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Analyzing the Performance Index for a Hybrid Electric Vehicle

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Abstract— **The definition of a performance index for the optimization design and optimal control problem of a Hybrid Electric Vehicle is not often considered and analyzed explicitly. In literature, there is no study about proposing a method of building or evaluating whether a performance index is appropriate. In this paper a method of objectively analyzing the performance index for the optimal control problem of a parallel Hybrid Electric Vehicle is introduced. The correlations and interdependencies among the objectives of the performance index are addressed by using the Singular Value Decomposition method. It is found that a simplified performance index consisting of fuel consumption and comfort can be obtained without sacrificing the vehicle performance compared to the case with the original one including fuel consumption, comfort and driveability.**

I. INTRODUCTION

In a Hybrid Electric Vehicle (HEV), the number of design and control objects is larger than that of a conventional vehicle due to the presence of the secondary power source which is mostly an electric machine powered by the battery system. The optimization and optimal control design problem have been actively chosen to achieve the objectives and optimal parameters for HEVs. Coordination between the design objectives and the design parameters for the hybrid propulsion system in general will bring better fuel economy, emissions, performance driveability and comfort, etc. The design process can be performed through a 'parametric optimization procedure', see [1], to investigate the system's parameter variations on a certain design objective and as a result, the optimal set of the design parameters can be obtained. Regardless of the chosen method for the design and control problems, the starting point is to define a performance index or a cost functional which consists of a single or multiple objectives. For the case of multi-objective problems, the cost functional is built up by summing all the objectives multiplied by the corresponding weight factors. However, defining an appropriate cost functional with the optimum weight factors for a specific hybrid powertrain is a complicated task. This is due to the fact that there might be strong interdependencies of the design parameters and the unknown sensitivities of the design parameters to the design objectives, [1]. So a cost functional is appropriate when it satisfies the tradeoffs among objectives and it can reduce the parameter interdependencies of the design process [1]–[3], [19].

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In literature, there is no study about designing and analyzing the performance index for a hybrid powertrain system. The cost functional is defined based upon the powertrain configurations, the method of parametric optimization design, the approach of designing the controller and the designers' subjective reasonings. The question of whether the proposed performance index is suitable is mostly not addressed. In this paper, a method of objectively analyzing the performance index of a HEV is proposed. The method uses the Singular Value Decomposition (SVD) technique to analyze the structural behavior of the powertrain system based on data obtained from the correspondingly optimal control problem.

The content of this paper is organized as follows: in Section II, the performance indexes for the optimization design and the optimal control problems of HEVs are reviewed. In Section III, a multi-objective optimal control problem is formulated for a parallel HEV. In Section IV, a data-based structural analysis method based on the SVD is discussed. Analysis of the interdependencies among objective is given. Application to the parallel HEV is addressed to show that a simplified cost functional can be attained for the optimal control problem. Finally, conclusions and future research will be outlined in Section V.

II. A REVIEW OF PERFORMANCE INDEXES

Optimal control problems for the Energy Management Systems (EMS) of HEVs are formulated by defining a cost functional or performance index describing the vehicle performances. The optimal solutions are obtained by minimizing or maximizing the defined cost functional over a specified drive cycle meanwhile fulfilling the vehicle dynamics and constraints. The most concerned performance index for the optimal controls of HEVs is the fuel consumption, see [1]–[5] and the references therein. The cost functional can be expressed as follows.

$$J = \sum_{k=0}^{N-1} \dot{m}_f(k)\Delta t$$

The EMSs usually require the battery State Of Charge (SOC) at the beginning and the end of the drive cycle to be equal. However this never happens in practice. Therefore, it is often assumed that the SOC needs to stay within a predefined operating range to avoid any damage to the battery. The cost functionals proposed in [6]–[9] took into account the SOC deviation as a penalty besides the fuel consumption.

Emission regulations have been increasingly restricted due to the environmental problem. Hence, the engine emissions such as CO_2, NOx, CO, PM, HC can be incorporated into the cost functional in order to achieve the optimum set of fuel economy and emissions, see in [2] and the references therein. Furthermore, the cost functionals consisting of the fuel consumption, the SOC deviation and the engine emissions can be seen in [2] and the references therein.

Driveability is a comprehensive terminology for vehicle responsiveness. It can be determined as the instant available engine power, or the instant available drive torque or the tracking ability of vehicle. In [10], the cost functional was proposed including the fuel consumption, the SOC deviation penalty and the tracking ability.

The main comfort indexes for passengers can be listed as a jerk, a shifting time or a shifting frequency cost, etc. The authors in [11]–[14] defined the cost functional for the optimal control problems of HEVs consisting of the fuel consumption, the equivalent energy consumption from battery, the emissions, the SOC deviation penalty and the comfort indexes.

The more complex cost functional consisting of the fuel consumption, the SOC deviation penalty, the driveability and the comfort indexes can be found in [15]. Due to the complexity of this cost functional, the authors suggested a method to solve for optimal solution by eliminating the driveability and comfort criteria out of the cost functional for a simplified performance index. Then, these criteria are checked and ensured after the optimal solution was already found.

III. OPTIMAL CONTROL FOR A PARALLEL HEV

A. Powertrain topology

The parallel hybrid powertrain topology under investigation in this paper is shown in Fig.1.

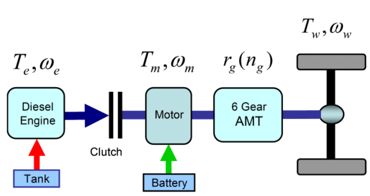

Fig. 1: A parallel hybrid electric vehicle topology.

With aiming at deriving an EMS based on optimal control, static models of the powertrain components are chosen when dynamics are faster than 1Hz. Hence, a discretized model with time step of one second is chosen for this vehicle model. The clutch system is considered as a switch to connect and to disconnect the engine immediately to and from the driveline.

The longitudinal motion of the vehicle is given by

$$\omega_w(k+1) = \omega_w(k) + \frac{1}{J_v}\big(T_w(k) - T_{load}(k)\big)\Delta t \tag{1}$$

with

$$\omega_e(k) = \omega_m(k); \quad \omega_w(k) = \frac{\omega_e(k)}{r_g\big(n_g(k)\big)}$$

where: $\omega_w(k)$ is the vehicle speed; $T_w(k)$ denotes the required torque at the wheel; $T_{load}(k)$ is the load torque acting on the vehicle; J_v is the vehicle inertia; $r_g\big(n_g(k)\big)$ denotes the discrete gear ratio depending on the gear position $n_g(k)$; Δt is the length of time step. The gearbox efficiency is assumed to be 100%.

1) Start-stop system: The start-stop system can be modeled by a decision variable $s_e(k)$ to control the engine on or off state. When engine is off, no fuel consumed by the engine.

$$s_e(k) = \begin{cases} 1, & \text{if engine on,} \\ 0, & \text{if engine off.} \end{cases} \tag{2}$$

2) Engine model: the engine fuel rate model $\dot{m}_f(k)$ is defined by interpolation from the static fuel rate map. Constraints on engine speed $\omega_e(k)$ and engine power $P_e(k)$ are as follows.

$$\omega_{e\_min} \leq \omega_e(k) \leq \omega_{e\_max} \tag{3a}$$
$$0 \leq P_e(k) \leq P_{e\_max}\big(\omega_e(k)\big) \tag{3b}$$

3) Electric machine: we assume the efficiency η is constant. Therefore the power flowing in and out of the electric machine in motoring mode and generating mode is expressed as follows.

$$P_m(k) = \eta P_{elec}(k), \quad \text{motoring} \tag{4a}$$
$$P_m(k) = \frac{1}{\eta}P_{elec}(k), \quad \text{generating} \tag{4b}$$

Constraints on electric machine speed $\omega_m(k)$ and electric machine power $P_m(k)$

$$0 \leq \omega_m(k) \leq \omega_{m\_max} \tag{5a}$$
$$P_{m\_min}\big(\omega_m(k)\big) \leq P_m(k) \leq P_{m\_max}\big(\omega_m(k)\big) \tag{5b}$$

4) Battery system: the battery dynamical system describing the battery state of energy $e_s(k)$ is assumed as a function of chemical power $P_s(k)$.

$$e_s(k+1) = e_s(k) + P_s(k)\Delta t \tag{6}$$

Constraints on the battery system are as follows.

$$P_{s\_min} \leq P_s(k) \leq P_{s\_max} \tag{7}$$
$$E_{s\_min} \leq e_s(k) \leq E_{s\_max} \tag{8}$$

The battery chemical power P_s is modeled as a quadratic function of electrical power P_{elec} expressed by (9).

$$P_s(k) \approx b_0 P_{elec}^2(k) + b_1 P_{elec}(k) + b_2 \tag{9}$$

5) Automated Manual Transmission (AMT): the next gear position $n_g(k+1)$ is expressed through the current gear position $n_g(k)$ and the shift command $u_g(k)$ as follows.

$$n_g(k+1) = n_g(k) + u_g(k) \tag{10}$$

The shift command at step k is as follows

$$u_g(k) = \begin{cases} -1, & \text{downshift} \\ 0, & \text{sustaining} \\ 1, & \text{upshift} \end{cases} \tag{11}$$

Constraint on gear position is as follows.

$$1 \leq n_g(k) \leq 6 \tag{12}$$

6) Power flow model: at any time step k, the power flow equilibrium in the prototype hybrid powertrain is expressed as

$$P_e(k) = P_w(k) + P_m(k) \tag{13}$$

B. Multi-objectives cost functional proposal

The operating points for the hybrid powertrain topology are governed by the gear position and the power split ratio between power produced by the diesel engine and the electric machine. Therefore, optimizing power split ratio and gear shifting improve fuel economy more than the case of optimizing only power split ratio. However, changing the operating points by extra optimizing the gear shifting would affect driveability (power reserve) and comfort (shifting frequency) of the powertrain system. The future trends of designing HEVs should be focused on multi-objective optimization and optimal control problems in which defining an appropriate cost functional is a core to achieve the vehicle performances.

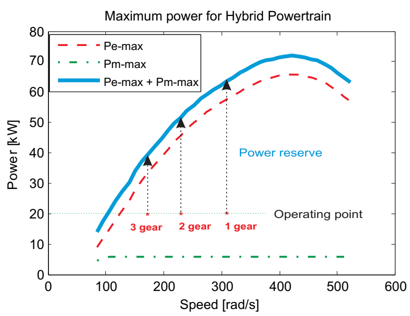

Fig. 2: Power reserve of the hybrid powertrain of a 68kW engine and a 6kW electric machine.

Demonstrated in Fig.2, the power reserve is decreased when vehicle performs up-shift and vice versa. Intuitively, when the driver allows more up-shifts, the engine operating points will be moved to a region of higher torque and lower speed which results a low power reserve. Vice verse, when reducing up-shifts, the engine operates at region of lower torque and higher speed which increases power reserve. Furthermore, fuel flow is a function of engine power, therefore engine power reserve has correlations with fuel consumption and shifting cost. To balance the tradeoffs among those objectives, the cost functional for the optimal control problem of this powertrain is proposed as follows.

$$\begin{aligned} J &= \sum_{k=0}^{N-1} L\big(x(k), u(k), k\big) \\ &= \sum_{k=0}^{N-1} \Big[\dot{m}_f(k) + w_1|u_g(k)| + w_2\big(\Delta P_e(k) + \Delta P_m(k)\big)^{-1}\Big]\Delta t \end{aligned} \tag{14}$$

In (14), the first term represents the fuel consumption, the second term represents the comfort related to the gear shift; the third term stands for the driveability related to the power reserve for both the engine and electric machine. The weight factors w_1 and w_2 are designed to meet the balances among the objectives.

C. Optimal control problem

The hybrid powertrain system dynamics and constraints described by (1)-(13) can be rewritten in a generic form as follows.

$$x(k+1) = x(k) + f\big(x(k), u(k), k\big) \tag{15}$$

$$C_{eq}\big(x(k), u(k)\big) = 0 \tag{16}$$

$$C_{in}\big(x(k), u(k)\big) \leq 0 \tag{17}$$

wherein: $x(k) = \big[s_e(k),\ \ e_s(k),\ \ n_g(k)\big]$; and $u(k) = \big[P_s(k),\ \ u_g(k)\big]$.

Problem: given a drive cycle $v(k)$ with time length of N, find an optimal control law $u^*(k)$ that minimizes the fuel consumption cost functional J as in (14) over the entire drive cycle, subject to:

$$(15)\text{-}(17)$$

$$x(N) = x(0) \tag{18}$$

For reasons of acceptable comfort, the set of discrete shift command values is chosen as $[-1, 0, 1]$ to avoid a large variation of engine speed for a certain shift at a certain time step k. One gear down or upshift or sustaining for each time step of one second are reasonably, because the average shifting time for a standard AMT is typically less than one second.

Dynamic Programming (DP) [20] is well known as a powerful tool to solve a non-linear optimization problem with constraints while obtaining a globally optimal time-variant, state feedback solution. To apply DP to the optimal control problem (14)-(18), we need to

- grid the state variables $x(k)$;
- grid the corresponding control variables $u(k)$;
- calculate the cost matrix for the whole drive cycle;

Then, the optimal cost-to-go path is defined as

Step $k = N$*:*

$$J_N^* = 0, \tag{19}$$

Step $k, \quad 0 \leq k < N$*:*

$$\begin{aligned} J_k^*\big(x_i(k), k\big) = \min_{u_{ij}(k)} \Big[& L\big(x_i(k), u_{ij}(k), k\big) + \\ & + J_{k+1}^*\big(x_j(k+1), k+1\big)\Big] \end{aligned} \tag{20}$$

Repeatedly solving the optimal cost-to-go path backwards until $k = 0$, the optimal solution $u^*(k)$ is obtained correspondingly with a specific initial value of $x(0)$.

D. Simulation results and discussions

1) Baseline problem: the baseline optimal control problem for the hybrid powertrain described in Section III-A is formulated with the cost functional consists of only the fuel consumption. DP is applied to solve for the globally optimal engine start decision and optimal power split ratio between the engine and the electric machine. The New European Drive Cycle (NEDC) is chosen for simulation. The prescribed gear shift schedule is used to shift the transmission, see Fig.3. Simulation result of the fuel consumption is 332.33gr.

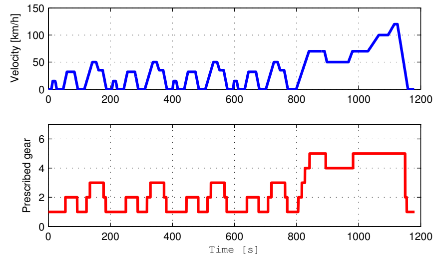

Fig. 3: NEDC and a prescribed gear shift schedule.

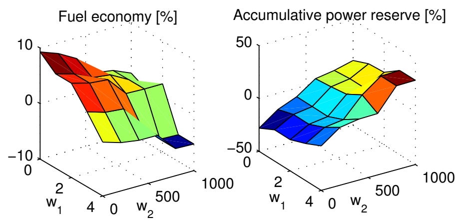

Fig. 4: Fuel economy and accumulative power reserve improvements for a HEV with mild configuration.

2) Simulation results and discussions: the primary power source is a diesel engine of maximum power of 68kW. The secondary power source is a 6kW electric machine with a battery system of 6Ah, 110V. The AMT type is of 6 gears. The sizings of the electric machine and battery pack classify the vehicle as a mild hybrid configuration.

At $w_1 = 0$ and $w_2 = 0$, the optimal control problem described in III-C optimizes the system for only fuel economy improvement up to 9.3% meanwhile the accumulative power reserve is degraded up to 28.1% compared to the baseline.

With the purpose of studying the effects among the fuel economy, comfort and driveability of the cost functional (14), the weight factors w_1 and w_2 are varied as in (21)-(22).

$$w_1 \in \{1, 2, 3, 4\} \tag{21}$$
$$w_2 \in \{200, 400, 600, 800, 1000\} \tag{22}$$

Simulation results of the fuel economy and the accumulative power reserve improvements for every combination of w_1 and w_2 are shown in Fig.4. It can be observed that by increasing the weight factors for comfort and driveability separately or simultaneously leads to a decrease of fuel economy and an increase of power reserve respectively. Apparently, the objectives in the cost functional (14) affect each other. Therefore tuning the weight factors to achieve the Pareto optimal set is not straightforward.

Reducing the cost functional to a simple form so that the weight factors can be tuned easily meanwhile the control algorithm can achieve the desired performances is the ultimate goal. In order to do so, we need to study the correlations and dependencies among the objectives of the performance index. Then the dependent ones can be eliminated so that a simplified cost functional can be obtained. Clearly, this boils down to a rank analysis which will be introduced next.

IV. DATA BASED STRUCTURAL ANALYSIS

SVD is a widely used method in the structural analysis and rank analysis of systems. A similar usage of rank conditions using the SVD is to measure the interactions between the inputs and the outputs of the MIMO systems [16], [17]. Furthermore, a special application of the SVD method is that it can be used to evaluate the system's performance by doing the data-based structural analysis [18]. A brief introduction of the SVD-based structural analysis is given as followings.

A. Singular value decomposition

Consider a linear algebraic equation as

$$Ax = b; \quad A \in \mathbf{R}^{NxM} \tag{23}$$

SVD of the matrix A is given

$$A = U\Sigma V^T$$

wherein:

$$U = \left[u_1, u_2, \cdots u_N\right] \in \mathbf{R}^{NxN}$$
$$V = \left[v_1, v_2, \cdots v_M\right] \in \mathbf{R}^{MxM} \tag{24}$$
$$\Sigma = diag(\sigma_i|0) \in \mathbf{R}^{NxM}$$

The column vectors v_i's of V and u_i's of U are known as the right input singular vectors and left output singular vectors of matrix A, respectively. They hold an important relation:

$$Av_i = \sigma_i u_i \tag{25}$$

This equation states that each right input singular vector is mapped by the system A onto the corresponding left output singular vector with the magnification factor being the corresponding singular value. From (25), if $\sigma_i = 0$, then $Av_i = 0$, i.e., v_i forms the null space of A. Thus any change made in the direction corresponding to the singular value $\sigma_i = 0$ maps into the zero vector in the output space. In other words, any input in that direction is not reflected in the output space.

By using (24), the solution x to the (23) is directly obtained as linear combination of the scaled right input singular vectors

$$x = \sum_i^{rank(A)} \frac{u_i^T \cdot b}{\sigma_i} v_i = \sum_i^{rank(A)} q_i v_i \tag{26}$$

Any target vector b can be achieved by a linear combination of the scaled left output singular vectors as

$$b = \sum_i^{rank(A)} \sigma_i q_i u_i = \sum_i^{rank(A)} (u_i^T \cdot b) u_i \tag{27}$$

B. Structural analysis

In the structural analysis of this study, v_i is called input mode vector and u_i is call output mode vector. From (26) the control input vector x is a linear combination of input mode vectors v_i's multiplied by the corresponding input weight factors q_i's. So, the designer can assess the control input distribution pattern through the input weight factors q_i's.

Similarity, we can use the output mode vectors u_i's to assess tradeoffs among the contributions of them to the target vector b as in (27).

For example, consider the input weight factor q_1 in (27) to be much larger than the other weights in the linear combination. Since σ_1 is the largest singular value, $q_1\sigma_1$ is dominant among the coefficients of the linear combination then the actual target vector b will be largely dependent on the first output mode vector u_1.

Input mode vectors v_i's and output mode vectors u_i's obtained from SVD are paired by the corresponding singular values σ_i's. Thus, truncating any input mode vector v_i would affect the corresponding output mode vector u_i, and vice versa. Truncating the output mode vector u_i that is part of the linear combination of the target vector b would degrade the total performance. If, however, the truncated output mode vector contributes little to the target vector b, then there would be little performance degradation and it would be possible to save control effort equal in size to the weighted input mode vector that is paired with the truncated output mode vector.

Cutting off any input mode vector v_i that consumes large control input, i.e., one with a large weight q_i, but contributes little to the total performance of target vector b, will reduce the complexity of control system design with little performance degradation.

From (27), it is clear that the output mode vector u_i that lines up more towards the target vector b will contribute more to b. Therefore, the value of inner product of $(u_i^T \cdot b)$ can be used to measure the performance dependency of the target vector b on each output mode vector u_i. This is called the *collinearity* of two vector u_i and b. Using this indicator, we can identify which output mode vector u_i is dominant in achieving the target vector b and which output mode vector u_i contributes little.

C. Application to the hybrid powertrain system

The structural analysis of the cost functional of the hybrid powertrain system will be carried out by SVD method. The analysis is not based on the system model but is based on data obtained from simulating the system model governed by the optimal control algorithm described in Section III. The data is the discrete optimal values of the cost fucntional along the drive cycle.

1) Cost functional analysis: the cost functional (14) is rewritten as follows.

$$J = \sum_{k=1}^{N} \Big[m_{fe}(k) + m_{sh}(k) + m_{pr}(k) \Big] \tag{28}$$

wherein:

$$m_{fe}(k) \triangleq \dot{m}_f(k)\Delta t$$
$$m_{sh}(k) \triangleq w_1 |u_g(k)| \Delta t$$
$$m_{pr}(k) \triangleq w_2 \big(\Delta P_e(k) + \Delta P_m(k)\big)^{-1} \Delta t$$

The discrete optimal cost values of (28) are stacked along the discrete time dimension of the drive cycle to create a

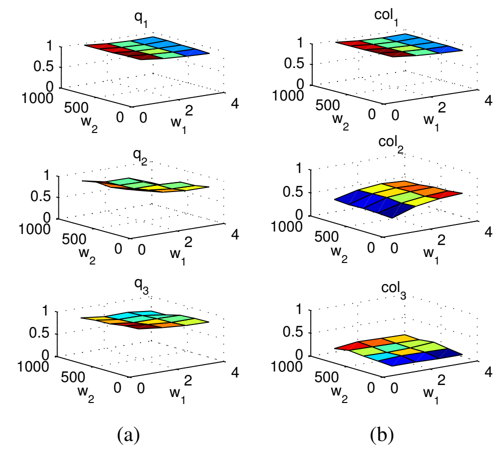

Fig. 5: The input weight factor q_i in (a); the collinearity col_i in (b).

linear algebraic equation as

$$Ax = b \tag{29}$$

with

$$A = \begin{bmatrix} m_{fe}(1) & m_{sh}(1) & m_{pr}(1) \\ m_{fe}(2) & m_{sh}(2) & m_{pr}(2) \\ \vdots & \vdots & \vdots \\ m_{fe}(N) & m_{sh}(N) & m_{pr}(N) \end{bmatrix}, \quad b = \begin{bmatrix} J(1) \\ J(2) \\ \vdots \\ J(N) \end{bmatrix}$$
$$x = \begin{bmatrix} 1 & 1 & 1 \end{bmatrix}^T \tag{30}$$

In (29), the vector x acts as the control input vector imposed on the weighted objectives, so it is chosen as $[1 \quad 1 \quad 1]^T$ to respect the original optimal values of the objectives. The target vector b contains the optimal values of the performance index at every time step. Note that the weight factors of the objectives varied in the certain ranges as in (21)-(22) are merged into the matrix A.

SVD method presented in Section IV is applied to the linear algebraic system (29)-(30) to study the correlations and interdependencies among the objectives for all combinations of the weight factors w_1 and w_2.

2) Simulation results and discussions: the computational results of SVD analyses for the input weight factors q_i's and the normalized collinearities col_i's for all combinations of w_1 and w_2 are shown in Fig.5.

In Fig.5a, q_1 varies from 0.86 to 0.97, q_2 varies from 0.5 to 0.94 and q_3 varies from 0.67 to 0.9. By looking at the values of collinearities in Fig.5b, we can recognize that the values of col_i's are totally different each other. col_1 approximately varies from 0.86 to 0.97; col_2 approximately varies from 0.23 to 0.51 whereas col_3 roughly ranges between 0.05 and 0.14 respectively.

We can observe that: the maximum contributions of output mode vectors u_2 and u_3 to the target vector b considered on the whole ranges of w_1 and w_2 are 26% $(= 0.51^2 * 100\%)$

and 2% $(=0.14^2*100\%)$ respectively. Meanwhile, the contributions of input mode vectors v_2 and v_3 to the control input vector x are almost equivalent to that of v_1 (observed from q_1, q_2 and q_3). This gives very important information on the fact that the output mode vector u_3 pays only a very small contribution to the target vector b. Therefore, truncating the input mode vector v_3 will save a large amount of control effort without serious performance degradation.

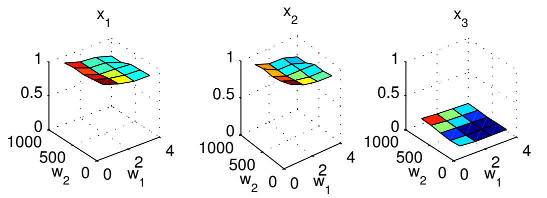

Fig. 6: Three directions x_1, x_2 and x_3 of the reconstructed x.

From the observations and analyses above, we can come up with a design decision to truncate the input mode vector v_3 out of the control input vector x. Using (26), we can get x as follows.

$$x = q_1 \cdot v_1 + q_2 \cdot v_2 \triangleq \begin{bmatrix} x_1 & x_2 & x_3 \end{bmatrix}^T \tag{31}$$

Using the obtained simulation results, we can reconstruct the control input vector x as depicted in the Fig.6. We can see that x_1 and x_2 are almost equal to 1. Meanwhile, x_3 is very small to be approximated as zero. Or the control input vector x can be re-designed as follows.

$$x \approx \begin{bmatrix} 1 & 1 & 0 \end{bmatrix}^T \tag{32}$$

for the whole ranges of of w_1 and w_2.

This means that we can eliminate the third objective or the power reserve term from the cost functional (28). In other words, the cost functional for the optimal control problem in Section III is simplified to a new one J_{new} as

$$J_{new} = \sum_{k=0}^{N-1} \Big[\dot{m}_f(k) + w_1 |u_g(k)| \Big] \Delta t \tag{33}$$

It would be easy to tune only one weight factor w_1 such that the Pareto optimal value is achieved to satisfy the required vehicle's performance index.

V. CONCLUSIONS

A method of analyzing and evaluating the performance index of the optimal control problem for a prototype HEV was proposed. The cost functional was simplified towards an appropriate one without seriously affecting the concerned performance index of the hybrid powertrain system. Complexity of the optimal control problem was reduced. The simulation results show that the SVD method is an efficient tool for objectively analyzing the correlations and dependencies among the objectives in a cost functional.

The proposed method in this paper is not limited to a specific hybrid powertrain configuration. In principle, it can be extended to other types of vehicle configurations.

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