

# Sliding Mode/ $H_\infty$ Control of a Hydro-power Plant

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**Abstract**—This paper presents a new approach to Load Frequency Control (LFC) for a hydro-power plant. The control algorithm combines sliding mode control with  $H_\infty$  technique. The dynamic model of an entire hydro-power plant has been developed. The sliding hyperplane is constructed by  $H_\infty$  full state feedback control method. The ideal lossless turbine-penstock model is used in this paper to conduct the simulation. Results are compared with those from traditional PI and LQR designs. It is demonstrated that the proposed sliding mode control blended with  $H_\infty$  method improves the system performance under a load disturbance and parametric uncertainties.

## I. INTRODUCTION

Hydraulic power represents a major percentage of US renewable energy. Currently, it provides between 6 to 9 percents of total US electricity. Hydro-power plant usually consists of a mountain reservoir, penstock, wicket gate, turbine with a generator attached and draft tube. Some plants with long penstocks may have single or multiple surge tanks to reduce water hammer. Therefore, a hydro-power plant has complicated nonlinear dynamic characteristics. Since 1980s, there have been very few newly built hydro-power plants. But the turbines and the control systems have been constantly upgraded to meet the changes of demand [1]. Here, we will focus on the control system design. Load frequency control (LFC) is an important part of electric power system design. Since the load of the system is constantly changing, to provide quality power (constant frequency and voltage) a load frequency control system needs to be applied to the entire power system. That control goal is achieved through governors, which are used to control the wicket gate positions to regulate the water flow in order to adjust the power output to compensate the demand and stabilize the frequency. Generally, there are two types of governors, mechanical-hydraulic governor and electric-hydraulic governor [2]. Before 1960's, mechanical-hydraulic governors with flyball speed sensing were widely used. From 1970's, electric-hydraulic governors became popular with implied PID controller [3]. The classical PID controller methods have been developed in [4]-[6]. IEEE has published a guide for tuning PID coefficients based on system parameters such as water starting time constant in [7], [8]. Reference [9] indicated that coefficients of turbine model changed at different operating points. Later investigations [9]-[11] using state-space eigenvalue analysis showed that the boundary of a stable system depends not only on the system operating point but also on parameters of network [12] which it

connects to. Those parameters change from time to time. It is difficult to design a perfect PID controller for the system, since it might not be an optimal controller or might even be unstable controller in other situations. Above problems lead to the "Gain Schedule for PID" [2], [13] for which the controller has different coefficients at different operating points. But it does not guarantee performance against parametric uncertainties due to the wear and tear during the operation. In recent years, many advanced control methods such as fuzzy logic, adaptive control and neural networks [14], [15] have been studied. Reference [16] shows the application of  $H_\infty$  method to govern the system. There are also several studies on variable structure control (VSC) which is a more general method than the sliding mode [17]-[19]. Sliding mode control theory [20] is an effective robust control method. Similar to VSC, the sliding mode control method generates the control input which undergoes structure change to force the state trajectories of the system to reach the intersection of desired sliding hyperplanes and stay on it. This approach provides a robust performance and is insensitive to system uncertainties and external disturbances, provided certain matching conditions are satisfied [20].

In this paper, we propose a new control method for hydro-power system: sliding mode control (SMC) blended with robust optimal  $H_\infty$  theory, which is inspired by studies conducted in vibration area [20], [21], in which no assumptions on matching conditions are made. The  $H_\infty$  control provides the optimal choice of sliding surface and also minimizes the impact of external disturbances. We have studied the entire hydro-power plant system and a dynamic model is developed in MATLAB/Simulink software. The simulation results show the advantages of our method in comparison to other traditional control techniques.

## II. BASIC HYDRO-POWER PLANT DYNAMIC MODEL

The basic hydro-power plant scheme is shown in Fig. 1. The entire plant with the control system can be divided into four major components: Penstock, Electric-hydraulic servo system, Turbine and Generator. These components are modeled as follows. (Variables with bars are values normalized to their rated values)

### 1. Penstock:

General transfer function between turbine head and flow rate, which considers the elasticity of the pipe and water hammer, can be expressed as [2], [8]:

$$\frac{\Delta \bar{h}_t(s)}{\Delta \bar{u}_t(s)} = -[\bar{\phi}_p + z_p \tanh(t_{ep}s)] \quad (1)$$

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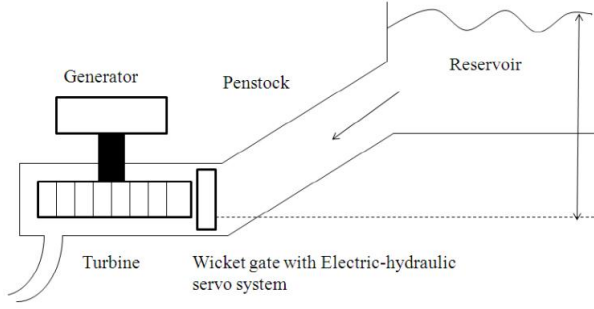


Fig. 1. Hydro-power plant scheme [2]

where  $\Delta\bar{h}_t$ : normalized value of change in turbine head (*p.u.*),  $\Delta\bar{u}_t$ : normalized value of change in turbine water velocity (*p.u.*),  $\bar{\phi}_p$ : penstock friction loss,  $z_p$ : hydraulic surge impedance of the penstock, and  $t_{ep}$ : elastic time constant of penstock (*sec.*). The symbol *p.u.* stands for per unit [2].

The function  $\tanh(t_{ep}s)$  is approximated as:

$$\tanh(t_{ep}s) = \frac{1 - e^{-2t_{ep}s}}{1 + e^{-2t_{ep}s}} = \frac{t_{ep}s \prod_{n=1}^{n=\infty} [1 + (\frac{t_{ep}s}{n\pi})^2]}{\prod_{n=1}^{n=\infty} [1 + (\frac{2t_{ep}s}{(2n-1)\pi})^2]} \quad (2)$$

If the penstock is short and water and pipe are considered to be incompressible, the simplified version of the transfer function (1) between water head and velocity is:

$$\frac{\Delta\bar{h}_t(s)}{\Delta\bar{u}_t(s)} = -t_w s \quad (3)$$

with  $t_w = \frac{lu_r}{a_g h_r}$

where  $t_w$ : water starting time constant at the rated load (*sec.*),  $u_r$ : rated value of velocity in the conduit (*m/s*),  $h_r$ : rated value of head in the conduit (*m*),  $a_g$ : acceleration due to gravity (*m/s<sup>2</sup>*),  $l$ : length of penstock (*m*).

2. Electric-hydraulic servo system [22]:

Since the hydro-power system requires a large force to move the wicket gate, the whole system usually contains two cascaded systems. One is pilot actuator and the other one is the main gate servo. The relationship between the input signal  $u$  (volt) and the change in the pilot actuator position  $\Delta\bar{x}_e$  is:

$$\frac{\Delta\bar{x}_e(s)}{u(s)} = \frac{1}{t_p s + 1} \quad (4)$$

where  $t_p$ : pilot valve and servomotor time constant (*sec.*)

The relationship between the pilot actuator output  $\Delta\bar{x}_e$  and the change in the gate servo (gate) position  $\Delta\bar{g}$  is:

$$\frac{\Delta\bar{g}(s)}{\Delta\bar{x}_e(s)} = \frac{1}{t_g s + 1} \quad (5)$$

where  $t_g$ : main servo time constant (*sec.*)

3. Turbine:

The simplified nonlinear hydro-turbine model has been obtained by treating the turbine as a valve [2], [8] as follows:

$$u_t = k_u g \sqrt{h_t} \quad (6)$$

$$p_m = k_p h_t u_t \quad (7)$$

where  $u_t$ : turbine water velocity (*m/s*),  $g$ : ideal gate opening,  $h_t$ : hydraulic head at gate (*m*),  $p_m$ : turbine power (*MW*),  $k_u, k_p$ : constant coefficients.

Equations (6) and (7) are linearized and normalized as follows:

$$\Delta\bar{u}_t = a_{11}\Delta\bar{h}_t + a_{12}\Delta\bar{g} \quad (8)$$

$$\Delta\bar{p}_m = a_{21}\Delta\bar{h}_t + a_{22}\Delta\bar{g} \quad (9)$$

with  $a_{11} = \frac{\partial u_t}{\partial h_t}$ ,  $a_{12} = \frac{\partial u_t}{\partial g}$ ,  $a_{21} = \frac{\partial p_m}{\partial h_t}$ , and  $a_{22} = \frac{\partial p_m}{\partial g}$

It should be noted that the partial derivatives with respect to rotor speed have been neglected because they are usually small. Values of coefficients in (8) and (9) are provided by Kundur [2]:  $a_{11} = 0.5$ ,  $a_{12} = 1.0$ ,  $a_{21} = 1.5$  and  $a_{22} = 1.0$ .

From (3), (8), and (9), the transfer function between power change and gate position change is:

$$\frac{\Delta\bar{p}_m(s)}{\Delta\bar{g}(s)} = \frac{1 - t_w s}{1 + 0.5t_w s} \quad (10)$$

The transfer function (10) is described as an ideal linear lossless turbine-penstock model.

4. Generator [2]:

The generator transfer function is described as:

$$\frac{\Delta\bar{\omega}(s)}{(\Delta\bar{p}_m(s) - \Delta\bar{p}_e(s))} = \frac{1}{ms + d} \quad (11)$$

where  $\Delta\bar{\omega}$ : normalized value of deviation of the rotor speed or frequency (*p.u.*),  $\Delta\bar{p}_e$ : normalized value of load deviation (*p.u.*),  $d$ : load-damping constant,  $m$ : total generator inertia constant (*MW - sec./MVA*).

The block diagram of different components of the system is shown in Fig. 2 with the conventional PI controller. If the plant contains surge tanks, the system is separated into two loops [2] such as ‘‘reservoir, tunnel, surge tank’’ and ‘‘surge tank, penstock, turbine’’; and then connected together to obtain the entire dynamic model.

### III. DESIGN OF $H_\infty$ /SLIDING MODE CONTROLLER

#### A. Sliding Mode Controller [20]

The fundamental concepts of sliding mode control theory can be found in [20], [23], [24]. Generally, there are two steps for sliding mode control design as follows.

First step is to determine the sliding hyperplane which is defined as follows:

$$s_l(t) = \mathbf{g}\mathbf{x}(t) = 0 \quad (12)$$

where  $\mathbf{x}$  is the state vector and  $\mathbf{g}$  is a vector which needs to be determined. It should be noted that there is only one sliding hyperplane as there is only one control input,  $u$ , equation (4).

Second step is to determine the control law.

The control input signal  $u$  which is the input voltage for the electric-hydraulic servo system is generated to achieve the following condition:

$$s_l \dot{s}_l < 0 \quad (13)$$

The condition (13) ensures that the system will reach the sliding hyperplane and remain on it.

## B. Problem Statement

Based on hydro-power plant dynamic model, the states for the entire plant model as shown in Fig. 1 can be described as

$$\mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = \begin{bmatrix} \int_0^t \Delta\bar{\omega} \\ \Delta\bar{x}_e \\ \Delta\bar{g} \\ \Delta\bar{p}_m \\ \Delta\bar{\omega} \end{bmatrix} \quad (14)$$

All of these states can be easily measured. Therefore, full state feedback sliding mode control will be designed.

The state-space model for the system without any controller is generated by (4), (5), (10), and (11):

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{b}u + \mathbf{d}\Delta\bar{p}_e \quad (15)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & 0 & 0 & 0 & 1 \\ 0 & -\frac{1}{t_p} & 0 & 0 & -\frac{1}{t_p r_p} \\ 0 & \frac{1}{t_g} & -\frac{1}{t_g} & 0 & 0 \\ 0 & -\frac{2}{t_g} & (\frac{2}{t_w} + \frac{2}{t_g}) & -\frac{2}{t_w} & 0 \\ 0 & 0 & 0 & \frac{1}{m} & -\frac{d}{m} \end{bmatrix} \quad (16)$$

$$\mathbf{b} = \begin{bmatrix} 0 \\ -\frac{1}{t_p} \\ 0 \\ 0 \\ 0 \end{bmatrix} \quad \text{and} \quad \mathbf{d} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ -\frac{1}{m} \end{bmatrix}$$

Since vectors  $\mathbf{b}$  and  $\mathbf{d}$  are independent, matching condition [20] is not satisfied. It will be assumed that

$$|\Delta\bar{p}_e| \leq q_u \quad (17)$$

where  $q_u$  is the upper bound of the load disturbance. Step 1. Constructing the sliding surface [20].

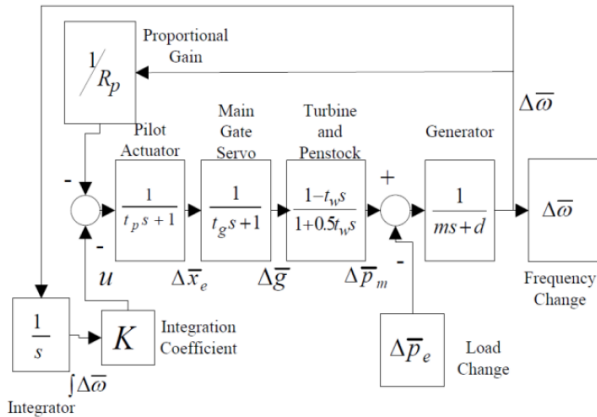


Fig. 2. PI control block diagram

First, the following similarity transformation is applied to the state vector  $\mathbf{x}(t)$ :

$$\mathbf{q}(t) = \mathbf{H}\mathbf{x}(t) \quad (18)$$

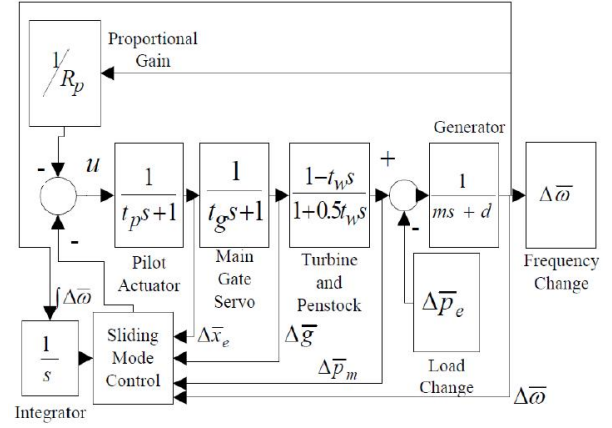


Fig. 3. Sliding mode control block diagram

where  $\mathbf{H} = [\mathbf{N} \quad \mathbf{b}]^T$  and  $\mathbf{N}$  is composed of basis vectors of the null space of  $\mathbf{b}^T$ . Combining (15) and (18).

$$\dot{\mathbf{q}} = \bar{\mathbf{A}}\mathbf{q}(t) + \bar{\mathbf{b}}u(t) + \bar{\mathbf{d}}\Delta\bar{p}_e \quad (19)$$

where  $\bar{\mathbf{A}} = \mathbf{H}\mathbf{A}\mathbf{H}^{-1}$ ,  $\bar{\mathbf{b}} = \mathbf{H}\mathbf{b}$  and  $\bar{\mathbf{d}} = \mathbf{H}\mathbf{d}$ .

Because of the definition of  $\mathbf{H}$  matrix, equation (19) can be written as

$$\dot{\mathbf{q}} = \begin{bmatrix} \bar{A}_{11} & \bar{A}_{12} \\ \bar{A}_{21} & \bar{A}_{22} \end{bmatrix} \begin{bmatrix} \mathbf{q}_1 \\ q_2 \end{bmatrix} + \begin{bmatrix} \mathbf{0} \\ \bar{b}_r \end{bmatrix} u + \begin{bmatrix} \bar{d}_1 \\ \bar{d}_2 \end{bmatrix} \Delta\bar{p}_e \quad (20)$$

where

$$\mathbf{q} = \begin{bmatrix} \mathbf{q}_1 \\ q_2 \end{bmatrix} \quad (21)$$

It should be noted that  $\mathbf{q}_1$  is a 4 dimensional vector where as  $q_2$  is a scalar. From (20),

$$\dot{\mathbf{q}}_1 = \bar{A}_{11}\mathbf{q}_1 + \bar{A}_{12}q_2 + \bar{d}_1\Delta\bar{p}_e \quad (22)$$

Here,  $\mathbf{q}_1$  and  $q_2$  are viewed as states and input for the construction of sliding hyperplane via full state feedback

$$q_2 = -\mathbf{k}\mathbf{q}_1 \quad (23)$$

where  $\mathbf{k}$  is the state feedback gain vector. In this case, sliding hyperplane becomes

$$s_l = [\mathbf{k} \quad 1] \mathbf{q} \quad (24)$$

Using equations (12) and (18), the sliding hyperplane vector  $\mathbf{g}$  is defined as:

$$\mathbf{g} = [\mathbf{k} \quad 1] \mathbf{H} \quad (25)$$

Because the matching condition [20] is not satisfied,  $\bar{d}_1 \neq 0$  and disturbance will affect the system response on the sliding hyperplane. The objective is to select the state feedback gain vector  $\mathbf{k}$  or equivalently the sliding hyperplane vector  $\mathbf{g}$  via  $H_\infty$  technique such that the effect of disturbance is minimized. For this purpose, the block diagram, Fig. 4, is developed, where the matrix  $\mathbf{M}$  is based on (22):

$$\begin{bmatrix} \dot{\mathbf{q}}_1 \\ e \\ \mathbf{y} \end{bmatrix} = \mathbf{M} \begin{bmatrix} \mathbf{q}_1 \\ \Delta\bar{p}_e \\ q_2 \end{bmatrix} \quad (26)$$

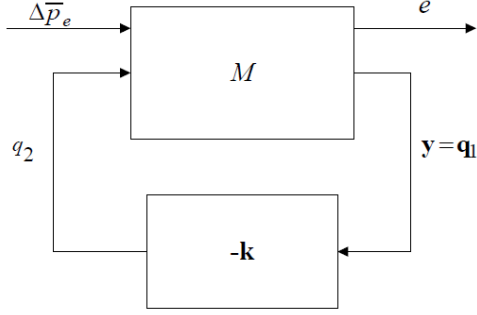


Fig. 4. A full state feedback control system

where

$$M = \begin{bmatrix} \bar{A}_{11} & \bar{\mathbf{d}}_1 & \bar{A}_{12} \\ C_1 & 0 & D_{12} \\ I_4 & 0 & 0 \end{bmatrix} \quad (27)$$

Matrices  $C_1$  and  $D_{12}$  are determined by cost function to be minimized. The objective  $\|T_{e\Delta\bar{p}_e}\|_{H_\infty} < \gamma$  ( $T_{e\Delta\bar{p}_e}$  is the transfer function from  $e$  to  $\Delta\bar{p}_e$ ) is equivalent to the minimization of the following objective function

$$J = \frac{1}{2} \int_0^\infty (e^T e - \gamma^2 \Delta\bar{p}_e^T \Delta\bar{p}_e) dt \quad (28)$$

where  $\gamma > 0$

Utilizing the expression of  $e$  from (26) and (27),

$$J = \frac{1}{2} \int_0^\infty (\mathbf{q}_1^T Q_{11} \mathbf{q}_1 + 2\mathbf{q}_1^T Q_{12} q_2 + q_2^T Q_{22} q_2 - \gamma^2 \Delta\bar{p}_e^T \Delta\bar{p}_e) dt \quad (29)$$

where  $Q_{11} = C_1^T C_1$ ,  $Q_{12} = C_1^T D_{12}$ ,  $Q_{22} = D_{12}^T D_{12}$ .

It should be noted that

$$e^T e = \mathbf{q}^T \begin{bmatrix} C_1^T \\ D_{12}^T \end{bmatrix} [ C_1 \quad D_{12} ] \mathbf{q} = \mathbf{x}^T \bar{Q} \mathbf{x} \quad (30)$$

where

$$\bar{Q} = H^T \begin{bmatrix} C_1^T \\ D_{12}^T \end{bmatrix} [ C_1 \quad D_{12} ] H \quad (31)$$

The gain matrix  $\mathbf{k}$  for the minimum value of  $J$  is [20]

$$\mathbf{k} = Q_{22}^{-1} (\bar{A}_{12} P + Q_{12}^T) \quad (32)$$

where  $P$  is the unique, symmetric, positive semi definite solution of the following ARE (Algebraic Riccati Equation):

$$P(\bar{A}_{11} - \bar{A}_{12} Q_{22}^{-1} Q_{12}^T) + (\bar{A}_{11}^T - Q_{12} Q_{22}^{-1} \bar{A}_{12}^T) P - P(\bar{A}_{12} Q_{22}^{-1} \bar{A}_{12}^T - \gamma^{-2} \bar{\mathbf{d}}_1 \bar{\mathbf{d}}_1^T) P + (Q_{11} - Q_{12} Q_{22}^{-1} Q_{12}^T) = 0 \quad (33)$$

MATLAB Robust Control Toolbox is used to solve the equation (33).

Step 2. Determine the control law to satisfy the reaching condition  $s_l \dot{s}_l < 0$ .

From (12),

$$\dot{s}_l = \mathbf{g}\dot{\mathbf{x}} = \mathbf{g}(A\mathbf{x} + \mathbf{b}u + \mathbf{d}\Delta\bar{p}_e) \quad (34)$$

Since  $s_l$  is a scalar,

$$s_l \dot{s}_l = s_l(\mathbf{g}A\mathbf{x} + \mathbf{g}\mathbf{b}u + \mathbf{g}\mathbf{d}\Delta\bar{p}_e) \quad (35)$$

TABLE I  
PARAMETER VALUES FOR SIMULATION

Symbol	Quantity	Unit
$t_w$	1.3	Second
$t_g$	0.2	Second
$t_p$	0.02	Second
$R_p$	0.5	Unit less
$m$	6	Second
$d$	1	Second

The input signal  $u$  can be divided into two parts.

$$u = u_{eq} + u_{un} \quad (36)$$

where

$$\mathbf{g}A\mathbf{x} + \mathbf{g}\mathbf{b}u_{eq} = 0 \quad (37)$$

Therefore,

$$u_{eq} = -(\mathbf{g}\mathbf{b})^{-1}(\mathbf{g}A\mathbf{x}) \quad (38)$$

And,  $u_{un}$  is used to make sure that the condition (13) is satisfied; i.e.,

$$s_l \dot{s}_l = s_l(\mathbf{g}\mathbf{b}u_{un} + \mathbf{g}\mathbf{d}\Delta\bar{p}_e) < 0 \quad (39)$$

or

$$u_{un} = -(\mathbf{g}\mathbf{b})^{-1} e_u \text{sgn}(s_l) \quad (40)$$

where  $e_u = |\mathbf{g}\mathbf{d}| q_u + \eta$  and  $\eta > 0$ .

In order to eliminate the chattering effect,  $\text{sgn}(s_l)$  is replaced by a saturation function [20]

$$\text{sat}(s_l) = \begin{cases} \text{sgn}(s_l) & \text{if } |s_l| > \rho \\ s_l/\rho & \text{otherwise} \end{cases} \quad (41)$$

where  $\rho$  is the boundary layer thickness around the sliding hyperplane.

#### IV. SIMULATION RESULTS

The values of our simulation model parameters are in Table 1. The controller is designed with  $q_u = 0.5$ ,  $\eta = 0.05$  and  $\rho = 0.05$ . Simulation is conducted with a step load change of 0.03 (*p.u.*).

Since the main objective is to minimize deviations in generator speed and change in the power output, the cost function  $\bar{Q}$  in (31) is chosen as:

$$\bar{Q} = \begin{bmatrix} 250 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 50 & 0 \\ 0 & 0 & 0 & 0 & 250 \end{bmatrix} \quad (42)$$

In Fig. 5,  $\Delta\bar{\omega}$  is plotted for three types of control systems: PI,  $H_\infty$ /sliding mode and LQR control ( $r = 1$ ) for which the objective function is defined as

$$J = \frac{1}{2} \int_0^\infty (\mathbf{x}^T \bar{Q} \mathbf{x} + r u^2) dt \quad (43)$$

It is obvious that the sliding mode blended with  $H_\infty$  method gives the shortest responding time and lowest overshoot value as shown in Fig. 5.

When the system is controlled by  $H_\infty$ /sliding mode method, the response of the system is quite insensitive to parametric uncertainties, Fig. 6 for -20% deviation in turbine model parameter  $t_w$  only and -10% difference on all parameters ( $t_w, t_g, t_p, m,$  and  $d$ ). Note that the robustness of the traditional PI controller is significantly less as shown in Fig. 7

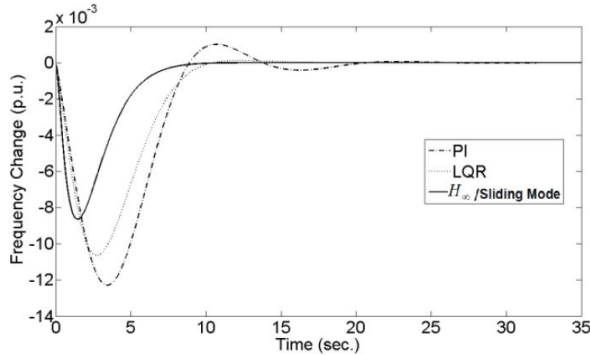


Fig. 5. Frequency deviation  $\Delta\omega$  versus time under constant load change

## V. CONCLUSIONS

A sliding mode controller blended with  $H_\infty$  full state feedback control has been developed for governing hydro-power plant. The sliding hyperplane is constructed based on the dynamic model of the plant by full state feedback  $H_\infty$  theory. The saturation function is used to eliminate the chattering effect. The performance of our control method in the presence of disturbance and parametric uncertainties has been evaluated via MATLAB/Simulink software. The results from simulation have shown the advantages which our method brings to the controller design. Currently, we are also investigating the performance of more complicated dynamic model under our proposed control method. Also, controllers will be designed by simultaneously considering parametric uncertainties and external disturbances.

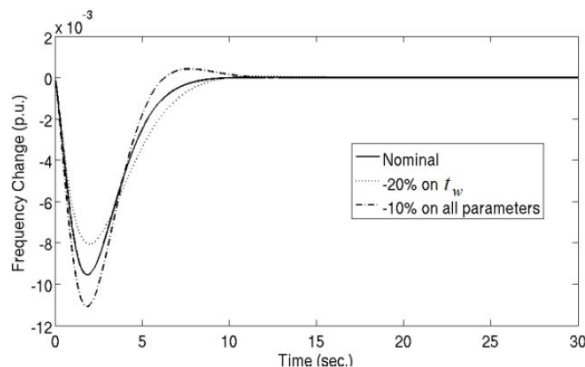


Fig. 6. Frequency deviation  $\Delta\omega$  with parametric uncertainties under  $H_\infty$ /sliding mode control

## VI. ACKNOWLEDGEMENT

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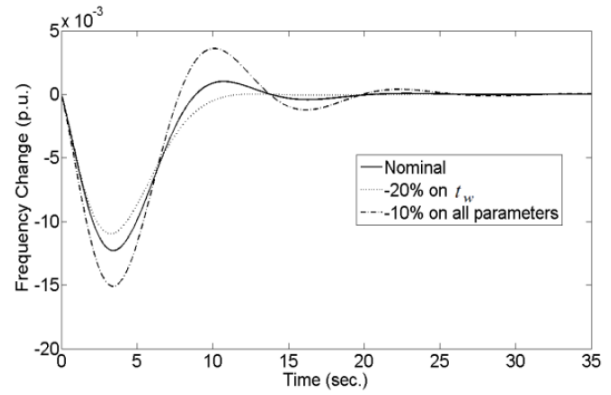


Fig. 7. Frequency deviation  $\Delta\omega$  with parametric uncertainties under PI control

gram.

## REFERENCES

- [1] T. Letcher *Future Energy: Improved, Sustainable and Clean Options for our Planet*. Elsevier Science, 2008.
- [2] P. Kundur, *Power System Stability and Control*. McGraw-Hill, Inc., 1994.
- [3] J. Jiang, "Design of an Optimal Robust Governor for Hydraulic Turbine Generating Units", *IEEE Trans Energy Conversion*, vol. 10, no.1, Mar. 1995.
- [4] H. M. Paynter, *A Palimpsest on the Electronic Analog Art*. George. A. Philbrick Research, Inc., 1955.
- [5] L. M. Hovey, "Optimum Adjustment of Hydro Governors on Manitoba Hydro System", *AIEE Trans*, vol. 81, pp. 581-586, Dec. 1962.
- [6] M. H. Chaudhry, "Governing Stability of a Hydro Electric Power Plant", *Water Power*, pp. 131-136, 1970.
- [7] IEEE Committee Report, "Dynamic Models for Steam and Hydro Turbines in Power System Studies", *IEEE Trans Power App. Syst.*, vol. 92, pp. 1904-1915, Nov./Dec. 1973.
- [8] Working Group on Prime Mover and Energy Supply Models for System Dynamic Performance Studies, "Hydraulic Turbine and Turbine Control Models for System Dynamic Studies", *IEEE Trans on Power Systems*, vol. 7, pp. 167- 179, Feb. 1992.
- [9] E. F. Hill and D. H. Thorne, "Field Testing and Simulation of Hydraulic Turbine Governor Performance", *IEEE Trans. Power App. Syst.*, vol. 93, pp. 1183-1191, Feb. 1973.
- [10] D. H. Thome and E. F. Hill, "Extensions of Stability Boundaries of a Hydraulic Turbine Generating Unit", *IEEE Trans Power App. Syst.*, vol. 94, pp. 1401-1408, Jul./Aug. 1975.
- [11] D. T. Phi, E. J. Bourque, D. H. Thome, and E. F. Hill, "Analysis and Application of the Stability Limits of a Hydro-generating Unit", *IEEE Trans. Power App. Syst.*, vol. 100, no. 7, pp. 3203-3211, Jul. 1981.
- [12] K. Natarajan, "Robust PID Controller Design for Hydroturbines", *IEEE Trans Energy Conversion*, vol. 20, no.3, Sep. 2005
- [13] G. Oreind, L. Wozniak, J. Medanic and T. Whittemore, "Optimal PID Gain Schedule for Hydrogenerators-design and Application", *IEEE Trans. on Energy Convers*, vol. 4, no. 3, pp. 300-307, Sep. 1989.
- [14] O. P. Malik and Y. Zeng, "Design of a Robust Adaptive Controller for a Water Turbine Governing System", *IEEE Trans. Energy Conversion*, vol. 10 no. 2, pp. 354-359, Jun. 1995.
- [15] G. K. Venayagamoorthy and R.G.Harley, "A Continually Online Trained Neurocontroller for Excitation and Turbine Control of a Turbogenerator", *IEEE Trans. Energy Conversion*, vol. 16, no. 3, pp. 261-269, Sep. 2001.
- [16] I. Eker, "Governors for Hydro-turbine Speed Control in Power Generation: a SIMO Robust Design Approach," *Energy Conversion and Management*, Nov. 2003.
- [17] A. Y. Sivaramakrishnan, M.V. Hariharan and M.C.Srisailam, "Design of Variable Structure Load-frequency Controller using Pole Assignment Technique", *Int. J. Control*, vol. 40, no. 3, pp. 487-498, Sep. 1984.

- [18] W. C. Chan and Y. Y. Hsu, "Optimal Control of Electric Power Generation Using Variable Structure Controllers", *Electric Power Systems Research*, vol. 6, pp. 269-278, Apr. 1983.
- [19] N. N. Bengiamin and W.C.Chan, "Variable Structure Control of Electric Power Generation," *IEEE Trans. Power App. Syst.*, vol. 101, no. 2 pp. 376- 380, Feb. 1982.
- [20] A. Sinha, *Linear Systems: Optimal and Robust Control*. CRC Press Taylor & Francis Group, 2007.
- [21] M. C. Pai and A. Sinha, "Active Control of Vibration in a Flexible Structure with Uncertain Parameters Via  $H_\infty/\mu$  and Sliding Mode Techniques", *ASME Vibration and Noise Control*, vol. DE-Vol. 97/DSC-Vol. 65, pp. 1-7, 1998.
- [22] J. Machowski, J. Bialek, and J. Bumby, *Power System Dynamics and Stability*. John Wiley & Sons, 1997.
- [23] V. Utkin, J. Guldner, and J. Shi, *Sliding Mode Control in Electro-Mechanical Systems*. CRC Press, 2009.
- [24] V. Utkin, *Sliding Modes in Control and Optimization*. Springer, 1992.