2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Mixed Parametric/Unstructured LFT Modelling for Robust Controller Design

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***Abstract*— The paper presents a general procedure to approximate a parametric Linear Fractional Representation (LFR) with a reduced order LFR. The error resulting from a multidimensional model reduction step is covered by an unstructured uncertainty and the reduced order LFR is further optimized such that the norm of the required unstructured uncertainty is minimized. The effectiveness of the described method is shown on the example of a μ-synthesis controller for a parametric model of the longitudinal motion of a missile.**

I. INTRODUCTION

A Linear Fractional Representation (LFR) of the form (1) can be considered a standard system representation for applying modern robust control methods, such as μ-analysis/synthesis [1].

$$\begin{aligned} \dot{x} &= A_{11}x + A_{12}e + B_1 u \\ e &= A_{21}x + A_{22}d + B_2 u \\ y &= C_1 x + C_2 d + Du \\ d &= \Delta e \\ \Delta &= \{\mathrm{diag}(\delta_1 I_{n_1\times n_1}, \ldots, \delta_p I_{n_p \times n_p}) \mid \delta_i \in \mathbb{R}\,,\ |\delta_i| \leq 1\} \end{aligned} \tag{1}$$

A system G described by (1) can be written as an upper linear fractional transformation (LFT) of the form

$$G = \mathcal{F}_u\left(\begin{bmatrix} A & B \\ C & D \end{bmatrix}, \begin{bmatrix} I/s & \\ & \Delta \end{bmatrix}\right) \tag{2}$$

The order m_{lfr} of such an LFR is defined as the dimension of Δ, i.e.

$$m_{lfr} = \sum_{i=1}^{p} n_i, \tag{3}$$

where p is the number of parameters in Δ and n_i the repetition of the parameter δ_i.

A large number of nonlinear dynamical systems can be approximated by such a parametric LFR, as for example shown in [2] or [3]. Since many LFR-based techniques are computationally expensive, having low order LFRs of a system is usually required for applying them. Another approach in literature is therefore the approximation of a dynamical system by an unstructured complex uncertainty model (see e.g. [4], [5]). Unlike parametric fits, such models are far simpler but also more conservative system descriptions.

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A general procedure to combine those two approaches (i.e. parametric and unstructured uncertainty modeling) to acquire a good tradeoff between the complexity and accuracy of a model is presented in this paper. Starting from a parametric LFR a numerical order reduction is performed [6]. The reduction error is then bounded by an unstructured uncertainty based on a modified version of the methods described in [4]. A further optimization problem can be employed to reduce the required norm of the unstructured cover.

In section II an algorithm for numerical order reduction of an LFR is described. In sections III and IV it will be shown how the error from the order reduction can be covered by an additional unstructured uncertainty. Then, an optimization problem for reducing the required norm of the unstructured cover is proposed. Finally, in section V the algorithms will be applied to a model of the longitudinal motion of a missile. It will be shown that a μ-controller designed for the reduced order LFR with the unstructured cover achieves almost the same performance than a controller designed for the full order, parametric LFR.

II. NUMERICAL ORDER REDUCTION FOR LFT SYSTEMS

The applied LFR order reduction, which has been proposed by [6], is based on a generalization of a coprime factor model reduction. In this paper just a brief outline of the computational approach is given. For a thorough treatment of the topic refer to [6]. Note, that the approach is only valid for strongly stabilizable and detectable systems.

1) Apply a bilinear transformation to discretize the LFR system G as described by (1), so that the LFR $\bar{G}$ considered for reduction has the form
$$\bar{G} = \mathcal{F}_u\left(\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix}, \bar{\Delta}\right)$$
with $\bar{\Delta} = \begin{bmatrix} z^{-1}I & \\ & \Delta \end{bmatrix}$.
2) Compute a right coprime factorization of the system $\bar{G}$ by calculating $P > 0$, $P\bar{\Delta} = \bar{\Delta}P$ satisfying
$$\bar{A}P\bar{A}^* - P - \bar{B}\bar{B}^* < 0. \tag{4}$$
A right coprime factorization is then given by
$$\mathcal{F}_u\left(\left[\begin{array}{c|c} \bar{A} + \bar{B}F & \bar{B} \\ \hline F & I \\ \bar{C} + \bar{D}F & \bar{D} \end{array}\right], \bar{\Delta}\right)$$
with $F = -(\bar{B}^* P^{-1}\bar{B})^{-1}\bar{B}^* P^{-1}\bar{A}$.

3) Find $Y > 0$, $Y\bar{\Delta} = \bar{\Delta}Y$ and $X > 0$, $X\bar{\Delta} = \bar{\Delta}X$ satisfying

$$\begin{aligned} \bar{A}_F Y \bar{A}_F^* - Y + \bar{B}\bar{B}^* < 0 \\ \bar{A}_F^* X \bar{A}_F - X + \bar{C}_F^* \bar{C}_F < 0 \end{aligned} \tag{5}$$

with $\bar{A}_F = \bar{A} + \bar{B}F$ and $\bar{C}_F = \begin{bmatrix} F \\ \bar{C} + \bar{D}F \end{bmatrix}$. X and Y are calculated using a matrix rank minimization approach as described in [7].

4) Simultaneously diagonalize Y and X and find a balancing similarity transformation T_{bal} for the right coprime factorization and the associated generalized singular values.

5) Based on the generalized singular values determine suitable truncation dimensions. The truncation matrix is defined as $P_T = \text{diag}([I_{r1}, 0], \dots, [I_{rp}, 0])$, where $r1$ is the dimension of the first entry of the reduced uncertainty block $\bar{\Delta}_r$. The reduced order LFR G_r is obtained by a reverse bilinear transformation of

$$\bar{G}_r = \mathcal{F}_u \left(\begin{bmatrix} P_T T_{bal} \bar{A} T_{bal}^{-1} P_T^T & P_T T_{bal} \bar{B} \\ C T_{bal}^{-1} P_T^T & \bar{D} \end{bmatrix}, \bar{\Delta}_r \right) \tag{6}$$

with $\bar{\Delta}_r = \{\text{diag}(z^{-1} I_{r1}, \delta_1 I_{r2}, \dots, \delta_p I_{rp+1}) \mid \delta_i \in \mathbb{R}\,,\ |\delta_i| \leq 1\}$, so that the final reduced model G_r is again a continuous time model. It has the form

$$G_r = \mathcal{F}_u \left(\begin{bmatrix} A_r & B_r \\ C_r & D_r \end{bmatrix}, \Delta_r \right) \tag{7}$$

with Δ_r containing also the continuous integrators I/s.

III. BOUNDING THE APPROXIMATION ERROR BY AN UNSTRUCTURED COMPLEX UNCERTAINTY

Given the full order LFR G (2) with n_y being the number of outputs and the reduced model G_r, the error between G and G_r can be bounded by an unstructured uncertainty based on a modified version of the procedure described in [4]. In the paper the algorithm is performed with an output multiplicative uncertainty, i.e. $G_{om} = \{(I + W_1 \Delta_c W_2) G_r : \|\Delta_c\|_\infty \leq 1\}$. The weighting functions W_1 and W_2 are restricted to be diagonal, i.e. $W_1 = \text{diag}(W_{11}, \dots, W_{1n_y})$ and $W_2 = \text{diag}(W_{21}, \dots, W_{2n_y})$ respectively. The error bounding is easily modified to other types of uncertainty description (e.g. input multiplicative and additive uncertainties).

The problem can be stated as finding weighting functions W_1 and W_2, such that $G \in G_{om}$. This condition is fulfilled if and only if $\exists \Delta_c \in \mathbb{C}^{n_y \times n_y}$ with $\|\Delta_c\|_\infty \leq 1$, such that

$$G = (I + W_1 \Delta_c W_2) G_r. \tag{8}$$

Lemma 1: Given complex matrices A, B, C of appropriate dimensions, there exists a Δ with $\|\Delta\|_\infty \leq 1$, such that $A = B\Delta C$, if and only if

$$\begin{bmatrix} BB^* & A \\ A^* & C^*C \end{bmatrix} \geq 0 \tag{9}$$

Proof: see [4]

Rewriting (8) yields

$$G - G_r = W_1 \Delta_c W_2 G_r, \tag{10}$$

which can be used in conjunction with Lemma 1 to find W_1 and W_2 that hold for each frequency and parameter values.

The procedure to determine small, feasible W_1 and W_2 can be described in the following way: First introduce variable $M_{W1} = W_1 W_1^*$ and $M_{W2} = W_2^* W_2$. Then, grid the frequency axis in the relevant bandwidth using $\{\omega_k\}_1^{n_\omega}$ and the parameter space using $\{\Delta_i\}_1^{n_{gp}}$, such that G_i represents the parametric uncertainty model evaluated at Δ_i and G_{ri} the reduced model respectively. At each frequency ω_k solve

$$\min_{M_{W1,2}} \text{Trace}(M_{W1}) + \text{Trace}(M_{W2}) \tag{11}$$

subject to

$$\begin{bmatrix} M_{W1} & (G_i - G_{ri}) \\ (G_i - G_{ri})^* & G_{ri}^* M_{W2} G_{ri} \end{bmatrix} \geq 0, \quad i = 1, \dots, n_{gp}. \tag{12}$$

Since W_1 and W_2 and therefore M_{W1} and M_{W2} are restricted to a diagonal form, the finite set obtained by solving (11) at each ω_k can be over-bounded by a stable, minimum-phase transfer function. To obtain $W_1(s)$ and $W_2(s)$ log-Chebyshev fitting techniques can be employed element wise on the optimal frequency-by-frequency values. Such methods are available in [8].

The uncertain model G_{om} can be rewritten as an LFR of the form

$$G_{om} = \mathcal{F}_u \left(\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{C} & \bar{D} \end{bmatrix}, \begin{bmatrix} \Delta_c & \\ & \Delta_r \end{bmatrix} \right).$$

It represents an over-bound version of the original system G, i.e. $G \in G_{om}$. Note, however, that the gridding, which is required in the procedure, has to be dense enough. Otherwise, it is possible that important points in the parameter space are missed and G_{om} is not an over-bound of the complete set G.

IV. FURTHER OPTIMIZATION OF THE REDUCED ORDER MODEL

Due to the model reduction and introduction of an unstructured complex uncertainty, a tradeoff can be achieved between the complexity of an LFR of a system and its accuracy. As the reduction method used in this paper bounds the approximation error in a coprime factorization sense, it cannot be directly related to the required norm of the proposed unstructured cover. Even more important is the fact that the reduced order model G_r is not optimally designed to minimize the norm of the required unstructured cover. Therefore, an a-posteriori optimization is introduced, which shall minimize the required norm of the unstructured cover for a fixed reduced order of the LFR.

The optimization problem proposed is

$$\begin{gathered} \min_{A_r, B_r, C_r, D_r} \max_i \left(\sum_{k=1}^{n_\omega} \gamma_{ik} \right)^{0.5} \\ \gamma_{ik} = \text{Tr}\left[(G_i(j\omega_k) - G_{ri}(j\omega_k))^* (G_i(j\omega_k) - G_{ri}(j\omega_k)) \right]. \end{gathered} \tag{13}$$

The matrices describing the reduced order system G_r are used as optimization parameters. As initial value G_r obtained in section II is utilized. Similar to the computation of the weighting functions described in the previous section, grids over the frequency axis (i.e. $\{\omega_k\}_1^{n_\omega}$) and the parameter space (i.e. $\{\Delta_i\}_1^{n_{gp}}$) are needed.

Unlike the optimization problem of the previous section, (13) is, however, not convex. Hence, the general optimization environment MOPS [9] is utilized for the minimization, which offers a wide range of built-in solvers for global optimization. It shall be noted that as in the previous section a gridding is required which has to be dense enough.

V. EXAMPLE: LONGITUDINAL MOTION OF A MISSILE

As an example for an application of the proposed methods a μ-controller for a simple nonlinear model of the longitudinal motion of a missile is designed. The model and the weighting structure for the controller design are taken from [10].

A. Model Description

The system is described by (14) and (15), in which α is the angle of attack, q the pitch rate, Ma the Mach number, η the elevator deflection angle and n_z the load factor in the z-direction. All other parameters are considered to be constant in this example and are given in table I.

K_1 =	0.0207	force coefficient
K_2 =	1.2320	torque coefficient
K_3 =	0.0116	load factor coefficient
z_3 =	19.347	C_z coefficients
z_2 =	-31.008	
z_1 =	-9.7174	
z_0 =	-1.9481	
m_3 =	40.485	C_m coefficients
m_2 =	-64.166	
m_1 =	2.9221	
m_0 =	-11.803	

TABLE I

NUMERICAL DATA FOR THE MISSILE MODEL

Since all relations are analytically known, the LFR can be directly derived from the systems equation by analytically linearizing the model and employing the tools of [11]. The procedure of deriving an exact LFR by symbolic calculations is described in detail in [12].

$$\begin{aligned}\dot{\alpha} &= K_1 Ma C_z(\alpha, Ma, \eta) cos(\alpha) + q \\ \dot{q} &= K_2 Ma^2 C_m(\alpha, Ma, \eta) \\ n_z &= K_3 Ma^2 C_Z(\alpha, Ma, \eta)\end{aligned} \tag{14}$$

$$\begin{aligned}C_Z(\alpha, Ma, \eta) =& z_3\alpha^3 + z_2\alpha^2 + z_1\left(2 - \frac{1}{3}Ma\right)\alpha + \\ &+ z_0\eta \\ C_m(\alpha, Ma, \eta) =& m_3\alpha^3 + m_2\alpha^2 + m_1\left(-7 + \frac{8}{3}Ma\right)\alpha + \\ &+ m_0\eta\end{aligned} \tag{15}$$

The nonlinear model is valid in the range $\alpha = [0, 20]$ and $Ma = [2, 4]$.

The following calculation are mainly taken from [12]. For the linearized model the states are chosen to be $x = [\alpha, q]^T$, the output is n_z and the input η. In addition, Ma and α are considered to be parameters on which the model depend (i.e. $\delta = [\alpha, Ma]^T$). Thus, (14) can be seen as

$$\begin{aligned}\dot{x} &= f(x, u, \delta) \\ y &= g(x, u, \delta).\end{aligned} \tag{16}$$

By analytically differentiating $f(x, u, \delta)$ and $g(x, u, \delta)$ the linearized model of the missile can be obtained. The index 0 represents the value at the equilibrium point in the following equations.

$$\dot{x} = \begin{bmatrix} a_{11} & 1 \\ a_{21} & 0 \end{bmatrix} x + \begin{bmatrix} K_1 Ma_0 \cos\alpha_0 z_0 \\ K_2 Ma_0^2 m_0 \end{bmatrix} u \tag{17}$$

$$\begin{aligned}a_{11} =& -K_1 Ma_0 C_{z,0} \sin\alpha_0 + K_1 Ma_0 (3z_3\alpha_0^2 + 2z_2\alpha_0 + \\ &+ z_1(2 - \frac{1}{3}Ma_0)) \cos\alpha_0 \\ a_{21} =& K_2 Ma_0^2 (3m_3\alpha_0^2 + 2m_2\alpha_0 + m_1(-7 + \frac{8}{3}Ma_0))\end{aligned} \tag{18}$$

$$y = \begin{bmatrix} c_{11} & 0 \\ 0 & 1 \end{bmatrix} x + \begin{bmatrix} K_3 Ma_0^2 z_0 \\ 0 \end{bmatrix} u \tag{19}$$

$$c_{11} = K_3 Ma_0^2 \left(3z_3\alpha_0^2 + 2z_2\alpha_0 + z_1\left(2 - \frac{1}{3}Ma_0\right)\right) \tag{20}$$

Note that (17) still depends on η_0 due to $C_{z,0}(\alpha_0, Ma_0, \eta_0)$. Setting $\dot{x}$ to zero η_0 can be calculated as a function of α_0 and Ma_0.

$$\eta_0 = -\frac{3m_3\alpha_0^3 + 2m_2\alpha_0^2 + m_1\left(-7 + \frac{8}{3}Ma_0\right)}{m_0} \tag{21}$$

For simplification the index 0 representing the value at the equilibrium point will be omitted. To allow a transformation into an LFR the trigonometric functions in (17) have been approximated by a Taylor series expansion. For the considered region of the angle of attack it is sufficient to use

$$\begin{aligned}\cos\alpha &= 1 - \alpha^2/2 \\ \sin\alpha &= \alpha\end{aligned} \tag{22}$$

For the generation of the low order LFRs, the LFR toolbox of [11] is used. The generation is carried out in three steps [13]:

- Symbolic preprocessing of (17) and (19)
- Object orientated LFR generation
- Numerical order reduction

By employing these sophisticated techniques the resulting LFR G has a dimension of 9 (i.e. $\Delta = \text{diag}(MaI_{5\times5}, \alpha I_{4\times4})$).

B. Mixed Parametric/Unstructured Model of the Missile

The procedure described in the previous sections is applied to the missile model to obtain simpler mixed parametric/unstructured models. The first step is the numerical order reduction of the model. The generalized singular values in this example are

$$\sqrt{\text{eig}(XY)} = [6.4847, 2.7323, 5.3948, 0.6053, 0.1688, 0.0498, 0.0204, 1.22, 0.1188, 0.0369, 0.0065]^T \tag{23}$$

The first two correspond to the states of G, the next five to Ma and the last four to α.

In order to show the possible tradeoff between the quality and complexity of mixed models, different truncations are chosen. The first reduced model G_{r1} is truncated at 0.01, so that it has an order of $m_{r1} = 8$, the second G_{r2} at 0.1 with $m_{r2} = 5$ and the last one G_{r3} at 0.5 with $m_{r3} = 3$. For validation, a complete unstructured model G_{r4} is also computed with the tools available in [8]. These tools are based on the work of [4]. Complete unstructured in this case means that the missile model is overbounded by an unstructured uncertainty model of the form

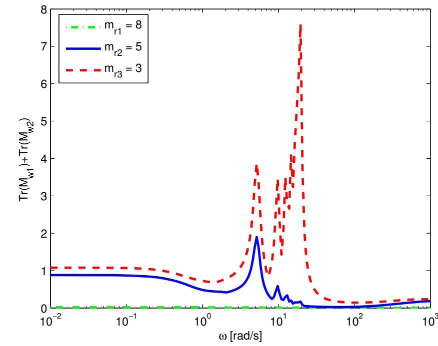
$$G_{r4} = (I + W_{r4,1}(s)\Delta W_{r4,2}(s))G_{r4,nom}(s), \tag{24}$$

where $G_{r4,nom}$ is a linear time invariant system and $\Delta \in \mathbb{C}^{n_y \times n_y}$ with $\|\Delta\|_\infty \leq 1$.

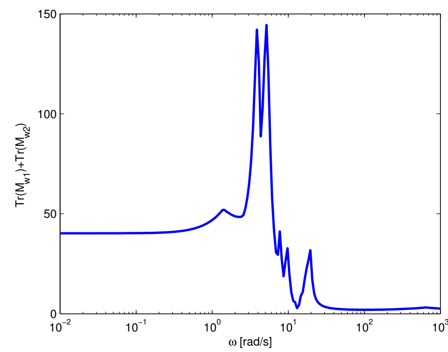
For these models covers in the form of output multiplicative uncertainties are computed by solving (11). The results of the cover fittings are presented in Fig. 1. In the figure the cost function of (11) is shown over the frequency. As can be seen, the size of the cover (measured in form of $\text{Trace}(M_{W1}) + \text{Trace}(M_{W2})$) increases as the order of the reduced models decreases. Note, that the size of G_{r1} is almost zero and therefore hardly noticeable in the plot. Even the worst mixed model G_{r3} with an LFR order of only three is still far better than the complete unstructured model G_{r4} shown in the lower figure.

Next, the effectiveness of the optimization of the state matrices described in section IV is demonstrated on the example of G_{r2}. In order to solve (13) the frequency axis is gridded up to $100 rad/s$. As is shown in Fig. 2, the size of the required cover is noticeable smaller after the optimization in the considered frequency range. Only at higher frequency, which are well beyond the bandwidth of the system, the a-posteriori optimization results in a worse cover.

The final step in the generation of mixed parametric/unstructured models is the approximation of the finite, frequency-by-frequency weighting functions by stable minimum phase transfer functions. In the present example first



(a) Mixed Parametric/Unstructured Models



(b) Unstructured Model

Fig. 1. Comparison of the Size of the Unstructured Cover for Different Reduced Models

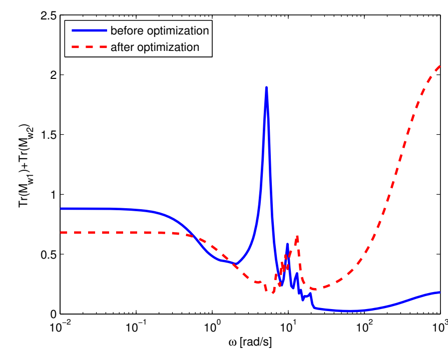


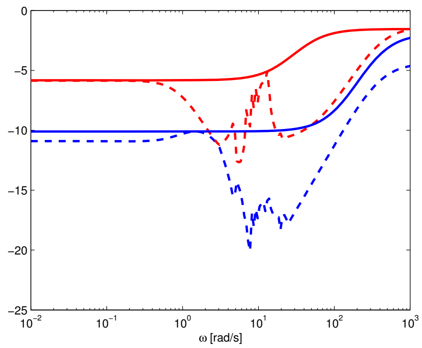
Fig. 2. Results of the Optimization for the Case $m_r = 5$

order functions are sufficient to overbound the finite weights for all reduced models. An example of such fitted transfer functions for the optimized system G_{r2} is given in Fig. 3. The solid lines represent the continuous transfer functions while the dashed lines show the original data from the gridding of the frequency axis.

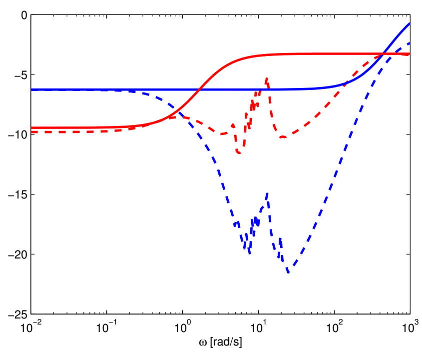
C. μ-Controller Synthesis

Finally, a μ-controller will be designed for the different LFR models. The structure for the μ-controller synthesis is taken from [10] and is depicted in Fig. 4. The following weighting functions are chosen:

- $W_{e1} = 0.7$



(a) Weighting Function W_1



(b) Weighting Function W_2

Fig. 3. Fitting of Stable Minimum Phase Transfer Functions for the Case $m_r = 5$

- $W_{e2} = 3$
- $W_\alpha = W_u = 0.001$
- $W_q = \frac{5000(s+1)}{s+5e6}$

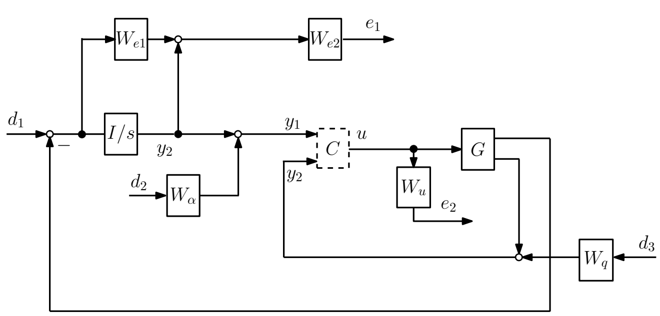


Fig. 4. Generalized Plant for the μ-Controller Synthesis

G in Fig. 4 is set to the full order as well as the various reduced order models for the synthesis. The performance of the different μ-controllers is then assessed based on a robust performance analysis [8] with the full order plant.

The results of the robust performance analysis are presented in Fig. 5. In comparison to the full order LFR of the missile the mixed models significantly reduce the computational demand of the controller synthesis, which is for most LFT-based methods growing rapidly with the size of the Δ-block. As is shown in the figure, the achievable level of robust performance is almost the same for G_{r1} and G_{r2} as when using the full order LFR for synthesis. Only G_{r3} has a noticeable worse performance, which is, however, still better than the performance of the unstructured model G_{r4}.

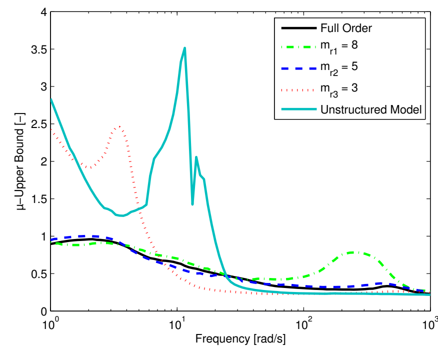


Fig. 5. Robust Performance of the Different μ-Controllers

VI. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

A general procedure to approximate a parametric LFR with a reduced order one including unstructured uncertainty has been developed. A numerical multidimensional order reduction method is applied to a parametric LFR and the reduction error is overbounded by an unstructured uncertainty. It has been shown that a tradeoff between the complexity and the quality of an LFR model can be achieved by using such a mixed parametric/unstructured modeling approach.

In the present work, the algorithm has been applied to a μ-controller synthesis problem for a model of the longitudinal motion of a missile. The quality of different mixed models has been studied and compared to a fully unstructured approximation method. Finally, it was possible to design a μ-controller with almost the same performance as the full order LFR model with a significantly simpler model.

B. Future Works

In the future, it is contemplated to improve the multidimensional reduction method used in the procedure. Since the coprime factorization used in the reduction method is not unique, there is still the open issue whether a better factorization can be found. As the a-posteriori optimization of the reduced order LFR to minimize the norm of the unstructured uncertainty can decisively improve the quality of the approximation, it can be assumed that a more suitable order reduction can be performed for finding mixed parametric/unstructured models.

VII. ACKNOWLEDGMENTS

The authors would like to thank the German Aerospace Center and the LFK-Lenkflugkoerper GmbH, especially Elmar Wallner, for their support.

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