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Inferential adaptive control for non-uniformly sampled-data systems

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Abstract— **An inferential adaptive control algorithm is developed for a class of non-uniformly sampled-data systems with fast and non-uniformly updated inputs and infrequently sampled outputs. The specific approach involves three steps: first, to derive the mathematical relationships between the transfer function model of the measurable output and that of the non-uniform missing outputs; second, to compute the models of the non-uniform missing outputs based on the derived mathematical relationships and the identified model of the measurable output; third, using the computed models to estimate the non-uniform missing outputs and further supply which for feedback control. The proposed control algorithm can generate fast rate control signals and has the property of minimum variance. An example is included and the simulation results illustrate the effectiveness of the proposed inferential control scheme.**

Index Terms: Multirate systems; inferential control; adaptive control; least squares; identification.

I. Introduction

In process industries, due to large measurement delays and high investment and maintenance costs of analyzers, many variables that indicate the product qualities (quality variables) are difficult to measure at frequent rate, such as product composition in a distillation column [1], Kappa number in pulping process [2], Melt index in polymer production process [3]. Therefore, the desired control performances may not be achieved by using the infrequent quality variables directly as controlled variables. In order to realize on-line quality control and enhance the control performances, the inferential control scheme is especially designed, the basic idea of which is to estimate the unmeasurable quality variables based on the process model, as well as other fast and continuously available variables (auxiliary variables), and then use the estimates for feedback control.

Inferential control has been an active research area in recent years. For example, Ogawa and his coworkers developed an inferential model to estimate the Melt index value in polyethylene process, and proposed a quality inferential control scheme by combining the inferential estimation with the quality control system [4]; Pannocchia et al. investigated the effects of choosing the auxiliary variables and the plant data collecting approaches to design the estimators for inferential control [5]; Kano et al. presented a predictive inferential control scheme to control the product composition in a distillation column [6]; Singh et al. also studied inferential control for distillation, the estimator of which is based on an artificial neural network [7].

In particular, for a single input single output system, when the sampling period of the quality variable is an integer multiple of the control signal, the system is termed as a dual-rate system [8]. In the field of dual-rate inferential control, Li et al. designed a dual-rate inferential model predictive control scheme, which obtained the fast rate model to estimate the unmeasured quality variables, and then supplied the estimates to implement the control algorithms at the fast rate [9]; and further studied the stability, tracking performance and robustness of the dual-rate inferential control system [10]. Motivated by the idea of inferential control, Ding and Chen established the mathematical model for dual-rate systems by using the polynomial transformation technique, and then proposed a least squares based and a gradient based adaptive control algorithms [11], [12].

This paper is to present an inferential adaptive control algorithm for a class of non-uniformly sampled-data systems, where the input updating instants are non-uniformly spaced within a frame period T, and the output sampling instant is a constant T. Such systems are quite general, if the input updating instants are uniformly spaced, which will reduce to dual-rate systems. However, since the non-uniformly sampled-data systems have no base periods, it is impossible to obtain any fast rate models or construct the mathematical models using the polynomial transformation technique, and consequently the dual-rate inferential control schemes mentioned before can not be applied. Briefly, the contributions of this paper are as follows:

- The mathematical relationships between the transfer function model of the measured output and that of the unmeasured outputs are derived. To our best knowledge, this research findings has not yet been reported.
- Using the recursive least squares (RLS) algorithm, the transfer function model of the measured output is identified based on the non-uniform updating inputs and infrequent sampling outputs; and the models of the unmeasured outputs are further obtained by applying the above relationships.
- Based on the obtained models, the estimates of the unmeasured outputs are computed. Using the estimates to replace the unknown true values and according to the minimum variance theory, the inferential adaptive control algorithm for non-uniformly sampled-data systems is presented.

The paper is organized as follows. Section II discusses

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the modeling issues related to the non-uniformly sampled-data systems, where the first contribution is formed. Section III identifies the model parameters by using the RLS algorithm, and computes the estimates of the missing outputs. Section IV presents the inferential adaptive control algorithm for non-uniformly sampled-data systems. Section V provides an example to demonstrate the effectiveness of the proposed algorithm. Finally, some concluding remarks are given in Section VI.

II. MODELING OF NON-UNIFORMLY SAMPLED-DATA SYSTEMS

Considering the non-uniformly sampled-data systems, as briefly illustrated in Fig. 1, where $\mathscr{H}_\tau$ is a non-uniform zero-

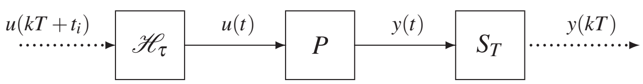

Fig. 1. The non-uniformly sampled-data systems

order hold with irregularly updating intervals $\{\tau_1, \tau_2, \cdots, \tau_r\}$, processing a discrete-time signal $u(kT+t_i)$ ($t_0 = 0$, $t_i = t_{i-1} + \tau_i$, $i = 1, 2, \cdots, r$) and producing the input $u(t)$ to the continuous-time process P; $y(t)$ is the process output and sampled by a sampler S_T with the frame period $T := \tau_1 + \tau_2 + \cdots + \tau_r = t_r$, yielding a discrete-time signal $y(kT)$.

The non-uniform sampling pattern is assumed to repeat with the frame period T, as shown in Fig. 2, where $\circ$

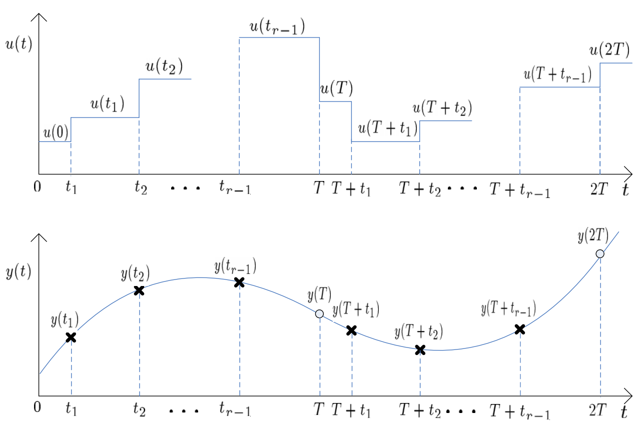

Fig. 2. The non-uniformly zero-order hold sampling pattern

denotes measurable outputs and $\times$ denotes unmeasurable outputs (or missing outputs). It is clear that the control input is non-uniformly updated r times during each period, but the output is sampled only once. Taking the first period $[0, T)$ for example, the available input-output data is $u(0)$, $u(t_1)$, $u(t_2), \cdots$, $u(t_{r-1})$ and $y(0)$, all the other outputs $y(t_1)$, $y(t_2), \cdots$, $y(t_{r-1})$ are missing.

Assume that the continuous-time process P in Fig. 1 has the following state-space representation:

$$P: \begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ y(t) = Cx(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, A, B, C, are matrices of appropriate dimensions. Discretizing system (1) with the frame period T gives

$$\begin{aligned} x(kT+T) &= \exp(AT)x(kT) \\ &\quad + \int_{kT}^{kT+T} \exp[A(kT+T-\tau)]Bu(\tau)\mathrm{d}\tau \\ &=: Gx(kT) + \sum_{j=1}^{r} F_j u(kT+t_{j-1}), \end{aligned} \tag{2}$$

$$G := \exp(AT) \in \mathbb{R}^{n\times n}, \tag{3}$$

$$F_j := \exp[A(T-t_j)] \int_0^{\tau_j} \exp(At)\mathrm{d}tB \in \mathbb{R}^n. \tag{4}$$

Hence, using (1) and (2), the discretization state-space model of the measurable output is

$$\begin{cases} x(kT+T) = Gx(kT) + \sum\limits_{j=1}^{r} F_j u(kT+t_{j-1}), \\ y(kT) = Cx(kT). \end{cases} \tag{5}$$

For the non-uniformly sampled-data systems, assume the frame period T is non-pathological and (C, A) in (1) is observable, then (C, G) in (5) is accordingly observable [13], [14]. Hence, there exists a nonsingular matrix Q_o such that the following linear transformation

$$\overline{x}(kT) = Q_o x(kT), \tag{6}$$

to make the state-space model (5) become the observability canonical form,

$$\begin{cases} \overline{x}(kT+T) = \overline{G}\ \overline{x}(kT) + \sum\limits_{j=1}^{r} \overline{F}_j u(kT+t_{j-1}), \\ y(kT) = \overline{C}\ \overline{x}(kT), \end{cases} \tag{7}$$

$$\overline{G} = Q_o G Q_o^{-1} = \begin{bmatrix} 0 & 0 & \cdots & 0 & -\alpha_n \\ 1 & 0 & \cdots & 0 & -\alpha_{n-1} \\ 0 & 1 & \cdots & 0 & -\alpha_{n-2} \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\alpha_1 \end{bmatrix}, \tag{8}$$

$$\overline{F}_j = Q_o F_j = \begin{bmatrix} \beta_{0,j}(n) \\ \beta_{0,j}(n-1) \\ \beta_{0,j}(n-2) \\ \vdots \\ \beta_{0,j}(1) \end{bmatrix}, \tag{9}$$

$$\overline{C} = C Q_o^{-1} = [0, 0, \cdots, 0, 1]. \tag{10}$$

Introducing a unit forward shift $z: z\overline{x}(kT) = \overline{x}(kT+T)$ or $z^{-1}\overline{x}(kT) = \overline{x}(kT-T)$, equation (7) can be rewritten in the following input-output representation,

$$\begin{aligned} y(kT) &= \sum_{j=1}^{r} \frac{z^{-n}\overline{C}\,\mathrm{adj}[zI-\overline{G}]\overline{F}_j}{z^{-n}\det[zI-\overline{G}]} u(kT+t_{j-1}), \\ &=: \frac{1}{\alpha(z)} \sum_{j=1}^{r} \beta_{0,j}(z) u(kT+t_{j-1}), \end{aligned} \tag{11}$$

$$\begin{aligned}\alpha(z) &= z^{-n}\det[zI-\overline{G}] \\ &= 1+\alpha_1 z^{-1}+\alpha_2 z^{-2}+\cdots+\alpha_n z^{-n}, \\ \beta_{0,j}(z) &= z^{-n}\overline{C}\,\mathrm{adj}[zI-\overline{G}]\overline{F}_j \\ &= \beta_{0,j}(1)z^{-1}+\beta_{0,j}(2)z^{-2}+\cdots+\beta_{0,j}(n)z^{-n}, \\ &\quad j=1,2,\cdots,r.\end{aligned}$$

Integrating (1) from kT to $kT+t_i$, $i=1,2,\cdots,r-1$, gives

$$\begin{aligned}x(kT+t_i) &= \exp(At_i)x(kT) \\ &\quad +\int_{kT}^{kT+t_i}\exp[A(kT+t_i-\tau)]Bu(\tau)\mathrm{d}\tau \\ &=: G_i x(kT)+\sum_{j=1}^{i}F_{ij}u(kT+t_{j-1}),\end{aligned} \tag{12}$$

$$G_i := \exp(At_i)\in\mathbb{R}^{n\times n}, \tag{13}$$

$$F_{ij} := \exp[A(t_i-t_j)]\int_0^{\tau_j}\exp(At)\mathrm{d}tB\in\mathbb{R}^n. \tag{14}$$

Comparing (13) and (14) with (3) and (4), respectively, the following relationships can be derived,

$$G_i = G^{t_i/T},\quad F_{ij}=G^{-1}G_iF_j. \tag{15}$$

From (1), using (12), the non-uniform missing output can be expressed as

$$y(kT+t_i)=CG_ix(kT)+\sum_{j=1}^{i}CF_{ij}u(kT+t_{j-1}). \tag{16}$$

Define

$$\overline{G}_i := \overline{G}^{t_i/T},\quad \overline{F}_{ij} := \overline{G}^{-1}\overline{G}_i\overline{F}_j. \tag{17}$$

Using (6), (8)-(10), (15) and (17), we have

$$\begin{aligned}\overline{C}\,\overline{G}_i\overline{x}(kT) &= CQ_o^{-1}[Q_oGQ_o^{-1}]^{t_i/T}\overline{x}(kT) \\ &= CQ_o^{-1}Q_oG^{t_i/T}Q_o^{-1}\overline{x}(kT) \\ &= CG_ix(kT),\end{aligned} \tag{18}$$

$$\begin{aligned}\overline{C}\,\overline{F}_{ij} &= CQ_o^{-1}[Q_oGQ_o^{-1}]^{-1}[Q_oGQ_o^{-1}]^{t_i/T}Q_oF_j \\ &= CQ_o^{-1}Q_oG^{-1}Q_o^{-1}Q_oG^{t_i/T}Q_o^{-1}Q_oF_j \\ &= CF_{ij}.\end{aligned} \tag{19}$$

Thus, using (18) and (19), equation (16) can be equivalently written as,

$$y(kT+t_i)=\overline{C}\,\overline{G}_i\overline{x}(kT)+\sum_{j=1}^{i}\overline{C}\,\overline{F}_{ij}u(kT+t_{j-1}),$$

and the corresponding transfer function model is

$$\begin{aligned}&y(kT+t_i) \\ &= \left\{\sum_{j=1}^{i}\frac{z^{-n}\overline{C}\,\overline{G}_i\,\mathrm{adj}[zI-\overline{G}]\overline{F}_j}{z^{-n}\det[zI-\overline{G}]}+\overline{C}\,\overline{F}_{ij}\right\}u(kT+t_{j-1}) \\ &\quad +\sum_{j=i+1}^{r}\frac{z^{-n}\overline{C}\,\overline{G}_i\,\mathrm{adj}[zI-\overline{G}]\overline{F}_j}{z^{-n}\det[zI-\overline{G}]}u(kT+t_{j-1}) \\ &=: \frac{1}{\alpha(z)}\sum_{j=1}^{r}\beta_{ij}(z)u(kT+t_{j-1}),\ i=1,2,\cdots,r-1,\end{aligned} \tag{20}$$

where $\alpha(z)$ is defined as before,

$$\begin{aligned}\beta_{ij}(z) &:= z^{-n}\overline{C}\,\overline{G}_i\,\mathrm{adj}[zI-\overline{G}]\overline{F}_j+\overline{C}\,\overline{F}_{ij}\alpha(z) \\ &= \beta_{ij}(0)+\beta_{ij}(1)z^{-1}+\beta_{ij}(2)z^{-2}+\cdots+\beta_{ij}(n)z^{-n}, \\ &\quad j=1,2,\cdots,i,\end{aligned} \tag{21}$$

$$\begin{aligned}\beta_{ij}(z) &:= z^{-n}\overline{C}\,\overline{G}_i\,\mathrm{adj}[zI-\overline{G}]\overline{F}_j \\ &= \beta_{ij}(1)z^{-1}+\beta_{ij}(2)z^{-2}+\cdots+\beta_{ij}(n)z^{-n}, \\ &\quad j=i+1,i+2,\cdots,r.\end{aligned} \tag{22}$$

III. Parameter identification and missing outputs estimation

The most difficult issue in designing inferential control scheme is to estimate the unmeasurable outputs, and it is no exception for the non-uniformly sampled-data systems. This Section is to identify the transfer function model of the measurable output based on the available input-output data, and then compute the estimates of the missing outputs.

In practice, the system outputs are always interfered by various disturbances. So the white noise $v(kT)$ is added to the deterministic input-output model and (11) becomes,

$$\alpha(z)y(kT)=\sum_{j=1}^{r}\beta_{0,j}(z)u(kT+t_{j-1})+v(kT). \tag{23}$$

Define the parameter vector θ_0 and the information vector $\varphi_0(kT)$ as

$$\begin{aligned}\theta_0 &:= [\alpha_1,\alpha_2,\cdots,\alpha_n,\beta_{0,1}(1),\beta_{0,1}(2),\cdots,\beta_{0,1}(n), \\ &\quad \beta_{0,2}(1),\beta_{0,2}(2),\cdots,\beta_{0,2}(n),\cdots, \\ &\quad \beta_{0,r}(1),\beta_{0,r}(2),\cdots,\beta_{0,r}(n)]^{\mathrm{T}}\in\mathbb{R}^{rn+n}, \\ \varphi_0(kT) &:= [-y(kT-T),-y(kT-2T),\cdots,-y(kT-nT), \\ &\quad \psi_0^{\mathrm{T}}(kT)]^{\mathrm{T}}\in\mathbb{R}^{rn+n}, \\ \psi_0(kT) &:= [u(kT-T),u(kT-2T),\cdots,u(kT-nT), \\ &\quad u(kT-T+t_1),u(kT-2T+t_1),\cdots, \\ &\quad u(kT-nT+t_1),\cdots,u(kT-T+t_{r-1}), \\ &\quad u(kT-2T+t_{r-1}),\cdots,u(kT-nT+t_{r-1})]^{\mathrm{T}}\in\mathbb{R}^{rn}.\end{aligned}$$

Then from (23), we have

$$y(kT)=\varphi_0^{\mathrm{T}}(kT)\theta_0+v(kT). \tag{24}$$

The information vector $\varphi_0(kT)$ in the identification model (24) only contains available input-output data $\{y(kT),u(kT+t_i):\ i=0,1,\cdots,r-1\}$, thus the parameter vector θ_0 can be identified by the following recursive least squares (RLS) algorithm:

$$\hat{\theta}_0(kT) = \hat{\theta}_0(kT-T)+L(kT)[y(kT)-\varphi_0^{\mathrm{T}}(kT)\hat{\theta}_0(kT-T)], \tag{25}$$

$$L_0(kT) = \frac{P_0(kT-T)\varphi_0(kT)}{1+\varphi_0^{\mathrm{T}}(kT)P_0(kT-T)\varphi_0(kT)}, \tag{26}$$

$$P_0(kT) = [I-L_0(kT)\varphi_0(kT)]P_0(kT-T), \tag{27}$$

$$\begin{aligned}\hat{\theta}_0(kT) &:= [\hat{\alpha}_1(kT),\hat{\alpha}_2(kT),\cdots,\hat{\alpha}_n(kT), \\ &\quad \hat{\beta}_{0,1}(1)(kT),\hat{\beta}_{0,1}(2)(kT),\cdots,\hat{\beta}_{0,1}(n)(kT), \\ &\quad \hat{\beta}_{0,2}(1)(kT),\hat{\beta}_{0,2}(2)(kT),\cdots,\hat{\beta}_{0,2}(n)(kT),\cdots, \\ &\quad \hat{\beta}_{0,r}(1)(kT),\hat{\beta}_{0,r}(2)(kT),\cdots,\hat{\beta}_{0,r}(n)(kT)]^{\mathrm{T}}.\end{aligned} \tag{28}$$

Based on the parameter estimates of $\hat{\alpha}_i(kT)$ and $\hat{\beta}_{0,j}(l)(kT)$, the estimates of the unknown parameters $\hat{\beta}_{ij}(l)(kT)$ and the missing outputs $y(kT+t_i)$, $i=1,2,\cdots,r-1$, in (20) can be computed accordingly. The computing steps are listed below:

1) From (8)-(10), let $\overline{C}=[0,0,\cdots,0,1]$, form the estimates of $\overline{G}$ and $\overline{F}_j$ by

$$\hat{\overline{G}}=\begin{bmatrix} 0 & 0 & \cdots & 0 & -\hat{\alpha}_n(kT) \\ 1 & 0 & \cdots & 0 & -\hat{\alpha}_{n-1}(kT) \\ 0 & 1 & \cdots & 0 & -\hat{\alpha}_{n-2}(kT) \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & -\hat{\alpha}_1(kT) \end{bmatrix}, \quad (29)$$

$$\hat{\overline{F}}_j=\begin{bmatrix} \hat{\beta}_{0,j}(n)(kT) \\ \hat{\beta}_{0,j}(n-1)(kT) \\ \hat{\beta}_{0,j}(n-2)(kT) \\ \vdots \\ \hat{\beta}_{0,j}(1)(kT) \end{bmatrix}. \quad (30)$$

2) From (17), the estimates of $\overline{G}_i$ and $\overline{F}_{ij}$ can be computed by

$$\hat{\overline{G}}_i=\hat{\overline{G}}^{t_i/T}, \quad \hat{\overline{F}}_{ij}=\hat{\overline{G}}^{-1}\hat{\overline{G}}_i\hat{\overline{F}}_j. \quad (31)$$

3) From (21) and (22), the parameters $\hat{\beta}_{ij}(l)(kT)$ can be estimated by

$$\begin{aligned}\hat{\beta}_{ij}(z)&=z^{-n}\overline{C}\hat{\overline{G}}_i\,\mathrm{adj}[zI-\hat{\overline{G}}]\hat{\overline{F}}_j+\overline{C}\hat{\overline{F}}_{ij}\hat{\alpha}(z) \quad (32)\\ &=\hat{\beta}_{ij}(0)(kT)+\hat{\beta}_{ij}(1)(kT)z^{-1}+\hat{\beta}_{ij}(2)(kT)z^{-2}\\ &\quad+\cdots+\hat{\beta}_{ij}(n)(kT)z^{-n},\ j=1,2,\cdots,i,\end{aligned}$$

$$\begin{aligned}\hat{\beta}_{ij}(z)&=z^{-n}\overline{C}\hat{\overline{G}}_i\,\mathrm{adj}[zI-\hat{\overline{G}}]\hat{\overline{F}}_j \quad (33)\\ &=\hat{\beta}_{ij}(1)(kT)z^{-1}+\hat{\beta}_{ij}(2)(kT)z^{-2}\\ &\quad+\cdots+\hat{\beta}_{ij}(n)(kT)z^{-n},\ j=i+1,i+2,\cdots,r.\end{aligned}$$

4) Using the estimates of $\hat{\beta}_{ij}(l)(kT)$ and from (20), the estimates of $y(kT+t_i)$ can be computed by the linear regression model,

$$\hat{y}(kT+t_i)=\hat{\varphi}_i^{\mathrm{T}}(kT)\hat{\theta}_i(kT)+\psi_i^{\mathrm{T}}(kT)\hat{\vartheta}_i(kT), \quad (34)$$

where

$$\begin{aligned}\hat{\varphi}_i(kT)&=[-\hat{y}(kT-T+t_i),-\hat{y}(kT-2T+t_i),\cdots,\\ &\qquad -\hat{y}(kT-nT+t_i),\psi_0^{\mathrm{T}}(kT)]^{\mathrm{T}},\\ \hat{\theta}_i(kT)&=[\hat{\alpha}_1(kT),\hat{\alpha}_2(kT),\cdots,\hat{\alpha}_n(kT),\\ &\qquad \hat{\beta}_{i,1}(1)(kT),\hat{\beta}_{i,1}(2)(kT),\cdots,\hat{\beta}_{i,1}(n)(kT),\\ &\qquad \hat{\beta}_{i,2}(1)(kT),\hat{\beta}_{i,2}(2)(kT),\cdots,\hat{\beta}_{i,2}(n)(kT),\cdots,\\ &\qquad \hat{\beta}_{i,r}(1)(kT),\hat{\beta}_{i,r}(2)(kT),\cdots,\hat{\beta}_{i,r}(n)(kT)]^{\mathrm{T}},\\ \hat{\vartheta}_i(kT)&=[\hat{\beta}_{i,1}(0)(kT),\hat{\beta}_{i,2}(0)(kT),\cdots,\hat{\beta}_{i,i}(0)(kT)]^{\mathrm{T}},\\ \psi_i(kT)&=[u(kT),u(kT+t_1),\cdots,u(kT+t_{i-1})]^{\mathrm{T}}.\end{aligned}$$

IV. The inferential adaptive control algorithm

The inferential adaptive control (IAC) scheme for the non-uniformly sampled-data systems is shown in Fig. 3, where $y_r(kT+t_i)$, $k=0,1,2,\cdots$, $i=0,1,\cdots,r-1$, denotes a sequence of deterministic reference inputs or desired outputs, $v(kT)$ is a random noise sequence with zero mean. For the sampling pattern in Fig. 2, the output $y(kT)$ is sampled infrequently with period T. In order to feed back to the controller a fast rate signal $y_f(kT+t_i)$, an periodic switch is applied. When the output is measured at times $t=kT$, $y_f(kT+t_i)$ connects to the true output $y(kT)$; otherwise connects to the estimates $\hat{y}(kT+t_i)$ at times $t=kT+t_i$, $i=1,2,\cdots,r-1$. Thus the output of the switch is a fast rate signal and expressed as

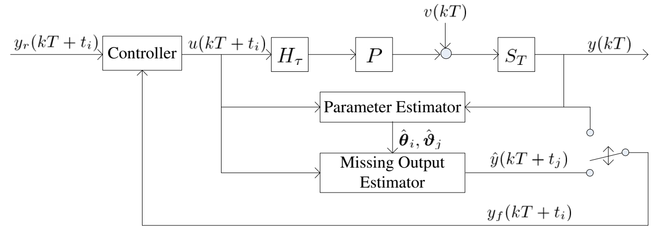

Fig. 3. The inferential adaptive control scheme

$$y_f(kT+t_i)=\begin{cases} y(kT), & i=0,\\ \hat{y}(kT+t_i), & i=1,2,\cdots,r-1.\end{cases}$$

The objective of this paper is to design an inferential adaptive controller based on the identified model and the estimated missing outputs, so as the output $y(kT+t_i)$ can track the given desired output $y_r(kT+t_i)$ by minimizing the tracking error criterion function:

$$\begin{aligned}&J[u(kT+t_{i-1})]\\ &\quad=\mathrm{E}\{[y_f(kT+t_i)-y_r(kT+t_i)]^2\},\ i=1,2,\cdots,r.\end{aligned}$$

Using (24) and (34), yields the control law of the form:

$$\begin{aligned}&y_r(kT+t_i)\\ &\quad=\begin{cases}\hat{\varphi}_i^{\mathrm{T}}(kT)\hat{\theta}_i(kT)+\psi_i^{\mathrm{T}}(kT)\hat{\vartheta}_i(kT),\ i=1,2,\cdots,r-1,\\ \varphi_0^{\mathrm{T}}(kT+T)\hat{\theta}_0(kT),\ i=r.\end{cases}\end{aligned}$$

Hence, the control signal $u(kT)$, $u(kT+t_1)$, $\cdots$, $u(kT+t_{r-1})$ can be computed in turn with the following recursive equations:

$$\begin{aligned}u(kT)&=1/\hat{\beta}_{11}(0)(kT)\times\Big[y_r(kT+t_1)\\ &\quad+\sum_{i=1}^{n}\hat{\alpha}_i(kT)\hat{y}(kT-iT+t_1)\\ &\quad-\sum_{j=1}^{r}\sum_{l=1}^{n}\hat{\beta}_{1j}(l)(kT)u(kT-lT+t_{j-1})\Big], \quad (35)\end{aligned}$$

$$\begin{aligned}u(kT+t_1)&=1/\hat{\beta}_{22}(0)(kT)\times\Big[y_r(kT+t_2)\\ &\quad+\sum_{i=1}^{n}\hat{\alpha}_i(kT)\hat{y}(kT-iT+t_2)-\hat{\beta}_{21}(0)(kT)u(kT)\\ &\quad-\sum_{j=1}^{r}\sum_{l=1}^{n}\hat{\beta}_{2j}(l)(kT)u(kT-lT+t_{j-1})\Big], \quad (36)\end{aligned}$$

$$
\begin{aligned}
&\vdots\\
&u(kT+t_{r-1}) = 1/\hat{\beta}_{0,r}(1)(kT)\times\Big[y_r(kT+T)\\
&\quad+\sum_{i=1}^{n}\hat{\alpha}_i(kT)y(kT+T-iT)\\
&\quad-\sum_{j=1}^{r-1}\sum_{l=1}^{n}\hat{\beta}_{0,j}(l)(kT)u(kT+T-lT+t_{j-1})\\
&\quad-\sum_{l=2}^{n}\hat{\beta}_{0,r}(l)(kT)u(kT+T-lT+t_{r-1})\Big]. \qquad (37)
\end{aligned}
$$

V. Example

Consider a non-uniformly sampled-data system with the following state-space representation,

$$
P: \begin{cases} \dot{x}(t) = \begin{bmatrix} 0 & -0.2500 \\ 1 & -0.0750 \end{bmatrix} x(t) + \begin{bmatrix} 0.2500 \\ 0.2500 \end{bmatrix} u(t), \\ y(t) = [0,\ 1]x(t). \end{cases}
$$

Let $r=2$, $\tau_1 = 1.5$ s, $\tau_2 = 1$ s, i.e., $t_1 = \tau_1 = 1.5$ s, $t_2 = t_1 + \tau_2 = T = 2.5$ s. From (5) and (16), we have

$$
\begin{cases}
x(kT+T) = \begin{bmatrix} 0.3551 & -0.4327 \\ 1.7310 & 0.2252 \end{bmatrix} x(kT) + \begin{bmatrix} 0.1098 & 0.2100 \\ 0.7273 & 0.3504 \end{bmatrix} \begin{bmatrix} u(kT) \\ u(kT+t_1) \end{bmatrix}, \\
\begin{bmatrix} y(kT) \\ y(kT+t_1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 1.2890 & 0.6447 \end{bmatrix} x(kT) + \begin{bmatrix} 0.0000 & 0.0000 \\ 0.5810 & 0.0000 \end{bmatrix} \begin{bmatrix} u(kT) \\ u(kT+t_1) \end{bmatrix}.
\end{cases}
$$

The corresponding transfer function model is given by

$$
\begin{aligned}
y(kT) &= \frac{0.72731z^{-1} - 0.068125z^{-2}}{1 - 0.58029z^{-1} + 0.82903z^{-2}} u(kT) \\
&\quad + \frac{0.35037z^{-1} + 0.23918z^{-2}}{1 - 0.58029z^{-1} + 0.82903z^{-2}} u(kT+t_1), \qquad (38)\\
y(kT+t_1) &= \frac{0.58097 + 0.27336z^{-1}}{1 - 0.58029z^{-1} + 0.82903z^{-2}} u(kT) \\
&\quad + \frac{0.49671z^{-1} - 0.10231z^{-2}}{1 - 0.58030z^{-1} + 0.82900z^{-2}} u(kT+t_1). \qquad (39)
\end{aligned}
$$

For the non-uniform sampling scheme, the transfer function model in (39) can not be identified directly based on the measured input-output data. As an alternative approach, we compute it from the parameters of the transfer function model in (38). From (8)-(10), using the parameters in (38) to form

$$
\overline{G} = \begin{bmatrix} 0 & -0.82903 \\ 1 & 0.58029 \end{bmatrix}, \quad \overline{F}_1 = \begin{bmatrix} -0.068125 \\ 0.72731 \end{bmatrix},
$$
$$
\overline{F}_2 = \begin{bmatrix} 0.23918 \\ 0.35037 \end{bmatrix}, \quad \overline{C} = [0,\ 1].
$$

From (17), we have

$$
\overline{G}_1 = \overline{G}^{t_1/T} = \begin{bmatrix} 0.47689 & -0.61755 \\ 0.74491 & 0.90916 \end{bmatrix},
$$
$$
\overline{F}_{11} = \overline{G}^{-1}\overline{G}_1\overline{F}_1 = \begin{bmatrix} 0.27336 \\ 0.58097 \end{bmatrix}.
$$

Substituting above equations into (20) gives the same representation as in (39), which demonstrates the algorithms in (29)-(34) to compute the estimates of the missing outputs can be realized.

Take the desired output in Fig. 3 to be

$$
y_r(250i+j) = (-1)^i, \quad i = 0,1,2,\cdots, \quad j = 1,2,\cdots,250.
$$

Assume $v(kT)$ is a white noise sequence with zero mean and variance $\sigma^2 = 0.05^2$. Applying the inferential adaptive control algorithm proposed in Section III and IV to compute the missing outputs $y(kT+t_1)$ and the control signals $u(kT)$ and $u(kT+t_1)$, the system output $y(t)$ and the desired output $y_r(t)$ are shown in Fig. 4, where $t = kT$ is the simulation time.

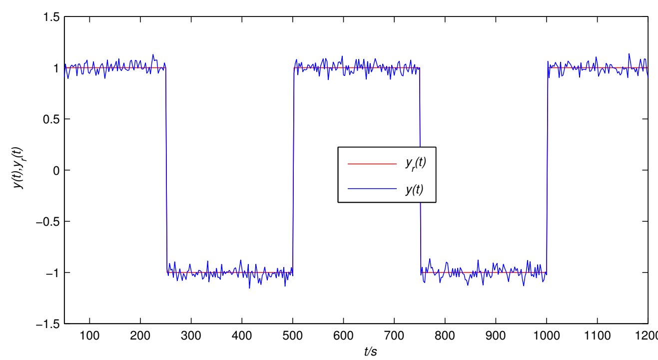

Fig. 4. $y(t)$ and $y_r(t)$ versus t (The IAC algorithm)

Assume the system output is sampled simultaneously with the control input, i.e., the available output data is $\{y(kT), y(kT+t_1),\ k = 0,1,\cdots\}$, and the parameters in (38) and (39) are known, then the minimum variance control (MVC) algorithm can be used, the results is shown in Fig. 5.

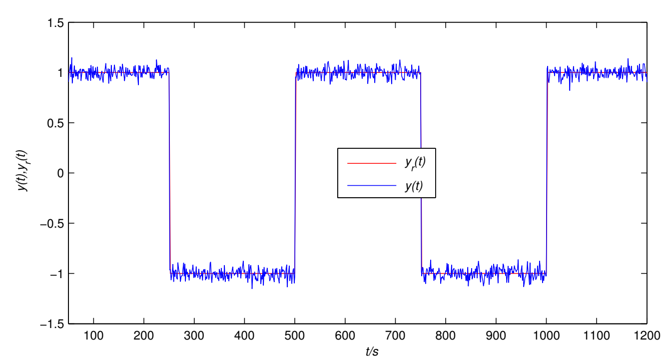

Fig. 5. $y(t)$ and $y_r(t)$ versus t (The MVC algorithm)

From Figs. 4 and 5, we can see that the proposed inferential adaptive control algorithm can achieve the tracking performance as the minimum variance control algorithm.

VI. Conclusions

Because of the non-uniform sampling pattern, the system output is sampled slowly, while the control input is updated fast and non-uniformly. In order to feed back the controller a synchronization signal with the control input, an inferential adaptive control algorithm is proposed by replacing the missing outputs with their corresponding estimates. The simulation example shows that the controlled output can well track the desired output and has the property of minimum variance. The inferential control algorithms for non-uniformly sampled-data systems with colored noises require further study.

The parameter estimation procedure embed in the proposed inferential adaptive control algorithm can also be completed by the multi-innovation identification methods [15]–[23], the iterative identification methods [24], [25] and some other identification methods [26]–[41].

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