j(t) - \sum_{j=n+1}^{n+s} a_{ij}f_j(t) \\ &= -\sum_{j=1}^{n+s} a_{ij}\{D_i(t)\text{sgn}[\psi_i(t)] + k_1\psi_i(t) + k_2\hat{v}_i(t)\} \\ &\quad + \sum_{j=1}^{n} a_{ij}\{D_j(t)\text{sgn}[\psi_j(t)] + k_1\psi_j(t) + k_2\hat{v}_j(t)\} \\ &\quad - \sum_{j=n+1}^{n+s} a_{ij}f_j(t) \\ &= -\sum_{j=1}^{n+s} a_{ij}\{D_i(t)\text{sgn}[\psi_i(t)] + k_1\psi_i(t)\} \\ &\quad -k_2\sum_{j=1}^{n+s} a_{ij}[v_i(t) + \bar{v}_i(0)] \\ &\quad + \sum_{j=1}^{n} a_{ij}\{D_j(t)\text{sgn}[\psi_j(t)] + k_1\psi_j(t)\} \\ &\quad + k_2\sum_{j=1}^{n+s} a_{ij}v_j(t) + k_2\sum_{j=1}^{n} a_{ij}\bar{v}_j(0) - \phi_i(t). \quad (8) \end{aligned}$$

Note that (8) can be rewritten in a vector form as

$$\begin{aligned} \ddot{\Psi}(t) &= -(L_1 \otimes I_m)D(t)\text{sgn}[\Psi(t)] - k_1(L_1 \otimes I_m)\Psi(t) \\ &\quad -k_2\dot{\Psi}(t) - k_2(L_1 \otimes I_m)\bar{v}(0) - \Phi(t), \quad (9) \end{aligned}$$

where $D(t)$ is a block diagonal matrix of all $D_i(t)$, $i = 1, \cdots, n$.

Consider the following Lyapunov function candidate

$$\begin{aligned} & V(t) \\ = \ & \frac{1}{2}[\Psi(t) + \dot{\Psi}(t)]^T (L_1^{-1} \otimes I_m)[\Psi(t) + \dot{\Psi}(t)] + V_1(t) \\ & + \frac{1}{2}\Psi^T(t)[k_1 I_{nm} + (k_2 - 1)(L_1^{-1} \otimes I_m)]\Psi(t) \\ & + \frac{1}{2}[D(t)\mathbf{1}_{nm} - k^*\mathbf{1}_{nm}]^T[D(t)\mathbf{1}_{nm} - k^*\mathbf{1}_{nm}], \end{aligned}$$

where k^* is a constant satisfying (7). Under Assumption 2.2, it follows from Lemma 3.1 that L_1 is symmetric positive definite, which means that L_1^{-1} is also symmetric positive definite. Because k^* satisfies (7), it follows from Lemma 3.6 that $V_1(t) \geq 0$. Because $k_1 > 0$ and $k_2 > 1$, we have that $k_1 I_{nm} + (k_2 - 1)(L_1^{-1} \otimes I_m)$ is symmetric positive definite. Therefore, $V(t)$ is symmetric positive definite with respect to $\Psi(t)$, $\dot{\Psi}(t)$ and $D(t)\mathbf{1}_{nm} - k^*\mathbf{1}_{nm}$.

From (6) we have that

$$\begin{aligned} \dot{d}_{il}(t) \ = \ & \{\sum_{j=1}^{n+s} a_{ij}[v_{il}(t) - v_{jl}(t)]\} \\ & \times \mathrm{sgn}\{\sum_{j=1}^{n+s} a_{ij}[x_{il}(t) - x_{jl}(t)]\} \\ & + \{\sum_{j=1}^{n+s} a_{ij}[x_{il}(t) - x_{jl}(t)]\} \\ & \times \mathrm{sgn}\{\sum_{j=1}^{n+s} a_{ij}[x_{il}(t) - x_{jl}(t)]\}, \\ & i = 1, \cdots, n, \quad l = 1, \cdots, m. \end{aligned}$$

It follows that

$$\begin{aligned} & [D(t)\mathbf{1}_{nm} - k^*\mathbf{1}_{nm}]^T \dot{D}(t)\mathbf{1}_{nm} \\ = \ & \sum_{i=1}^{n}\sum_{l=1}^{m}[d_{il}(t) - k^*]\dot{d}_{il}(t) \\ = \ & \sum_{i=1}^{n}\sum_{l=1}^{m}[d_{il}(t) - k^*]\left(\{\sum_{j=1}^{n+s} a_{ij}[x_{il}(t) - x_{jl}(t)]\}\right. \\ & \left. + \{\sum_{j=1}^{n+s} a_{ij}[v_{il}(t) - v_{jl}(t)]\}\right) \\ & \times \mathrm{sgn}\{\sum_{j=1}^{n+s} a_{ij}[x_{il}(t) - x_{jl}(t)]\} \\ = \ & [\Psi(t) + \dot{\Psi}(t)]^T[D(t) - k^* I_{nm}]\mathrm{sgn}[\Psi(t)]. \end{aligned}$$

Taking the derivative of $V(t)$, we have that

$$\begin{aligned} & \dot{V}(t) \\ = \ & [\Psi(t) + \dot{\Psi}(t)]^T (L_1^{-1} \otimes I_m)[\dot{\Psi}(t) + \ddot{\Psi}(t)] + \dot{V}_1(t) \\ + \ & \Psi^T(t)[k_1 I_{nm} + (k_2 - 1)(L_1^{-1} \otimes I_m)]\dot{\Psi}(t) \\ + \ & [D(t)\mathbf{1}_{nm} - k^*\mathbf{1}_{nm}]^T \dot{D}(t)\mathbf{1}_{nm} \\ = \ & [\Psi(t) + \dot{\Psi}(t)]^T (L_1^{-1} \otimes I_m)\{-(L_1 \otimes I_m)D(t) \\ \times \ & \mathrm{sgn}[\Psi(t)] - k_1(L_1 \otimes I_m)\Psi(t) - (k_2 - 1)\dot{\Psi}(t) \\ - \ & k_2(L_1 \otimes I_m)\bar{v}(0) - \Phi(t)\} \\ + \ & [\Psi(t) + \dot{\Psi}(t)]^T\{k^*\mathrm{sgn}[\Psi(t)] + (L_1^{-1} \otimes I_m)\Phi(t) \\ + \ & k_2\bar{v}(0)\} + \Psi^T(t)[k_1 I_{nm} + (k_2 - 1)(L_1^{-1} \otimes I_m)]\dot{\Psi}(t) \\ + \ & [\Psi(t) + \dot{\Psi}(t)]^T[D(t) - k^* I_{nm}]\mathrm{sgn}[\Psi(t)] \\ = \ & -k_1\Psi(t)^T\Psi(t) - (k_2 - 1)\dot{\Psi}(t)^T(L_1^{-1} \otimes I_m)\dot{\Psi}(t). \end{aligned} \tag{10}$$

Because $k_1 > 0$ and $k_2 > 1$, we have that $\dot{V}(t)$ is negative semi-definite. It follows that $V(t)$ is bounded, which implies that $\Psi(t)$, $\dot{\Psi}(t)$ and $D(t)$ are all bounded. Because $\bar{v}(0)$ and $\Phi(t)$ are also bounded, it follows from (9) that $\ddot{\Psi}(t)$ is bounded. From (10) we have that

$$\begin{aligned} \ddot{V}(t) \ = \ & -2k_1\Psi(t)^T\dot{\Psi}(t) \\ & -2(k_2 - 1)\dot{\Psi}(t)^T(L_1^{-1} \otimes I_m)\ddot{\Psi}(t). \end{aligned} \tag{11}$$

Therefore, $\ddot{V}(t)$ is bounded. By Barbalatt's Lemma we have that $\dot{V}(t) \to 0$ as $t \to \infty$, which implies that $\Psi(t) \to 0$ and $\dot{\Psi}(t) \to 0$ as $t \to \infty$. Therefore, we have that $(L_1 \otimes I_m)x_F(t) + (L_2 \otimes I_m)x_L(t) \to 0$ and $(L_1 \otimes I_m)v_F(t) + (L_2 \otimes I_m)v_L(t) \to 0$ as $t \to \infty$. It follows that $||x_F(t) - x_d(t)|| \to 0$ and $||v_F(t) - \dot{x}_d(t)|| \to 0$ as $t \to \infty$. It follows from Lemma 3.2 that $\inf_{y(t)\in\Omega(t)} \|x_i(t) - y(t)\| \to 0$ and $\inf_{y(t)\in\Upsilon(t)} \|v_i(t) - y(t)\| \to 0$ as $t \to \infty$. ■

IV. CONCLUSIONS

In this paper, the coordinated tracking problem has been investigated for multiple autonomous vehicles with double-integrator dynamics in the presence of multiple dynamic leaders. Two distributed tracking algorithms have been derived under different constraints. Different from the related results in the literature, the proposed algorithms use only the position measurements of the leaders and the followers. Therefore, they can be realized more easily. Future work will find algorithms that ensure collision avoidance between adjacent vehicles.

REFERENCES

[1] A. Jadbabaie, J. Lin, and A. S. Morse, "Coordination of groups of mobile autonomous agents using nearest neighbor rules," *IEEE Transactions on Automatic Control*, vol. 48, no. 6, pp. 988-1001, 2003.

[2] R. Olfati-Saber and R. M. Murray, "Consensus problems in networks of agents with switching topology and time-delays," *IEEE Transactions on Automatic Control*, vol. 49, no. 9, pp. 1520-1533, 2004.

[3] W. Ren and R. W. Beard, "Consensus seeking in multiagent systems under dynamically changing interaction topologies," *IEEE Transactions on Automatic Control*, vol. 50, no. 5, pp. 655-661, 2005.

[4] M. Cao, A. S. Morse, and B. D. O. Anderson, "Agreeing asynchronously," *IEEE Transactions on Automatic Control*, vol. 53, no. 8, pp. 1826-1838, 2008.

[5] L. Moreau, “Stability of multi-agent systems with time-dependent communication links,” *IEEE Transactions on Automatic Control*, vol. 50, no. 2, pp. 169-182, 2005.

[6] H. Su, X. Wang, and G. Chen, “Rendezvous of multiple mobile agents with preserved network connectivity,” *Systems and Control Letters*, vol. 52, no. 5, pp. 313-322, 2010.

[7] Y. Hong, G. Chen, and L. Bushnell, “Distributed observers design for leader-following control of multi-agent networks,” *Automatica*, vol. 44, no. 3, pp. 846-850, 2008.

[8] W. Ren, “Multi-vehicle consensus with a time-varying reference state,” *Systems and Control Letters*, vol. 56, no. 7-8, pp. 474-483, 2007

[9] Y. Cao, W. Ren, and Y. Li, “Distributed discrete-time coordinated tracking with a time-varying reference state and limited communication,” *Automatica*, vol. 45, no. 5, pp. 1299-1305, 2009.

[10] H. Su, X. Wang, and Z. Lin, “Flocking of multi-agents with a virtual leader,” *IEEE Transactions on Automatic Control*, vol. 54, no. 2, pp. 293-307, 2009.

[11] K. Peng and Y. Yang, “Leader-following consensus problem with a varying-velocity leader and time-varying delays,” *Physica A*, vol. 388, no. 2-3, pp. 193-208, 2009.

[12] Y. Cao and W. Ren, “Distributed coordinated tracking via a variable structure approach-part I: consensus tracking,” in *Proceedings of the American Control Conference*, Baltimore, MD, June-July 2010, pp. 4744-4749.

[13] Y. Cao, W. Ren, and Z. Meng, “Decentralized finite-time sliding mode estimators and their applications in decentralized finite-time formation tracking,” *Systems and Control Letters*, vol. 59, no. 9, pp. 522-529, 2010.

[14] M. Ji, G. Ferrari-Trecate, M. Egerstedt, and A. Buffa, “Containment control in mobile networks,” *IEEE Transactions on Automatic Control*, vol. 53, no. 8, pp. 1972-1975, 2008.

[15] Y. Cao and W. Ren, “Containment control with multiple stationary or dynamic leaders under a directed interaction graph,” in *Proceedings of the IEEE Conference on Decision and Control*, Shanghai, China, December 2009, pp. 3014-3019.

[16] Y. Cao, D. Stuart, W. Ren, and Z. Meng, “Distributed containment control for double-integrator dynamics: Algorithms and experiments,” in *Proceedings of the American Control Conference*, Baltimore, MD, June-July 2010, pp. 3830-3835.

[17] Z. Meng, W. Ren, and Z. You, “Distributed finite-time containment control for multiple Lagrangian systems,” in *Proceedings of the American Control Conference*, Baltimore, MD, June-July 2010, pp. 2885-2890.

[18] Y. Su, P. C. Mller, and C. Zheng, “A simple nonlinear observer for a class of uncertain mechanical systems,” *IEEE Transactions on Automatic Control*, vol. 52, no. 7, pp. 1340-1345, 2007.

[19] A. Dávila, J. A. Moreno, and L. Fridman, “Optimal Lyapunov funtion selection for reaching time estimation of super twisting algorithm,” in *Proceedings of the IEEE Conference on Decision and Control*, Shanghai, China, December 2009, pp. 8405-8410.

[20] A. Levant, “Principles of 2-sliding mode design,” Automatica, vol. 43, no. 4, pp. 576-586, 2007.

Appendix *Proof:* The proof is motivated by the proof of Lemma 1 in [18]. Because $k^* > \| (L_1^{-1}\otimes I_m)\Phi(t) \|_\infty + k_2\|\bar{v}(0)\|_\infty$, we have that

$$\begin{aligned} & \int_0^t \dot{\Psi}^T(\tau)\{k^*\mathrm{sgn}[\Psi(\tau)] + (L_1^{-1}\otimes I_m)\Phi(\tau) \\ & +k_2\bar{v}(0)\}d\tau \\ = & \int_0^t \{(L_1^{-1}\otimes I_m)\Phi(\tau) + k_2\bar{v}(0)\}d\Psi(\tau) \\ & + \int_0^t k^*\dot{\Psi}^T(\tau)\mathrm{sgn}[\Psi(\tau)]d\tau \\ = & \Psi^T(\tau)\{k^*\mathrm{sgn}[\Psi(\tau)] + (L_1^{-1}\otimes I_m)\Phi(\tau) + k_2\bar{v}(0)\}|_0^t \\ & - \int_0^t \Psi(\tau)d\{(L_1^{-1}\otimes I_m)\Phi(\tau) + k_2\bar{v}(0)\} \\ = & \Psi^T(t)\{k^*\mathrm{sgn}[\Psi(t)] + (L_1^{-1}\otimes I_m)\Phi(t) + k_2\bar{v}(0)\} \\ & -V_2 - \int_0^t \Psi^T(\tau)(L_1^{-1}\otimes I_m)\dot{\Phi}(\tau)d\tau \\ \geq & -V_2 - \int_0^t \Psi^T(\tau)(L_1^{-1}\otimes I_m)\dot{\Phi}(\tau)d\tau, \end{aligned} \tag{12}$$

where we have used the fact that

$$\begin{aligned} & \Psi^T(t)\{k^*\mathrm{sgn}[\Psi(t)] + (L_1^{-1}\otimes I_m)\Phi(t) + k_2\bar{v}(0)\} \\ \geq & k^*\Psi^T(t)\mathrm{sgn}[\Psi(t)] - [\| (L_1^{-1}\otimes I_m)\Phi(t) \|_\infty \\ & +k_2\|\bar{v}(0)\|_\infty]\Psi^T(t)\mathrm{sgn}[\Psi(t)] \\ \geq & 0. \end{aligned} \tag{13}$$

Because $k^* > \| (L_1^{-1}\otimes I_m)[\Phi(t) - \dot{\Phi}(t)] \|_\infty + k_2\|\bar{v}(0)\|_\infty$, it then follows that

$$\begin{aligned} V_1(t) \geq & V_2 + \int_0^t \Psi^T(\tau)\{k^*\mathrm{sgn}[\Psi(\tau)] \\ & +(L_1^{-1}\otimes I_m)\Phi(\tau) + k_2\bar{v}(0)\}d\tau \\ & -V_2 - \int_0^t \Psi^T(\tau)(L_1^{-1}\otimes I_m)\dot{\Phi}(\tau)d\tau \\ = & \int_0^t \Psi^T(\tau)\{k^*\mathrm{sgn}[\Psi(\tau)] \\ & +(L_1^{-1}\otimes I_m)[\Phi(\tau) - \dot{\Phi}(\tau)] + k_2\bar{v}(0)\}d\tau \\ \geq & 0, \end{aligned} \tag{14}$$

where we have again used the fact that

$$\begin{aligned} & \Psi^T(t)\{k^*\mathrm{sgn}[\Psi(t)] + (L_1^{-1}\otimes I_m)[\Phi(t) - \dot{\Phi}(t)] \\ & +k_2\bar{v}(0)\} \\ \geq & k^*\Psi^T(t)\mathrm{sgn}[\Psi(t)] - \{\| (L_1^{-1}\otimes I_m)[\Phi(t) - \dot{\Phi}(t)] \|_\infty \\ & +k_2\|\bar{v}(0)\|_\infty\}\Psi^T(t)\mathrm{sgn}[\Psi(t)] \\ \geq & 0. \end{aligned} \tag{15}$$