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Design of low order dynamic pre-compensators using convex methods

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***Abstract*— Pre-compensators are used in multivariable control systems to reduce (or eliminate) open loop system interactions. Classical methods for the design of pre-compensator are traditionally based on static designs. Static pre-compensators are preferred for their simplicity, but are highly unsystematic in the amounts of achievable diagonal dominance. In many applications, only the more powerful dynamic pre-compensators are able to deliver the desired amounts of decoupling. This paper proposes a new method for the design of dynamic pre-compensators which is based on a Quadratic Programming (QP) optimization. Using the proposed approach the total pre-compensator is found through several smaller optimization problems, one for each column. The application and effectiveness of the QP dynamic design is demonstrated on a Distributed Generation unit (DG) case study.**

I. INTRODUCTION

Rosenbrock's contribution to the design of control systems for linear multivariable plants inspired much activity in the development of techniques for achieving diagonal dominance [1]. The primary objective of all such techniques is to reduce plant interactions by the introduction of a multivariable pre-compensator so that the control system design can then be completed by using classical techniques to synthesise a set of single-loop controllers for the compensated plant [2][3].

Traditional techniques developed for the achievement of diagonal dominance by the use of static pre-compensators are the pseudo-diagonalisation [4][5], the function-minimisation method using conjugate-direction optimisation [6], and the ALIGN algorithm developed initially in conjunction with characteristic-locus methods [7]. More recently, advent of powerful optimization algorithms paved the way for development of improved techniques based on Evolution Strategies [8], $\mathcal{H}_2$-norm [9], and $\mathcal{H}_\infty$-norm [10].

Dynamic pre-compensation offers the opportunity to not only aim to routinely achieve diagonal dominance for a wide range of plants, but also achieve far higher levels of diagonal dominance. Methods for the design of dynamic pre-compensators should strike the right balance between achievable performance and pre-compensator complexity. This is not always easily obtained. For example while Chughtai and Munro [10] extend their static formulation for dynamic design, the dynamical order of the eventual dynamic pre-compensator will be very high (each element would be equal to the order of the entire plant plus the weighting functions). Conversely, in [11] a method is proposed which can produce low order pre-compensators, but the method is not systematic and comes at the expense of a considerable design effort required for m^2 curve fitting problems in case of an $m \times m$ system. One of the more powerful recent approaches in solving the dynamic problem has been with the use of Evolutionary Algorithms [8]. An evolutionary optimization offers large design flexibility including the ability to set each element of the pre-compensator to have a desired order. Alas, these user benefits are countered by two important obstacles; a huge computational effort, and the 'curse of dimensionality'. The latter is especially inhibiting when it comes to problems of larger size or higher dynamical order.

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This paper aims to draw upon the main benefits of the previous techniques to propose a practical and usable method for the design of dynamic pre-compensators. Using the proposed algorithm, a separate design problem is solved for each column of the pre-compensator. Each element of the pre-compensator can be either static or have arbitrary dynamical complexity, but it may not be set to zero.

II. A QUADRATIC PROGRAMME APPROACH FOR DYNAMIC DECOMPOSITION

A. *Problem Statement*

Consider the system to be a stable LTI transfer function matrix (TFM) $G(s) = [g_{ij}(s)] \in \mathbb{R}^{m\times m}$ where $\mathbb{R}^{m\times m}$ is the set of $m\times m$ rational transfer functions. The design problem is to find a dynamic precompensator $K(s) = [k_{ij}(s)] \in \mathbb{R}^{m\times m}$ such that $Q(j\omega) = G(j\omega)K(j\omega)$ is dominant over a set of frequencies $\Omega = \{\omega_k : k = 1, \ldots, N\}$ [4], where

$$q_{ij}(j\omega) = \sum_{l=1}^{m} g_{il}(j\omega)k_{lj}(j\omega). \quad (1)$$

For design considrations it is desirable not to impose any restrictions on the dynamical complexity of $K(s)$ so that the desired order for each element of the pre-compensator may be set independently of others. The pre-compensator is thus defined as,

$$k_{lj}(s) = \sum_{r=0}^{o_{lj}} \alpha_{lj(o_{lj}-r+1)} s^{o_{lj}-r} \quad (2)$$

where $O \in \mathbb{N}^{m\times m}$ is a matrix of integers. Elements of $O = [o_{lj}]$ determine the respective order of the (l, j) element of the pre-compensator $K(s)$. In using this technique, either column or row dominance can be used; but in this paper the design problem will be solved in the case of column dominance used for direct frequency designs. The column dominance problem solved in this paper is to determine a dynamic pre-compensator $K(s)$ such that the cost function,

$$\Gamma(K,\Omega)=\sum_{j=1}^{m}\eta_j(j\omega)=\sum_{j=1}^{m}\sum_{k=1}^{N}\frac{\sum_{\substack{i=1\\ i\neq j}}^{m}|q_{ij}(j\omega_k)|}{|q_{jj}(j\omega_k)|} \tag{3}$$

is minimised. There are two important features of (3) which may be exploited for a reduced problem size. Let $Q(s)=[q_1(s),\ldots,q_m(s)]$ and $K(s)=[k_1(s),\ldots,k_m(s)]$. It will then follow from (3) that,

$$\min_{K(s)}\Gamma(K,\Omega)=\min_{K(s)}\sum_{j=1}^{m}\eta_j(j\omega)=\sum_{j=1}^{m}\min_{k_j(s)}\eta_j(j\omega) \tag{4}$$

It is therefore possible to solve m different problems, one for each column of $K(s)$. If the row dominance measure was used instead, the same could be shown with respect to the row. Secondly, note that diagonal scaling does not alter the value of (3). That is $\Gamma(K,\Omega)=\Gamma(K*D,\Omega)$, where $D=diag(d_1,\ldots,d_m), D\in\mathbb{R}^{m\times m}$. It will therefore always be possible to scale $K(s)$ such that,

$$\alpha_{ii(1)}=1, \tag{5}$$

where $\alpha_{ij(p+1)}$ is the coefficient of pth term (s^p) of the polynomial $k_{(ij)}(s)$ (see (2)). Conversely,

$$\min_{K(s)}\Gamma(K,\Omega)=\left\{\sum_{j=1}^{m}\min_{k_j(s)}\eta_j(j\omega)\mid\alpha_{ii(1)}=1\right\} \tag{6}$$

Imposing (5) on (2) will mean that minimization of (3) will no longer result in the trivial zero solution by ensuring $q_{ii}(s)$ will be nonzero. Thus, instead of minimizing the ratio defined in (3), the same solution $k_j(s)$ can be obtained by minimizing the modulus of the off-diagonal terms of $q_j(s)$, subject to (5). This leads to the following optimization problem for the j^{th} column of the precompensator,

$$\min_{k_j(s)}\sum_{k=1}^{N}\sum_{i=1,j\neq i}^{m}|q_{ij}(j\omega_k)|^2 \tag{7}$$

subject to,

$$\alpha_{ii(0)}=1 \tag{8}$$

where the pre-compensator is defined according to (2).

B. The QP optimization problem

To form the QP problem, we create a modified system whose $\mathcal{H}_2$ norm represents the summed interactions of the original system as represented by (7). The $\mathcal{H}_2$ norm problem can then be easily converted into a QP. The vectorization procedure is similar to one laid out for static pre-compensators in Lemma 3 of [12], extended here for dynamic designs. Application of the vectorization allows (7) to be rewritten as,

$$\min_{k_j(s)}||q_j(s)||_2=\min_{k_j(s)}\left|\left|\tilde{k}_j\tilde{G}\right|\right|_2 \tag{9}$$

where,

$$\tilde{k}_j=(\alpha_{1j},\alpha_{2j},\ldots,\alpha_{mj}) \tag{10}$$

and,

$$\alpha_{lj}=\left(0\in\mathbb{R}^{1\times O_m-o_{lj}},\alpha_{lj(o_{lj}+1)},\alpha_{lj(o_{lj})},\ldots,\alpha_{lj(1)}\right) \tag{11}$$

In (11), O_m is defined as,

$$O_m=\max_i\{o_{ij}\} \tag{12}$$

Similarly, we define $\tilde{G}$ as,

$$\tilde{G}=\left(\tilde{G}_1,\ldots,\tilde{G}_m\right) \tag{13}$$

The dimensions of $\tilde{G}$ are $(m\times O_m)\times(m-1)$ and the $\tilde{G}_i$ are defined as follows,

$$\begin{aligned}&\tilde{G}_i=M\mid_j\\ &M=\begin{pmatrix}M_1\\ \vdots\\ M_m\end{pmatrix},\quad M_i=(S^T(s)g_{il}(s))\quad,\quad i=1,...,m\end{aligned} \tag{14}$$

and,

$$S(s)=\left(s^{O_m},\ldots,1\right) \tag{15}$$

In (14), $M\mid_j$ denotes the matrix M with its M_j^{th} block deleted. This will correspond to removing the diagonal entries of $Q(s)$ from the minimization as specified in (7). To proceed with the QP formulation, note that,

$$\left|\left|\tilde{k}_j\tilde{G}\right|\right|_2^2=\tilde{k}_j\tilde{G}\tilde{G}^H\tilde{k}_j^T \tag{16}$$

Each vector $\tilde{k}_j$ will contain a single 1 (imposed by the constraint (5)) and a series of zeros (see (11)). Let P be a permutation matrix which will bring $\tilde{k}_j$ into this form,

$$\begin{aligned}\tilde{k}_j^{'}=\tilde{k}_jP=(&\underbrace{1,0,\ldots,0}_{u},\underbrace{\alpha_{1j(o_{ij}+1)},\ldots,\alpha_{1j(1)}}_{y},\ldots,\\ &\ldots,\alpha_{mj(o_{mj}+1)},\ldots,\alpha_{mj(1)})\end{aligned} \tag{17}$$

since any permutation matrix $PP^T=I$, then from (16) we have,

$$\tilde{k}_j\tilde{G}\tilde{G}^H\tilde{k}_j^T=\tilde{k}_j^{'}\underbrace{P^T\tilde{G}\tilde{G}^HP}_{\tilde{Q}}\tilde{k}_j^{'T} \tag{18}$$

$$\begin{aligned}&=\begin{pmatrix}u & y\end{pmatrix}\Re(\tilde{Q})\begin{pmatrix}u^T\\ y^T\end{pmatrix}\\ &=u\tilde{Q}_{11}u^T+y\tilde{Q}_{21}u^T+u\tilde{Q}_{12}y^T+y\tilde{Q}_{22}y^T\end{aligned} \tag{19}$$

since u is constant the first term does not effect the minimization. Hence the solution is obtained by minimizing,

$$2u\tilde{Q}_{12}y^T+y\tilde{Q}_{22}y^T \tag{20}$$

which is in the form of a QP. Since $\tilde{Q}$ is a Hermitian matrix with complex entries. Therefore it is only necessary to consider the real part of $\tilde{Q}$. Alternatively, note that since the $\mathcal{L}_2$ norm is real, then $\tilde{k}_j^{'}\Im(\tilde{Q})\tilde{k}_j^{'T}$ will always be necessarily zero. In summary, the problem of designing the j^{th} column of the dynamic pre-compensator (7) is solved by the following QP problem,

$$\min_{\alpha_{lj}} \frac{1}{2} y\tilde{Q}_{22}y^T + u\tilde{Q}_{12}y^T \tag{21}$$

A separate QP is solved for each column of $K(s)$. For a row dominance measure, a QP would be required for each row of $K(s)$.

III. Algorithm Verification Example

To demonstrate the essential features of the algorithm, we shall first consider a single model design problem. The 3×3 system to consider is given by,

$$G(s) = \begin{pmatrix} (s^2+5s+5)/(s^3+9s^2+17s+9) & (2s^2+7s+6)/(s^4+10s^3+26s^2+26s+9) & (-s^2-7s-8)/(s^4+10s^3+26s^2+26s+9) \\ -1/(s^4+12s^3+45s^2+68s+36) & -3/(s^5+13s^4+57s^3+113s^2+104s+36) & (s+7)/(s^5+13s^4+57s^3+113s^2+104s+36) \\ (-2s-3)/(s^3+11s^2+33s+27) & (s^2+2s)/(s^4+12s^3+44s^2+60s+27) & 1/(s^3+9s^2+17s+9) \end{pmatrix} \tag{22}$$

The Nyquist Array of $G(s)$ is shown in Figure 1. The first column is already dominant, but the second and third columns are clearly not dominant at all. We begin be setting,

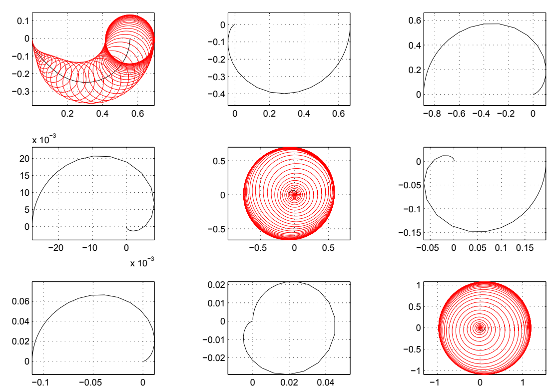

Fig. 1. Nyquist Array for uncompensated $G(s)$

$$O^1 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \tag{23}$$

This will correspond to a completely static pre-compensator (zero order for all elements). The solution in this case was found to be,

$$K^1 = \begin{pmatrix} 1 & -0.1308 & -0.2449 \\ 2 & 1 & 1.459 \\ 1 & 0.6837 & 1 \end{pmatrix} \tag{24}$$

The Nyquist Array of $G(s)K^1$ is shown in Figure 2. Notice that the first column has been completely decoupled, but the second two columns retain some interactions. We may therefore wish to increase the order of the elements of the second and third columns and set,

$$O^2 = \begin{pmatrix} 0 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix} \tag{25}$$

Using O^2, the pre-compensator (26) is obtained,

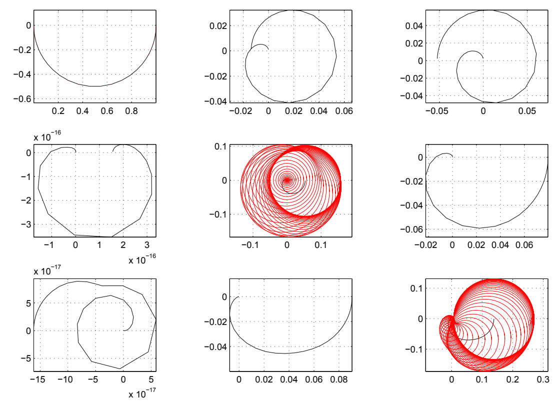

Fig. 2. Nyquist Array for $G(s)K^1$

$$K^2(s) = \begin{pmatrix} 1 & -0.001184s + 1.042 & -2 \\ 2 & -0.01757s + 1 & s+3 \\ 1 & 0.96s + 1.385 & 1 \end{pmatrix} \tag{26}$$

Even though all elements of the third column were allowed to be up to first order, only the $k_{2,3}$ element is first order in $K^2(s)$ and the other two elements in the third column remain static. Clearly, from the user point of view this is important since if the order of an element is set higher than it ought to be, the algorithm will automatically return a null coefficient. The Nyquist Array of $G(s)K^2(s)$ is shown in Figure 3. The third column has now also been completely decoupled. Since both the first and third columns have been completely decoupled, if we wish to reduce the interactions further, the only option left is to increase the order of the

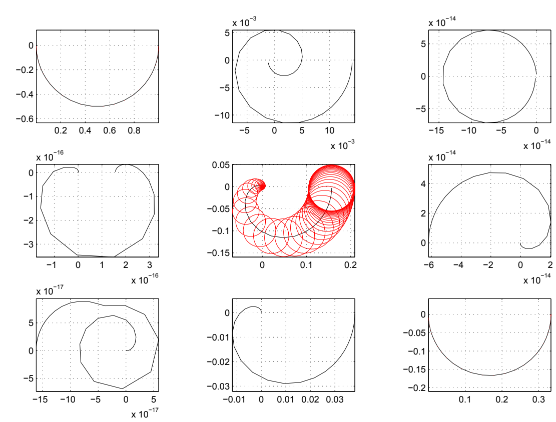

Fig. 3. Nyquist Array for $G(s)K^2$

second column. We find that for,

$$O^3 = \begin{pmatrix} 0 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 2 & 0 \end{pmatrix} \tag{27}$$

The solution which is obtained will completely decouple the system. The pre-compensator in this case is,

$$K^3(s) = \begin{pmatrix} 1 & s+2 & -2 \\ 2 & s+1 & s+3 \\ 1 & s^2+3s+2 & 1 \end{pmatrix} \tag{28}$$

The decomposition can be verified by forming the product $G(s)K^3(s)$ as shown below,

$$G(s)K^3(s) = \begin{pmatrix} 1/(s+1) & 0 & 0 \\ 0 & 1/(s^2+4s+4) & 0 \\ 0 & 0 & 1/(s+3) \end{pmatrix} \tag{29}$$

IV. The Distributed Generation (DG) case study

A. System Description

The use of distributed generation (DG) units such as photo-voltaic arrays, wind turbines, and fuel cells provides several advantages for the utility distribution grid. For instance, the DG systems decrease the cost of energy production, increase power quality of the distribution system, and reduce the environmental and economical problems. Figure 4 shows the schematic of an electronically-coupled DG unit. The DG unit is represented by a DC voltage source, a power electronics converter (VSC) which has a fast dynamic response and used as an interface to connect a DG unit to a utility grid, a series filter, and a step-up transformer. R_t and L_t represent both the series filter and the step-up transformer parameters. The local load is represented by a balanced three-phase parallel RLC network at the point of common coupling (PCC). Parameters of the system shown in Figure 4 are summarized in Table I.

A DG unit normally operates in a grid-connected mode when the CB switch in Figure 4 is closed, i.e., the DG unit and its dedicated load are part of the distribution grid. In the grid-connected mode, the host grid assumes a supervisory role and determines the voltage amplitude and frequency values of the load at the PCC. Consequently, the DG unit is only responsible for control of its real/reactive power components. Often this is based on the well-known dq-current control methodology [13]. The DG unit and the local load form an islanded system when switch CB is open. In this mode, grid control of voltage and frequency is no longer present, leading to possible instabilities [14]. Evidently, to maintain voltage and frequency stability (and desired response characteristics) it is necessary to activate replacement control systems.

B. DG Model and linearization

Application of Kirchhoff's voltage and current laws for the islanded DG system (Figure 4) gives rise to a nonlinear set

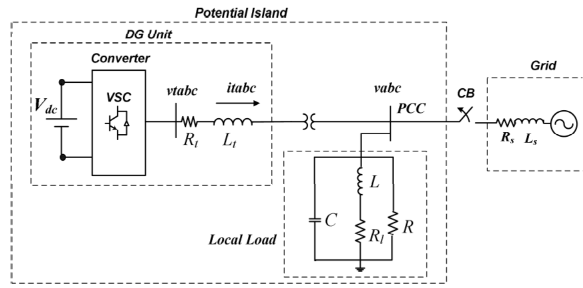

Fig. 4. Schematic diagram of an islanded DG unit

TABLE I

PARAMETERS OF THE STUDY SYSTEM OF FIG. 4

Quantity	Value
VSC filter resistance, R_t	1.5 $m\Omega$ (0.010 pu)
VSC filter inductance, L_t	300 μH (0.785 pu)
VSC terminal voltage, base voltage	600 V (l-l) (1 pu)
PWM carrier frequency	1980 Hz
DC bus voltage, V_{dc}	1500 V
VSC rated power	2.5 MW
Load nominal resistance, R	76 Ω (1 pu)
Load nominal inductance, L	111.9 mH (0.554 pu)
Load nominal capacitance, C	62.86 μF (1.805 pu)
Frequency set-point	60 Hz
Voltage set-point	11267

of equations. The equations are described by,

$$\begin{cases} \dfrac{dI_{td}}{dt} &= \omega\, I_{tq} + \dfrac{V_{td}}{L_t} - \dfrac{R_t}{L_t} I_{td} - \dfrac{V_d}{L_t} \\ \dfrac{dI_{tq}}{dt} &= -\omega\, I_{td} + \dfrac{V_{tq}}{L_t} - \dfrac{R_t}{L_t} I_{tq} \\ \dfrac{dV_d}{dt} &= \dfrac{I_{td}}{C} - \dfrac{I_{Ld}}{C} - \dfrac{V_d}{RC} \\ \dfrac{dI_{Ld}}{dt} &= \omega\, I_{Lq} + \dfrac{1}{L} V_d - \dfrac{Rl}{L} I_{Ld} \\ \dfrac{dI_{Lq}}{dt} &= -\omega\, I_{Ld} - \dfrac{Rl}{L} I_{Lq} \\ \omega &= \dfrac{I_{tq} - I_{Lq}}{CV_d} \end{cases} \tag{30}$$

The state vector, x, control input, u, and control output, y, are

$$x = [I_{td} \quad I_{tq} \quad V_d \quad I_{Ld} \quad I_{Lq}]^T$$
$$u = [V_{td} \quad V_{tq}]^T, \quad y = [V_d \quad \omega]^T$$

The state space equations of (30) represent a multi-input, multi-output nonlinear autonomous forced system. The chief requirement of the controller would be to regulate the output at around the same value (11267 volts and 60 Hz). The fact that the control problem is only a regulatory one permits the use of linear techniques provided it can be demonstrated that a linear and nonlinear model are matched reasonably well for typical expected variations around the specified output set-points. Therefore, the approach taken here is to linearize the model at the nominal output and load values. This gives rise to the following linear set of equations,

$$\begin{aligned} \dot{x}(t) &= A\, x(t) + B\, u(t) \\ y(t) &= C\, x(t) \end{aligned} \tag{31}$$

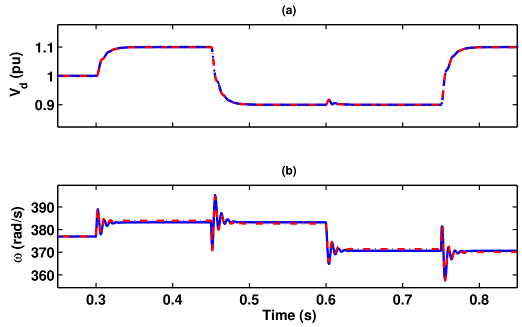

Fig. 5. Comparison of the linear approximation (dashed) with the nonlinear model (solid) with respect to step changes in control inputs

where $A \in \mathbb{R}^{5\times5}$, $B \in \mathbb{R}^{5\times2}$ and $C \in \mathbb{R}^{2\times5}$ are constant matrices as follows,

$$\mathbf{A} =$$
$$\begin{bmatrix} -\frac{R_t}{L_t} & \frac{(2I_tq - I_Lq)}{CV_d} & \frac{(I_Lq-I_tq)I_tq}{C(V_d)^2} - \frac{1}{L_t} & -\frac{I_tq}{CV_d} & 0 \\ \frac{(I_Lq-I_tq)}{CV_d} & -\frac{I_td}{CV_d} - \frac{R_t}{L_t} & \frac{(I_tq-I_Lq)I_td}{C(V_d)^2} & \frac{I_td}{CV_d} & 0 \\ \frac{1}{C} & 0 & -\frac{1}{RC} & 0 & -\frac{1}{C} \\ 0 & -\frac{I_Ld}{CV_d} & \frac{(I_tq-I_Lq)I_Ld}{C(V_d)^2} & \frac{I_Ld}{CV_d} - \frac{R_l}{L} & \frac{(I_Lq-I_tq)}{CV_d} \\ 0 & \frac{I_Lq}{CV_d} & \frac{(I_Lq-I_tq)I_Lq}{C(V_d)^2} + \frac{1}{L} & \frac{(I_tq-2I_Lq)}{CV_d} & -\frac{R_l}{L} \end{bmatrix}$$
$$\mathbf{B} = \begin{bmatrix} \frac{1}{L_t} & 0 \\ 0 & \frac{1}{L_t} \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}, \mathbf{C} = \begin{bmatrix} 0 & 0 & 1 & 0 & 0 \\ 0 & \frac{1}{CV_d} & \frac{(I_Lq-I_tq)}{C(V_d)^2} & -\frac{1}{CV_d} & 0 \end{bmatrix} \tag{32}$$

To verify the accuracy of the linearized models, independent set-point changes around the specified point are. The voltage is subjected to a $\pm10\%$ change whilst the change in the frequency is $\pm1Hz$. These are well within permissible ranges of allowed power fluctuations. The results are shown in Figure 5. It is clear that the linear model follows the non-linear response very well. Subsequent analysis of the open-loop linear model show that the system is open-loop stable, but with significant amounts of interaction especially in the frequency loop. In order to proceed, preliminary processing is carried out in accordance with standard multivariable design steps. Thus for example; the inputs and outputs are paired according to the RGA criteria, and the model's inputs and outputs are made dimensionless to reveal the true extend of system interactions [15][3].

C. Pre-compensator design for the DG system

The range of frequency point ω for the design of the pre-compensator is chosen to be 50 logarithmically spaced points between 10^{-2} and 10^{2} which adequately cover the entire bandwidth of the system. The aim is to keep the order as low as possible in this case. It was found that a purely static design was not sufficient to achieve decent levels of dominance. However a precompensator with complexity matrix,

$$O = \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \tag{33}$$

was found to give acceptable performance. The TFM of the optimizing QP pre-compensator was found to be,

$$\mathbf{K}(s) = \begin{pmatrix} 0.03806s + 1 & 0.002343s + 0.1189 \\ -0.9271 & 1 \end{pmatrix} \tag{34}$$

which is transformed by column scaling into the rational form given below,

$$\mathbf{K}_{QP}^{nl}(s) = \begin{pmatrix} -1.0786 & 0.1189 \\ \frac{1}{0.03806s+1} & \frac{1}{0.0197s+1} \end{pmatrix} \tag{35}$$

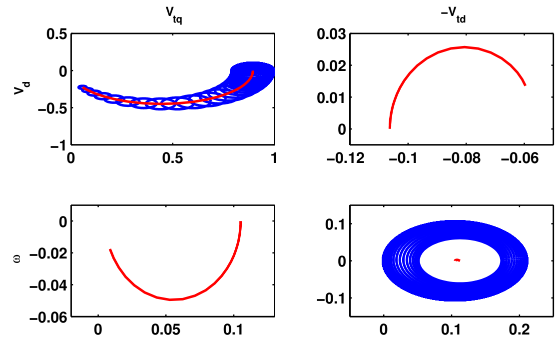

Fig. 6. Nyquist Array of uncompensated system

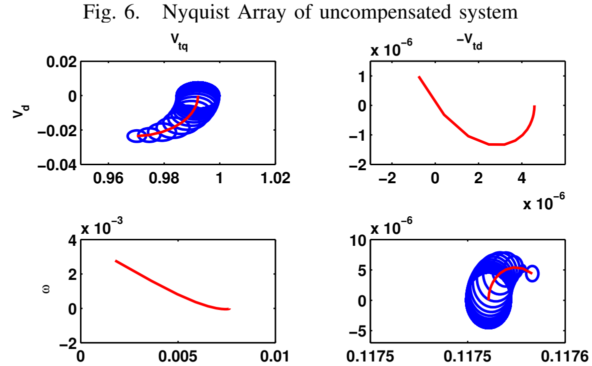

Fig. 7. Nyquist Array of the DG system compensated with (35)

Figures 6 and 7 respectively show the Nyquist Array of the DG before and after application of pre-compensator (35). The figures clearly show the amount of gained dominance. To compare the QP design, we also use Evolutionary Algorithms for parameter optimization [8] of a dynamic pre-compensator with a similar structure to (35) . Each candidate pre-compensator transfer function matrix is encoded in a chromosome comprising m^2 concatenated sub-chromosomes that each represents an element of the $m \times m$ compensator matrix with its respected dynamical complexity. Entire populations of chromosomes of such candidate pre-compensator matrices are caused to evolve subject to the actions of mutation, crossover, and selection: the measure of fitness used in such algorithms in the present context is simply the reciprocal of the cost function defined in equation (3). The solution was found to be,

$$\mathbf{K}_{EA}^{nl}(s) = \begin{pmatrix} -0.9997 & 0.1189 \\ \dfrac{1}{0.03968s+1} & \dfrac{1}{0.01971s+1} \end{pmatrix} \quad (36)$$

Comparison of (36) and (35) reveals that the QP solution is extremely close to the global optimum of the original nonlinear dominance ratio cost function. The second column is almost identical with minor differences in the first column. Of course the QP was computed significantly faster than the EA solution. To see the differences consider Figure 8 which shows the plot of the Perron root of the uncompensated plant, together with those compensated by the EA and QP pre-compensators. The column dominance ratios for the three cases are also shown in Figures 9 and 10.

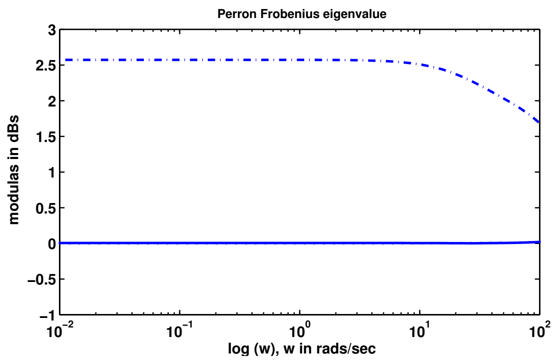

Fig. 8. Perron Frobenius eigenvalue of plant (dashed), and compensated plant with QP (solid) and EA algorithms (dotted)

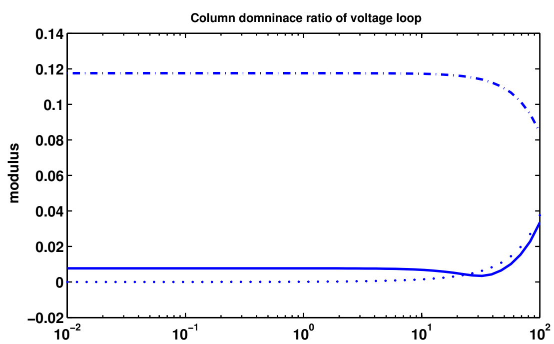

Fig. 9. Column dominance ratio of voltage loop -plant (dashed), QP (solid), EA (dotted)

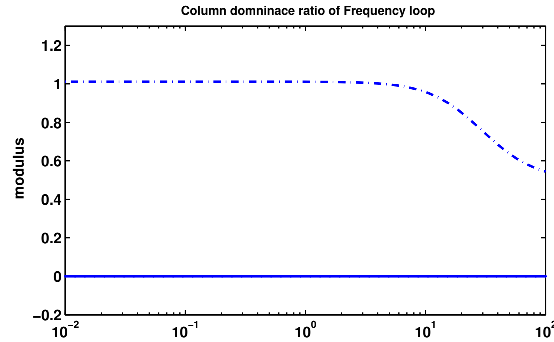

Fig. 10. Column dominance ratio of frequency loop -plant (dashed), QP (solid), EA (dotted)

V. Conclusion

This paper has proposed a new technique for the design of dynamic pre-compensators. We believe the technique represents a significant and important contribution to the existing family of methods. In particular it combines the design versatility and flexibility of direct optimization methodologies (such as EAs or optimization by PSQ, SA, etc..), while at the same time offering the efficiency of computation offered by the convex optimization methods (such as LMIs). The technique has been demonstrated through application to two examples. In the design verification example it was shown how the designer is able to gradually increase the order of each element as required and until sufficient or desired amounts of diagonal dominance are achieved. The method was also able to find the globally optimal design that completely decoupled the system (but there are no guarantees this is always possible). In the DG case study, a design comparison with Evolutionary Algorithms demonstrated that the QP solution is extremely close to the global optimum of the original column dominance ratio minimization. The method has also been applied to models of other real-life systems with larger size and complexity with similar excellent results.

References

[1] H.H Rosenbrock. Design of multivariable control systems using the inverse Nyquist array. *Proc. IEE*, 116:1929–1936, 1969.

[2] R.V. Patel and N. Munro. *Multivariable System Theory and Design*. Pergamon Press, Oxford, 1982.

[3] J.M. Maciejowski. *Multivariable Feedback Design*. Addison-Wesley, 1989.

[4] D.J. Hawkins. Multifrequency version of pseudodiagonalisation. *Electronic Letters*, 8(19):473–474, 1972.

[5] D.J. Hawkins. Pseudodiagonalisation and the inverse Nyquist array method. *Proc IEE*, 119:337–342, 1972.

[6] G.G Leininger. Diagonal dominance for multivariable Nyquist array methods using function minimisation. *Automatica*, 15:339–345, 1979.

[7] B Kouvaritakis. *Characteristic Locus Methods for Multivariable Feedback System Design*. PhD thesis, University of Manchester, 1974.

[8] A. Nobakhti, N. Munro, and B. Porter. Evolutionary achievement of diagonal dominance in linear multivariable plants. *Electronic Letters*, 39(1):165–166, 2003.

[9] A. Nobakhti and H. Wang. On a new method for h_2 -based decomposition. *IEEE Transactions on Automatic Control*, 51(12):1956–1961, 2006.

[10] S.S. Chughtai and N. Munro. Diagonal dominance using lmis. *IEE Proc control Theory and Application*, 151(2), March 2004.

[11] A. Nobakhti and N. Munro. Achieving diagonal dominance by frequency interpolation. *American Control Conference*, 2004. Boston, USA.

[12] A. Nobakhti and N. Munro. A new method for singular value loop shaping in design of multiple-channel controllers. *IEEE Transactions on Automatic Control*, 49(2):249–253, Feb 2004.

[13] P. Piagi and R.H. Lasseter. Autonomous control of microgrids. *IEEE Power Engineering Society General Meeting*, 2006.

[14] H. Karimi, H. Nikkhajoei, and M.R. Iravani. Control of an electronically-coupled distributed resource unit subsequent to an islanding event. *IEEE Transactions on Power Delivery*, 23(1):493–501, 2008.

[15] S. Skogestad and I. Postlethwaite. *Multivariable Feedback Control: Analysis and Design*. Wiley, 1996.