# Market-Based Control of Shear Structures Utilizing Magnetorheological Dampers

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Abstract—The use of magnetorheological (MR) dampers for control of structures subject to seismic, wind, and/or other excitations has been an extensive field of study for over a decade. Many of the proposed feedback control laws have been based on modern linear systems control theory, e.g. linear quadratic Gaussian (LQG) control. Alternatively, this paper presents a nonlinear controller that explicitly handles the dynamic force saturation limits of MR dampers, a feature not available in the design of linear controllers. The nonlinear controller builds on an agent-based control (ABC) architecture with a diverse agent population. Agents can be characterized as buyers or sellers capable of sensing and control respectively. These agents participate in a competitive market place trading control energy in a way that leads to Pareto optimal agent utilities at each control time step. The ABC architecture allows for easy implementation with inexpensive partiallydecentralized large-scale wireless sensing and control networks. This novel controller is validated with a numerical simulation of a seismically excited six story shear structure with MR dampers at the base of V-braces installed on each story. The controller, deemed a 'market-based controller' (MBC) due to the optimization of agent utilities in control force markets, is compared against a benchmark LQG controller in a variety of test cases.

#### I. INTRODUCTION

Nontrol of the dynamic response of civil structures has been studied at length since the 1970's, yet a limited number of building owners are choosing structural control over more traditional passive design methodologies [1]. The high costs of the feedback control system components (i.e. sensors, actuators, centralized controllers, and associated wiring) required to execute the control law are partially responsible. These costs may be reduced by implementing partially decentralized control architectures that utilize wireless controllers. Fortunately, the advances in low-cost microcontrollers have recently led to the development of inexpensive wireless controllers with collocated sensing, actuation, communication, and

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Fig 1. 6-story partial-scale single-bay steel structure used for validation of MR damper control laws on the NCREE shake-table.

computation abilities [2]. Researchers have demonstrated in simulation and experimentation the ability of low cost wireless controllers, with varying degrees of decentralization, to effectively control civil structures using MR dampers [2-9]. MR dampers belong to the class of semiactive control devices characterized by their ability to effectively control the response of a stable mechanical system without the possibility of destabilization, and were chosen for this study due to their low power requirements, small size, high dynamic range, and relatively quick dynamics [10].

Structures utilizing concentrically braced frames as a lateral force resisting system can be modified to allow for the installation of MR dampers as shown in Fig 1. Controlling such a system of MR dampers with a centralized controller requires extensive lengths of cables routed to a central computer. If the central computer were to fail, the entire system would see a detrimental drop in system performance. Alternatively, the control architecture could be completely decentralized, without the need for expensive signal cabling. However decentralized controllers cannot offer the same level of system performance as centralized controllers due to their decentralized sparse information constraints [11].

Partially decentralized control architectures balance the tradeoffs between centralization and decentralization. Wireless units can sense and/or control the response of the structure utilizing embedded computing along with a wireless transceiver to share information with other 'local' controllers. The definition of 'local' can vary from the entire network, for small and reliable wireless environments, to only a single unit, in spatially large and unfavorable wireless environments. In order to achieve the best possible

performance, the controller should adapt to the time-varying definition of 'local'.

In this study an MBC law is formulated to control the nonlinear response of a six story shear structure outfitted with MR dampers subject to seismic disturbances (see Fig. 1). The control law, abbreviated MR-MBC, is specifically formulated to account for the non-linear nature of the MR dampers as described by a Bouc-Wen hysteresis model.

## II. A FORMULATION OF MARKET BASED CONTROL

The analogy between the distributed resource allocation found in economies and distributed control of dynamic systems is so strong that the well established microeconomic theories are the foundation for the contemporary field of market-based control of dynamic physical [12] and computer [13] systems. The system described herein seeks to emulate a market in which control force is traded between actuators and sensors at each discrete control time step resulting in an instantaneous market equilibrium that is optimal in some sense and reduces the response of the structure to external loading.

## A. Control systems as a distributed allocation problem

Dynamic systems can often be described by the discretetime state space equation presented in (1), where the system has N dynamic degrees of freedom, M inputs r(k), and timestep k of length  $\Delta t$ . The state of the system can be fully described by the Nx1 state vector x(k) at each time step.

$$\begin{aligned} x(k+1) &= f(x(k), r(k), k), \quad x(0) = x_0 \\ y(k) &= g(x(k), r(k), k) \end{aligned}$$
(1)

Let each controllable input be represented by an agent j that has a non-negative cost,  $K_j$ , associated with the production of each unit of force produced  $c_j(k)$ . Similarly, the *n* outputs y(k) to be controlled are represented by agents *i* that receive a utility,  $\Phi_i$ , associated with each unit of a good  $u_i(k)$  received by the agent. The goal of the centralized resource allocation problem is the solution to the optimization problem:

$$\max \sum_{k=1}^{\infty} J(u(k), c(k), x(k), k)$$
  
s.t.  $\sum_{i=1}^{n} u_i(k) = \sum_{j=1}^{M} c_j(k), \ c_j(k) \ge 0 \ \forall j \ , k.$  (2)

 $J(\bullet)$  is an objective function describing the efficiency of the consumption and production of a resource subject to mechanical and economic constraints. Due to the complex and often non-linear nature of large-scale control problems, the solution to (2) is often very difficult to find. Alternatively, Voos and Litz [12] proposed to individually maximize each agents objective function separately (3), a task that can be completed in real-time by agents using their onboard computing, wireless transceivers, and a set of market rules.

Equation (3), while formulated as an optimal control problem, is also in the form of a special distributed resource allocation problem that has a set of Pareto optimal solutions at each step in time k. Pareto optimal solutions are those solutions to resource allocation problems that occur when an agent's utility cannot increase without simultaneously decreasing another agent's objective function [14].

$$\max_{J_{1}(c_{1}(k), k)} \\ \vdots \\ \max_{J_{M}(c_{M}(k), k)} \\ \max_{J_{1}(u_{1}(k), x_{1}(k), k)} \\ \vdots \\ \max_{J_{n}(u_{n}(k), x_{n}(k), k)} \\ \text{s.t. } \sum_{i=1}^{n} u_{i}(k) = \sum_{j=1}^{M} c_{j}(k), \ c_{j}(k) \ge 0 \ \forall j \end{cases}$$
(3)

The optimal solution to (3) is computed when agents maximize their objective function following a set of market rules. First, the objective function of each agent must be formulated to map the current state of an agent to its desire to purchase or supply a quantity of control force at a particular price. Second, a set of rules must be given to each agent to optimize its objective function by trading with other agents. One set of rules could require each agent to send out its entire objective function to all other agents. Afterwards each agent solves the centralized resource distribution problem. If information transfer is limited, the rules could instead require each agent to send only a limited amount of information only to certain agents. In the case of limited information transfer, the convergence to the Pareto optimal allocation may require iterative negotiation between agents.

# B. Single degree of freedom (SDOF) formulation

The goal of this paper is to present an MR-MBC law that can be implemented on a network of wireless control units utilizing MR dampers. This implementation environment, with limited communication and computational capacity, restricts the agents' ability to be omniscient and to individually solve the centralized control problem. Additionally, the communication rate of wirelessly networked controllers restricts the number of iterative negotiations. One possible solution to these problems, which serves as the basis of this paper, formulates the agents' utility based on simple heuristics. The formulation of the heuristics, rules, and objective functions will be presented for a single SDOF system with a single buyer and seller. The concepts will then be extended to more complex MDOF systems.

## 1) The Supplier's Utility

The goal of the supply agent should be to minimize the amount of actuation supplied, and thus minimize the power consumed. The cost should increase as the specified amount of force increases up to some saturation limit. Unfortunately the force saturation limit of MR dampers is a dynamic property that changes with respect to the damper's velocity and hysteresis. In order to capture the nonlinear dynamics of the MR damper, each supply agent employs a Bouc-Wen hysteretic model which has been shown to adequately model MR dampers [15].

The cost of control force is based on the heuristic that the magnitude of damper force will increase with increasing voltage, V(k), and will saturate at  $F_j(V_{max}, k)$  (abbreviated as  $F_{max}$ ). The supplier's cost function when defined by (5) quantifies the supplier's heuristic and abides by the constraints in (4) which help to guarantee a Pareto optimal solution exists.

$$\frac{dK_j(F_j(k+1))}{dF_j(k+1)} > 0 \quad and \left. \frac{dK_j}{dF_j} \right|_{F_j=0} = 0$$

$$\frac{d^2K_j(F_j(k+1))}{dF_j^2(k+1)} \ge 0 \quad \forall F_j(k+1) > 0$$
(4)

$$K_{j}(F_{j}(V(k+1), k+1)) = -\mu \ln \left(1 - \frac{F_{j}(V(k+1), k+1)}{F_{j}(V_{max}, k+1)}\right) + \frac{\mu F_{j}(V(k+1), k+1)}{F_{j}(V_{max}, k_{1})}$$
(5)

The cost function utilizes a one-step ahead prediction of the maximum possible control force magnitude to find the asymptote of the negative logarithmic relationship between a specified control force  $F_j(V(k+1), k+1)$  and the cost  $K_j$ . The tuning variable  $\mu$  is a non-negative real number that the designer chooses to adjust the rate of convergence to the asymptote. Suppliers aim to maximize their profit by producing control force at the market price  $p^*$  that solves

$$\max_{F_j \le F_j, \max} p^* F_j - K_j(F_j). \tag{6}$$

# 2) The Buyer's Utility

Buyers in MR-MBC aspire to minimize the response of the structure by purchasing control force. In this study, the buyers are the wireless sensors measuring the response of the structure. With a heuristic stating an increase in damper force should decrease the system response, the agent strives to maximize its utility. The utility function,  $\Phi$ , of the buyer as defined by (8) is a twice differentiable function bounded by the constraints in (7) resulting in a utility abiding to microeconomic theory.

$$\frac{d\Phi(F(k))}{dF(k)} > 0 \quad and \ \Phi(0) = 0$$

$$\frac{d^2 \Phi(F(k))}{dF(k)^2} < 0 \quad \forall F(k) > 0$$

$$\Phi(F(k)) = -\frac{0.5 \ F^2(k)}{T \ |y(k)| + Q \ |y(k)|} + F(k) \ \tau \ w(k)$$
(8)

The amount of utility an agent receives from each unit of control force increases with increases in inter-story drift, velocity, or buyer's wealth, w(k). The extent of increase is determined by the control parameters T, Q, and  $\tau$ , respectively. It should be noted that the utility function is only based on an approximate heuristic that clearly may not hold for all possible system states. This inaccuracy has the advantages that the utility function is not based on a model, with possible modeling error, and is simple enough that agents can easily compute and transmit their utility. Additionally, the simplicity of the utility function allows for it to be described at any point in time with only two values,  $(T |y(k)| + Q | \dot{y}(k)|)$  and  $(\tau w(k))$ . This will aid the system in finding the market equilibrium.

#### 3) Equilibrium

Just as in physics, equilibrium is the state of a system where opposing forces are balanced. In the case of markets, the forces are the buyer's push to make the prices lower in order to increase their utility, and the supplier's push to raise the market price in order to increase revenue. Due to the physical constraints on the system, all of the control force produced must be equal to the control force consumed. The



Fig 2. Graphical depiction of the supply and demand curves for the SDOF MBC formulated above.

market equilibrium that satisfies these two constraints is computed as the solution to (9) which analytically represents the intersection of the supply and demand curves of Fig. 2.

$$\max_{F_{demand} \ge 0} \Phi(F_{demand}) - p^* F_{demand} + w$$

$$\max_{F_{supply} \ge 0} p^* F_{supply} - K(F_{supply})$$
(9)
$$F_{supply} = F_{supply} = F_{supply}$$

s.t.  $F_{demand} = F_{supply} = F$ 

The utility function of the buying agents was conveniently formulated such that the first maximum in (9) is uniquely identified by locating the zero of its first derivative. Similarly, the second maximization in (9) is also strictly concave on the interval and the maximization can be found at the zero of its derivative. These two observations lead to the following simplification in finding  $F_{eq}$  and  $p_{eq}^*$ .

$$\frac{-F}{T |y| + Q |\dot{y}|} + \tau w = p_{eq}^* \quad and \quad \frac{-\mu}{F_{eq} - F_{max}} + \frac{\mu}{F_{max}} = p_{eq}^* \quad (10)$$

A solution must exist to the two simultaneous equations in (10) for  $F \ge 0$  due to constraints (4) and (7). Therefore the Pareto optimal solution for the SDOF case of the MR-MBC is guaranteed and occurs when an equilibrium quantity (11) is traded at a price determined by (10) with  $F = F_{eq}$ .

$$F_{eq} = \frac{a_1 a_2 F_{max} - \mu a_2 + F_{max}^2 + \sqrt{a_3}}{2 F_{max}}$$
  
where:  $a_1 = \tau w \quad a_2 = T |y| + Q |\dot{y}|$   
 $a_3 = a_1^2 a_2^2 F_{max}^2 - 2 a_1 a_2^2 F_{max} \mu - 2 a_1 a_2 F_{max}^3$   
 $+ \mu^2 a_2^2 + 6 \mu a_2 F_{max}^2 + F_{max}^4$  (11)

The existence of (11) as an explicit function makes it possible for agents to send messages,  $M \in \{[S \in \mathbb{R}^+, D \in \mathbb{R}^+]^T\}$ , to negotiate an equilibrium price only if all agents utility functions are in the form of (5) or (8). The messages sent by the supply agent should be within the message space described by (12), while the demand agent's message space is described by (13).

$$M_{supply} = \begin{bmatrix} S & \mu & F_{max} \end{bmatrix}$$
(12)

$$M_{demand} = \begin{bmatrix} D & (T \mid y \mid + Q \mid \dot{y} \mid) \quad \tau w \end{bmatrix}$$
(13)

Using (11) to compute equilibrium by sending messages within (12) and (13) requires that the agents are cooperative in that they do not lie to other agents about their utility and follow the market rules.

- Rule 1. Every agent shall formulate their utility functions in the form of (5) or (8).
- Rule 2. Every agent shall transmit a message M to every other agent in the form of (12) or (13).
- Rule 3. Every agent shall compute the equilibrium market quantity by (11) and price by (10).

Since every agent is required to follow rules R1-R3, each agent will choose a Pareto optimal action by simply optimizing their own utility. As the control steps occur, the buying agent takes part of their initial endowment of wealth  $w(0)=w_0$  and gives it to the supplier in exchange for damper force. Eventually the buying agents will run out of wealth if there is not some method for redistributing the wealth from the suppliers back to the buyers. To alleviate this problem for SDOF MR-MBC the buyer receives a re-endowment at the beginning of each step equal to the amount spent during the previous step.

#### C. Extension to multiple degrees of freedom (MDOF)

The MDOF shear structure used as a case study is controlled by installing MR dampers as depicted in Fig. 1. The MR damper of floor *j* is selling in the market the control force  $F_i(k)$  at each control step k. Similarly each floor has an agent *i* measuring inter-story drift  $y_i(k)$  and velocity  $\dot{y}_i(k)$  that will try to maximize its utility  $\Phi_i(k)$  by purchasing control force from its supplier. For simplicity sake, an explicit formulation between the minimization of  $v_i(k)$  and  $\dot{v}_i(k)$  w.r.t. the control force at any arbitrary floor j was not developed for use in the agents demand function. Instead, the buyers use a heuristic that says only the force from supplier *j* with *j* = *i* is capable of effecting  $y_i(k)$  or  $\dot{y}_i(k)$ . The buying agent also assumes that the action made by any other supplier has no effect on  $y_i(k)$  or  $\dot{y}_i(k)$ . Similar to the heuristic simplification used in the SDOF formulation, this heuristic minimizes the amount of computation required by the agents. The heuristic is quantified by (14) as the buyers utility function.

$$\Phi_i(F_j(k)) = \frac{-0.5 F_j^2(k)}{T|y_i(k)| + Q|\dot{y}_i(k)|} + F_j(k)\tau w_i(k)$$

$$\forall i \le n, where \ j = i$$
(14)

The utility functions of the agents are unchanged in MDOF MR-MBC as are the processes in which the markets act. The key change is in the redistribution of wealth after each control step. Buyers, initially deployed with initial wealth  $w_0$ , can gain more wealth by reducing the response of the whole structure better than other agents who as a result loose wealth such that to total wealth is constant. The wealth redistribution rule takes the total amount of wealth transferred in all markets  $\Gamma$  in the previous step and distributes  $s \cdot \Gamma$  directly back to the agents that spent it. The remaining is sent to each agent *i* as a payment  $P_i$  governed by (15) representing how much agent *i* helped to improve the total state of the structure.

$$P_{i} = (1 - s) \Gamma \left( L \frac{\bar{Y}_{i} + \left| \min_{n \le N} \bar{Y}_{n} \right|}{1 + N \left| \min_{n \le N} \bar{Y}_{n} \right|} + M \frac{\dot{Y}_{i} + \left| \min_{n \le N} \dot{Y}_{n} \right|}{1 + N \left| \min_{n \le N} \dot{Y}_{n} \right|} \right)$$
(15)  
where:  $0 \le s \le 1$ 

$$\bar{Y}_i = \frac{y_i^2(k) - y_i^2(k-1)}{\sum_{n=1}^N \left(y_n^2(k) - y_n^2(k-1)\right)} \quad \dot{\bar{Y}}_i = \frac{\dot{y}_i^2(k) - \dot{y}_i^2(k-1)}{\sum_{n=1}^N \left(\dot{y}_n^2(k) - \dot{y}_n^2(k-1)\right)}$$

The execution of each control step starts with the agents formulating their utility function either in the form of (5) or (14). Buying agents transmit a message in the form of (13) or (12). The equilibrium point in each market as computed by (11) determines the amount of control force the supplier should generate, and the amount of wealth the buyer should pay. At the conclusion of each step, every supplier should inform the wealth distribution agent of the amount of wealth received such that it can be redistributed according to (15). When all the agents follow the rules described in this formulation of MR-MBC, a locally Pareto optimal control force is generated at each MR damper. Also, wealth is transferred between markets such that more efficient markets may obtain more wealth to purchase more control force over the time trajectory of the system.

# III. NUMERICAL SIMULATION AND VALIDATION

The National Center for Research on Earthquake Engineering (NCREE) in Taipei, Taiwan has graciously provided the authors with a model of the six-story steel structure as seen in Fig. 1 that was installed on their shaking table in March of 2010. The partial scale, single-bay structure has a floor-to-floor height of 1.0 m, bays 1.0 m square, and 15 cm x 2.5 cm steel columns oriented in their weak flexural direction. Lord Corporation RD-1005-3 MR dampers are installed on each story at the base of H100x100x6x8 steel V-braces. A 0-0.8 V signal amplified by a 24 V, 2 A VCCS is used for semi-active control of the six MR dampers. The MR dampers have a maximum force capacity of ±2.0 kN and a maximum stroke of 20 mm. NCREE has also supplied the authors with coefficients of a discrete time Bouc-Wen model used to simulate the response of the MR dampers.

#### A. Benchmark Controllers

Due to the difficultly of mathematically proving bounds for non-linear market-based control laws, the effectiveness of MR-MBC was determined by a parametric study of the simulated controlled response of the structure subjected to single direction ground motions. The MR-MBC controlled response was compared against the structure's response under identical ground motions and network environments when controlled by a clipped-optimal linear quadratic Gaussian (LQG) controller and maximum (i.e. passive on) and zero (i.e. passive off) damper voltages. Clipped-optimal LQG was chosen as the benchmark control law due to its previously successful use as a control law for the control of structures similar to the one used in this study using MR dampers [6, 16-17].

The LQG control law, schematically shown in Fig 3, generates an optimal, in the  $\mathcal{L}_2$  sense assuming ideal actuators, desired control force U(t) for the structure represented by a state variable model (SVM) with story position and velocity relative to ground as states X(t). The system states are estimated as  $\hat{X}(t)$  by a Kalman filter with noisy absolute acceleration feedback Y(t). The LQR was



Fig 3. Block diagram of LQG control of MR dampers.

designed to minimize inter-story drift by specifying the weighting variables Q and R according to the method proposed by Bryson and Ho [18]. The proposed method specifies Q and R as a function of the percent change from the maximum expected values of inter-story drift and control force respectively. A non-negative constant  $\rho$  can be multiplied with Q to allow for a greater or less weighting of the response versus control force.

The output of the control law U(t) must be compared against the estimated damper force as computed by the Bouc-Wen model for 10 discrete damper voltages. The voltage that results in an estimate closest to the desired control force is applied to the damper at each time step. The control law becomes suboptimal when the desired control force U(t) varies in sign or magnitude from the actual damper force.

# B. Metrics

The performance of the MR-MBC against the three benchmark controllers is judged by eight cost functions J1-J8 similar to those developed in [19]. The first three measure the root-mean-squared (rms) response of the structure, while the 4<sup>th</sup> through 7<sup>th</sup> measure the peak response. The 8<sup>th</sup> cost function is a measure of the mean electrical power consumed by the MR damper averaged over the simulation length. Formulas for the cost functions are presented in Table I.

TABLE I COST FUNCTIONS TO COMPARE CONTROLLER RESPONSES.

$J1^{a,b} = \frac{y_{rms:Controlled}}{y_{rms:Uncontrolled}}$	$J5^{c} = \frac{\max_{Floor,t}  \dot{x}_{controlled} }{\max_{Floor,t}  \dot{x}_{uncontrolled} }$
$J2^{a,c} = \frac{\ddot{x}_{rms:Controlled}}{\ddot{x}_{rms:Uncontrolled}}$	$J6^d = \max_{Floor,t}  F $
$J3^{a,d} = F_{rms}$	$J7^{c,f} = \max_{t} \frac{ \ddot{x}_{controlled}W }{ \ddot{x}_{uncontrolled}W }$
$J4^{b} = \frac{\max_{Floor,t} y_{controlled} }{\max_{Floor,t} y_{uncontrolled} }$	$J8^{e} = \frac{1}{6 t_{end} V_{max}} \sum_{n=1}^{6 Floors} \left( \int_{0}^{t_{end}} (V_n(t)) dt \right)$

The comparison between controlled and passive cases applies consistent ground motion type and magnitude.



#### C. Simulation Cases and Results

The simulations were run with MATLAB® in a simulation environment developed by the authors. The simulator utilizes a set of 'real' system properties, while the controllers within the simulator use separately specified properties. In this way one can easily study the effect of model uncertainty on the controller. The parametric study to determine the effectiveness of MR-MBC considered the effect of six different environmental parameters; controller



Fig 4. (a) and (b) partial results of the parametric study for the MR-MBC and LQG controller respectively. The cost functions *J1-J8*, defined in Table I, scaled such that the largest value of each cost function across all tests has a value equal to one. (c) MR-MBC, LQG, and passive on control laws all perform similarly in controlling inter-story drift.

type, failure of sensors & actuators, variation in ground motion magnitude and type, controller delay, and model uncertainty.

The first study compares the response of the structure under the best-case scenario of controller environment. In this case the controllers are given the same model parameters that are used in the simulation. The controllers are allowed to update their output voltage at 100 Hz. The structure is excited by a scaled unidirectional 100 Gal peakground-acceleration (p.g.a.) NS record of the 1940 El Centro (Imperial Valley Irrigation District Station) earthquake.

Study 2 evaluates the response of the controllers to a power failure on the 3<sup>rd</sup> floor resulting in grounded damper voltage and sensor reading. Since the MR-MBC is agent based, the remaining agents can quickly realize agent 3 has stopped responding and refrain from including the lost agent in their calculations. On the other hand the LQG controller would unrealistically have to update the optimal control gain by solving an algebraic Riccati equation. Instead, the LQG controller must rely on the Kalman filter, with an errant measurement, to estimate the current state.

The third study simulates the structure excited by the ground motions other than El Cento, the ground motion for which the MR-MBC and LQG controllers were tuned. The 1999 Chichi, Taiwan earthquake (station TCU076-NS) and white noise acceleration were chosen as the alternative ground motions. Both records were scaled to a PGA of 100 Gal.

Since both the control law and actuators in MR-MBC are non-linear, it is of interest to study the response of the controlled structure excited by motions of different magnitude. Study 4 examines the response of the structure to a test similar to study one with PGA of 50 Gal and 200 Gal respectively.

Wireless control networks will inherently have a delay larger than their wired counterparts. To study the effect of the cost saving adoption of wireless controllers, the fifth set of simulations increases the controller delays to 50 Hz and then to 20 Hz. Previous work has shown that wireless control networks of this size can currently communicate at rates up to 50 Hz [6].

The final series of simulations studies the effect of model uncertainty on the response of the different control laws. In this study an extra 100 kg is added to each story in the simulated structure, however the controllers are unaware of the added mass. This amounts to approximately a 15% modeling error in the mass of the structure. The structure is excited by the same 100 Gal motion as study one.

A total of twenty-two simulations were conducted as part of the parametric study. The three plots in Fig. 4 represent a portion of the results that capture well the performance of the four different control laws in a variety of tests. Due to the non-linear characteristics of both the MR dampers and the MR-MBC a quantitative analysis of the results is difficult and not required. Instead, a qualitative analysis is presented in the conclusion describing the efficacy of MR-MBC.

# IV. CONCLUSION

This paper presented a non-linear closed-loop controller for the semi-active control of shear structures using MR dampers. A parametric study was undertaken to compare the effectiveness of the MR-MBC against a LQG controller and two passive open-loop controllers. The results of the parametric study, qualitatively presented in Fig. 4, show that MR-MBC can successfully limit the response of structures during seismic events of different types and magnitudes. In the metrics that both the MR-MBC and LQG controllers were designed to control, *J1*, *J4*, *J8*, the MR-MBC was shown to be just as effective and in some cases marginally better than LQG control.

Not apparent in the results presented was the problems associated with designing a linear LQG controller for control of a linear system with nonlinear MR damper actuators. During study 1, the LQG controller frequently desired over 400% of the possible control force and desired more control force than was achievable over 60% of the time. The clipping of the LQG controller led to suboptimal performance. However, the MR-MBC always recognized the force capacity of each MR damper resulting in power savings over the LQG during most tests.

In conclusion, a decentralized architecture for semi-active control of civil structures has been proposed. While the heuristics utilized by the agents sacrifice accuracy for computational efficiency, the resulting decentralized controllers perform on par with centralized LQG solutions. Future work may include the development of physics based heuristics that account for effect of a single damper on every story of the structure. This would require the solutions to the decentralized resource allocation problem referred to by economists as the public goods problem. The development of these stronger heuristics along with the experimental validation in a realistic packet-loosing wireless network may show the true efficacy of MR-MBC.

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