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3D Cooperative Localization and Mapping: Observability Analysis

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Abstract— **In this paper it is presented an analysis on observability properties of systems of cooperative flying robots equipped with inertial (IMU) and bearing sensors. Relating to the cooperative localization and mapping problem, the main issue of the present work is to enstablish which quanties are observable and thus can be estimated by the robots.**

I. INTRODUCTION

In recent years, flying robotics has received significant attention from the automation and robotics community. The ability to fly allows easily avoiding obstacles and quickly having an excellent birds eye view. These navigation facilities make flying robots the ideal platform to solve many tasks like exploration, mapping, reconnaissance for search and rescue, environment monitoring, security surveillance, inspection etc. In the framework of flying robotics, micro aerial vehicles (MAV) have a further advantage. Due to the small size they can also be used in narrow out- and indoor environment and they represent only a limited risk for the environment and people living in it. A fundamental prerequisite for many applications is the capability to perform localization. However, systems navigating on GPS information only are not sufficient any more. Fully autonomous operation in cities or other dense environments requires the MAV to fly at low altitude or indoors where GPS signals are often shadowed and to actively explore unknown environments while avoiding collisions and creating maps. A main problem to investigate is to understand if it is possible to perform localization by only fusing monocular vision and inertial measurements. Localization is an estimation problem. The first issue to be addressed in any estimation problem is the observability property of the system. In control theory, a system is defined as observable when it is possible to reconstruct its initial state by knowing, in a given time interval, the control inputs and the outputs [7]. The observability property has a very practical meaning. It is easy to realize from the definition that when a system is observable it contains all the necessary information to perform the estimation with an error which is bounded [7]. Regarding the localization problem, this means that the observability implies a bound error in the localization. The value of this bound obviously depends on the accuracy of the sensors. Regarding the localization problem, the observability analysis was carried out from several authors. Roumeliotis [17] presented it for a multi robots system equipped with encoder and sensors able to provide an observation consisting of the relative configuration between each pair of robots. The analysis was performed through the linear approximation. The main result of this observability analysis was that the system is not observable and it becomes observable when at least one of the robots in the team has global positioning capabilities. Bonnifait and Garcia considered the case of one robot equipped with encoders and sensors able to provide the bearing angles of known landmarks in the environment [3]. The observability analysis was carried out by linearizing the system (as in the previous case) and by applying the *observability rank condition* introduced by Hermann and Krener in [6] for nonlinear systems. As in many nonlinear systems, they found that in some cases while the associated linearized system is not observable, the system is observable. Bicchi and collaborators extended this result to the Simultaneous Localization and Mapping (SLAM) problem ([2], [10]). They considered one robot equipped with the same bearing sensors of the previous case. They considered in the environment landmarks with a priori known position and landmarks whose position has to be estimated. They found that two landmarks are necessary and sufficient to make the system observable. Furthermore, they applied optimal control methods in order to minimize the estimation error. In particular, in [10] they maximized the Cramer-Rao lower bound as defined in [9].

In [16] a distributed GPS-based localization algorithm for 3D cooperative multi robot systems has been presented by the authors; in particular it turns out that the entire system is observable for such measurements. On the other hand, the unavailability of GPS-data could lead to a lack of observability.

In this paper we present an observability analysis for 3D cooperative localization and mapping. We consider a system of cooperative flying robots equipped with bearing sensors (cameras) and inertial sensors (accelerometers, gyroscopes). It will be taked into account three main issues: relative localization between two robots, localization with respect to a common global frame, mapping. We have found that, while the considered systems are completely observable for relative localization and mapping, in the case of global localization an invariance with respect to rotations around the vertical axis arises. In particular it is not possible to estimate the yaw angle of all the robots of the team.

II. PROBLEM STATEMENT

We provide here a mathematical description of our system. We introduce a global frame, whose z-axis is the vertical one. Let us consider a robot equipped with IMU proprioceptive sensors (an accelerometer and a gyroscope) as well as some suitable exteroceptive sensors (GPS, range sensors). In this

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paper we assume that the IMU data are unbiased. From a practical point of view, unbiased data can be obtained by continuously calibrating the IMU sensors (see for instance [8]). The configuration of the robot is described by a vector $(r, v, \theta) \in \mathbf{R}^9$ where $r = (r_x, r_y, r_z) \in \mathbf{R}^3$ is the position, $v = (v_x, v_y, v_z) \in \mathbf{R}^3$ is the speed and $\theta = (\theta_\mathrm{r}, \theta_\mathrm{p}, \theta_\mathrm{y}) \in \mathbf{R}^3$ assignes the robot orientation: θ_r is the roll angle, θ_p is the pitch angle and θ_y is the yaw angle. We will adopt lower case letters to express a quantity in the global frame, while capital letters for the same quantity exptressed in the local frame (i.e. the one attached to the robot).The system description can be simplified adopting a quaternions framework. We recall that the quaternions space $\mathbf{H}$ is the noncommutative set of elements

$$\mathbf{H} = \{q_t + q_x i + q_y j + q_z k : \quad q_t, q_x, q_y, q_z \in \mathbf{R}, \\ i^2 = j^2 = k^2 = ijk = -1\}.$$

For an arbitrary quaternion $q = q_t + q_x i + q_y j + q_z k$, we define the conjugate element $q^* = q_t - q_x i - q_y j - q_z k$ and the norm $||q|| = \sqrt{qq^*} = \sqrt{q^* q} = \sqrt{q_t^2 + q_x^2 + q_y^2 + q_z^2}$.

Denoting by A, Ω the accelerometer and the gyroscope values respectively and by a_g the gravity acceleration (i.e. $a_g = -(0, 0, g)$ with $g \simeq 9.81 m/s^2$), the continuous-time dynamics of the robot is given by the following system of ordinary differential equations

$$\begin{cases} \dot{r} = v \\ \dot{v} = q \cdot A \cdot q^* + a_g \\ \dot{q} = \frac{1}{2} q \cdot \Omega \end{cases} \tag{1}$$

where r, v, Ω, A are purely imaginary quaternions, while q is a unitary quaternion. The following relations for roll, pitch and yaw angles $\theta_\mathrm{r}, \theta_\mathrm{p}, \theta_\mathrm{y}$ hold

$$\theta_\mathrm{r} = \frac{q_t q_x + q_y q_z}{1 - 2(q_x^2 + q_y^2)}$$

$$\theta_\mathrm{p} = q_t q_y - q_x q_z$$

$$\theta_\mathrm{y} = \frac{q_t q_z + q_y q_x}{1 - 2(q_y^2 + q_z^2)}.$$

Let us consider a single feature environment and suppose to have a robot equipped with a camera. As shown in [14], the dimension of the maximal observable subsystem is 9: the camera measurements together with the IMU sensors do not provide enough information in order to estimate the yaw angle θ_y. Following [13], we can reformulate the above proposition saying that $\mathcal{S} = (-2r_y, 2r_x, 0, -2v_y, 2v_x, 0, -q_z, -q_y, q_x, q_t)$, corresponding to a rotation around the z-axis, constitutes the only continuous symmetry of the system with bearing observations.

When systems composed by more than one robot are considered, the observability properties should change. Two main issues to address are the observability of the relative system (placing a local frame on one of the robots) as well as the observability in the presence of features (mapping process). To this purpose, in the present paper we will discuss the answers to the following questions:

1) Let consider a system composed by two robots performing relative observation via cameras. Does the system contains enough information to estimate the relative pose of the robots?

2) Let us consider a system composed by several robots equipped with cameras and a fixed landmark associated to a global reference frame. Which are the observable quantities of the problem?

3) When, during the mapping process, the state vector is increased by the addition of some new feature coordinates, is the augmented state observable?

Sections III, IV and V are, respectively, dedicated to the answers of such questions. For linear control systems there exists a closed-formula to express the observable subspace and the observability properties can be checked by simple algebraic conditions; nonlinear observability is a harder task to deal with. Following the results by Hermann and Krener and by Isidori ([6], [7]), we can ensure the system observability providing that, among the set of all Lie derivatives of the observation functions, it is satisfied the so called *observability rank condition*: the dimension of the maximal observable subspace is equal to the rank of the matrix having as rows the differantials of Lie derivatives of all robots observations.

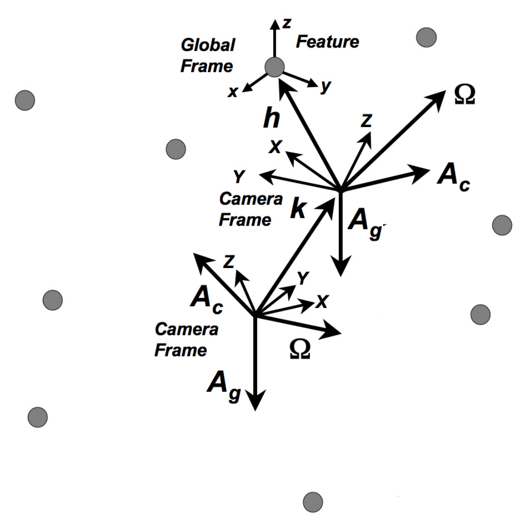

Fig. 1. Two robots scenario: cooperative localization and mapping.

III. LOCAL FRAME OBSERVABILITY

Let us suppose that $N = 2$ and assume that the robots perform only relative observation, i.e. no feature measurements are involved in the estimation process. The aim of this section is to investigate the observability of the system when the relative dynamics is considered. Let us assume the first robot configuration as the reference frame and express the pose of the second robot using relative coordinates. We obtain a system very similar to (1); the key difference is the dependence on several control functions. Adopting the notations $r = r_2 - r_1$, $v = v_2 - v_1$, $q = q_2 \cdot q_1^*$, the following equations system describes the relative dynamics:

$$\begin{cases} \dot{r} = v \\ \dot{v} = qA_2q^* - A_1 \\ \dot{q} = \frac{1}{2}\left(q\Omega_2 + \Omega_1^* q^*\right) \end{cases} \tag{2}$$

The camera on the second robot provides the observation

$$h = [h_1, h_2]^T = \left[\frac{(q^*rq)_x}{(q^*rq)_z}, \frac{(q^*rq)_y}{(q^*rq)_z}\right]^T, \tag{3}$$

Remark 3.1: It is worth to note that $q = q_2 \cdot q_1^*$ is still a unitary quaternion:

$$q \cdot q^* = q_2 \cdot q_1^* \cdot (q_2 \cdot q_1^*)^* = q_2 \cdot q_1^* \cdot q_1 \cdot q_2^* = 1.$$

As a consequence, the quaternion norm

$$h_0 = q \cdot q^* \tag{4}$$

can be treated as a further observation In the following we will use the notations

$$A_i = [0, A_i^x, A_i^y, A_i^z] \quad \Omega_i = [0, \Omega_i^x, \Omega_i^y, \Omega_i^z] \qquad i = 1, 2.$$

We can define the vector fields $F_\star : \mathbb{R}^{10} \to \mathbb{R}^{10}$

$$F_0 = (v_x, v_y, v_z, 0, 0, 0, 0, 0, 0, 0)$$

$$F_{A_1^x} = (0, 0, 0, -1, 0, 0, 0, 0, 0, 0)$$

$$F_{A_1^y} = (0, 0, 0, 0, -1, 0, 0, 0, 0, 0)$$

$$F_{A_1^z} = (0, 0, 0, 0, 0, -1, 0, 0, 0, 0)$$

$$F_{A_2^x} = (0, 0, 0, q_t^2 + q_x^2 - q_y^2 - q_z^2, 2(q_xq_y + q_tq_z), 2(q_xq_z - q_yq_t), 0, 0, 0, 0)$$

$$F_{A_2^y} = (0, 0, 0, 2(q_xq_y - q_tq_z), q_t^2 + q_y^2 - q_x^2 - q_z^2, 2(q_xq_t + q_yq_z), 0, 0, 0, 0)$$

$$F_{A_2^z} = (0, 0, 0, 2(q_xq_z + q_tq_y), 2(q_yq_z - q_tq_x), q_t^2 + q_z^2 - q_y^2 - q_x^2, 0, 0, 0, 0)$$

$$F_{\Omega_1^x} = \frac{1}{2}(0, 0, 0, 0, 0, 0, -q_x, q_t, q_z, -q_y)$$

$$F_{\Omega_1^y} = \frac{1}{2}(0, 0, 0, 0, 0, 0, -q_y, -q_z, q_t, q_x)$$

$$F_{\Omega_1^z} = \frac{1}{2}(0, 0, 0, 0, 0, 0, -q_z, q_y, -q_x, q_t)$$

$$F_{\Omega_2^x} = \frac{1}{2}(0, 0, 0, 0, 0, 0, -q_x, -q_t, -q_z, q_y)$$

$$F_{\Omega_2^y} = \frac{1}{2}(0, 0, 0, 0, 0, 0, -q_y, q_z, -q_t, -q_x)$$

$$F_{\Omega_2^z} = \frac{1}{2}(0, 0, 0, 0, 0, 0, -q_z, -q_y, q_x, -q_t)$$

The system dynamics can be rewritten as

$$(\dot{r}, \dot{v}, \dot{q}) = F_0 + \sum_{\substack{i = 1, 2 \\ \star \in \{x, y, z\}}} \left(F_{A_i^\star} A_i^\star + F_{\Omega_i^\star} \Omega_i^\star\right). \tag{5}$$

The above equation is dependant on 10 independent variables. The following theorem states that system is fully observable in the local frame.

Theorem 3.1: It is given the equation (5) together with the observation function $h = [h_1, h_2]^T$ assigned by (3). The system verifies the observability rank condition, i.e. all the system variables are observable quantities.

Proof: Let us consider the Lie derivatives of the functions h_1, h_2 along the vector fields $F_\star$. We need to prove that, within the family of all Lie derivatives, a subset can be choosen such that the whole state space is spanned by the corresponding differentials. It is easy to verify that the requested property is satisfied, for example, by

$$\mathcal{L}^0 h_1, \quad \mathcal{L}^1_{F_0} h_1, \quad \mathcal{L}^1_{\Omega_1^x} h_1, \quad \mathcal{L}^1_{\Omega_1^y} h_1, \quad \mathcal{L}^1_{\Omega_2^x} h_1, \quad \mathcal{L}^1_{\Omega_2^z} h_1,$$

$$\mathcal{L}^2_{A_1^x F_0} h_1, \quad \mathcal{L}^2_{A_2^x F_0} h_1, \quad \mathcal{L}^2_{\Omega_2^x F_0} h_1, \quad \mathcal{L}^1_{F_0} h_2.$$

The explicit computation, as it is long but trivial, is omitted. ■

Compared to the problem studied in [14], the robot to which is attached the local frame, is not treated as a simple feature: due to the presence of IMU sensors of both robots, it is possible to estimate the (relative) yaw angle too. This means that, having two or more robots equipped with cameras, the cooperative relative localization problem can be completely solved.

IV. GLOBAL FRAME OBSERVABILITY

Consider now the case of two robots whose coordinates are expressed in a common global frame identified with the position of a fixed landmark. For sake of notation clarity let us denote with (r, v, q) the pose of the first robot and with (p, w, b) the pose of the second one. The camera on the first robot observes the feature at the origin while the second robot performs a relative observation. The dynamics

dynamics is described by the following systems.

First robot:

$$\begin{cases} \dot{r} = v \\ \dot{v} = q \cdot A_1 \cdot q^* + a_g \\ \dot{q} = \frac{1}{2} q \cdot \Omega_1 \end{cases} \tag{6}$$

$$h = [h_1, h_2]^T = \left[\frac{(q^* r q)_x}{(q^* r q)_z}, \frac{(q^* r q)_y}{(q^* r q)_z} \right]^T, \tag{7}$$

$$h_0 = q \cdot q^*. \tag{8}$$

Second robot:

$$\begin{cases} \dot{p} = w \\ \dot{w} = b \cdot A_2 \cdot b^* + a_g \\ \dot{b} = \frac{1}{2} b \cdot \Omega_2 \end{cases} \tag{9}$$

$$k = [k_1, k_2]^T = \left[\frac{(b^*(p-r)b)_x}{(b^*(p-r)b)_z}, \frac{(b^*(p-r)b)_y}{(b^*(p-r)b)_z} \right]^T, \tag{10}$$

$$k_0 = b \cdot b^*. \tag{11}$$

We can rewrite the equations, as for the local frame case, as

$$(\dot{r}, \dot{v}, \dot{q}, \dot{p}, \dot{w}, \dot{b}) = F_0 + \sum_{\substack{i = 1,2 \\ \star \in \{x,y,z\}}} \left(F_{A_i^\star} A_i^\star + F_{\Omega_i^\star} \Omega_i^\star \right).$$

Following the previous case, the expressions for the vector fields $F_\star : \mathbb{R}^{20} \to \mathbb{R}^{20}$ can be easily derived; for example we have

$$F_0 = (v_x, v_y, v_z, 0, ..., 0) \oplus (w_x, w_y, w_z, 0, ..., 0),$$

$$F_{A_1^x} = (0, 0, 0, q_t^2 + q_x^2 - q_y^2 - q_z^2, 2(q_x q_y + q_t q_z), 2(q_x q_z - q_y q_t), 0, 0, 0, 0) \oplus (0, 0, ..., 0)$$

$$F_{\Omega_2^z} = (0, 0, ..., 0) \oplus \frac{1}{2}(0, 0, 0, 0, 0, 0, -b_x, b_t, b_z, -b_y),$$

where we have adopted the notation

$$(u_1, ..., u_{n_1}) \oplus (\omega_1, ..., \omega_{n_2}) = (u_1, ..., u_{n_1}, \omega_1, ..., \omega_{n_2}).$$

As mentioned before (see [14]) the first system is observable up to an invariance with respect to the z-axis (associated to yaw angle). It is worth to note that nine Lie derivatives can be choosen independent without involving the control Ω_1. These are for example:

$$L_1 = \Big\{ \mathcal{L}^0 h_1, \quad \mathcal{L}^0 h_2, \quad \mathcal{L}^1_{F_0} h_1, \quad \mathcal{L}^1_{F_0} h_2, \quad \mathcal{L}^2_{A_1^x F_0} h_1, \quad \mathcal{L}^2_{F_0 F_0} h_1, \quad \mathcal{L}^2_{F_0 F_0} h_2, \quad \mathcal{L}^3_{F_0 A_1^x F_0} h_1, \quad \mathcal{L}^0 h_0 \Big\}.$$

Combining the observations performed by both robots, we are able to estimate all state vector components but one; there is again an invariance with respect to rotation of the entire system around the vertical axis This invariance is due to the absence of an absolute orientation reference. We can state the following result

Theorem 4.1: Consider the two robots system given by equations (6)-(11). The maximal observable subspace has dimension 19, that is, all system variables but one can be estimated; in particular the system configuration is invariant with respect to rotations around the z-axis.

Proof: The maximum number of independent Lie derivatives is 19. This can be verified by direct inspection; for example, the diffentials of following set of Lie derivatives together with those given in L_1, constitute a 19×20 matrix with maximum rank, i.e. $rk(\mathrm{d}L_1 \cup \mathrm{d}L_2)) = 19$:

$$L_2 = \Big\{ \mathcal{L}^0 k_1, \quad \mathcal{L}^0 k_2, \quad \mathcal{L}^1_{F_0} k_1, \quad \mathcal{L}^1_{F_0} k_2, \quad \mathcal{L}^2_{A_1^x F_0} k_1, \quad \mathcal{L}^2_{A_1^y F_0} k_1, \quad \mathcal{L}^2_{F_0 F_0} k_1, \quad \mathcal{L}^2_{A_1^y F_0} k_1, \quad \mathcal{L}^2_{A_z^x F_0} k_2, \quad \mathcal{L}^0 k_0 \Big\}.$$

As before the explicit computation is omitted. The invariance with respect to rotations around the z-axis can be obtained observing that the null space of the matrix generated by all Lie derivatives differentials is spanned by the vector

$$\omega = (-2r_y, 2r_x, 0, -2p_y, 2p_x, 0, -2v_y, 2v_x, 0, -2w_y, 2w_x, 0, -q_z, -q_y, q_x, q_t, -b_z, -b_y, b_x, b_t).$$

■

Let us point out that the continuous symmetry ω (see [13]) has the same structure of the symmetry $\mathcal{S}$ obtained for a single robot.

Corollary 4.1: It follows from the structure of the Lie derivatives set $L_1 \cup L_2$, that the observability is guaranteed without the explicit employing of the data from the gyroscopes Ω_1, Ω_2.

V. OBSERVABILITY AND MAPPING

In the previous section a simplified two-robots scenario has been analized; nevertheless, the derived observability analysis can be applied straightforward to general multi robot systems. In particular, considering a N robots system and therefore a 10N dimensional state vector, 10N-1 independent observable quantities can be found. This can be seen noting that the observations of the form (10) are independent one to each other if performed by different robots, this meaning that any of the components of the team is able to provide up to 10 independent observable quantities. These considerations lead us to the statement that, in the cooperative localization problem, all variables but one are observable quantities. In this section we will analize the problem of observability in cooperative localization and mapping. In particular we are interested in the observability of the system when a new feature is observed and as a consequence the dimension of the state vector, which contains also the environment map, has to be increased. We will not discuss the algorithm used to perform the estimation process.

The state of the system, composed by the poses of N robots and the coordinates of M observed features is given by

$$X = (r_1, v_1, q_1, , ..., r_N, v_N, q_N, x_1, y_1, z_1, ..., x_M, y_M, z_M).$$

When a new feature is observed, the state vector is changed by the rule

$$X \rightarrow X \oplus (x_{M+1}, y_{M+1}, z_{M+1}).$$

The feature observation by robot j, with respect to the global frame reference is expressed by the equation

$$h^{(M+1)} = [h_1^{(M+1)}, h_2^{(M+1)}]^T =$$

$$= \left[\frac{(q_j^*(r_j - \varrho_{M+1})q_j)_x}{(q_j^*(r_j - \varrho_{M+1})q_j)_z}, \frac{(q_j^*(r_j - \varrho_{M+1})q_j)_y}{(q_j^*(r_j - \varrho_{M+1})q_j)_z} \right]^T, \quad (12)$$

where $\varrho_{M+1} = (0, x_{M+1}, y_{M+1}, z_{M+1})$ is the position of the feature in the global frame. Each time a new feature is added to the map, the vector fields $F_\star$ are modified as follows

$$F_\star \rightarrow F_\star \oplus (0, 0, 0).$$

Proposition 5.1: The observation function $h^{(M+1)}$ allows estimating of the variables $(x_{M+1}, y_{M+1}, z_{M+1})$.

Proof: The validity of the proposition is straightforward. It is sufficient to consider the Lie derivatives

$$L_{M+1} = \left\{ \mathcal{L}^0 h_1^{(M+1)}, \quad \mathcal{L}^0 h_2^{(M+1)}, \quad \mathcal{L}^1_{F_0} h_1^{(M+1)} \right\};$$

set the operator $\nabla_{\varrho M+1} = (\partial_{x_{M+1}}, \partial_{y_{M+1}}, \partial_{z_{M+1}})$ and define the matrix $\mathcal{H}_{M+1}$ as

$$\mathcal{H}_{M+1} = \begin{pmatrix} \nabla_{\varrho M+1} \mathcal{L}^0 h_1^{(M+1)} \\ \nabla_{\varrho M+1} \mathcal{L}^0 h_2^{(M+1)} \\ \nabla_{\varrho M+1} \mathcal{L}^1_{F_0} h_1^{(M+1)} \end{pmatrix}$$

Since $rk(\mathcal{H}_{M+1}) = 3$ and $\nabla_{\varrho M+1} \mathcal{L}^0 h_\star = 0$ for any $h_\star \neq h_1^{(M+1)}, h_2^{(M+1)}$, we see that the observability rank condition in satisfied. ■

We can conclude our analysis with the following statement:

Corollary 5.1: In the cooperative SLAM problem for N flying robots the number of observable independent quantities is given by

$$10N - 1 + 3M,$$

where M is the number of observed features.

Remark 5.1: A key issue in the simoultaneous localization and mapping problem (SLAM) is the so called loop-closure, this meaning the re-observation of a feature already stored in the estimated map. In particular in the presence of a loop closure, a good SLAM algorithm is asked to perform a correction over the whole estimated state. Nevertheless such critical situation does not change anything on the system observability properties.

VI. CONCLUSIONS

In this paper we have focused our analysis on the obserbaility properties for the problem of cooperative localization and mapping using measurements obtained from bearing sensors and inertial sensors. We have addressed the following points:

- *Relative localization*: The problem of relative localization between two robots has been considered; using only cameras and inertial sensors measurements the system is observable, i.e. the relative configuration can be completelty estimated.
- *Global-frame localization*: The obserbability analysis has been extended to cooperative localization of N robots with respect to a common global frame (assumed to be attached to a known feature). In this case, employing the measurements obtained by cameras and accelerometers, all the system variables but one can be estimated, i.e. since the configuration of each robot is characterized by 10 independent variables, the maximal observable subsystem has dimension $10N - 1$. The absolute yaw angle is not observable, since the entire system configuration is invariant with respect to rotations around the vertical axis.
- *Mapping*: It has been showed that, if the state vector is augmented by adding the coordinates of a new observed feature, the observations provided by the camera are able to estimate such new variables without violating the observability properties of the system; in particular, the mapping problem is observable.

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