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Games, Deception, and Jones' Lemma

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Abstract— **Deception is pervasive in adversarial situations. Here we present a formulation of deception using a two-player game setting. One of the two players deploys a sensor network to gather information on the opponent who in turn can employ deception tactics. We solve the resulting general game using linear programs. We pose an illustrative example and develop closed form solutions for special cases. Finally, we show how our solutions capture the well-known "Jones' Lemma" from the deception literature.**

I. INTRODUCTION

Deception plays an important role in a large variety of adversarial situations ranging from competition in nature [1] to warfare [2], [3]. Importance of deception in autonomous systems has been emphasized by McEneaney [4] where it is stated that "Deception is a critical component of real-world games in complex and imperfectly observed environments. However, even the basic mathematical definitions of issues in deception are not complete. This an important practical problem, which is natural to humans, but presents deep difficulties." Game theoretic methods have been leveraged to incorporate secrecy and deception into defensive strategies in many studies of deception. In [5], Brown et al. present a two-sided optimization model for planning the placement of defensive missile interceptors and examine the beneficial role secrecy and deception can play for either side. In a similar scenario involving potential terrorist attacks, Zhuang and Bier [6] explore whether the first mover in a two-step game should disclose the allocation of defensive resources or attempt to provide false information in the hopes of misleading the attacker in the second stage of the game. In [7], a general asymmetric zero-sum, two-player game is analyzed, and deception is used by one player to trick the second player into selecting a non-optimal action.

In most of the previous works, the player that falls victim to the deception is assumed to be ignorant of the possibility of deception. However, in many cases, it is common knowledge that one's opponent may be trying to implement some form of deception. This does not mean that all information should be ignored, but instead, the risk that a particular piece of information may be compromised needs to be balanced with the potential advantage that the information provides. An example of such a scenario can be found in [8] where the authors examine a particular two-player game in which one player utilizes cost-free, passive deception through concealment or disclosure of defensive resource allocations in order to neutralize the opponent's informational advantage. There have also been some results on the detection of deception within repeated games [9].

In this paper, we pose a generic two-player, zero-sum game in which a stochastic sensor network provides one player, Player B, an informational advantage over its opponent, Player A. Simultaneously, Player A possesses the ability to corrupt the sensor network output, at a cost, in an attempt to manipulate Player B's actions. The possible use of deception allows Player A to neutralize the informational advantage of Player B and shift the game's equilibrium value closer to the solution of the game where the information network is removed. It is assumed that Player B knows of the possibility of deception, but if the risk of deception is small enough, Player B will still utilize the information provided by its sensor network. We propose a utility function for the game which takes into account the effects that the deceptive tactics have on the sensor network and its corresponding cost. The solution to this game consists of the optimal strategies for each of the players and the corresponding value of the utility function. With respect to Player A, the optimal strategy represents the best mix of actions along with the complimentary deceptive tactic. The optimal strategy of Player B represents the best stochastic control law based on the measured sensor network output. Utilizing the relationship between the minimax theorem and the strong duality theorem of linear programing, we show that the solution of the zero-sum game can be computed by solving a pair of dual linear programming problems.

Using this framework, we examine an illustrative example, which can be modified to represent a large range of scenarios. In our example, Player A must select one of the two locations in order to store or hide a high value item. There are a number of information channels that provide Player B a noisy estimate of the location of the item, which it can then use to determine which location to attack or search. It is common knowledge that Player A can corrupt this information network, but the corruption is not cost free and the cost of corruption is incorporated into the value function.

We then analytically develop closed form solutions to two special cases of this game. The resulting equilibrium player strategies and utility values very nicely capture a well-known, qualitative principle in the deception field known as the "Jones' Lemma". This maxim is attributed to R. V. Jones who is considered to be the "father of scientific intelligence". It states, "Deception becomes more difficult as the number

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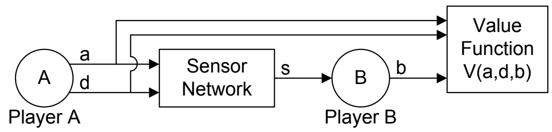


Fig. 1. Overall Game Structure

of channels available to the target increases. Nevertheless, within limits, the greater the number of channels that are controlled by the deceiver, the greater the likelihood that the deception will be believed."[3] We further explore Jone's Lemma for a range of parameter values by numerically solving the general linear program solution.

II. Game Formulation

In this section, we develop a zero-sum game with two players, Player A and Player B, attempting to minimaximize a von Neumann-Morgenstern utility function.

A. Player Actions and Deception Tactics

Within the game, each player must select a single action from their respective sets of possible actions. Player A selects an action $a \in A$, where $A := \{a_1, a_2, \ldots, a_l\}$ is the set of its possible actions. Similarly, Player B selects action $b \in B$, where $B := \{b_1, b_2, \ldots, b_m\}$ is its own set possible actions. Player A must also select a deception tactic, d from the set of possible deception tactics $D := \{d_1, d_2, \ldots, d_p\}$. Player A's selected action and deception tactic along with Player B's selected action are passed into the games value function $V(a, d, b)$, which generates the value that both players strive to minimaximize.

B. Sensor Network

Player A's action and deception tactic are passed into a stochastic sensor network. The stochastic sensor network produces a sensor value $s \in S$, where $S := \{s_1, s_2, \ldots, s_N\}$ is the set of N possible sensor values. The sensor values within S can represent object classifications, strategy predictions, or raw sensor measurements. The probability that the sensor network will produce a particular sensor value s given that Player A has played action a and deception tactic d is determined by the conditional probability distribution $P_{S|A,D}(s|a, d)$. The conditional probability distribution fully defines the sensor network characteristics and is common knowledge within the game, i.e., both players know $P_{S|A,D}(s|a, d)$.

C. Player Strategies

In this game, a player's strategy is defined as the probability distribution representing the likelihood of selecting a particular action from its action set. Because Player A must select an action and a deceptive tactic, Player A's strategy is defined as the joint probability distribution $P_{A,D}(a, d)$. Player B is allowed to measure the output of the sensor network before selecting its action. Due to the possible dependence of the measured sensor value, Player B's strategy is represented by conditional probability mass function $P_{B|S}(b|s)$.

D. Utility Function

Using the player strategies and value function, we can define the von Neumann-Morgenstern utility function [10]:

$$U(P_{A,D}(a,d), P_B(b)) := \sum_{A,D,S,B} P_{A,D}(a,d) P_{S|A,D}(s|a,d) P_{B|S}(b|s) V(a,d,b) \quad (1)$$

The utility function represents the expected value when each player implements their respective strategies. The opposing goals of the players lead to the following zero-sum game in which Player A strives to minimize the utility function while Player B simultaneously attempts to maximize.

$$U^* := \min_{P_{A,D}(a,d)} \max_{P_{B|S}(b|s)} U(P_{A,D}(a,d), P_B(b)) \quad (2)$$

Although, the game possesses a sequential structure, Player B only has information generated by the sensor network and does not posses direct knowledge of Player A's selected action. Therefore, the selection of Player B's optimal strategy in terms of s, can be generated at the same time Player A develops its optimal strategy.

III. General Game Solution

We begin developing the solution by parameterizing Player A and Player B's strategies using matrices $\boldsymbol{\alpha} = [\alpha_{ij}]$ and $\boldsymbol{\beta} = [\beta_{ij}]$ respectively, where $\alpha_{ij} = P_{A,D}(a_i, d_j)$ and $\beta_{ij} = P_{B|S}(b_i|s_j)$. The value function $V(a, d, b)$ is parameterized in matrix form using $\mathbf{V} = [V_{l(j-1)+i,k}]$, where $V_{l(j-1)+i,k} = V(a_i, d_j, b_k)$. The conditional probability functions that describe the sensor network are placed in matrix $\boldsymbol{\sigma} = [\sigma_{l(j-1)+i,k}]$ such that $\sigma_{l(j-1)+i,k} = p(s_k|a_i, d_j)$. Using these matrix parameterizations, we can rewrite the utility function (1) in matrix form:

$$\begin{aligned} W(\boldsymbol{\alpha}, \boldsymbol{\beta}) &:= \text{vec}(\boldsymbol{\alpha})^T \mathbf{V}_s \text{vec}(\boldsymbol{\beta}) && (3) \\ &= U(P_{A,D}(a,d), P_B(b)) && (4) \end{aligned}$$

where the symbol $\text{vec}(\mathbf{A})$ represents a column vector formed by stacking the columns of matrix $\mathbf{A}$ below one another. The matrix $\mathbf{V}_s$ is defined as the row-wise Kronecker product of $\boldsymbol{\sigma}$ and $\mathbf{V}$:

$$\mathbf{V}_s := \begin{bmatrix} \boldsymbol{\sigma}_{1*} \otimes \mathbf{V}_{1*} \\ \boldsymbol{\sigma}_{2*} \otimes \mathbf{V}_{2*} \\ \vdots \\ \boldsymbol{\sigma}_{lp*} \otimes \mathbf{V}_{lp*} \end{bmatrix} \quad (5)$$

where $\boldsymbol{\sigma}_{i*}$ and $\mathbf{V}_{i*}$ represent the i$^{\text{th}}$ row of $\boldsymbol{\sigma}$ and $\mathbf{V}$ respectively and the symbol $\otimes$ indicates the Kronecker product. From (4), the original game (2) can be rewritten:

$$\begin{aligned} W(\boldsymbol{\alpha}, \boldsymbol{\beta})^* &:= \min_{\boldsymbol{\alpha}} \max_{\boldsymbol{\beta}} \text{vec}(\boldsymbol{\alpha})^T \mathbf{V}_s \text{vec}(\boldsymbol{\beta}) \\ &= \min_{P_{A,D}(a,d)} \max_{P_{B|S}(b|s)} U(P_{A,D}(a,d), P_B(b)) \\ &= U^* && (6) \end{aligned}$$

with the constraints

$$\alpha_{ij} \geq 0 \quad \text{and} \quad \sum_{i,j} \alpha_{ij} = 1 \quad (7)$$

$$\beta_{ij} \geq 0 \quad \text{and} \quad \sum_i \beta_{ij} = 1 \, \forall j = 1, 2, \ldots, n \tag{8}$$

The constraints (7) and (8) ensure that the parameterizations $\boldsymbol{\alpha}$ and $\boldsymbol{\beta}$ represent valid probability mass functions.

The solution to this game is the pair of equilibrium strategies $\boldsymbol{\alpha}^*$ and $\boldsymbol{\beta}^*$ and the corresponding equilibrium utility U^* that satisfy the following condition

$$W(\boldsymbol{\alpha}^*, \boldsymbol{\beta}) \leq W(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*) = U^* \leq W(\boldsymbol{\alpha}, \boldsymbol{\beta}^*) \tag{9}$$

Using the strong duality theorem of linear programming, the solution can be found by solving a set of dual linear programming problems [10], [11]. We will denote by $\boldsymbol{e}$ and $\mathbf{0}$ column vectors containing all ones and all zeros respectively. Vector inequalities are evaluated element-wise.

Theorem 1: Consider the game defined in (6):

$$U^* = \min_{\boldsymbol{\alpha}} \max_{\boldsymbol{\beta}} \operatorname{vec}(\boldsymbol{\alpha})^T \mathbf{V}_s \operatorname{vec}(\boldsymbol{\beta}). \tag{10}$$

The equilibrium strategies, $\boldsymbol{\alpha}^*$ and $\boldsymbol{\beta}^*$, and the resulting equilibrium value, U^*, are given by the solutions of the following dual linear programming problems:

Solving for $\boldsymbol{\alpha^*}$ and $\mathbf{u}^*$

$$\boldsymbol{\alpha}^*, \mathbf{u}^* = \operatorname*{argmin}_{\boldsymbol{\alpha}, \mathbf{u}} \begin{bmatrix} \mathbf{0}^T & \boldsymbol{e}^T \end{bmatrix} \begin{bmatrix} \operatorname{vec}(\boldsymbol{\alpha}) \\ \mathbf{u} \end{bmatrix} \tag{11}$$

$$\text{s.t.} \quad \begin{array}{c} \mathbf{V}_s^T \operatorname{vec}(\boldsymbol{\alpha}) - (\mathbf{u} \otimes \boldsymbol{e}) \leq \mathbf{0} \\ \operatorname{vec}(\boldsymbol{\alpha}) \geq \mathbf{0} \quad \text{and} \quad \sum_{i,j} \alpha_{i,j} = 1 \end{array} \tag{12}$$

Solving for $\boldsymbol{\beta}^*$ and v^*

$$\boldsymbol{\beta}^*, v^* = \operatorname*{argmax}_{\boldsymbol{\beta}, v} \begin{bmatrix} \mathbf{0}^T & 1 \end{bmatrix} \begin{bmatrix} \operatorname{vec}(\boldsymbol{\beta}) \\ v \end{bmatrix} \tag{13}$$

$$\text{s.t.} \quad \begin{array}{c} v\boldsymbol{e} - \mathbf{V}_s \operatorname{vec}\boldsymbol{\beta} \leq \mathbf{0} \\ \boldsymbol{\beta} \geq \mathbf{0} \quad \text{and} \quad \sum_i \beta_{ij} = 1 \, \forall j = 1, 2, \ldots, n+1 \end{array} \tag{14}$$

Equilibrium Value U^*

$$U^* = \boldsymbol{e}^T \mathbf{u}^* = v^* \tag{15}$$

IV. Two-Action Game with Identical Information Channels

A. Motivating Scenario

Consider a scenario in which Player A represents the leader of an illegal drug distribution network. He knows that there is an impending raid on one of two possible locations where the drugs are hidden, and he knows that it has not yet been decided which site will be targeted by the law enforcement organization whose leader is represented by Player B. The first location is a local community center and the other location is Player A's warehouse. It is relatively easy to hide the drugs at his warehouse, but if they are found there, it will be difficult to deny involvement and Player A will most likely be convicted. This is considered the best possible outcome from the law enforcement's perspective.

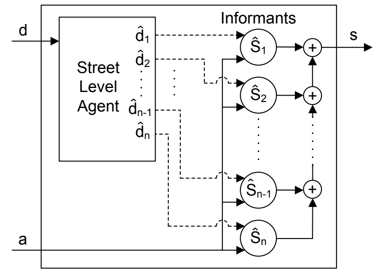


Fig. 2. Information Network for Two-Action Game

Hiding the drugs at the community center will be initially more costly; however, if the community center is raided while the drugs are present, there will be less evidence linking Player A to the drugs, and it will be easier to avoid prosecution. On the other hand, if the community center is raided and the drugs are not there, there will be significant community outcry against Player B, which will hinder future operations against Player A. From Player A's perspective, this is the best possible outcome.

It is also common knowledge that there are several informants within the community that are willing to provide information to the law enforcement about the location of the drugs. These informants are not always accurate though and can only correctly identify the location with a probability of p_{id}. It is common knowledge, that Player A is able to persuade these informants to provide false information if he is willing to pay a particular price, c_d. Player A does not directly pick which informants to pay off, but tells a lower level agent the total number of informants to pay off. It is assumed that any resources used to pay these informants are resources not used to produce more drugs, which is beneficial from the law enforcement's view.

The drug network leader must decide on the location to hide the drugs and how many informants should be paid off in an attempt to mislead Player B. On the other hand, Player B must determine which location to raid based on the information provided by the informants while taking into account that these informants could possibly be corrupt.

B. Game Model and Description

Both Player A and Player B have two possible actions to choose from: $A := \{a_1, a_2\}$ and $B := \{b_1, b_2\}$. Actions a_1 and a_2 represent Player A's choices to hide the drugs at the community center or his warehouse, respectively. Actions b_1 and b_2 represent Player B's choices to raid the community center or the leader's home, respectively.

The sensor network consists of n identical informants and is depicted graphically in Figure 2. Each informant produces a scalar value $\hat{s}$. When an informant is not corrupted, it correctly identifies Player A's action with a likelihood of p_{id} by outputting a zero for a_1 and a one for a_2. The informant may be corrupted by the deception signal $\hat{d}_i$, where $\hat{d}_i = 1$ forces the informant to zero; $\hat{d}_i = 2$ does not corrupt the channel; and $\hat{d}_i = 3$ forces the informant to one.

Although each informant can be manipulated through its corresponding deception signal $\hat{d}$, Player A is only allowed to select the total number of informants to corrupt through its deception strategy d. It is assumed that Player A must force all corrupted informants to the same value. For example, Player A may choose to set five informants to zero, but Player A is not allowed to simultaneously set two informants to zero and three other informants to one. Therefore, Player A has the option of forcing up to n informants to zero, forcing up to n informants to one, or not manipulating any informants for a total of $2n+1$ different deception tactics. A particular deception tactic $d_j \in D$, where $D := \{d_1, \ldots, d_{2n+1}\}$, defines the number of corrupted informants and their corresponding value according to the following rule.

$$\begin{array}{cc} \text{Force } ((n+1)-j) \text{ channels to zero} & j < n+1 \\ \text{No Corruption} & j = n+1 \\ \text{Force } (j-(n+1)) \text{ channels to one} & j > n+1 \end{array} \quad (16)$$

The outputs of the individual informants are added together, and the sum is used as the sensor network output $s \in S := \{s_1, \ldots, s_{n+1}\}$, where $s_i = i-1$. The value function $V(a_i, d_j, b_k) = V_{ik} + c_d|n+1-j|$, where c_d is the cost to corrupt a single informant. It is assumed that

$$V_{21} \leq V_{12} \leq V_{11} \leq V_{22}. \quad (17)$$

C. Game Parameterization

In this section, we parameterize the game in order to solve it using the method described in Theorem 1. The player strategies are parameterized the same as in Section III, where we represent Player A and Player B's strategies using $\boldsymbol{\alpha} = [\alpha_{i,j}]$ and $\boldsymbol{\beta} = [\beta_{i,j}]$, respectively.

The elements of the sensor network characteristic matrix $\boldsymbol{\sigma}$ are calculated as

$$\sigma_{2(j-1)+i,k} = \left\{ \begin{array}{cc} B(j, p(a_i), k) & k \leq j, j \leq N \\ 0 & k > j, j < N \\ 0 & k < j-N, j > N \\ B(2N-j, p(a_i), k-j+N) & k \geq j-N, j > N \end{array} \right. \quad (18)$$

where

$$B(n,p,k) = \binom{n}{k} p^k (1-p)^{n-k} \quad (19)$$

and

$$p(a) = \left\{ \begin{array}{cc} (1-p_{id}) & a = a_1 \\ p_{id} & a = a_2 \end{array} \right. \quad (20)$$

The elements of value matrix $\mathbf{V}$ are

$$V_{2(j-1)+i,k} = V(a_i, d_j, b_k) = V(a_i, b_k) + c_d|n-j|. \quad (21)$$

D. Standard Game with No Deception and No Sensor Network

If there are no informants, the sensor network has only one possible value s_1 representing "no data". Player B's dependency on s is trivial, and $\boldsymbol{\beta}$ is simply a column vector representing the probability of playing each action b. Since there are no informants to corrupt, Player A only has one deception strategy d_1, which is the degenerative "no deception" tactic. Therefore Player A's strategy matrix $\boldsymbol{\alpha}$ also reduces to a column vector representing the probability of each action. This situation results in a scalar sensor characteristic matrix $\boldsymbol{\sigma} = 1$. Therefore, the games value function matrix is defined as $\mathbf{V_s} = \mathbf{V}$. These simplifications convert this game into a standard two-action zero-sum game whose solution follows easily from standard results [10].

Theorem 2: Assuming that $n = 0$ and the value function $V(a, d, b)$ possesses the structure defined by (17) and (21), the equilibrium strategies and resulting equilibrium value are given as follows.

Equilibrium Strategies

$$\boldsymbol{\alpha}^* = \begin{pmatrix} \frac{V_{22}-V_{21}}{V_{11}-V_{12}-V_{21}+V_{22}} & \frac{V_{11}-V_{12}}{V_{11}-V_{12}-V_{21}+V_{22}} \end{pmatrix}^T \quad (22)$$

$$\boldsymbol{\beta}^* = \begin{pmatrix} \frac{V_{22}-V_{12}}{V_{11}-V_{12}-V_{21}+V_{22}} & \frac{V_{11}-V_{21}}{V_{11}-V_{12}-V_{21}+V_{22}} \end{pmatrix}^T \quad (23)$$

Equilibrium Utility

$$U^* = W(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*) = V_s := \frac{V_{11}V_{22}-V_{21}V_{12}}{V_{11}-V_{12}-V_{21}+V_{22}} \quad (24)$$

This equilibrium represents the baseline value for Player A. As more informants are added to the game, Player B gains more information about Player A's selected action, and the value of the game increases. Player A can attempt to corrupt the informants in order to reduce the information content of the sensor network output and thereby hold the equilibrium value closer to the standard game equilibrium. Player A cannot reduce the equilibrium value below the basic game equilibrium by corrupting the sensors because Player B can always guarantee at least this value by playing the standard game mixed equilibrium strategy for each sensor value.

E. The Case of Perfect Informants $p_{id} = 1$

In the case where the informants can perfectly identify the drug location, $p_{id} = 1$, the sensor values are deterministically dependent on Player A's action and deception tactic. In other words, each combination of action and deception tactic can result in only one sensor value. However, the resulting sensor values are not necessarily unique. For instance, the combinations (a_1, d_2) and (a_2, d_{n+2}) both result in the same sensor value s_{n-1}. Therefore, Player A now has control over which sensor value Player B receives, but Player B cannot uniquely determine Player A's implemented action or deception tactic from the measured sensor value. This special case can be represented as a sequential game with imperfect information from Player B's perspective. The resulting closed form solution to this game is described in Theorem 3.

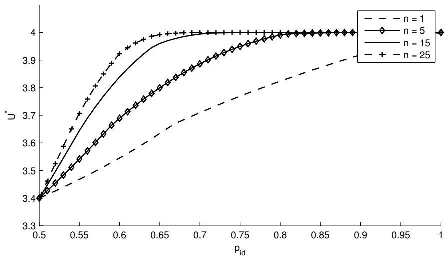


Fig. 3. Equilibrium Utility vs p_{id} with no Deception

Theorem 3: Suppose that $n > 0$, $p_{id} = 1$, and the value function $V(a, d, b)$ possesses the structure defined by (17) and (21). Let V_s be as in (24) and define the Jones' Cost J_c and the Jones' Ratio γ respectively as

$$J_c := \frac{(V_{11}-V_{12})nc_d}{V_{11}-V_{21}-V_{12}+V_{22}} \quad \text{and} \quad \gamma := \frac{V_{11}-V_{21}}{nc_d}. \tag{25}$$

Then, the equilibrium strategies and resulting equilibrium value are given as follows.

Equilibrium Strategies

$$\alpha^*_{ij} = P^*_{A,D}(a_i, d_j) = \begin{cases} \hat{\alpha} & i = 1, j = n+1 \\ 1-\hat{\alpha} & i = 2, j = 1 \\ 0 & \text{otherwise} \end{cases} \tag{26}$$

$$\beta^*_{ij} = P^*_{B|S}(b_i, s_j) = \begin{cases} \hat{\beta} & i = 1 \\ 1-\hat{\beta} & i = 2 \end{cases} \tag{27}$$

where

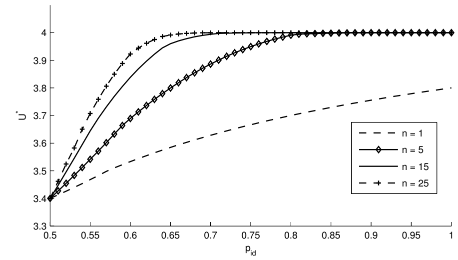
$$\hat{\alpha} = \begin{cases} \frac{V_{22}-V_{21}}{V_{11}-V_{21}-V_{12}+V_{22}} & \gamma \geq 1 \\ 1 & \gamma < 1 \end{cases} \tag{28}$$

$$\hat{\beta} = \begin{cases} \frac{V_{22}-V_{12}+nc_d}{V_{11}-V_{21}-V_{12}+V_{22}} & \gamma \geq 1 \\ 1 & \gamma < 1 \end{cases} \tag{29}$$

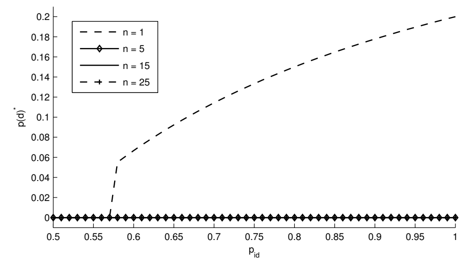
Equilibrium Utility

$$U^* = U(\boldsymbol{\alpha}^*, \boldsymbol{\beta}^*) = \begin{cases} V_s + J_c & \gamma \geq 1 \\ V_{11} & \gamma < 1 \end{cases} \tag{30}$$

In this special case, the equilibrium value for the game (30) is split into two terms. The value J_c, which we will refer to as the *Jones' Cost* (see Section V for connection to Jones' Lemma), represents the additional cost to Player A for mixing in the deceptive tactic d_1. The Jones Cost is critically dependent on the the number of informants and the cost of corruption. As either the number of information channels or the cost of corruption increases, the expected utility increases as well. Once the number of informants or the cost of corruption have increased such that $\gamma < 1$, the cost of the deceptive tactic d_1 outweighs any benefit the Player A would receive. Therefore, Player A falls back to a non-deceptive safety strategy (a_1, d_{n+1}). We will refer to γ as the *Jones' Ratio*. The value of the Jones' Ratio indicates whether or not to engage in deceptive tactics. This thresholding behavior leads us to the next section where we examine the effects of the number of informants and their corresponding corruption costs for all values of p_{id}.



(a) Equilibrium Utility Value vs p_{id}



(b) Probability of Player A Corrupting Informants

Fig. 4. Results for $c_d = 2$

V. Jones' Lemma and the Number of Informants

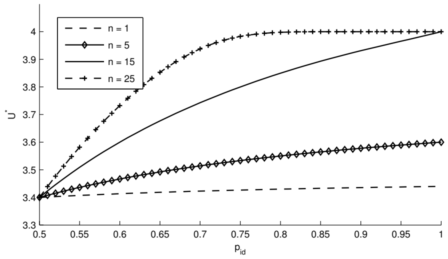
The total number of informants is a very influential parameter in the determination of optimal strategies. Theorem 3 showed that there is a limit to Player A's willingness to pay the large cost of corruption for several informants. When the Jones' Ratio exceeds one, deception is an effective tactic for Player A, but when the Jones' Ratio is less then one, the costs of deception outweigh any benefit received. The impact of the number of information channels and the cost of corruption on the success of deception is captured in a maxim from the intelligence literature known as Jones' Lemma [12]:

Deception becomes more difficult as the number of channels available to the target increases. Nevertheless, within limits, the greater the number of channels that are controlled by the deceiver, the greater the likelihood that the deception will be believed [3].

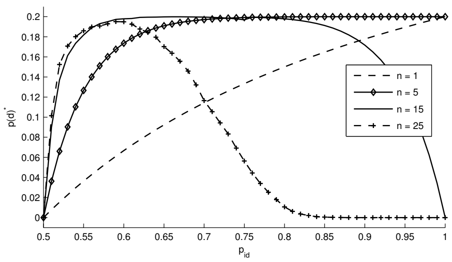
In this game, Player A must corrupt a portion of the available informants in order to conceal his action from Player B. If there is a large number of informants, even a small corruption cost for each informant can prohibit effective concealment. On the other hand, if the corruption cost is low enough, Player A can quickly compromise a large segment of the information network causing it to become unreliable.

Theorem 3 provides a closed form solution which analytically formalized the Jones' Lemma in the special case $p_{id} = 1$. We use numerical solutions using Theorem 1 for the general case where $0 \leq p_{id} \leq 1$, $n > 0$, and $c_d > 0$. In the following simulations, the values of V_{ij} are $V_{11} = 4$, $V_{12} = 3$, $V_{21} = 1$, and $V_{22} = 5$. Using these values, the standard equilibrium value V_s equals 3.4.

Figure 3 shows four curves depicting the equilibrium utility as p_{id} sweeps from .5 to 1 for the game where Player A is denied the opportunity to corrupt any informants. As p_{id}

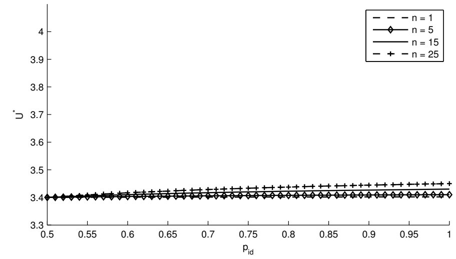


(a) Equilibrium Utility Value vs p_{id}

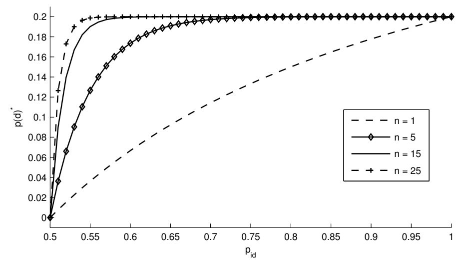


(b) Probability of Player A Corrupting Informants

Fig. 5. Results for $c_d = .2$



(a) Equilibrium Utility Value vs p_{id}



(b) Probability of Player A Corrupting Informants

Fig. 6. Results for $c_d = .01$

increases, all situations eventually force Player A to choose his safety strategy resulting in the safety value of 4. It can be seen that as the number of informants increases, the game reaches the safety value for smaller values of p_{id}.

Introducing the ability to corrupt the informants has little effect if the costs are too high for large numbers of informants as seen in Fig. 4a. Figure 4b shows the probability of Player A implementing some form of deception. Attempting deception is a waste of resources for every case except $n = 1$. Figure 5a shows that by lowering the cost of deception, $c_d = .2$, Player A can hold the value close to the original Nash equilibrium for $n = 1$ and can slow the rate of increase for $n = 5$ and $n = 15$, but a large number of informants still results in the safety value. In Fig. 5b, it can be seen that Player A starts using deception when the informants are relatively inaccurate, but abandons deceptive tactics for large numbers of informants as informant accuracy increases.

By allowing Player A to cheaply corrupt the informants, he can slow the increase substantially and hold the equilibrium value very close to the original Nash equilibrium value even with many informants that are highly accurate, which can be seen in Fig. 6a. The more informants there are in the information network, the sooner deceptive tactics play a role as the informants become more accurate as seen in Fig. 6b.

VI. Conclusions

In this paper, we have formulated a general two-player, zero-sum game, that takes into account the possibility that Player A may implement deception to neutralize Player B's information. It was shown that the solution of this game can be found by solving a pair of dual linear programming problems. An illustrative example was posed, and the closed form solutions were developed for special cases of this game. The resulting equilibrium strategies were qualitatively similar to general strategic guidelines described within the deception community. We also showed how our results capture the principle enunciated in the Jones' Lemma. In the future, we plan to extend these formulations to the setting of autonomous systems, dynamic games, and repeated games.

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