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Adaptive Output Consensus Tracking of Uncertain Multi-agent Systems

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Abstract— **In this paper, we consider the adaptive output consensus tracking of a class of higher-order parametric strict-feedback systems with mismatched uncertainties. The consensus reference is taken as a virtual leader, whose output is available to the leader of the group. The proposed approach is based on the estimation of consensus reference and decentralized tracking control, where each agent communicates with their neighbors, estimates the consensus reference, and make a control decision based on the estimated consensus reference. Challenges of the approach include the design of decentralized adaptive tracking control assuming each agent has knowledge of the consensus reference in the presence of mismatched uncertainties, and the design of estimators that allow agents to make correct estimations of the consensus reference. Sufficient conditions are given to ensure output consensus tracking is achieved. Simulation results show satisfactory performances.**

I. INTRODUCTION

Increasing attention has been paid recently to cooperative control of multi-agent systems due to its numerous potential applications in space-based interferometers, combat, surveillance, reconnaissance systems, hazardous material handling, and distributed reconfigurable sensor networks. Consensus, to achieve an agreement on certain quantities of interest, is a critical problem in cooperative control of multi-agent systems. Based on eigenvalue analysis, the consensus problem was studied in [5], [17], [15], [11], [19], [13]. The passivity-based framework in [1] provides an explicit way of finding Lyapunov functions on undirected communication graphs. These results are based on the fact that the consensus equilibrium is a weighted average or a weighted power mean of the initial conditions of all agents' states. Consensus with a constant reference is studied in [7] with undirected switching inter-vehicle communications, and in [10], [8] under a directed fixed interaction topology. Consensus algorithm with a time-varying reference is proposed in [6] with a variable undirected interaction topology. In [14], taking consensus reference as a virtual leader, consensus tracking algorithms are proposed to track a time-varying consensus reference with a directed topology. The dynamics of agents considered in these work are single-integrators or double-integrators. In [18], the consensus problem of multi-agent systems with higher-order dynamics is studied. However, the same linear model applies to each agent. In [16], the authors study lth-order ($l \leq 3$) consensus algorithms, present the idea of higher-order consensus with a leader, and introduce the concept of an lth-order model-reference consensus problem.

In practice, consensus may face uncertainties or disturbance from the model, communication and measurement. The uncertainties or disturbances may destroy consensus of multi-agent systems. In [9], a model transformation is used to transform the original system into a reduced-order system so that a sufficient condition can be obtained for all agents to reach consensus with a desired H_∞ performance. Matched uncertainties are considered therein. In [2], robust redesign is proposed for the consensus problem with undirected communication topology in the presence of matched uncertainties. The consensus of multi-agent systems with mismatched uncertainties remains an open problem.

In this paper, we consider the adaptive output consensus tracking of a class of high-order parametric strict-feedback systems with mismatched uncertainties. The consensus reference is taken as a virtual leader, whose output is available to the leader of the group. The proposed approach is based on an estimation of consensus reference and decentralized tracking control, where each agent communicates with their neighbors, estimates the consensus reference, and make a control decision based on the estimated consensus reference. Challenges of the approach include the design of decentralized adaptive tracking control assuming each agent has knowledge of the consensus reference in the presence of mismatched uncertainties, and the design of estimators that allow agents to make correct estimations of the consensus reference. Sufficient conditions are given to ensure output consensus tracking is achieved. Simulation results show satisfactory performances.

The subsequent sections are organized as follows: Section II introduces the related graph theory preliminaries. In Section III, the statement of the output consensus tracking problem is provided. In Section IV, main results are given on adaptive output consensus tracking. Section V shows the simulation results. In Section VI, we present the conclusions.

II. PRELIMINARIES AND NOTATIONS

A directed graph $\mathcal{G}$ consists of a triple $(\mathcal{V}, \mathcal{E}, \mathcal{A})$, where $\mathcal{V}$ is a finite nonempty set of nodes, $\mathcal{E} \in \mathcal{V}^2$ is a set of ordered pairs of nodes defined as edges, and a weighted adjacency matrix $\mathcal{A} = [a_{ij}]$ with nonnegative adjacency elements a_{ij}. The node indexes belong to a finite index set $\mathcal{I} = \{1, 2, ..., n\}$. An edge of $\mathcal{G}$ is denoted by $e_{ij} = (v_i, v_j)$, with the weight a_{ij}, that is $e_{ij} \in \mathcal{E} \Longleftrightarrow a_{ij} > 0$, and we assume $a_{ii} = 0$ and $a_{ij} = 1, i \neq j$ for all $i, j \in \mathcal{I}$. The set of neighbors of node v_i is denoted by $N_i = \{v_i \in \mathcal{V} : (v_i, v_j) \in \mathcal{E}\}$.

A directed path in a digraph is a sequence of edges as $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots, (v_{i_m}, v_{i_{m+1}})$, where $v_{i_j} \in \mathcal{V}$ and $e_{i_j i_{j+1}} \in \mathcal{E}, j = 1, \ldots, m$. A directed graph has a directed

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spanning tree if there exists at least one node that all the other node could reach it following directed path directions.

The *graph Laplacian* associated with the graph $\mathcal{G}$ is defined as

$$\mathcal{L}(\mathcal{G}) = L = \Delta - \mathcal{A} \tag{1}$$

The diagonal matrix $\Delta = [\Delta_{ij}]$ where $\Delta_{ij} = 0$ for all $i \neq j$ and $\Delta_{ii} = \deg_{\text{out}}(v_i)$. Since every row sum is zero, the Laplacian matrix always has a zero eigenvalue with the right eigenvector of one. We denote it as

$$\lambda_1 = 0, \quad w_r = \mathbf{1} = (1, 1, \ldots, 1)^T \tag{2}$$

Lemma 1: If a directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ has a spanning tree and with a Laplacian matrix L, there exists a non-singular matrix M such that $L = M^{-1}JM$, where J is the Jordan block with $J = \text{diag}\{J_1, 0\}$ where $-J_1$ is a $(n-1) \times (n-1)$ Hurwitz matrix.

Proof: Since the directed graph $\mathcal{G} = (\mathcal{V}, \mathcal{E}, \mathcal{A})$ has a spanning tree, one of eigenvalues of L is zero and others are greater than zero. The Jordan Canonical Form Theorem ([12]) guarantees that there exists a non-singular matrix M such that $L = M^{-1}JM$, where J has the form $J = \text{diag}\{J_1, 0\}$. ■

The following notations are used in this paper: For a function $f(x, y)$, we denote the notation $f|_{y=0}$ as $f(x, 0)$. For a matrix $A \in R^{n \times n}$, we denote $A > 0$ as a positive-definite matrix.

III. Problem Formulation

We study the agents that are described as follows:

$$\begin{aligned}
\dot{x}_{i1} &= \varphi_{i1}^T(x_{i1}, t)\,\theta_i + x_{i2} + \psi_{i1}(x_{i1}), \\
\dot{x}_{i2} &= \varphi_{i2}^T(x_{i1}, x_{i2}t)\,\theta_i + x_{i3} + \psi_{i2}(x_{i1}, x_{i2}), \\
&\vdots \\
\dot{x}_{ir} &= \varphi_{ir}^T(x_{i1}, \cdots, x_{ir}, t)\,\theta_i + b_i u_i + \psi_{ir}(x_{i1}, \cdots, x_{ir}),
\end{aligned} \tag{3}$$

where $x_i = [x_{i1}, \cdots, x_{ir}]^T \in R^r$, the vector $\theta_i \in R^p$ is constant and unknown, $\varphi_i^j \in R^p$, ψ_{ij}, $j = 1, \cdots, r$, are known nonlinear functions, the high frequency gain b_i is an unknown constant. The system output is $y_i = x_{i1}$.

Assumption 1: The sign of b_i, $i = 1, \cdots, N$ are known and the nonlinear functions φ_i^j, $j = 1, \cdots, r$, $i = 1, \cdots, N$ are of class $\mathcal{C}^{r-1}$.

We study N agents described by (3). Each agent is considered as a node and the link between two nodes is seen as an edge. In this way, N agents constitutes a directed graph $\mathcal{G}$. We make the following assumption about this directed graph:

Assumption 2: The communication graph contains a spanning tree.

Assumption 3: Suppose that the consensus reference, denoted by ξ_d, satisfies

$$\begin{aligned}
\dot{\xi}_d &= f_0(t, \xi_d) \\
y_r &= \xi_d
\end{aligned} \tag{4}$$

where $f_0(\cdot, \cdot)$ is in $\mathcal{C}^{r-1}$ and $\frac{d^{r-1} f_0}{dt}(t, \xi_d(t))$ is piecewise continuous and bounded. The output of reference is available to at least one root agent of the spanning tree of the directed graph $\mathcal{G}$.

Adaptive Output Consensus Tracking problem: Under Assumptions 1, 2 and 3, design distributed control laws, $u_i(x_i, x_j)$, $j \in \mathcal{N}_i$, $i = 1, \ldots, N$, such that the outputs y_i of all agents converge to the time-varying reference y_r.

IV. Main Results

We design an estimator for each agent:

$$\begin{aligned}
\dot{\xi}_{i1} &= \xi_{i2}, \\
&\vdots \\
\dot{\xi}_{ir} &= v_i,
\end{aligned} \tag{5}$$

where the output is $y_{oi} = \xi_{i1}$, which acts as an output reference of the ith agent, and v_i is a distributed control to be chosen later, which uses its neighbors' states x_j, where $j \in \mathcal{N}_i$, for $i = 1, 2, \cdots, N$. We are facing the following challenges:

1) The design of the decentralized control u_i so that we have $y_i \to y_{oi}$, as $t \to \infty$ for $i = 1, 2, \cdots, N$;
2) The design of the distributed control $v_i(x_i, x_j)$ where $j \in \mathcal{N}_i$ so that we have $y_{oi} \to y_r$ as $t \to \infty$ for $i = 1, 2, \cdots, N$.

A. The Design of Adaptive Decentralized Control

The backstepping technique is applied for the design of the adaptive decentralized control. We introduce the change of coordinates

$$\begin{aligned}
\eta_{i1} &= x_{i1} - \xi_{i1} \\
\eta_{ij} &= x_{ij} - \alpha_{i,j-1} - \xi_{ij}, \quad j = 2, 3, \cdots, r
\end{aligned} \tag{6}$$

where α_{ij} are virtual controllers. Denote that $\eta_i = [\eta_{i1}, \cdots, \eta_{ir}]^T$. The iterative design procedure is described as follows:

Step 1. The virtual control α_{i1} is chosen as

$$\alpha_{i1} = -k_{i1}\eta_{i1} - \varphi_{i1}^T\hat{\theta}_i - \psi_{i1}, \tag{7}$$

where k_{i1} is a positive constant and $\hat{\theta}_i$ is the estimate of unknown parameters vector θ_i that will be chosen later. We consider the Lyapunov function candidate

$$V_{i1} = \frac{1}{2}\eta_{i1}^2 + \frac{1}{2}\tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i, \tag{8}$$

where Γ_i is a positive definite matrix and $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$. Taking derivative of Lyapunov function candidate (8) gives

$$\dot{V}_{i1} = -k_{i1}\eta_{i1}^2 + \eta_{i1}\eta_{i2} + \tilde{\theta}_i^T\left(\tau_{i1} - \Gamma_i^{-1}\dot{\hat{\theta}}_i\right), \tag{9}$$

where $\tau_{i1} = \varphi_{i1}\eta_{i1}$ is the first tuning function.

Step 2. The following Lyapunov function candidate is chosen as:

$$V_{i2} = V_{i1} + \tfrac{1}{2}\eta_{i2}^2. \tag{10}$$

We choose the virtual control

$$\begin{aligned}
\alpha_{i2} &= -k_{i2}\eta_{i2} - \eta_{i1} - \psi_{i2} + \tfrac{\partial\alpha_{i1}}{\partial x_{i1}}(x_{i2} + \psi_{i1}) \\
&\quad -\hat{\theta}_i^T\left(\varphi_{i2} - \tfrac{\partial\alpha_{i1}}{\partial x_{i1}}\varphi_{i1}\right) + \tfrac{\partial\alpha_{i1}}{\partial\hat{\theta}_i}\Gamma_i\tau_{i2} + \tfrac{\partial\alpha_{i1}}{\partial\xi_{i1}}\xi_{i2},
\end{aligned} \tag{11}$$

where $\tau_{i2} = \tau_{i1} + \left(\varphi_{i2} - \frac{\partial \alpha_{i1}}{\partial x_{i1}}\varphi_{i1}\right)\eta_{i2}$, k_{i2} is a positive constant and τ_{i2} is the second tuning function. Taking derivative of the Lyapunov function candidate (10) gives

$$\begin{aligned}\dot{V}_{i2} &= -k_{i1}\eta_{i1}^2 - k_{i2}\eta_{i2}^2 + \eta_{i2}\eta_{i3} \\ &\quad + \eta_{i2}\frac{\partial \alpha_{i1}}{\partial \hat{\theta}_i}\left(\Gamma_i\tau_{i2} - \dot{\hat{\theta}}_i\right) + \tilde{\theta}_i^T\left(\tau_{i2} - \Gamma_i^{-1}\dot{\hat{\theta}}_i\right).\end{aligned} \tag{12}$$

Step 3. We choose the virtual control α_{i3} as follows

$$\begin{aligned}\alpha_{i3} &= -k_{i3}\eta_{i3} - \eta_{i2} - \psi_{i3} \\ &\quad + \frac{\partial \alpha_{i2}}{\partial x_{i1}}\left(x_{i2} + \psi_{i1}\right) + \frac{\partial \alpha_{i2}}{\partial x_{i2}}\left(x_{i3} + \psi_{i2}\right) \\ &\quad + \left(\frac{\partial \alpha_{i1}}{\partial \hat{\theta}_i}\Gamma_i\eta_{i2} - \hat{\theta}_i^T\right)\left(\varphi_{i3} - \frac{\partial \alpha_{i2}}{\partial x_{i1}}\varphi_{i1} - \frac{\partial \alpha_{i2}}{\partial x_{i2}}\varphi_{i2}\right) \\ &\quad + \frac{\partial \alpha_{i2}}{\partial \hat{\theta}_i}\Gamma_i\tau_{i3} + \frac{\partial \alpha_{i2}}{\partial \xi_{i1}}\xi_{i2} + \frac{\partial \alpha_{i2}}{\partial \xi_{i2}}\xi_{i3},\end{aligned} \tag{13}$$

where $\tau_{i3} = \tau_{i2} + \left(\varphi_{i3} - \frac{\partial \alpha_{i2}}{\partial x_{i1}}\varphi_{i1} - \frac{\partial \alpha_{i2}}{\partial x_{i2}}\varphi_{i2}\right)\eta_{i3}$, k_{i3} is a positive constant and τ_{i2} is the third tuning function. The Lyapunov function candidate is chosen as

$$V_{i3} = V_{i2} + \tfrac{1}{2}\eta_{i3}^2. \tag{14}$$

Taking derivative of Lyapunov function candidate (14) gives

$$\begin{aligned}\dot{V}_{i3} &= -\sum_{j=1}^{3} k_{ij}\eta_{ij}^2 + \eta_{i3}\eta_{i4} + \tilde{\theta}_i^T\left(\tau_{i3} - \Gamma_i^{-1}\dot{\hat{\theta}}_i\right) \\ &\quad + \left(\eta_{i2}\frac{\partial \alpha_{i1}}{\partial \hat{\theta}_i} + \eta_{i3}\frac{\partial \alpha_{i2}}{\partial \hat{\theta}_i}\right)\left(\Gamma_i\tau_{i3} - \dot{\hat{\theta}}_i\right).\end{aligned} \tag{15}$$

Step j $j = 4, \cdots, r-1$: Repeating the procedure in a recursive manner, we choose the virtual control α_{ij} as follows

$$\begin{aligned}\alpha_{ij} &= -k_{ij}\eta_{ij} - \eta_{i,j-1} - \psi_{ij} + \sum_{l=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial x_{il}}\left(x_{i,l+1} + \psi_{il}\right) \\ &\quad + \left(\sum_{l=2}^{j-1}\eta_{il}\frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_i}\right)\Gamma_i\left(\varphi_{ij} - \sum_{l=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial x_{il}}\varphi_{il}\right) \\ &\quad - \hat{\theta}_i^T\left(\varphi_{ij} - \sum_{l=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial x_{il}}\varphi_{il}\right) \\ &\quad + \frac{\partial \alpha_{i,j-1}}{\partial \hat{\theta}_i}\Gamma_i\tau_{ij} + \sum_{l=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial \xi_{il}}\xi_{i,l+1},\end{aligned} \tag{16}$$

where $\tau_{ij} = \tau_{i,j-1} + \left(\varphi_{ij} - \sum_{l=1}^{j-1}\frac{\partial \alpha_{i,j-1}}{\partial x_{il}}\varphi_{il}\right)\eta_{ij}$, k_{ij} is a positive constant and τ_{ij} is the jth tuning function. The Lyapunov function candidate is chosen as

$$V_{ij} = V_{i,j-1} + \tfrac{1}{2}\eta_{ij}^2. \tag{17}$$

Taking time derivative of Lyapunov function candidate (17) gives

$$\begin{aligned}\dot{V}_{ij} &= -\sum_{l=1}^{j} k_{il}\eta_{il}^2 + \eta_{ij}\eta_{i,j+1} + \tilde{\theta}_i^T\left(\tau_{ij} - \Gamma_i^{-1}\dot{\hat{\theta}}_i\right) \\ &\quad + \left(\sum_{l=2}^{j}\eta_{il}\frac{\partial \alpha_{il}}{\partial \hat{\theta}_i}\right)\left(\Gamma_i\tau_{ij} - \dot{\hat{\theta}}_i\right).\end{aligned} \tag{18}$$

In the last step r, the actual control input u appears. We design control u_i and update laws $\dot{\hat{\theta}}_i$ and $\hat{p}_i$ as

$$u_i = \hat{p}_i\bar{u}_i \tag{19}$$
$$\bar{u}_i = \alpha_{ir} + v_i \tag{20}$$
$$\dot{\hat{\theta}}_i = \Gamma_i\tau_{ir} \tag{21}$$
$$\dot{\hat{p}}_i = -\gamma_i \operatorname{sign}(b_i)\,\bar{u}_i\eta_{ir}, \tag{22}$$

where

$$\begin{aligned}\alpha_{ir} &= -k_{ir}\eta_{ir} - \eta_{i,r-1} - \psi_{ir} + \sum_{l=1}^{r-1}\frac{\partial \alpha_{i,r-1}}{\partial x_{il}}\left(x_{i,l+1} + \psi_{il}\right) \\ &\quad + \left(\sum_{l=2}^{r-1}\eta_{il}\frac{\partial \alpha_{i,l-1}}{\partial \hat{\theta}_i}\right)\Gamma_i\left(\varphi_{ir} - \sum_{l=1}^{r-1}\frac{\partial \alpha_{i,r-1}}{\partial x_{il}}\varphi_{il}\right) \\ &\quad - \hat{\theta}_i^T\left(\varphi_{ir} - \sum_{l=1}^{r-1}\frac{\partial \alpha_{i,r-1}}{\partial x_{il}}\varphi_{il}\right) \\ &\quad + \frac{\partial \alpha_{i,r-1}}{\partial \hat{\theta}_i}\Gamma_i\tau_{ir} + \sum_{l=1}^{r-1}\frac{\partial \alpha_{i,r-1}}{\partial \xi_{il}}\xi_{i,l+1},\end{aligned}$$

γ_i is a positive constant, $\hat{p}_i$ is an estimate of $p = 1/b_i$, and v_i is to be determined. Note that

$$b_iu_i = b_i\hat{p}_i\bar{u}_i = \bar{u}_i - b_i\tilde{p}_i\bar{u}_i, \tag{23}$$

where $\tilde{p}_i = p_i - \hat{p}_i$. We choose the Lyapunov function

$$V_i = \sum_{l=1}^{r} k_{il}\frac{1}{2}\eta_{il}^2 + \frac{1}{2}\tilde{\theta}_i^T\Gamma_i^{-1}\tilde{\theta}_i + \frac{|b_i|}{2\gamma_i}\tilde{p}_i^2. \tag{24}$$

Taking time derivative of Lyapunov function (24) gives

$$\dot{V}_i \leq -\sum_{l=1}^{r} k_{il}\eta_{il}^2. \tag{25}$$

Thus, we obtain the following Lemma:

Lemma 2: Under Assumptions 1 and 2, the adaptive decentralized control law (19) with parameters update laws (21) and (22) ensures the global boundedness of η_i, $\hat{\theta}_i$, and $\hat{p}_i$, and asymptotic tracking $\lim_{t\to\infty}(y_i - y_i^o) = 0$.

Proof: From the Lasalle's Theorem, the fact that $\dot{V}_i \leq 0$ shows the proof of uniform stability, such that $\eta_{i1}, \cdots, \eta_{ir}$, $\hat{\theta}_i$, and $\hat{p}_i$ are bounded and $\eta_{ij} \to 0$, $j = 1, \cdots, r$. This further implies that $\lim_{t\to\infty}(y_i - y_{oi}) = 0$. ■

B. Distributed Output Consensus Tracking Control

Based on Assumption 2, the directed graph has a spanning tree. The corresponding Laplacian matrix is L and the set of neighbors of ith agent is $\mathcal{N}_i$. From Assumption 3, the consensus reference is available to at least one root agent of the spanning tree. We introduce the consensus reference (4) as a virtual leader of the group estimators with the output y_{or}. We name the virtual leader as the $(N+1)$th agent without loss of generality. Prim's algorithm ([4]) can be applied to find all root agents to a spanning tree. By Assumption 2, the $(N+1)$th agent is the root agent of the expanded spanning tree, which means that the $(N+1)$th agent does not use any information from the other agents. Thus, the Laplacian matrix $\hat{L} = \left[\hat{l}_{ij}\right]$ corresponding to the new graph is a $(N+1)\times(N+1)$ matrix with $\hat{l}_{i(N+1)}=0$, $i=1,\ldots,N$. We assume that the set of neighbors of agent i associated with the new graph is $\hat{\mathcal{N}}_i$.

Denoting $y_{o,n+1}=y_{or}=\xi_d$, the dynamics

$$y_{o,N+1}^{(r)} \quad = v_{N+1} \tag{26}$$

is the same as the dynamics of tracking reference (4) with $v_{N+1}=\frac{d^{(r-1)}f_0}{dt}(t,\xi_d)$. Note that if the tracking reference is a constant, $\frac{d^{(r-1)}f_0}{dt}(t,\xi_d)$ in (4) is zero. Thus, v_{n+1} is zero. Let $\hat{y}_o=[y_{o1},\ldots,y_{oN},y_{o,N+1}]^T\in R^{N+1}$. The dynamics of the output y of all $N+1$ agents is

$$\hat{y}_o^{(r)} \quad = \bar{v}, \tag{27}$$

where $\bar{v}=[v_1,\ldots,v_{N+1}]^T$. From Lemma 1, there exists a non-singular matrix $\hat{M}\in R^{n+1}$ such that $\hat{L}=\hat{M}^{-1}\hat{J}\hat{M}$, where $\hat{J}$ is the Jordan form with $\hat{J}=\text{diag}\{\hat{J}_1,0\}$ where $-\hat{J}_1$ is a $N\times N$ Hurwitz matrix. From the definition of $\hat{J}$, it is easy to see that

$$\hat{J}=\begin{bmatrix} I_N \\ \mathbf{0}_{1\times N}\end{bmatrix}\hat{J}_1\begin{bmatrix} I_N & \mathbf{0}_{N\times 1}\end{bmatrix}. \tag{28}$$

Thus, the Lapalacian matrix $\hat{L}$ can be represented by

$$\hat{L}=\hat{M}^{-1}\begin{bmatrix} I_N \\ \mathbf{0}_{1\times N}\end{bmatrix}\hat{J}_1\begin{bmatrix} I_N & \mathbf{0}_{N\times 1}\end{bmatrix}\hat{M}. \tag{29}$$

We employ a dimension-reduced transformation

$$\hat{z}=\begin{bmatrix} I_N & \mathbf{0}_{N\times 1}\end{bmatrix}\hat{M}\hat{y}_o. \tag{30}$$

Differentiating it r times with respect to time gives

$$\hat{z}^{(r)}=\begin{bmatrix} I_N & \mathbf{0}_{N\times 1}\end{bmatrix}\hat{M}\hat{y}_o^{(r)}. \tag{31}$$

Substituting (27) into (31) yields

$$\hat{z}^{(r)}=\begin{bmatrix} I_N & \mathbf{0}_{N\times 1}\end{bmatrix}\hat{M}\hat{v}. \tag{32}$$

Let

$$\hat{v}=\hat{M}^{-1}\begin{bmatrix} I_N \\ \mathbf{0}_{1\times N}\end{bmatrix}\hat{J}_1\hat{v}+\hat{M}^{-1}\begin{bmatrix} \mathbf{0}_{N\times 1} \\ 1\end{bmatrix}\frac{1}{d}\frac{d^{(r-1)}f_0}{dt}, \tag{33}$$

where d is the $N+1$th element of the vector $\hat{M}^{-1}\begin{bmatrix} \mathbf{0}_{N\times 1} \\ 1\end{bmatrix}$. We obtain that

$$\hat{z}^{(r)}=B\hat{v}, \tag{34}$$

where $B=\hat{J}_1$. Note that the second term in (33) picks a particular element in the null space of the one-dimension-reduced transformation (30) in order to have $v_{N+1}=\frac{d^{(r-1)}f_0}{dt}$. To solve the output consensus tracking problem, a distributed control is required. Ideally, the distributed control is chosen as

$$\begin{aligned}\hat{v}_i^* &= -\sum_{l=1}^{r}k_{lc}\sum_{j\in\hat{\mathcal{N}}_i}(\xi_{il}-\xi_{jl}) \\ &= \begin{cases} -\sum_{l=1}^{r}k_{lc}\sum_{j\in\mathcal{N}_i}(\xi_{il}-\xi_{jl}) \\ \qquad\qquad \text{as } \left\{i|\hat{\mathcal{N}}_i=\mathcal{N}_i\right\} \\ -\sum_{l=1}^{r}k_{lc}\left(\xi_{il}-\xi_d^{(l-1)}+\sum_{j\in\mathcal{N}_i}(\xi_{il}-\xi_{j,l})\right) \\ \qquad +\frac{d^{(r-1)}f_0}{dt} \\ \qquad\qquad \text{as } \left\{i|\hat{\mathcal{N}}_i\neq\mathcal{N}_i\right\}\end{cases},\end{aligned} \tag{35}$$

for $i=1,\cdots,N$. Note that $v_{N+1}=\frac{d^{(r-1)}f_0}{dt}$. However, in our problem, the distributed control v_i only use neighbors' states x_j, where $j\in\mathcal{N}_i$. Thus, we have to use the following distributed control instead

$$\hat{v}_i = \begin{cases} -k_{1c}\sum_{j\in\mathcal{N}_i}(x_{i1}-x_{j1}) \\ -\sum_{l=2}^{r}\sum_{j\in\mathcal{N}_i}k_{lc}\left(x_{il}-\alpha_{i,l-1}|_{\eta_i=0}-x_{jl}\right. \\ \quad \left.+\alpha_{j,l-1}|_{\eta_i=0}\right) \\ \qquad\qquad \text{as } \left\{i|\hat{\mathcal{N}}_i=\mathcal{N}_i\right\} \\ -k_{1c}\sum_{j\in\mathcal{N}_i}(x_{i1}-x_{j1})-k_{1c}(x_{i1}-\xi_d) \\ -\sum_{l=2}^{r}\sum_{j\in\mathcal{N}_i}k_{lc}\left(x_{il}-\alpha_{i,l-1}|_{\eta_i=0}-x_{jl}\right. \\ \quad \left.+\alpha_{j,l-1}|_{\eta_i=0}\right) \\ -\sum_{l=2}^{r}k_{lc}\left(x_{il}-\alpha_{i,l-1}|_{\eta_i=0}-\xi_d^{(l-1)}\right) \\ \quad +\frac{d^{(r-1)}f_0}{dt} \\ \qquad\qquad \text{as } \left\{i|\hat{\mathcal{N}}_i\neq\mathcal{N}_i\right\}\end{cases} \tag{36}$$

From (7), we have

$$\alpha_{i1}-\alpha_{i1}|_{\eta_{i1}=0}=-k_{i1}\eta_{i1}. \tag{37}$$

From (11), we have

$$\alpha_{i2}-\alpha_{i2}|_{\eta_{i1}=\eta_{i2}=0} \quad =-k_{i2}\eta_{i2}-\eta_{i1}. \tag{38}$$

Proceeding to calculate from (16), we have

$$\alpha_{ij}-\alpha_{ij}|_{\eta_{i1}=\cdots=\eta_{ij}=0}=-k_{ij}\eta_{ij}-\eta_{i,j-1}. \tag{39}$$

We define a $(N+1)\times(N+1)$ matrix $H=\text{diag}\{h_i\}$ as $h_i=1$ if $\hat{L}(i,(N+1))\neq 0$, $h_i=0$ if $\hat{L}(i,(N+1))=0$ and $h_{N+1}=1$. Thus (36) has the following form:

$$\begin{aligned}\hat{v} &= \hat{M}^{-1}\begin{bmatrix} I_N \\ \mathbf{0}_{1\times N}\end{bmatrix}\hat{J}_1\left(-\sum_{l=1}^{r}k_{lc}\hat{z}^{l-1}+\hat{\Delta}(\eta_{rl},\cdots,\eta_{rr})\right) \\ &\quad +\begin{bmatrix} I_N & \mathbf{0}_{N\times 1}\end{bmatrix}\hat{M}H\hat{M}^{-1}\begin{bmatrix} \mathbf{0}_{N\times 1} \\ 1\end{bmatrix}\frac{d^{(r-1)}f_0}{dt}\end{aligned} \tag{40}$$

where

$$\hat{\Delta}(\eta_{r1},\cdots,\eta_{rr}) = \sum_{l=1}^{r} k_{lc} \begin{bmatrix} I_N & \mathbf{0}_{N\times 1} \end{bmatrix} \hat{M} \left[(k_{rl}\eta_{rl} - \eta_{rl-1})^T, 0 \right]^T \tag{41}$$

From (41), it is easy to see that there exist constant scalars c_{ij}, $j = 1,\cdots,r$ such that $\|\Delta\|^2 \le \sum_{j=1}^{r}\sum_{i=1}^{N} c_{ij}\eta_{ij}^2$. Note that c_{ij} depends on k_{jc} and $k_{i,j-1}$, which means that we can choose c_{ij} after choosing k_{jc} and $k_{i,j-1}$ (with $k_{i0}=0$). Substituting the controller (40) into (31) and considering the relationship $H = I-(I-H)$ gives

$$\hat{z}^{(r)} = Bv + B\Delta + P_\omega \omega \tag{42}$$

where $\omega = \frac{d^{(r-1)}f_0}{dt}$, and $P_\omega \in R^n = \begin{bmatrix} I_N & \mathbf{0}_{N\times 1} \end{bmatrix} \hat{M}(\mathbf{I}_{N+1} - H)\hat{M}^{-1}[\mathbf{0}_{1\times N}, 1]$.
We rearrange the coordinate in a compact form $\hat{Z} = [\hat{z}^T,\cdots,(\hat{z}^{(r-1)})^T]^T \in R^{r\times N}$, (42) can be written into:

$$\dot{\hat{Z}} = A_Z\hat{Z} + B_Z u + B_Z\Delta + P_Z\omega \tag{43}$$

where $P_Z = \left[\mathbf{0}^T, \mathbf{0}^T, \cdots, P_\omega^T\right]^T$. Since $\hat{Z}=0$, $\hat{L}y_o = 0$. Thus, to achieve consensus of y_o, we focus on input ω to output $\hat{z} = \begin{bmatrix} I_N & \mathbf{0}_{N\times 1} \end{bmatrix}$ stability of the systems (43).

Theorem 1: Under Assumptions 1, 2, and 3, if there exist scalars γ, k_{ij} and k_{jc} and a symmetric positive definite matrix P satisfying

$$\begin{aligned} \begin{bmatrix} \Theta + C_Z^T C_Z & PB_Z & PP_Z \\ 0 & B_Z^T P & -\epsilon \\ P_Z^T P & 0 & -\gamma^2 \end{bmatrix} &< 0 \\ k_{ij} - \epsilon c_{ij} &> 0 \\ j = 1,\cdots,r. \end{aligned} \tag{44}$$

then the adaptive control law (19) with parameters updates (21)and (22), and the distributed control (36) ensures that global boundedness of $x_i(t)$, $\hat{\theta}_i$ and $\hat{p}_i$, and solves the adaptive output consensus tracking problem with a bounded tracking error.

Proof: Choose a Lyapunov function candidate as follows:

$$V(t) = \sum_{i=1}^{N} V_i + \hat{Z}^T(t)P\hat{Z}(t). \tag{45}$$

We discuss the performance of the system (43) with disturbance $\omega(t)$ by taking derivative of $V(t)$ along with the solutions of (43) with the distributed control (36) with respect to t gives

$$\begin{aligned} \dot{V}(t) \le& -\sum_{i=1}^{N}\sum_{l=1}^{r}(k_{il} - \epsilon c_{il})\eta_{il}^2 + \frac{1}{\gamma^2}PP_ZP_Z^TP \\ &+ \hat{Z}^T(t)\left(\Theta + \tfrac{1}{\epsilon}PB_Z^TPB_Z\right)\hat{Z}(t) \\ &-\gamma^2\left\|\omega - \tfrac{1}{\gamma^2}P_Z^TP\hat{Z}(t)\right\|_2^2 + \gamma^2\|\omega\|_2^2. \end{aligned} \tag{46}$$

By Schur Complement Formula [3], (44) is equivalent to

$$\Theta + C_Z^TC_Z + \gamma^{-2}PP_ZP_Z^TP + \frac{1}{\epsilon}PB_Z^TPB_Z < 0. \tag{47}$$

Substituting (47) into (46) yields

$$\dot{V}(t) \le \gamma^2\|\omega\|_2^2 - \|\hat{z}\|_2^2 - \sum_{i=1}^{N}\sum_{l=1}^{r}(k_{il} - \epsilon c_l)\eta_{il}^2. \tag{48}$$

Thus, η_i is bounded. Also, note that the left-hand side of (48) is the derivative of V along the trajectories of the system (43). Integrating (48) yields

$$\begin{aligned} &2V\left(\hat{Z}(\tau)\right) - 2V\left(\hat{Z}(0)\right) \\ &\le \gamma^2\int_0^\tau \|w\|_2^2\,dt - \int_0^\tau \|z\|_2^2\,dt, \end{aligned} \tag{49}$$

where $\hat{Z}(t)$ is the solution of (43) for a given $\omega \in \mathcal{L}_2[0,\infty)$. Using $V\left(\hat{Z}\right) \ge 0$, we obtain

$$\int_0^\tau \|\hat{z}\|_2^2\,dt \quad \le \gamma^2\int_0^\tau \|w\|_2^2\,dt + 2V\left(\hat{Z}_0\right), \tag{50}$$

which implies that the mapping from ω to $\hat{z}$ has finite L_2-gain γ. Thus, the system (43) is input-to-output bounded. From the definition of $\hat{z}$, we have $\hat{L}y_o = \begin{bmatrix} I_N & \mathbf{0}_{N\times 1} \end{bmatrix}\hat{J}_1\hat{z}$. Thus, we have $\left\|\hat{L}y_o\right\|_2 \le \lambda_{max}\left(\hat{J}_1\right)\gamma\|\omega\|_2$. we obtain that the output y_{oi} tracks the reference y_r in a small bound. From the boundedness of η_i, it is also obtained that y_i also tracks the reference y_r in a small bound. ■

V. Simulations

Consider a group of agents with mismatched uncertainties:

$$\begin{aligned} \dot{x}_{i1} &= \sin(x_{i1})\theta_i + x_{i2}, \\ \dot{x}_{i2} &= \cos(x_{i2})\theta_i + u_i \\ y_i &= x_{i1}, \end{aligned} \tag{51}$$

where θ_i are unknown parameters, and $i = 1,2,3,4,5$. We consider that the communication topology shown in

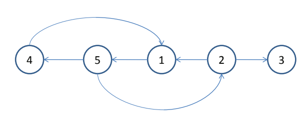

Fig. 1. Communication Topology with Spanning Trees

Fig. 1. We assume that both leaders are able to access the reference as shown in Figure 2. From Lemma 1, we obtain that $k_{1c} = 25$, and $k_{2c} = 10$ satisfy (44). The adaptive control law (19) with parameters updates (21) and (22), and the distributed control (36) are applied. Fig. 3 shows that all agents' reference estimators reach an agreement: proving the same reference signal to each agent. Fig. 4 shows that the proposed controller guarantees stably tracking to the estimated reference. Furthermore, the outputs of the system (3) achieve consensus simultaneously in Fig. 5. Moreover, Fig. 6 shows that the states of each agent are bounded.

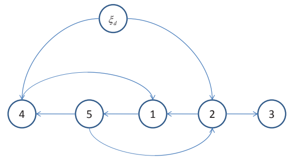

Fig. 2. The communication topology after adding a virtual leader

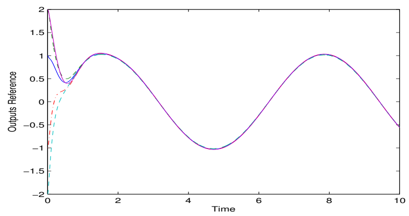

Fig. 3. The outputs of five agents' reference estimators

VI. Conclusions

In this paper, we consider the adaptive output consensus tracking of a class of parametric strict-feedback systems with mismatched uncertainties. Taking the consensus reference as a virtual leader, the adaptive output consensus tracking problem is transformed to output consensus with one more agent. The estimator is used for each agent to estimate the consensus reference by communicating with it's neighbors. Thus, the output consensus problem with uncertainties is transformed to the classic adaptive tracking control problem. Established control methods such as backstepping can be used to solve the adaptive output consensus tracking with mismatched uncertainties.

VII. Acknowledgement

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References

[1] M. Arcak. Passivity as a design tool for group coordination. *IEEE Trans. on Automatic Control*, 52(8):1380–1390, 2007.

[2] H. Bai and M. Arcak. Instability mechanisms in cooperative control. In *the 47th IEEE Conference on Decision and Control*, pages 357–362, Cancun, Mexico, 2008.

[3] B. Boyd, L.E. Ghaoui, E. Feron, and V. Balakrishnan. *Linear Matrix Inequalities in System and Control Theory*. SIAM, Philadelphia, PA, 1994.

[4] D. Cherition and R.E. Tarjan. Finding minimum spanning trees. *SIAM Journal on Computing*, 5.

[5] J.A. Fax and R.M. Murry. Information flow and cooperative control of vehicle formation. *IEEE Trans. on Auotomatic Control*, 49(9):1465–1476, 2004.

[6] Y. Hong, J. Hu, and L. Gao. Tracking control for multi-agent consensus with an active leader and variable topology. *Automatica*, 42(7):1177–1182, 2006.

[7] A. Jadbabaie, J. Lin, and A. S. Morse. Coordination of groups of mobile autonomous agents using nearest neighbor rules. *IEEE Trans. on Automatic Control*, 48(6):988–1001, 2003.

[8] Z. Jin and R.M. Murray. Consensus controllability for coordinated multiple vehicle control. In *Proc. of The Sixth Int. Conf. on Cooperative Control and Optimization*, Gainesville, FL, 2006.

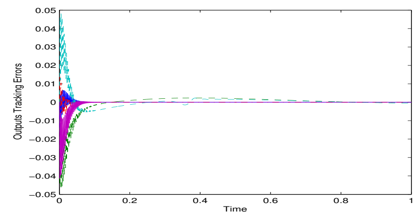

Fig. 4. Five agents' tracking errors

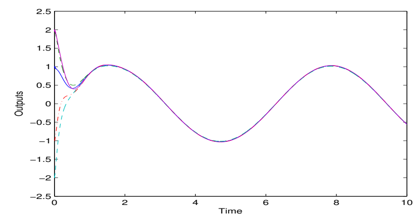

Fig. 5. The outputs of five agents with uncertainties

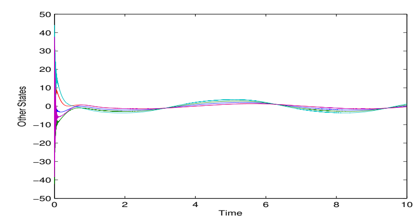

Fig. 6. Five agents' states x_{i2}

[9] P. Lin and Y.M. Jia. Robust H_∞ consensus analysis of a class of second-order multi-agent systems with uncertainty. *IET Control Theory Appl.*, 4(3):487–498, 2010.

[10] K. Moore and D. Lucarelli. Decentralized adaptive scheduling using consensus variables. *Int. J. Robust Nonlinear Control*, 17(10):921–940, 2007.

[11] L. Moreau. Stability of multi-agent systems with time-dependent communication links. *IEEE Trans. on Automatic Control*, 50(2):169–182, 2005.

[12] V.V. Prasolov. *Problems and Theorems in Linear Algebra*. American Mathematical Society, 1994.

[13] Z. Qu. *Cooperative Control of Dynamical Systems, Application to Autonomous Vehicles*. Springer-Verlag, London, 2009.

[14] W. Ren. Consensus tracking under directed interaction topologies: Algorithms and experiments. *IEEE Trans. Control Systems Technology*, 18(1):230–237, 2010.

[15] W. Ren and E. Atkins. Distributed multi-vehicle coordinated control via local information exchange. *Int. J. Robust Nonlinear Control*, 17(10):1002–1033, 2007.

[16] W. Ren, K. L. Moore, and Y. Chen. High-order and model reference consensus algorithms in cooperative control of multi-vehicle systems. *ASME Journal of Dynamic Systems, Measurement, and Control*, 129(5):678–688, 2007.

[17] R.O. Saber and R.M. Murry. Consensus problems in networks of agents with switching topology and time-delays. *IEEE Trans. on Auotomatic Control*, 49(9):1520–1533, 2004.

[18] J. Wang, Z. Liu, and X. Hu. Consensus of high order linear multi-agent systems using output error feedback. In *Joint 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, Shanghai, P.R. China, 2009.

[19] C.W. Wu and L.O. Chua. Synchronization in an array of linearly coupled dynamical systems. *IEEE Trans. on Circuits and Systems - I: Fundamental Theory and Applications*, 42(8):430–446, 1995.