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Adaptive Robust Actuator Fault-Tolerant Control in Presence of Input Saturation

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Abstract— **In this article, we solve the problem of unknown actuator fault accommodation for a class of uncertain nonlinear systems, with explicit consideration of input saturation. A review of the existing literature reveals that fault-tolerant controllers are often designed without any regard to actuator saturation, and it is assumed that they will not saturate in spite of faults. In reality, however, considerable amount of control effort needs to be expended for suppressing the transients due to actuator faults, which can easily saturate the working actuators. In the present work, an indirect adaptive robust fault-tolerant controller is proposed which explicitly takes into account the actuator limits. Furthermore, the indirect design ensures that adaptation mechanism is not affected adversely due to actuator saturation. Finally, simulation studies performed on a nonlinear hypersonic aircraft model are presented to demonstrate the effectiveness of the proposed scheme in dealing with actuator faults in presence of input saturation.**

I. Introduction

In complex systems like chemical plants, nuclear reactors, flight control systems etc., reliability is as important as performance. As the performance of any control system depends on the condition of actuators, any fault that affects actuators can have serious consequences on the system performance. In fact, actuator faults can not only lead to degraded system performance, but may even result in overall system instability. Therefore, it is important that the control system possess a degree of fault tolerance with respect to actuator faults. Among various approaches, adaptive control based fault-tolerant schemes [1], [2] have found popularity among researchers, as they have the ability to learn the change in system parameters due to actuator faults. However, input magnitude constraint - one of the most important factors which can limit the performance of any control system, has been largely overlooked in the literature.

The harmful effect of actuator faults on the system response increases manifold in presence of actuator saturation. In order to understand how actuator faults and actuator saturation negatively reinforce their destructive effects, we must first understand the inter-play of actuator fault-tolerant control and actuator faults. The effect of a large class of actuator faults e.g., loss in actuator efficiency, hard over etc. can be captured as sudden jump in system parameters, which can degrade the system performance in two chief ways - (i) undesirable transients and, (ii) unacceptably large steady-state tracking errors. If the controller is designed without any regard to saturation, it may generate large control signals to attenuate the effect of undesired transients after a fault, which may saturate the actuators. As the actuators saturate, the tracking-error further increases, which in turn increases the commanded control input, so on and so forth. Thus, the actuator cannot pull out of the saturation, and the closed-loop system may eventually become unstable. Another problem which direct adaptive schemes suffer from is unreliable parameter estimation in presence of input saturation. After an actuator fault, the state-estimation error is a *cumulative effect of two causes* – actuator faults and insufficient control input due to saturation. As in direct adaptive schemes the parameter estimation algorithm cannot differentiate between these two causes, the estimated parameters are not accurate.

From the preceding discussion, it should be clear that if a fault-tolerant controller can be designed which (a) relaxes the performance criteria when the actuator saturates, and (b) ensures reliable adaptation in spite of actuator faults, then the design will become more practical. Although, many results have been proposed in the past two decades to address the problem of input saturation, the problem of actuator fault-tolerant control in presence of saturation still remains largely unexplored. Similarly, in spite of the growing volume of literature on adaptive control based fault-tolerant designs, the effect of saturation has largely been overlooked. A novel model reference adaptive control (MRAC) based scheme for accommodating actuator faults in linear systems was proposed in [3]. For feedback linearizable nonlinear systems, an effective neural-networks based adaptive scheme was proposed in [4]. Note that most of the adaptation based fault-tolerant designs fall under the category of direct adaptive control, and primarily rely on two techniques to counter the harmful effect os saturation – pseudo control hedging (PCH) [4] and training signal hedging (TSH) [5]. The first technique PCH involves artificially modifying the reference model to bring the desired trajectory to a level where it can be tracked without saturating the actuators. It should be noted that this implies the trajectory being tracked is no longer the desired trajectory generated by the reference model and some of the properties which are a result of plant-reference model matching in MRAC cannot be guaranteed anymore e.g., stability of the overall system. The second technique TSH modifies the error signal used for parameter estimation. The main idea behind TSH is to remove the error due to saturation from the total error, such that the resulting error correlates only to the parameter mismatch. This leads

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to improved parameter estimation. This technique, although effective, needs to be extended to nonlinear systems and cases where reference models may not be linear.

In the present work, we attempt to combine nested saturation functions, indirect adaptive control and a modified backstepping based approach to solve the actuator fault-tolerant problem in presence of saturation. In the proposed approach, bounded feedback control laws are designed such that performance is compromised when the error-variables are far away from zero and more emphasis is laid on bringing the error-variables to a region where the controller is unsaturated. This ensures unrealistically high control inputs are not demanded of the saturating actuators after a fault. Once within the unsaturated region, the desired performance can be recovered. In order to deal with unreliable parameter estimates due to saturation, an indirect adaptive scheme is proposed. The indirect scheme automatically accounts for the actuator saturation, and no artificial modification to the reference model or error is required. Furthermore, the indirect scheme does not require the system or reference model to be linear. Additionally, it will be shown that the proposed controller is ISS with respect to estimation error in the unsaturated region. This allows us to show that good final tracking accuracy – asymptotic tracking in presence of parametric uncertainties only – can be achieved using the proposed design. The main contributions of the proposed work lies in - (i) design of a control law that accommodates unknown actuator faults with desired closed-loop performance in the unsaturated region, (ii) explicit consideration of actuator limits in the controller design, which allows the controller to pull out of saturation phase after faults and, (iii) reliable adaptation regardless of saturation.

II. Problem formulation

In this paper, we consider systems which can be written as

$$\begin{aligned} \dot{x} &= f(x) + g(x)[b_1 u_1 + \cdots + b_q u_q] + w(x)d(t) \\ y &= h(x), \quad |u_j(t)| \le u_M, \quad j = 1, ..., q \end{aligned} \tag{1}$$

where $w(x)$ is the distribution matrix for the disturbance. It will be assumed that $f(x)$, $g(x)$, $w(x)$ and $h(x)$ are smooth functions. The plant parameters b_i, which are assumed to be unknown, belong to a known region i.e., $b_i \in [b_{i,\min}, b_{i,\max}]$. In the present analysis, we focus our attention to the class of systems for which the following assumption holds.

A1: System (1) has a well-defined relative degree m with respect to the output $y = h(x)$ such that there exists a diffeomorphism $T(x)$ which transforms the state-vector x to $[\zeta, \eta]'$ coordinates as follows

$$\begin{bmatrix} \zeta \\ \eta \end{bmatrix} = \begin{bmatrix} \zeta_1 \\ \zeta_2 \\ \vdots \\ \zeta_m \\ \eta_1 \\ \vdots \\ \eta_{n-m} \end{bmatrix} = T(x) = \begin{bmatrix} h(x) \\ L_f h(x) \\ \vdots \\ L_f^{m-1} h(x) \\ T_1(x) \\ \vdots \\ T_{n-m}(x) \end{bmatrix} \tag{2}$$

Thus, the dynamics can be rewritten as

$$\begin{aligned} \dot{\zeta}_1 &= \zeta_2 \\ \dot{\zeta}_2 &= \zeta_3 \\ &\vdots \\ \dot{\zeta}_m &= L_f^m h(x) + L_g L_f^{m-1} h(x)[b_1 u_1 + \cdots + b_q u_q] \\ &\quad + L_w L_f^{m-1} h(x) d(t) \\ \dot{\eta} &= f_\eta(\zeta, \eta, d(t)) \end{aligned} \tag{3}$$

where it is assumed that the disturbance distribution matrix is such that $L_w L_f^j = 0$ for all $j = 1, ..., m-2$ and $L_w L_f^{m-1} \neq 0$, and $L_f^m h(x) \neq 0$, for all $x \in \mathbb{R}^n$.

In the present work, we will also make the following assumption about the zero-dynamics.

A2: The η-dynamics is input to state stable (ISS) with respect to ζ and $d(t)$ as inputs.

Assumption A2 guarantees the boundedness of all closed-loop signals when a stable controller can be designed for the ζ-dynamics.

In this work, we will consider actuator faults which can be modeled as

$$u_j(t) = \begin{cases} \bar{u}_j, \forall t \ge T_f, \\ \quad \text{if } j^{th} \text{ actuator gets stuck at } T_f \\ \eta_{jj} u_j^*(t), \forall t \ge T_f, \\ \quad \text{if } j^{th} \text{ actuator loses efficiency at } T_f \end{cases} \tag{4}$$

where $u_j^*(t)$ represents the control command to the j^{th} actuator, $\bar{u}_j$ is an unknown constant value at which the actuator gets stuck, T_f is the unknown instant of failure and η_{jj} represents actuator loss in efficiency with $\eta_{jj} \in [(\eta_{jj})_{min}, 1]$, $(\eta_{jj})_{min} \ge 0$. It will be assumed that $\bar{u}_j$ belongs to a known interval i.e., $\bar{u}_j \in [\bar{u}_{j,\min}, \bar{u}_{j,\max}]$.

For the system described by (1), subjected to unknown actuator faults (4) and bounded disturbances, the objective is to design a control law such that the output tracking error converges to a sufficiently small neighborhood of zero where the controller is unsaturated, and has desired closed-loop performance and disturbance attenuation properties within the unsaturated region. It is also desired that the tracking error asymptotically converges to zero in absence of disturbances.

III. Adaptive Robust Actuator Fault-Tolerant Control

Denote $f_\zeta(x) = L_f^m h(x)$, $g_\zeta(x) = L_g L_f^{m-1} h(x)$ and $w_\zeta = L_w L_f^{m-1} h(x)$. In the present work, we will assume that control commands to all the actuators are the same i.e., $u_1^* = \cdots = u_q^* = u_0$. With this control input, and fault model described by (4), the healthy and faulty actuators can be parameterized in the following way

$$\begin{aligned} u_j(t) &= \eta_{jj}(1 - \sigma_{jj}) u_0(t) + \sigma_{jj} \bar{u}_j \\ \Rightarrow \sum_{j=1}^{q} b_j u_j(t) &= \kappa u_0(t) + \mu \end{aligned} \tag{5}$$

where

$$\sigma_{jj} = \begin{cases} 0 & \text{before } j^{th} \text{ actuator gets stuck} \\ 1 & \text{after } j^{th} \text{ actuator gets stuck} \end{cases}$$
$$\eta_{jj} = \begin{cases} 1 & \text{before } j^{th} \text{ actuator loses efficiency} \\ [(\eta_{jj})_{min}, 1] & \text{after } j^{th} \text{ actuator loses efficiency} \end{cases}$$
$$\kappa = \sum_{j=1}^{q} b_j \eta_{jj}(1-\sigma_{jj}), \quad \mu = \sum_{j=1}^{q} b_j \sigma_{jj} \bar{u}_j$$

We will make the following practical assumption regarding the extent of uncertainties present in the system.

A3: The unknown parameters $\theta \triangleq [\kappa, \mu]^T$ and disturbance satisfy,

$$\theta \in \Omega_\theta \triangleq \{\theta : \theta_{\min} \leq \theta \leq \theta_{\max}\} \tag{6}$$
$$w_\zeta(x)d(t) \in \Omega_d \triangleq \{w_\zeta(x)d(t) : |w_\zeta(x)d(t)| \leq d_M\} \tag{7}$$

Now, that we have established a parametric fault-model, we are ready to present the bounded control laws to be used in the backstepping based design. We will use the coordinate transformation $\tilde{x}_i = \zeta_i - y_d^{(i-1)}$ to simplify the analysis. The designed virtual control law α_i and the corresponding error z_i are given by

$$\begin{aligned} z_i &= \tilde{x}_i - \alpha_{i-1}(z_{i-1}), \\ \alpha_i &= -\sigma_i(z_i), \\ \dot{\tilde{x}}_i &= z_{i+1} + \alpha_i, \quad i = 1, ..., n-1 \end{aligned} \tag{8}$$

where $\sigma_i(z_i)$ are saturation functions (see fig. 1 in [6]). The saturation functions are defined as

$$(a) \quad z_i \sigma_i(z_i) \geq 0, \forall z_i \tag{9}$$
$$(b) \quad \sigma_i(z_i) = k_i z_i, \forall |z_i| \leq l_i, \quad \sigma_i(z_i) = (sign(z_i))M_i, \forall |z_i| \geq L_i \tag{10}$$
$$(c) \quad |\sigma_i(z_i)| \leq M_i, \forall z_i \tag{11}$$
$$(d) \quad \frac{\partial \sigma_i}{\partial z_i} \leq k_i, \forall z_i \tag{12}$$

Also, $l_i = \beta_i L_i$ with $\beta_i \leq 1$, and $M_i = k_i l_i(1+\gamma_i)$ with $\gamma_i > 0$. The interval for z_i is divided into three different regions - $\Omega_1^i = \{z_i : |z_i| \leq l_i\}$, $\Omega_2^i = \{z_i : |z_i| \leq L_i\}$ and $\Omega_3^i = \{z_i : |z_i| > L_i\}$. Note that the nonlinear transition region of the saturation function ($\Omega_2^i \backslash \Omega_1^i$) needs to be at least second order differentiable, as the backstepping design involves taking derivatives of σ_i. σ_m, however, need not have a smooth transition region, as this appears in the last step. The l_m and L_m parameters used in the definition of σ_m depend on the extent of uncertainties, and will be defined later.

Now, substituting (8) in (3), the error dynamics can be written as

$$\begin{aligned} \dot{z}_1 &= z_2 - \sigma_1(z_1) \\ &\vdots \\ \dot{z}_i &= z_{i+1} - \sigma_i(z_i) \\ &+ \sum_{j=1}^{i-1} \left\{ \left[\prod_{r=1}^{j} \frac{\partial \sigma_{i-r}}{\partial z_{i-r}} \right] (z_{i-j+1} - \sigma_{i-j}(z_{i-j})) \right\} \end{aligned} \tag{13}$$
$$\begin{aligned} &\vdots \\ \dot{z}_m &= f_\zeta(x) + \kappa g_\zeta(x) u_0 + \mu g_\zeta(x) + w_\zeta(x) d(t) - y_d^{(m)} \\ &+ \sum_{j=1}^{m-1} \left\{ \left[\prod_{r=1}^{j} \frac{\partial \sigma_{m-r}}{\partial z_{m-r}} \right] (z_{m-j+1} - \sigma_{m-j}(z_{m-j})) \right\} \end{aligned} \tag{14}$$

A. Parameter Estimation

We will use x-swapping lemma (Ch.6, [7]) to implement least-squares estimation scheme, and then use discontinuous projection to ensure that the parameters stay within a known region in presence of disturbances. Note that as all unknown parameters appear in the mth channel in the ζ coordinates, we need only $\dot{\zeta}_m$ dynamics for estimation purposes.

The ζ_m-dynamics can rewritten as

$$\begin{aligned} \dot{\zeta}_m &= f_\zeta(x) + \kappa g_\zeta(x) u_0 + \mu g_\zeta(x) + w_\zeta(x) d(t) \\ &= f_\zeta(x) + \phi(x,u)^T \theta + w_\zeta(x) d(t) \end{aligned} \tag{15}$$

where $\theta \triangleq [\kappa, \mu]^T$ and $\phi(x,u)^T = [g_\zeta(x) u_0, g_\zeta(x)]$. Following the standard steps of x-swapping, we define the following filters

$$\dot{\Omega}_0 = A(\Omega_0 + \zeta_m) - f_\zeta(x) \tag{16}$$
$$\dot{\Omega}^T = A\Omega^T + \phi(x,u)^T \tag{17}$$

Define the prediction error as $\epsilon = \zeta_m + \Omega_0 - \Omega^T \hat{\theta}$, which is calculable. It is shown in [7] that ϵ can be rewritten as

$$\epsilon = \Omega^T \tilde{\theta} + \tilde{\epsilon} \tag{18}$$

where $\tilde{\epsilon}$ is governed by $\dot{\tilde{\epsilon}} = A\tilde{\epsilon}$, which exponentially converges to zero. Thus, we have a static model (18), that is linearly parameterized in terms of $\tilde{\theta}$ with an additional term $\tilde{\epsilon}$ which exponentially decays to zero. With this static model, various estimation algorithms can be used to estimate the unknown parameters. In the following, we present the least-squares estimation scheme which will be used in the present work.

$$\begin{aligned} \dot{\hat{\theta}} &= \Gamma\tau, \quad \tau = \frac{\Omega\epsilon}{1 + \nu \text{tr}\{\Omega^T \Gamma \Omega\}} \\ \dot{\Gamma} &= -\Gamma \frac{\Omega\Omega^T}{1 + \nu \text{tr}\{\Omega^T \Gamma \Omega\}} \Gamma, \quad \Gamma(0) = \Gamma(0)^T > 0, \nu \geq 0 \end{aligned} \tag{19}$$

In order to guarantee certain desired properties, we will use the following discontinuous projection algorithm.

$$\dot{\hat{\theta}} = \text{Proj}_{\hat{\theta}}(\Gamma\tau) \tag{20}$$
$$\text{Proj}_{\hat{\theta}_i} = \begin{cases} 0 & \text{if } \hat{\theta}_i = \theta_{i,max} \text{ and } \bullet_i > 0 \\ 0 & \text{if } \hat{\theta}_i = \theta_{i,min} \text{ and } \bullet_i < 0 \\ \bullet_i & \text{otherwise} \end{cases} \tag{21}$$

This guarantees that the parameters do not drift away and stay within known bounded region even in presence of disturbances.

B. Controller design

In this section, we present the proposed adaptive robust fault-tolerant controller and prove the overall stability of the system using the following steps:

1) In the first step, it is shown that for any set of initial conditions $z_i(0)$, all error-variables can be driven to an invariant region where the controller is unsaturated, as long as a set of controller parameters exist which satisfy certain inequalities.
2) Next, sufficient and necessary conditions for the existence of the controller parameters is proposed and proved.
3) In the last step, asymptotic convergence of the adaptive system is proved in absence of disturbances within the unsaturated region.

Define $u_a = -f_\zeta(x) - \hat{\mu} g_\zeta(x)$. Control law to be used

$$u_0 = \sigma_m \left[\frac{1}{g_\zeta(x)\hat{\kappa}} (\sigma_a(u_a) + y_d^{(m)} - k_m z_m) \right] \tag{22}$$

where

$$\sigma_a(u_a) = \begin{cases} u_a, & \text{for} \quad |u_a| \leq M_a \\ \text{sign}(u_a) M_a, & \text{for} \quad |u_a| > M_a \end{cases}$$

and

$$\sigma_m(u_0) = \begin{cases} u_0, & \text{for} \quad |u_0| \leq u_M \\ \text{sign}(u_0) u_M, & \text{for} \quad |u_0| > u_M \end{cases}$$

Note that as system (3) may not necessarily be stable, the model-compensation component u_a can easily become unbounded. In the context of bounded control, some assumptions must be made on the growth rate of the nonlinearities and the extent of uncertainties to make the stabilization/tracking problem feasible.

A4: In the present work, it will be assumed that the nonlinearities are such that the following conditions are satisfied within a finite time of occurrence of the fault
(i) the deficit in model-compensation i.e., $\sigma_a(u_a) = M_a$, can always be bounded above by a known constant i.e., $|\sigma_a(u_a) - u_a| \leq \tilde{u}_{aM}$
(ii) the extent of uncertainty due to fault-parameter estimation mismatch can be bounded above by a known constant i.e., $|\phi^T(x,u)\tilde{\theta}| \leq h_M$.
(iii) $g_\zeta(x)$ is such that $0 < g_{\zeta,\min} \leq |g_\zeta(x)| \leq g_{\zeta,\max}$.

Consider the $\zeta-$dynamics of system (3) along with assumptions (A1-A4). The following theorem states that in spite of unknown actuator faults (5), the error dynamics can be driven to small neighborhood around zero, where the controller is unsaturated and desired closed-loop performance can be recovered.

Theorem 1. Consider the error-dynamics represented by (14). With the control law given by (22), and the chosen parameter update law (19), if a set of controller parameters can be chosen such that

$$\begin{aligned} k_i l_i &> l_{i+1} + k_{i-1} N_i, \quad i = 1, ..., m-1 && (23) \\ k_m l_m &> k_{m-1} N_m + h_M + d_M + \tilde{u}_{aM}, && (24) \\ k_m l_m &\leq \hat{\kappa}_{\min} g_{\zeta,\min} u_M - (M_a + \lambda_M) && (25) \end{aligned}$$

where

$$N_i = L_i + M_i + \sum_{j=1}^{i-2} \left[\left(\prod_{r=1}^{j} k_{i-1-r}\right)(L_{i-j} - M_{i-1-j}) \right]$$

and $k_0 = 0$ then, the error variables z_i reach a region where the controller is unsaturated in a finite time (i.e., $z \in \bigcap_{j=1}^{m} \Omega_1^j$), for any set of initial conditions and any fault pattern.

Proof. The proof follows along the same line as outlined in [6] and has been omitted due to space restrictions. ∇

C. Controller Parameter Selection

1) Necessary and sufficient conditions for the existence of controller parameters: After a series of derivations, (23)-(24) can be rewritten in a matrix form

$$AL < D, \tag{26}$$

where $L = [l_1, l_2, \cdots, l_{m-1}, l_m]^T$, $D = [0, 0, \cdots, 0, -(h_M + d_M + u_{aM})]^T$. And A is a function of k_i given by

$$A = \begin{bmatrix} -k_1 & 1 & 0 & \cdots & 0 \\ k_1^2(1+\gamma_1) & \left(\frac{k_1}{\beta_2} - k_2\right) & 1 & \cdots & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & \left(\frac{k_{m-1}}{\beta_n} - k_m\right) \end{bmatrix}, \tag{27}$$

where

$$\begin{aligned} &a_{ij} = k_{i-1} k_j \prod_{r=1}^{i-j-1} k_{i-r-1}(1+\gamma_j) + k_{i-1} \prod_{r=1}^{i-j} k_{i-r-1} \tfrac{1}{\beta_j}, \ \forall i > j. \\ &a_{ij} = -k_i + \tfrac{k_{i-1}}{\beta_i}, \ \forall i = j \\ &a_{ij} = 1, \ \forall j = i+1 \\ &a_{ij} = 0, \ \forall j > i+1 \end{aligned} \tag{28}$$

For any $\gamma_i > 0$ and $0 < \beta_i \leq 1$, if we fix a set of positive k_is, then the control law is **feasible** if and only if the there exist l_1, l_2,..., $l_m > 0$ such that (inequality3) and (26) are satisfied. In other words, at least one solution to (inequality3) and (26) should lie in the region $\{(l_1, \cdots, l_m) : \ l_i > 0\}$. The following theorem gives the necessary and sufficient condition for k_is such that the control law is feasible.

Theorem 2. For any $\gamma_i > 0$ and $0 < \beta_i \leq 1$, with a set of positive k_is, at least one solution to (inequality3) and (26) lie in the region $L \in \{(l_1, \cdots, l_m) : \ l_i > 0\}$ **iff** the k_is satisfy the following set of inequalities:

$$\begin{aligned} k_1 &> 0, \\ k_2 &> \frac{a_{21}p_1+\frac{k_1}{\beta_2}p_2}{p_2} = (1+\gamma_1+\frac{1}{\beta_2})k_1, \\ &\cdots \\ k_i &> \frac{\sum_{j=1}^{i-1} a_{ij}p_j+\frac{k_{i-1}}{\beta_i}p_i}{p_i},\ \forall i < m \\ &\cdots \\ k_m &> \frac{\hat{\kappa}_{\min} g_{\zeta,\min} u_M-(M_a+\lambda_M)}{\hat{\kappa}_{\min} g_{\zeta,\min} u_M-(M_a+h_M+\tilde{u}_{aM}+\lambda_M+d_M)} \cdot \\ &\quad \frac{\sum_{j=1}^{n-1} a_{nj}p_j+\frac{k_{n-1}}{\beta_n}p_n}{p_n}, \end{aligned}$$

where, the coefficients p_is are computed recursively using the formula

$$\begin{aligned} p_1 &= 1 \\ p_i &= -\sum_{j=1}^{i-1} a_{i-1j}p_j \end{aligned} \tag{29}$$

Proof. The proof has been omitted due to space restrictions, but can be obtained from the authors upon request.

2) Controller gain selection: a recursive root-locus design: Please refer to [6] for an outline of recursive root-locus design technique.

D. Asymptotic Stability

In the linear unsaturated region, the dynamics can be represented as

$$\dot{z} = A_{cl}z + B_{\tilde{\theta}}\tilde{\theta} + B_d w_\zeta(x)d(t) \tag{30}$$

where

$$A_{cl} = \begin{bmatrix} -k_1 & \cdots & 0 \\ \vdots & \cdots & \vdots \\ -k_{i-1}...k_1^2 & \cdots & 0 \\ \vdots & \cdots & \vdots \\ -k_{i-1}...k_1^2 & \cdots & -(k_m-k_{m-1}) \end{bmatrix} \tag{31}$$

$$B_{\tilde{\theta}} = \begin{bmatrix} 0 & 0 \\ \vdots & \vdots \\ 0 & 0 \\ \phi_1(x,u) & \phi_2(x,u) \end{bmatrix}, \quad B_d = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}$$

From the preceding discussion, we know A_{cl} is stable and $\|\phi(x,u)^T\|$ is bounded, as $|g_\zeta(x)|$ is bounded below and above by known positive constants, $|u| \leq u_M$. Thus, it can be easily verified using a Lyapunov function $V_z = z^T z$ that $z(t)$ is Input to State Stable (ISS) with respect to the inputs $\tilde{\theta}$ and $w_\zeta(x)d(t)$. Now we are ready to state an important result which shows that the proposed controller preserves the desired property of an adaptive controller - asymptotic tracking in presence of parametric uncertainties only.

Theorem 3. In presence of parametric uncertainties only i.e., when $d(t) = 0$, by using the control law given by (22), and the parameter update law (19) along with filters (16-17), asymptotic output tracking is also achieved i.e., $z \to 0$ as $t \to \infty$.

Proof. The proof follows from the fact that the controller is ISS w.r.t $\tilde{\theta}$ and $\tilde{\theta} \in \mathcal{L}_\infty$ for least-square estimation using x-swapping lemma (see (Ch.6, [7])). This, in turn implies that $\dot{z} \in \mathcal{L}_\infty$, and from Barbalat's lemma, we obtain $z \to 0$ as $t \to \infty$. ■

IV. SIMULATION EXAMPLE

An adaptive robust control (ARC) based approach was recently proposed to solve the unknown actuator fault accommodation for linear [8], [9] and nonlinear [10], [11] systems. The superior performance of an ARC based approach in achieving desired transient response, as well as small steady-state tracking error over a robust adaptive control based design was demonstrated through comprehensive simulation studies. In the present work, we compare the performances of the proposed scheme and the recently developed ARC based fault-tolerant controller in presence of saturation. A nonlinear longitudinal reduced-order model of hypersonic aircraft cruising at a velocity of 15 mach, at an altitude of $110,000$ feet is used to test the effectiveness of the proposed scheme. Below, we only present the final form of the system model after coordinate transformation and refer the reader to [11] for the details.

$$\begin{aligned} \dot{\zeta}_1 &= \dot{y} = x_2 + a_1 y + a_2\sin(y) + a_3 y^2\sin(y) + a_4\cos(x_3) \\ \dot{\zeta}_2 &= a_5 y^2 + a_6 y + (a_7 + a_8 y + a_9 y^2)x_2 \\ &+ (a_1 + a_2\cos(y) + 2a_3 y\sin(y) + a_3 y^2\cos(y))\zeta_2 \\ &- a_4\sin(x_3)(a_{10}\cos(x_3) - a_1 y - a_2\sin(y)) \\ &\qquad +b_1u_1 + b_2u_2 + \Delta(t) \end{aligned} \tag{32}$$

where $a_1 = -0.0427, a_2 = -3.4496\times10^{-4}, a_3 = 5\times10^{-5}, a_4 = 0.0014, a_5 = -4.2006, a_6 = 1.0821, a_7 = -3.6896, a_8 = 0.1637, a_9 = -0.1242, a_{10} = 0.0014, b_1 = 0.8, b_2 = 0.8$ and $\Delta(t) = 0.02\sin(3t)$ represents the input disturbance.

As can be seen from fig. (1), in ARC based technique, once the controller saturates, it cannot return to the unsaturated region of controller operation. On the other hand, as seen in fig. (2), in the proposed design the controller saturates temporarily, but returns to the unsaturated region. This can be explained as follows. Following an actuator fault, the performance requirements necessitate that the transients be suppressed using large feedback action. However, as the control input is limited, the transients cannot be suppressed effectively which further increases the transient error. This, in turn, demands larger control input, leading to a controller saturation scenario, which is unsalvageable. In the proposed approach, on the other hand, when the error is large, we sacrifice the model compensation to certain extent, and use the available control input to supply maximum possible feedback action $-k_i z_i$, such that $\dot{z}_i$ can be made negative, and the error can be reduced to an extent which allows the controller to be unsaturated. This can also reduce the required model-compensation. The problem of unreliable parameter estimation in presence of saturation should also be evident from the simulation results.

V. CONCLUSIONS

In this article, an indirect adaptive robust scheme was proposed to accommodate unknown actuator faults in presence of actuator magnitude constraints. The proposed scheme combines indirect adaptation and nested-saturation functions in a modified backstepping-like framework, such that if the actuator saturates after fault, the control-system ensures that it can pull out of the saturation. Furthermore, the indirect adaptation ensures that adaptation mechanism is not affected

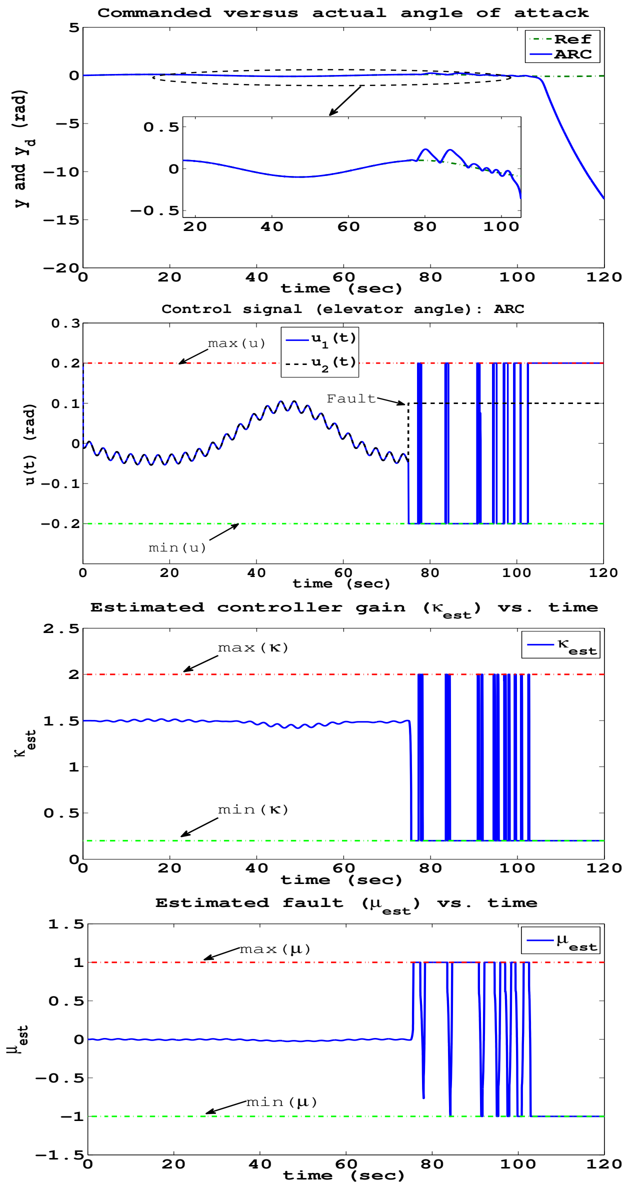

Fig. 1. Comparative results for stabilization in absence of disturbances

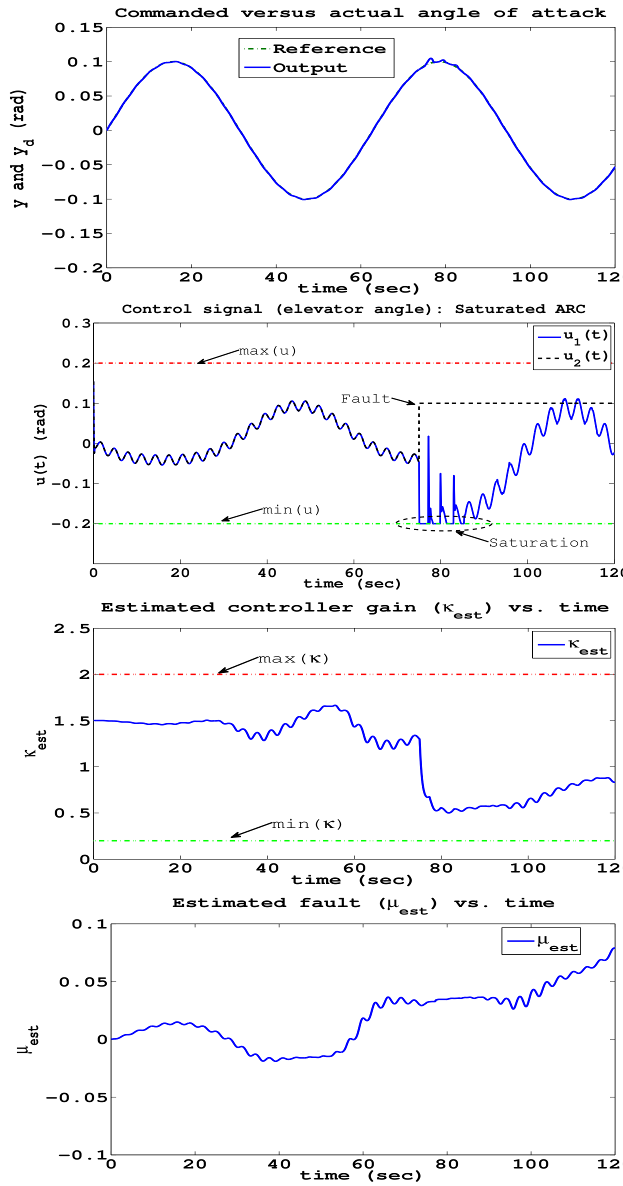

Fig. 2. Comparative results for stabilization in presence of disturbance

adversely in spite of faults. Comparative studies proved the efficiency of the proposed scheme in dealing with actuator faults in presence of saturation.

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