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Comparison of the DOB Based Control, A Special Kind of PID Control and ADRC

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***Abstract*— In this paper, the methods for estimating uncertainties in the disturbance observer (DOB), a special kind of PID (SPID) and the active disturbance rejection control (ADRC) are discussed. The stability of ADRC is proven for a class of SISO nonlinear systems with unknown dynamics and disturbance. The comparison study and the analysis for ADRC show that under some conditions, DOB and SPID can be generalized for nonlinear systems with mixed internal and external uncertainties.**

***Index Terms*— disturbance observer (DOB), A special kind of PID, active disturbance rejection control (ADRC).**

I. INTRODUCTION

Usually, one of the main objects of control is to deal with the uncertainties including internal (parameter or unmodeled dynamics) uncertainties and external (disturbance) uncertainties. The idea of the invariant principle [1] gives some suggestions for the problem of controlling uncertain systems: the uncertainties causing changes in the controlled variable can be used to generate an activating signal which will tend to cancel the effect of the same uncertainties, no matter they are internal or external. Obviously, the activating signal to attenuate uncertainties can be easily constructed if the uncertainties are measurable. However, most uncertainties are not measurable. Hence how to estimate uncertainties by the known information, for example, the control input and the output of system, become a significant problem. In the past years, lots of approaches have been proposed to estimate uncertainties from the input-output data, such as the disturbance accommodation control (DAC) [2], the unknown input disturbance observer (UIDO) [3], the disturbance observer (DOB)[4]-[9], some special PID (SPID) [10]-[11] and the active disturbance rejection control (ADRC) [12]-[16]. DAC, UIDO and DOB are initially proposed to deal with external disturbance for linear time-invariant systems. The survey in [4] addressed that by the transfer function from disturbance to its estimation, DAC and UIDO can be viewed as a special form of DOB. SPID was proposed to deal with internal dynamics uncertainty for some kinds of linear and nonlinear systems. The ADRC was proposed by Han to deal with the nonlinear systems with mixed uncertain dynamics and disturbances [12]-[13]. The common idea of these methods is to divide the process of controller design into two parts: one is to compensate for the uncertainties, which is reconstructed by the input-output data via certain kind of observer, the other is to realize the desired performance (tracking or regulating) for the compensated system.

ADRC's framework does not set strict mathematical constraints on the uncertainties to be estimated. There have been lots of application researches [17]-[22] and some stability analysis [23]-[25] since ADRC was introduced. In this paper, the methods for estimating uncertainties in DOB, SPID and ADRC are compared. The comparison illustrates that for a simplified benchmark plant, DOB, SPID and ADRC employ similar structure for estimating uncertainties. Furthermore, the paper proves that ADRC can stabilize a general class of nonlinear system with unknown dynamics and external disturbance. Hence, the frame of ADRC suggests the possibility to generalize DOB and SPID for estimating both internal and external uncertainties for nonlinear systems under some conditions. Although this statement has been indicated in some literature [6]-[9], the rigorous theoretical analysis has not been given.

The rest of paper is organized as follows. In Section II, the basic ideas and some theoretical results for DOB, SPID and ADRC are briefly introduced. Then these controllers are compared in Section III. Some simulations are given in Section IV. The proof of Theorem 1 which presents the performance and stability of ADRC is given in Section V, and Section VI are conclusions.

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II. THE BASIC IDEAS OF DOB, SPID AND ADRC

Consider the following SISO nonlinear system

$$\begin{cases} \dot{x} = Ax + B(f(x,t) + g(x,t)u \\ y = x_1 \end{cases} ,t \geq t_0 \tag{1}$$

where $x = \begin{bmatrix} x_1 \\ x_2 \\ \dots \\ x_n \end{bmatrix}, A = \begin{bmatrix} 0 & 1 & & \\ & 0 & & \\ & & \dots & 1 \\ & & & 0 \end{bmatrix}, B = \begin{bmatrix} 0 \\ \dots \\ 0 \\ 1 \end{bmatrix}$, $f(x,t)$, $g(x,t)$ can be linear or nonlinear, time-varying or time-invariant functions which may contain unknown dynamics and external disturbance, u is the control input and y is the measured output.

Next the basic ideas and some stability results of DOB, SPID and ADRC will be introduced.

A. Disturbance Observer (DOB)

Usually, the Disturbance Observer (DOB) is designed for the following LTI system with an unknown disturbance $d(t)$

$$\begin{cases} \dot{x} = Ax + B\left(a_1x_1 + \ldots + a_nx_n + b(d(t) + u)\right) \\ y = x_1 \end{cases}, t \geq t_0 \quad (2)$$

which can be seen as a special case of (1) with

$$f(x,t) = a_1x_1 + \ldots + a_nx_n + bd(t), \quad g(x,t) = b.$$

The open-loop transfer function for (2) is:

$$\frac{y}{u} = \frac{b}{s^n - a_ns^{n-1} - \ldots - a_1} \triangleq P(s).$$

Let $P_n(s)$ denote the nominal system and u_D denote the DOB based control. The following DOB can be designed to estimate $d(t)$ by the information of $P_n(s)$, the control input u_D and the output y.

$$\hat{d}_D = Q(s)(P_n^{-1}(s)y - u_D) \quad (3)$$

where $Q(s)$ is a low-pass filter. To guarantee the realization of (3), $Q(s)$ should satisfy $\partial Q(s) \geq \partial P_n(s)$, where $\partial Q(s)$ (or $\partial P_n(s)$) stands for the order of $Q(s)$ (or $P(s)$). Then u_D can be designed as

$$u_D = -\hat{d}_D + u_n \quad (4)$$

where $u_n = K_n(s)y$ can stabilize the nominal system $P_n(s)$.

In most research on DOB, $P_n(s)$ is assumed to be equal to $P(s)$ and DOB is used mainly for estimating the external disturbance. However, in practice, the parameters in $P(s)$ usually contain some uncertainty. In some literature [6]-[9], it's declared that the estimation of the plant uncertainties can be included into $\hat{d}_D$, and [7] analyzed the case of $P_n \neq P$ for some special kinds of system. However, in general case, how to choose the nominal system P_n to assure the closed-loop stability is still an open problem.

B. A Special Kind of PID (SPID)

[10]-[11] proposed a special PID control for a class of minimal-phase system. To simplify the analysis, we consider the following SISO system without zero dynamics:

$$\begin{cases} \dot{x} = Ax + B\left(f_1(x) + g_1(x)u\right) \\ y = x_1 \end{cases}, t \geq t_0 \quad (5)$$

which can also be a special form of (1) with

$$f(x,t) = f_1(x), \quad g(x,t) = g_1(x)$$

Rewrite the nth equation of (5) as

$$\dot{x}_n = d_S(x,u) + u \quad (6)$$

where

$$d_S(\cdot) = f_1(x) + (g_1(x) - 1)u \quad (7)$$

is an unknown term in (6). If x can be available, the influence of $d_S(\cdot)$ can be rejected by the following PID controller [10]-[11]:

$$u_S = -\sum_{i=1}^{n} k_ix_i - \hat{d}_S \quad (8)$$

where $k_1 + k_2s + \ldots + k_ns^{n-1} + s^n$ is a stable polynomial, and $\hat{d}_S$ is given by

$$\begin{cases} \hat{d}_S = \xi + \sum_{i=0}^{n-1} q_ix_{i+1} \\ \dot{\xi} = -q_{n-1}\xi - q_{n-1}\sum_{i=0}^{n-1} q_ix_{i+1} - \sum_{i=0}^{n-2} q_ix_{i+2} - q_{n-1}u \end{cases} \quad (9)$$

where $q_i, i = 0, \ldots, n-2$ are arbitrary constant and $q_{n-1} = \sigma(g_1(x))\mu$ with $\sigma(g_1(x))$ being the sign of $g_1(x)$ and $\mu > 0$ being a suitable constant.

The stability analysis of the system (5) with the control (8)-(9) can be shown by the following lemma.

Lemma 1 [11] . Under the assumptions:

A.1 $x_1, \ldots, x_n$ can be available;

A.2 $f_1(x), g_1(x)$ are pth differentiable, where $p > 0$ being a suitable integer;

A.3 $g_1(x)$ satisfies $g_1(x) \neq 0, |g_1(x)| \geq \underline{b} > 0$ where $\underline{b}$ is a constant and $\sigma(g_1(x))$ is known in the domain of interest,

there exists a constant $\mu^* > 0$ such that if $\mu > \mu^*$, then the closed-loop system (5), (8)-(9) is asymptotically stable.

Lemma 1 shows that the internal dynamics uncertainty $d_S(\cdot)$ satisfying A.2-A.3 can be estimated by $\hat{d}_S(\cdot)$ of the observer (9).

C. Active Disturbance Rejection Control (ADRC)

The key of ADRC is to design an extended state observer (ESO) to estimate not only states but also the "total disturbance" which contains internal uncertain dynamics and external disturbance [12]-[16].

Let $\hat{g}(t)$ be the estimation of $g(x,t)$, then the following ESO, which employs a linear structure, is designed for (1),

$$\begin{cases} \dot{\hat{x}}_1 = -\beta_1(\hat{x}_1 - x_1) + \hat{x}_2 \\ \dot{\hat{x}}_2 = -\beta_2(\hat{x}_1 - x_1) + \hat{x}_3 \\ \ldots \\ \dot{\hat{x}}_n = -\beta_n(\hat{x}_1 - x_1) + \hat{x}_{n+1} + \hat{g}(t)u \\ \dot{\hat{x}}_{n+1} = -\beta_{n+1}(\hat{x}_1 - x_1) \end{cases} \quad (10)$$

where the parameters β_i are designed to satisfy

$$\beta_i = \frac{\bar{\beta}_i}{\varepsilon^i}, i \in \underline{n+1}, \quad \varepsilon \in (0,\infty).$$

and

$$L(s) \triangleq s^{n+1} + \sum_{i=1}^{n+1} \bar{\beta}_is^{n+1-i} = \prod_{i=1}^{n+1}(s + \lambda_i), \quad Re(\lambda_i) < 0 \quad (11)$$

is the characteristic polynomial of ESO (10) when $\varepsilon = 1$. The bandwidth of ESO (10) can be adjusted by ε.

When ESO (10) is well tuned, its output $\hat{x} = [\hat{x}_1, \hat{x}_2, \ldots \hat{x}_n]^T$ and $\hat{x}_{n+1}$ can be used as the estimation of x and the total disturbance

$$x_{n+1} \triangleq f(x,t) + (g(x,t) - \hat{g}(t))u. \quad (12)$$

For ESO (10), when ε is small, the peaking phenomenon will happen if the observer's initial errors $\hat{e}_i \neq 0 (i \in \underline{n})$. In this paper, the following scheme is designed in the initial phase of the control to avoid peaking:

$$u = \begin{cases} 0, & t_0 \leqslant t < t_u \\ \dfrac{-K^T\hat{x} - \hat{x}_{n+1}}{\hat{g}(t)}, & t \geq t_u \end{cases} \tag{13}$$

where $K = [k_1 \quad k_2 \quad .. \quad k_n]^T$ and t_u is a constant to be designed.

Under the following assumptions:

A.4 $\frac{\partial f(x,t)}{\partial x}, \frac{\partial f(x,t)}{\partial t}, \frac{\partial g(x,t)}{\partial x}, \frac{\partial g(x,t)}{\partial t}$ are locally Lipschitz with respect to (x,t), and $\forall x \in \{x| \|x\| \leq \rho\}$, there is

$$|f(x,t)| \leq \mathscr{F}_1(\rho), |\frac{\partial f(x,t)}{\partial x}| \leq \mathscr{F}_2(\rho), |\frac{\partial f(x,t)}{\partial t}| \leq \mathscr{F}_3(\rho),$$
$$|g(x,t)| \leq \mathscr{F}_4(\rho), |\frac{\partial g(x,t)}{\partial x}| \leq \mathscr{F}_5(\rho), |\frac{\partial g(x,t)}{\partial t}| \leq \mathscr{F}_6(\rho),$$

where $\mathscr{F}_i, i \in \underline{6}$ are known functions dependent on ρ;

A.5

$$|g(x,t)| \geq \alpha_1 > 0, \tag{14}$$

where α_1 is constant,

the performance of the closed-loop system (1), (10)-(13) can be presented by the following theorem.

Theorem 1. Consider the system (1) with the control (10)-(13) under the assumption A.4-A.5. Design $\hat{g}(t)$ to satisfy

$$|\dot{\hat{g}}(\cdot)| \leq \alpha_2, \tag{15}$$

$$\|\frac{\bar{\beta}_{n+1}}{L(s)}\|_\infty \cdot |\Lambda(\cdot)| \leq \alpha_\Lambda < 1 \tag{16}$$

where $\Lambda(\cdot) = \frac{g(\cdot)-\hat{g}(\cdot)}{\hat{g}(\cdot)}$ and α_2, α_Λ are positives. Then there exists $\varepsilon^* > 0$ such that for $\varepsilon \in (0, \varepsilon^*]$

1). $$\|x(t) - x^*(t)\| \leq O(|\varepsilon ln\varepsilon|), \quad t \in [t_0, \infty)$$

2). $$\lim_{t\to\infty} \|x(t)\| \leq O(\varepsilon),$$

where $x^*(t)$ is the trajectory of the reference system

$$\begin{cases} \dot{x}^*(t) = f(x^*, t), & t \in [t_0, t_u) \\ \dot{x}^*(t) = A_c x^*, & t \in [t_u, \infty) \end{cases} \tag{17}$$

with $x(t_0) = x^*(t_0)$ and $A_c = A + BK^T$ being Hurwitz. The proof of Theorem 1 will be given in Section V.

The result 1) of Theorem 1 shows that the performance of closed-loop system can be controlled to be close to the reference system (17) by tuning ε. And the result 2) of Theorem 1 means the ultimate bound of the states can be small enough by tuning ε.

Remark 1. If the state x is available, then a reduced order ESO can be designed as follows to estimate x_{n+1}

$$\begin{cases} \dot{\hat{x}}_n = -\beta_n(\hat{x}_n - x_n) + \hat{x}_{n+1} + \hat{g}(t)u \\ \dot{\hat{x}}_{n+1} = -\beta_{n+1}(\hat{x}_n - x_n) \end{cases} \tag{18}$$

Furthermore, the peaking phenomenon will not happen in ESO (18). Hence, the control can be simply designed as

$$u = \frac{-K^T x - \hat{x}_{n+1}}{\hat{g}(t)}. \tag{19}$$

III. A COMPARISON FOR DOB, SPID AND ADRC

From the above introduction, DOB faces the problem that how to choose $P_n(s), Q(s)$ to guarantee the stability of closed-loop system, and SPID can only be implemented under the assumption that $x_1, ..., x_n$ are all available. ADRC provides a most systematic frame to estimate both external disturbance and internal uncertainty.

Next using the system (2) with the following assumptions:

A.6 $x_1, ..., x_n$ can all be available;

A.7 $\sigma(b)$ is known and $a_1, .., a_n$, $d(t), b$ are unknown,

as a benchmark, we will give a further comparison for DOB, SPID and ADRC. Since the discussion for $\sigma(b) = 1$ and $\sigma(b) = -1$ is similar, let $\sigma(b) = 1$ in the following analysis.

Let $k_1 + k_2 s + ... + k_n s^{n-1} + s^n$ be a stable polynomial. Set $P_n(s) = \frac{1}{s^n}$. In this setting, the disturbance to be handed by DOB will be

$$d_D(\cdot) = a_1 x_1 + ... + a_n x_n + bd(t) + (b-1)u. \tag{20}$$

Form (3)-(4), the DOB based control can be given by

$$u_D = -(k_1 + k_2 s + ... + k_n s^{n-1})y - \hat{d}_D \tag{21}$$
$$\hat{d}_D = Q(s)(P_n^{-1}(s)y - u_D) = Q(s)(sx_n - u_D) \tag{22}$$

where $\hat{d}_D$ is utilized as an estimation for the disturbance $d_D(\cdot)$. To guarantee the realization of (22), $Q(s)$ is a low-pass filter satisfying $\partial Q(s) \geq 1$. However, the stability of the DOB based control (21)-(22) by setting $P_n(s) = \frac{1}{s^n}$ has not been studied for the general case.

Next consider the SPID design. According to (8)-(9), the SPID based control for system (2) is equal to:

$$u_S = -\sum_{i=1}^{n} k_i x_i - \hat{d}_S \tag{23}$$

$$\hat{d}_S = \frac{q_{n-1}}{s + q_{n-1}}(sx_n - u_S) = \frac{\mu}{s + \mu}(sx_n - u_S) \tag{24}$$

where the uncertain term to be approximated by $\hat{d}_S$ is

$$d_S(\cdot) = a_1 x_1 + ... + a_n x_n + bd(t) + (b-1)u \tag{25}$$

which is the same as $d_D(\cdot)$ in (20). However, Lemma 1 dose not cover this case due to $d_S(\cdot)$ is time-varying.

Now, let us analyze ADRC. Since x are available, the ESO (18) with $\hat{g} = 1$ is used. Considering the peaking will not happen, the ADRC (18) and (19) for the system (2) can be simplified as:

$$u_A = -\sum_{i=1}^{n} k_i x_i - \hat{d}_A \tag{26}$$

$$\hat{d}_A = \frac{\beta_2}{s^2 + \beta_1 s + \beta_2}(sx_n - u_A) \tag{27}$$

where the uncertain term to be approximated by $\hat{d}_A$ is

$$d_A(\cdot) = a_1 x_1 + ... + a_n x_n + bd(t) + (b-1)u \tag{28}$$

which equals to $d_D(\cdot)$ and $d_S(\cdot)$ in (20) and (25).

(21)-(27) clearly show that for this simple benchmark plant, the difference in DOB, SPID and ADRC lies in the difference in the observers which are designed for the same uncertainty $d_D = d_S = d_A$:

$$\begin{cases} \hat{d}_D = Q(s)(sx_n - u_D) \\ \hat{d}_S = \dfrac{\mu}{s+\mu}(sx_n - u_S) \\ \hat{d}_A = \dfrac{\beta_2}{s^2+\beta_1 s+\beta_2}(sx_n - u_A) \end{cases} \tag{29}$$

Then some meaningful conclusions can be obtained from the above comparison:

1). Although the DOB based control (21) lacks theoretical base, Theorem 1 provides one for DOB when the nominal model P_n is simply chosen to be a chain of integrator and $Q(s) = \frac{\beta_2}{s^2+\beta_1 s+\beta_2}$. Furthermore, Theorem 1 suggests the possibility of generalizing DOB to the system (1) where $f(x,t), g(x,t)$ are nonlinear and time-varying as follows: if y is available and $\dot{y},\ldots,y^{(n-1)}$ are known, the following generalized DOB based control can be developed:

$$\begin{aligned} &\hat{d}_D(s) = Q(s)(P_n^{-1}(s)y - \tilde{U}(s)), \\ &P_n(s) = \frac{1}{s^n}, \quad Q(s) = \frac{\beta_{n+1}}{s^{n+1}+\beta_1 s^n+\ldots+\beta_{n+1}}, \end{aligned} \tag{30}$$

where $\tilde{U}(s)$ is the Laplace transform of $\hat{g}(t)u(t)$, and

$$u(t) = \frac{1}{\hat{g}(t)}(-\hat{d}_D(t) - k_1 y - k_2\dot{y} - \ldots - k_n y^{(n-1)}). \tag{31}$$

2) Theorem 1 also presents the enlightenment of generalizing SPID to the system (1).

2.1) If x is available, SPID can be used for the system (1). However, if only y is available and the the system order is greater than 2, then SPID no longer works.

2.2) [23] proved that ESO (10) can be independently used as a filter to estimate uncertainty if the uncertainty or its derivative is bounded. However, the observer (9) only works in the closed-loop system and can not be used as a filter when $\sigma(g(\cdot)) = -1$. However, when x is available, a first-order ESO can be designed as follows:

$$\begin{cases} \dot{z} = -\beta z - \beta^2 x_n - \beta\hat{g}(t)u \\ \hat{x}_{n+1} = \beta x_n + z \end{cases} \tag{32}$$

where $\beta > 0$ and $\sigma(\hat{g}) = \sigma(g)$. (32) can be viewed as another design of SPID which can be used as an independent filter.

IV. Simulation

Consider the following nonlinear time-varying system:

$$\begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = f(x_1,x_2,t) + b(x_1,x_2,t)u \\ y = x_1 \\ x_1(0) = 1, x_2(0) = 0 \end{cases} \tag{33}$$

where the uncertainties are set to be

$$\begin{cases} f(x_1,x_2,t) = sin(0.25\pi t)x_1 + x_2^2 + 0.2cos(0.2\pi t), \\ g(x_1,x_2,t) = -1 + 0.2sin(0.2\pi x_2). \end{cases} \tag{34}$$

The control object is $x_1 \to 0$. Although neither DOB nor SPID in its old frame can handle this problem, from the discussion in Section III, the following generalized DOB and SPID can be utilized to deal with the system (33).

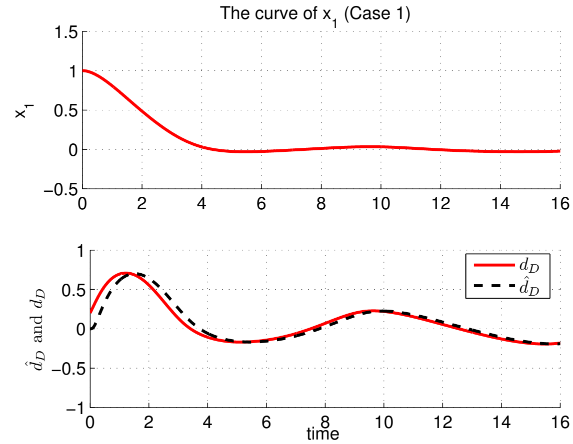

Fig. 1. The simulation result for DOB based control (36)-(37)

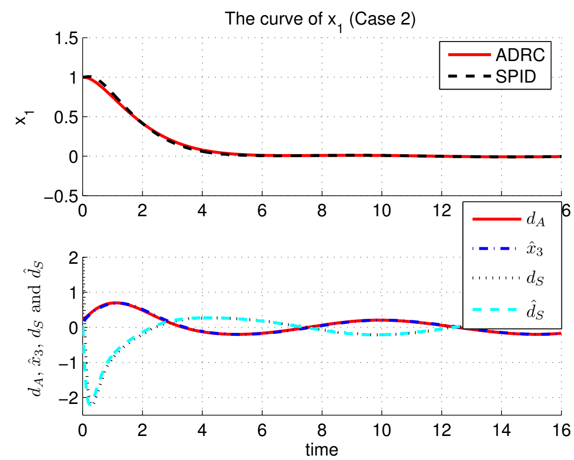

Fig. 2. The simulation result for SPID (38) (40), and ADRC (39) (41)

Case 1). Only $y = x_1$ is available.

Set $P_n(s) = \frac{1}{s^n}, \hat{g}(t) = -1$, from (30), the following generalize DOB based control is designed,

$$\hat{d}_D(s) = \frac{\beta_3}{s^3+\beta_1 s^2+\beta_2 s+\beta_3}(s^2 y + u) \tag{35}$$

where $\beta_1 = 3/\varepsilon, \beta_2 = 3/\varepsilon^2, \beta_3 = 1/\varepsilon^3, \varepsilon = 0.1$ and $\hat{d}_D$ is an estimation of the total disturbance $d_D = f(x_1,x_2,t) + (g(x_1,x_2,t)+1)u$. Rewrite (35) in the form of state space,

$$\begin{cases} \dot{\hat{x}}_1 = -\beta_1(\hat{x}_1 - x_1) + \hat{x}_2 \\ \dot{\hat{x}}_2 = -\beta_2(\hat{x}_1 - x_1) + \hat{d}_D - u \\ \dot{\hat{d}}_D = -\beta_3(\hat{x}_1 - x_1) \end{cases} . \tag{36}$$

Obviously, (36) is equal to the 3rd-order ESO, and $\hat{x}_1, \hat{x}_2$ are the estimation of x_1, x_2. Choose the initial condition as $\hat{x}_1(0) = 0, \hat{x}_2(0) = 0, \hat{d}_D(t_0) = 0$ and take $K = [2 \quad 1]$, then we can get the following control based on DOB (36)

$$u = \hat{d}_D + x_1 + 2\hat{x}_2. \tag{37}$$

The fig. 1 gives the simulation results for system (33) with the control (36)-(37).

Case 2). x_1, x_2 are available.

Set $q_1 = -\mu, q_0 = 0, \mu > 0$ in (8)-(9) and $\hat{g}(t) = -1$ in (32), the following SPID and 1st-order ESO are designed for the system (33), respectively:

$$\text{SPID:} \qquad \begin{cases} \hat{d}_S = \xi - \mu x_2, \\ \dot{\xi} = \mu\xi - \mu^2 x_2 + \mu u, \end{cases} \tag{38}$$

$$\text{1st-order ESO:} \qquad \begin{cases} \dot{z} = -\beta z - \beta^2 x_2 + \beta u, \\ \hat{x}_3 = \beta x_2 + z, \end{cases} \tag{39}$$

where $\hat{d}_S$ and $\hat{x}_3$ are the estimations of the total disturbance $d_S = f(\cdot) + (g(\cdot) - 1)u$ and $d_A = f(\cdot) + (g(\cdot) + 1)$, respectively. The controls based on (38) and (39) are taken as

$$\text{SPID:} \qquad u = -x_1 - 2x_2 - \hat{d}_S, \tag{40}$$

$$\text{ADRC:} \qquad u = x_1 + 2x_2 + \hat{x}_3. \tag{41}$$

Take $\mu = 10, \beta = 10$ and $\hat{d}_S(0) = 0, \hat{x}_3(0) = 0$. The fig. 2 is the simulation results for system (33) with SPID and ADRC.

Fig. 1 and Fig. 2 illustrate both generalized DOB and SPID can stabilize the nonlinear uncertain system (33), which validates our theoretical results.

V. The Proof of Theorem 1

Before the proof of Theorem 1, we introduce a transformation

$$\hat{E}_e = T_1(\varepsilon)\xi_e \tag{42}$$

where $\hat{E}_e = \begin{bmatrix} \hat{e}_1 \\ \cdots \\ \hat{e}_{n+1} \end{bmatrix} = \begin{bmatrix} x_1 - \hat{x}_1 \\ \cdots \\ x_{n+1} - \hat{x}_{n+1} \end{bmatrix}, \xi_e = \begin{bmatrix} \xi_1 \\ \cdots \\ \xi_{n+1} \end{bmatrix}$ and $T_1(\varepsilon) = diag\{\varepsilon^n, \ldots, \varepsilon, 1\}$. Using the control (10)-(13) and the transformation (42), we get the closed-loop system of (x, ξ_e) for $t < t_u$ and $t \geq t_u$, respectively:

$$\dot{x} = Ax + Bf(x,t), \quad t < t_u \tag{43}$$

$$\dot{\xi}_e = \frac{1}{\varepsilon}\tilde{A}_e\xi_e + B_2\eta_1(x,t), \quad t < t_u \tag{44}$$

$$\dot{x} = A_c x + BK_e^T\xi_e, \quad t \geq t_u \tag{45}$$

$$\begin{aligned} \dot{\xi}_e = &\frac{1}{\varepsilon}A_e\xi_e + B_2\eta_2(x,t) \\ &+ B_2\left(\eta_3(x,t,\varepsilon)\xi_e + \xi_e^T\eta_4(x,t,\varepsilon)\xi_e\right), t \geq t_u \end{aligned} \tag{46}$$

where

$$\tilde{A}_e = \begin{bmatrix} -\bar{\beta}_1 & 1 & & \\ & & \cdots & \\ -\bar{\beta}_n & & & 1 \\ -\bar{\beta}_{n+1} & & & \end{bmatrix}, A_e = \begin{bmatrix} -\bar{\beta}_1 & 1 & & \\ & & \cdots & \\ -\bar{\beta}_n & & & 1 \\ -\bar{\beta}_{n+1}(1+\Lambda) & & & \end{bmatrix},$$

$$\eta_1 = \frac{\partial f}{\partial x}(Ax + Bf) + \frac{\partial f}{\partial t}, B_2 = \begin{bmatrix} 0 \\ B^T \end{bmatrix}, K_e = \begin{bmatrix} K \\ 1 \end{bmatrix},$$

$$\begin{aligned} \eta_2 = &\frac{\partial f}{\partial x}A_c x + \frac{\partial f}{\partial t} - \frac{1}{g}\left(\frac{\partial g}{\partial x}A_c x + \frac{\partial g}{\partial t} - \dot{\hat{g}} - \Lambda(\cdot)\dot{\hat{g}}\right)\left(K^T X + f\right) \\ &- \Lambda(\cdot)K^T A_c x, \end{aligned}$$

$$\begin{aligned} \eta_3 = &\frac{1}{g}\left(\frac{\partial g}{\partial x}(A_c x - BK^T x - Bf) + \frac{\partial g}{\partial t} - \dot{\hat{g}} - \Lambda(\cdot)\dot{\hat{g}}\right)K_e^T T_1(\varepsilon) \\ &+ \frac{\partial f}{\partial x}BK_e^T T_1(\varepsilon) - \Lambda(\cdot)K^T BK_e^T T_1(\varepsilon) - \Lambda(\cdot)K^T T_2(\varepsilon), \end{aligned}$$

$$\eta_4 = \frac{1}{g}\frac{\partial g}{\partial x}BT_1(\varepsilon)K_e K_e^T T_1(\varepsilon),$$

$$T_2(\varepsilon) = \begin{bmatrix} \bar{\beta}_1\varepsilon^{n-1} & -\varepsilon^{n-1} & 0 & \ldots \\ \bar{\beta}_2\varepsilon^{n-2} & 0 & -\varepsilon^{n-2} & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ \bar{\beta}_n & 0 & \ldots & 1 \end{bmatrix}.$$

Suppose the initial condition be

$$\|x(t_0)\| = \rho_0, \|\hat{E}_e(t_0)\| = \rho^*.$$

Design t_u such that

$$x(t) \in \{x | \|x\| \leq \rho_1\}, \quad t \in [t_0, t_u), \tag{47}$$

where $\rho_1 > \rho_0$ is the scope accepted in practice.

Using A.4, we get for $\forall x \in \{x | \|x\| \leq \rho_1\}$

$$|\eta_1(\cdot)| \leq \gamma_1(\rho_1)$$

where $\gamma_1(\rho_1) \triangleq \mathscr{F}_2(\rho_1)(\|A\|\rho + \mathscr{F}_1(\rho_1)) + \mathscr{F}_3(\rho_1)$. And similarly

$$\begin{aligned} &|\eta_2(\cdot) + \eta_3(\cdot)\xi_e + \xi_e^T\eta_4(\cdot)\xi_e| \leq \\ &\qquad \gamma_2(\rho_1, \varepsilon) + \gamma_3(\rho_1, \varepsilon)\|\xi_e\| + \gamma_4(\rho_1, \varepsilon)\|\xi_e\|^2 \end{aligned} \tag{48}$$

where $\gamma_i, i = 2,3,4$ are positives dependent on (ρ_1, ε) and are analytical with respect to ε.

As for the properties of $A_c, \tilde{A}_e, A_e$, we have the following Lemmas.

Lemma 2.[26] If $A_c, \tilde{A}_e$ are Hurwitz, then there exist positive matrices $P_1, \tilde{P}_2$ and positives $c_{11}, c_{12}, \tilde{c}_{21}, \tilde{c}_{22}$ such that

$$\begin{aligned} A_c^T P_1 + P_1 A_c = -I, \quad c_{11}I \leq P_1 \leq c_{12}I, \\ \tilde{A}_e^T\tilde{P}_2 + \tilde{P}_2\tilde{A}_e = -I, \quad \tilde{c}_{21}I \leq \tilde{P}_2 \leq \tilde{c}_{22}I. \end{aligned}$$

Lemma 3.[27]-[28] If $\tilde{A}_e$ is Hurwitz and the condition (16) is satisfied, then there exist positive matrix P_2, and positives c_0, c_{21}, c_{22} such that

$$A_e^T P_2 + P_2 A_e \leq -c_0 I, \quad c_{21}I \leq P_2 \leq c_{22}I.$$

Proof of Theorem 1.

Define $V_1 = x^T P_1 x, \tilde{V}_2 = \xi_e^T\tilde{P}_2\xi_e, V_2 = \xi_e^T P_2\xi_e$ and $E = x - x^*$. According to (17) and (43), if $x(t_0) = x^*(t_0)$, then

$$E(t) = 0, t \in [t_0, t_u).$$

When $t \geq t_u$, since

$$\dot{E} = A_c E + BK_e\xi_e, \quad E(t_u) = 0, \tag{49}$$

there is

$$\dot{V}_1(E) \leq -\|E\|\left(\|E\| - 2\|P_1 BK_e^T\|\|\xi_e\|\right). \tag{50}$$

Next the property of ξ_e will be analyzed by two steps.

Step 1: *The bounds of* $\xi_e(t)$ *in* $[t_0, t_u)$

We will illustrate $\exists t_{p0}(\varepsilon) \le t_u$ such that when $t \ge t_{p0}$, $\xi_e(t)$ can converge to a small region.

By Lemma 2, the derivative of the Lyapunov function $\tilde{V}_2(\xi_e)$ along the trajectories of (44) will be

$$\dot{\tilde{V}}_2(\xi_e) \le -\frac{1}{\varepsilon}\|\xi_e\|^2 + \gamma_5(\rho_1)\|\xi_e\|, \quad t \in [t_0, t_u) \tag{51}$$

where $\gamma_5 = 2\|P_2B_2\|\gamma_1(\rho_1)$.

Then

$$\begin{aligned}\frac{d}{dt}\sqrt{\tilde{V}_2(\xi_e)} &= \frac{\dot{\tilde{V}}_2(\xi_e)}{2\sqrt{\tilde{V}_2(\xi_e)}} \le -\frac{1}{\varepsilon}\frac{\|\xi_e\|^2}{2\sqrt{\tilde{V}_2(\xi_e)}} + \frac{\gamma_5\|\xi_e\|}{2\sqrt{\tilde{V}_2(\xi_e)}} \\ &\le -\frac{1}{\varepsilon}\frac{\sqrt{\tilde{V}_2(\xi_e)}}{2\tilde{c}_{22}} + \frac{\gamma_5}{2\sqrt{\tilde{c}_{21}}}.\end{aligned} \tag{52}$$

According to Gronwall-Bellman inequality, we can obtain

$$\sqrt{\tilde{V}_2(\xi_e(t))} \le \frac{\tilde{c}_{22}\gamma_5}{\sqrt{\tilde{c}_{21}}}\varepsilon + \sqrt{\tilde{V}_2(\xi_e(t_0))}e^{-\left(\frac{1}{2\varepsilon\tilde{c}_{22}}\right)(t-t_0)} \tag{53}$$

Since $\hat{E}_e(t_0) = T_1(\varepsilon)\xi_e(t_0)$, the second term of (53) satisfies

$$\sqrt{\tilde{V}_2(\xi_e(t_0))}e^{-\left(\frac{1}{2\varepsilon\tilde{c}_{22}}\right)(t-t_0)} \le \sqrt{\tilde{c}_{22}}\|\varepsilon^n T_1^{-1}(\varepsilon)\|\frac{\rho^*}{\varepsilon^n}e^{-\left(\frac{t-t_0}{2\varepsilon\tilde{c}_{22}}\right)} \tag{54}$$

where $\varepsilon^n T_1^{-1}(\varepsilon)$ is analytical with respect to ε.

When $t_{p0}(\varepsilon) \triangleq \max\{t_0 - 2\tilde{c}_{22}(n+1)\varepsilon\ln\varepsilon, t_0\}$, there is

$$e^{-\left(\frac{t-t_0}{2\varepsilon\tilde{c}_{22}}\right)} \le \varepsilon^{n+1}, t \ge t_{p0}(\varepsilon). \tag{55}$$

Since $\lim\limits_{\varepsilon\to 0} t_{p0}(\varepsilon) = t_0$, there exists ε_1 such that $\forall\varepsilon \in (0, \varepsilon_1]$,

$$t_{p0}(\varepsilon) \le t_u.$$

Remark 2. If $\varepsilon > 1$, then $t_{p0}(\varepsilon) = t_0$. Otherwise, $t_{p0}(\varepsilon) = t_0 - 2\tilde{c}_{22}(n+1)\varepsilon\ln\varepsilon$.

From (53), (54) and (55), we can obtain

$$\sqrt{\tilde{V}_2(\xi_e(t))} \le \varepsilon\zeta_0(\rho_1, \rho^*), \quad t \in [t_{p0}, t_u] \tag{56}$$

where $\zeta_0 = \max\limits_{\varepsilon\in(0,\varepsilon_1]}\left(\frac{\tilde{c}_{22}\gamma_5(\rho_1)}{\sqrt{\tilde{c}_{21}}} + \sqrt{\tilde{c}_{22}}\left\|\varepsilon^n T_1^{-1}(\varepsilon)\right\|\rho^*\right)$.

Combining (56) and (47), it can be concluded that $\forall\varepsilon \in (0, \varepsilon_1]$

$$\begin{cases}\|x(t)\| \le \rho_1, & t \in [t_0, t_u) \\ \|\xi_e(t)\| \le \frac{\zeta_0}{\sqrt{\tilde{c}_{21}}}\varepsilon, & t \in [t_{p0}, t_u)\end{cases}. \tag{57}$$

Step 2: *The bounds of* $\xi_e(t)$ *in* $[t_u, \infty)$

According to A.4,

$$\|x(t_u)\| \le \rho_1, \quad \|\xi(t_u)\| \le \frac{\zeta_0}{\sqrt{\tilde{c}_{21}}}\varepsilon. \tag{58}$$

where $\xi = \begin{bmatrix}\xi_1 & \dots & \xi_n\end{bmatrix}^T$. According to (12) and (13),

$$\begin{aligned}\|\xi_{n+1}(t_u)\| &= \|x_{n+1}(t_u) - \hat{x}_{n+1}(t_u)\| \\ &= \|f(t_u) + \Lambda(t_u)(-K\hat{x}(t_u) - \hat{x}_{n+1}(t_u)) - \hat{x}_{n+1}(t_u)\| \\ &= \|f(t_u) - \Lambda(t_u)K\hat{x}(t_u) - (\Lambda(t_u)+1)\hat{x}_{n+1}(t_u)\|\end{aligned} \tag{59}$$

where $\hat{x}(t_u)$ can be written as

$$\hat{x}(t_u) = x(t_u) - diag\{\varepsilon^n, ..., \varepsilon\}\xi(t_u) \tag{60}$$

and $\hat{x}_{n+1}$ satisfies

$$\begin{aligned}\|\hat{x}_{n+1}(t_u)\| &= \|\lim_{t\to t_u}\hat{x}_{n+1}(t)\| \\ &= \|\lim_{t\to t_u^-} f(x,t) - \lim_{t\to t_u^-}\xi_{n+1}(t)\| \le \mathscr{F}_1(\rho_1) + \frac{\zeta_0}{\sqrt{\tilde{c}_{21}}}\varepsilon.\end{aligned} \tag{61}$$

Form (58)-(61), there exists $\gamma_6(\rho_1)$ such that $\forall\varepsilon \in (0, \varepsilon_1]$

$$\|\xi_e(t_u)\| = \|\xi(t_u)\| + \|\xi_{n+1}(t_u)\| \le \gamma_6(\rho_1)$$

Thus the initial condition $(x(t_u), \xi_e(t_u))$ satisfies:

$$\|x(t_u)\| \le \rho_1, \quad \|\xi_e(t_u)\| \le \gamma_6. \tag{62}$$

Define $\rho_2 \triangleq \sqrt{c_{12}}\max\{\rho_1, 2\|P_1BK_e^T\|\frac{\sqrt{c_{22}}}{\sqrt{c_{21}}}\gamma_6\}$. Next we will design ε such that

$$\Omega_2 = \{(x, \xi_e)|\sqrt{V_1(x)} \le \rho_2, \sqrt{V_2(\xi_e)} \le \sqrt{c_{22}}\gamma_6\}$$

is a positive invariant set for (45)-(46) in $t \in [t_u, \infty)$. The proof can be achieved by the following 2 steps.

i) Let

$$\sqrt{V_1(x)} = \rho_2, \quad \sqrt{V_2(\xi_e)} \le \sqrt{c_{22}}\gamma_6, \tag{63}$$

then along the trajectories of (45)

$$\begin{aligned}\dot{V}_1(x) &\le -\|x\|\left(\|x\| - 2\|P_1BK_e^T\|\|\xi_e\|\right) \\ &\le -\frac{1}{\sqrt{c_{12}}}\|x\|\left(\rho_2 - 2\|P_1BK_e^T\|\frac{\sqrt{c_{12}}\sqrt{c_{22}}}{\sqrt{c_{21}}}\gamma_6\right) \le 0.\end{aligned} \tag{64}$$

ii) Let

$$\sqrt{V_1(x)} \le \rho_2, \quad \sqrt{V_2(\xi_e)} = \sqrt{c_{22}}\gamma_6, \tag{65}$$

then

$$\begin{aligned}\dot{V}_2(\xi_e) \le &-\|\xi_e\|\frac{c_0\|\xi_e\|}{\varepsilon} + 2\|\xi_e\|\|P_2B_2\||\eta_2(\cdot)| \\ &+ 2\|\xi_e\|\|P_2B_2\||\eta_3(\cdot)\xi_e + \xi_e^T\eta_4(\cdot)\xi_e| \\ \le &-\|\xi_e\|\frac{c_0\|\xi_e\|}{\varepsilon} + 2\|\xi_e\|\|P_2B_2\|\gamma_2(\rho_2, \varepsilon) \\ &+ 2\|\xi_e\|\|P_2B_2\|(\gamma_3(\rho_2, \varepsilon)\|\xi_e\| + \gamma_4(\rho_2, \varepsilon)\|\xi_e\|^2).\end{aligned} \tag{66}$$

By $\sqrt{V_2(\xi_e)} = \sqrt{c_{22}}\gamma_6$, we can find ε_2 such that $\forall\varepsilon \in (0, \varepsilon_2]$

$$\dot{V}_2(\xi_e) \le 0. \tag{67}$$

Using (64) and (67), we can get Ω_2 is a positive invariant set for (45)-(46).

Define

$$t_{p1}(\varepsilon) \triangleq \max\{t_u - \frac{2c_{22}}{c_0}(n+1)\varepsilon\ln\varepsilon, t_u\}. \tag{68}$$

Similar to the deduction from (51)-(56), there exist $\zeta_1(\rho_2)$ and $\varepsilon_3 \le \varepsilon_2$ such that for $\forall\varepsilon \in (0, \varepsilon_3]$

$$\sqrt{V_2(\xi_e(t))} \le \zeta_1\varepsilon, \quad t \in [t_{p1}, \infty). \tag{69}$$

Since $\xi_e \in \Omega_2$ and ξ_e can be constrained by (69), according to (50) and Gronwall-Bellman inequality, we have

$$\sqrt{V_1(E(t))} \leq \sqrt{c_{12}}\|P_1BK_e^T\| \int_{t_u}^{t} e^{-\frac{c_{11}(t-\tau)}{2}} \|\xi_e(\tau)\| d\tau$$
$$\leq \sqrt{c_{12}}\|P_1BK_e^T\| \int_{t_u}^{t_{p1}} e^{-\frac{c_{11}}{2}(t-\tau)} \|\xi_e(\tau)\| d\tau$$
$$+ \sqrt{c_{12}}\|P_1BK_e^T\| \int_{t_{p1}}^{t} e^{-\frac{c_{11}}{2}(t-\tau)} \|\xi_e(\tau)\| d\tau \tag{70}$$
$$\leq \sqrt{c_{12}}\|P_1BK_e\| \left(2(n+1)\frac{c_{22}\sqrt{c_{22}}}{c_0\sqrt{c_{21}}}\gamma_6|\varepsilon \ln \varepsilon| + \frac{2\zeta_1}{c_{11}\sqrt{c_{21}}}\varepsilon \right).$$

Since $O(|\varepsilon ln\varepsilon|) \geq O(\varepsilon)$, from (70) there exists a positive $\gamma_7(\rho_2)$ such that $\sqrt{V_1(E(t))} \leq |\varepsilon ln(\varepsilon)|\gamma_7$ for $t \in [t_u, \infty)$. Therefore,

$$\|x(t) - x^*(t)\| \leq O(|\varepsilon ln\varepsilon|), \quad t \in [t_u, \infty). \tag{71}$$

Since

$$\lim_{t\to\infty} \sqrt{c_{12}}\|P_1BK_e^T\| \int_{t_u}^{t_{p1}} e^{-\frac{c_{11}}{2}(t-\tau)} \|\xi_e(\tau)\| d\tau = 0,$$
$$\sqrt{c_{12}}\|P_1BK_e^T\| \int_{t_{p1}}^{t} e^{-\frac{c_{11}}{2}(t-\tau)} \|\xi_e(\tau)\| d\tau \leq \frac{2\zeta_1}{c_{11}\sqrt{c_{21}}}\varepsilon,$$

and $\lim_{t\to\infty} \|x^*(t)\| = 0$, from (70), there is

$$\lim_{t\to\infty} \|x(t)\| = \lim_{t\to\infty} \|E(t)\| \leq O(\varepsilon). \tag{72}$$

VI. CONCLUSION

In this paper, the strategies of DOB, SPID and ADRC for estimating the uncertainties are compared. Originally, DOB is usually used to deal with external disturbances, SPID is proposed for nonlinear time-invariant systems with internal uncertainties, and ADRC is proposed for more general kinds of nonlinear systems with mixed internal uncertainties and external disturbances. The comparison study and Theorem 1 show that ADRC provides a base for generalizing both DOB and SPID for nonlinear system with mixed uncertainties. In other words, ADRC is a breakthrough in the research of controlling uncertain systems. The examples validates the efficiency of the generalized DOB and SPID which are designed according to the frame of ADRC.

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