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Online Estimation of an Electric Vehicle Lithium-Ion Battery Using Recursive Least Squares with Forgetting

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Abstract—**A battery model that is suitable for real-time State-of-Charge (SOC) estimation of a Lithium-Ion battery is presented in this paper. The battery open circuit voltage (OCV) as a function of SOC is described by an adaptation of the Nernst equation. The analytical representation can facilitate Kalman filtering or observer-based SOC estimation methods. A zero-state hysteresis correction term is used to depict the hysteresis effect of the battery. A parallel resistance-capacitance (RC) network is used to depict the relaxation effect of the battery. A linear discrete-time formulation of the battery model is derived. A recursive least squares algorithm with forgetting is applied to implement the online parameter calibration. Validation results show that the calibrated model can accurately simulate the dynamic voltage behavior of the Lithium-Ion battery for two different experimental data sets.**

I. Introduction

Hybrid electric vehicles (HEVs) and battery electric vehicles (BEVs) are being actively developed by automotive companies to reduce the carbon footprint of ground personal transportation. Plug-in Hybrids (PHEVs) potentially can take advantage of renewable electricity sources and reduce reliance on fossil fuels and are widely viewed as an important transitional technology toward sustainable transportation. Traction battery packs, critical sub-systems of PHEVs, are currently the performance and cost bottlenecks of PHEVs. Due to the transient and demanding vehicle operations in daily driving, a battery management system (BMS) is required to ensure safe and reliable battery operations. The BMS needs to provide accurate knowledge of the states of the traction battery pack to operate the battery reliably and efficiently. A critical variable that must be estimated (because no direct measurement is available) is the battery SOC. In the literature, techniques such as Kalman filters [1-4, 8, 13, 14], sliding mode observers [5,6] and others have been used. The performance of these estimation methods relies heavily on the accuracy of the battery model. Therefore, developing a good battery model is of vital significance for the development of PHEVs and BEVs.

For real-time battery management and control applications, a good battery model should accurately relate the output voltage of a battery yet maintain moderate complexity. Many equivalent circuit-based models [5-12] and simplified electrochemistry-based models [1,3,4,13,14] were developed in the literature. However, open circuit voltages (OCV) of most of the above mentioned models have to be beforehand measured or estimated at several specified SOC values in a special experiment in order to make the remaining resistance-capacitance (RC) parameters identifiable. This method to determine OCV as a function of SOC is time-consuming, laborious and error-prone, especially for batteries with a flat voltage-SOC characteristic. The error in OCV can reduce the accuracy of estimates of the RC parameters. Additionally, the form of representing OCV is not convenient and straightforward for Kalman filtering or observer-based SOC estimation techniques to which an explicit OCV-SOC relation is desirable. The simplified electrochemical battery models use an empirical function to describe the relationship between OCV and SOC so that the model parameters can be identified in any battery loading conditions without specific OCV-determination experiments. Their ability to describe the battery relaxation effect, however, is not as good as equivalent circuit-based models with RC networks. Although a low-pass filter integrated into the simplified electrochemical battery model is helpful for simulating the relaxation effect, the structure of the so called "enhanced self-correcting" (ESC) model is complicated [1,2]. On the other hand, the parameters of most of the models were identified using batch approaches, sometimes leading to a poor robustness. Finally, to be suitable for real-time implementation, the model parameters must be estimated recursively for better computation efficiency.

In this paper, a battery model suitable for real-time implementation is proposed. The model consists of three parts. The first part is an adaptation of the Nernst equation [1] which describes the relationship between OCV and SOC. The second part is a zero-state hysteresis correction term [1]. The third part is a first-order RC network which is used to simulate the battery transient response, such as the relaxation effect. Based on a linear discrete-time model form, a

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recursive least squares algorithm with a forgetting factor is used to estimate the parameters of the model. The performance of the algorithm will be demonstrated using two experimental datasets.

I. MODEL STRUCTURE

The battery model structure is shown in Fig.1. V represents the output voltage and I is the current; the RC network (R_1, C_1) is used to simulate the relaxation effect; R_2 is the ohmic resistance; OCV is depicted by an adaptation of the Nernst equation with three parameters K_0, K_1 and K_2. sM is used for the hysteresis effect. M is a correction term that needs to be identified. And s is a function of the sign of the current described as follows [1]:

$$s(k)=\begin{cases} 1, & I(k)>\varepsilon, \\ -1, & I(k)<-\varepsilon, \\ s(k-1), & |I(k)|\le\varepsilon, \end{cases} \tag{1}$$

where ε is a small positive number and k is the time index.

The discrete-time state equations of the battery model can be described as follows:

$$\mathrm{SOC}(k+1)=\mathrm{SOC}(k)-\frac{\eta I(k)\Delta t}{C_n}, \tag{2}$$

$$U_1(k+1)=\exp\left(\frac{-\Delta t}{R_1C_1}\right)U_1(k)+R_1\left[1-\exp\left(\frac{-\Delta t}{R_1C_1}\right)\right]I(k), \tag{3}$$

where η is the Coulombic efficiency which is assumed to be 1 for discharge and 98% for charging. C_n is the nominal capacity of the battery, U_1 represents the voltage across the capacitor C_1 and Δt is the sampling interval. The battery output voltage is then:

$$\begin{aligned} V(k)=K_0+K_1\ln(\mathrm{SOC}(k))+K_2\ln(1\text{-}\mathrm{SOC}(k)) \\ +s(k)M-I(k)R_2-U_1(k). \end{aligned} \tag{4}$$

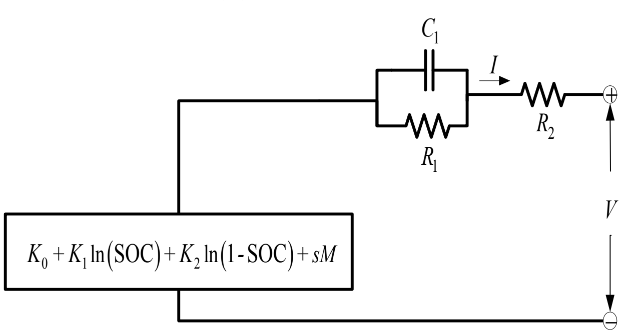

Fig. 1. The proposed battery model structure

II. PARAMETER IDENTIFICATION ALGORITHM

A. Linear identifiable formulation of the battery model

Substituting (3) into (4) leads to the following linear identifiable formulation for recursive algorithms:

$$\begin{aligned} V(k)=K_0(k)+K_1(k)\ln(\mathrm{SOC}(k))+K_2(k)\ln(1\text{-}\mathrm{SOC}(k)) \\ +s(k)M(k)-I(k)R_2(k)-A(k)U_1(k-1)-B(k)I(k-1) \end{aligned}, \tag{5}$$

where

$$A(k)=\exp\left(\frac{-\Delta t}{R_1(k)C_1(k)}\right), \tag{6}$$

$$B(k)=R_1(k)\left[1-\exp\left(\frac{-\Delta t}{R_1(k)C_1(k)}\right)\right], \tag{7}$$

$$\begin{aligned} U_1(k-1)=K_0(k-1)+K_1(k-1)\ln(\mathrm{SOC}(k-1))+s(k-1)M(k-1) \\ +K_2(k-1)\ln(1\text{-}\mathrm{SOC}(k-1))-I(k-1)R_2(k-1)-V(k-1),\ k>1, \end{aligned} \tag{8}$$

$$U_1(0)=0. \tag{9}$$

Given the output voltage, current and SOC of the battery and (8) and (9), the parameters $K_0(k)$, $K_1(k)$, $K_2(k)$, $M(k)$, $R_2(k)$, $A(k)$ and $B(k)$ in (5) can be calibrated recursively.

B. Recursive least squares algorithm with forgetting for the battery model

Based on the standard recursive least squares method, forgetting can be used to give less weight to older data and more weight to more recent data, which is frequently appropriate for online parameter identifications [15]. The recursive least squares algorithm with forgetting for (5) can be illustrated as follows:

$$\boldsymbol{G}(k)=\frac{\boldsymbol{P}(k-1)\boldsymbol{\varphi}(k)}{\lambda+\boldsymbol{\varphi}^T(k)\boldsymbol{P}(k-1)\boldsymbol{\varphi}(k)}, \tag{10}$$

$$\boldsymbol{\theta}(k)=\boldsymbol{\theta}(k-1)+\boldsymbol{G}(k)\left[V(k)-\boldsymbol{\varphi}^T(k)\boldsymbol{\theta}(k-1)\right], \tag{11}$$

$$\boldsymbol{P}(k)=\frac{\boldsymbol{P}(k-1)-\boldsymbol{G}(k)\boldsymbol{\varphi}^T(k)\boldsymbol{P}(k-1)}{\lambda}, \tag{12}$$

where

$$\boldsymbol{\varphi}(k)=\left[1,\ln(\mathrm{SOC}(k)),\ln(1\text{-}\mathrm{SOC}(k)),s(k),-I(k),U_1(k-1),-I(k-1)\right]^T \tag{13}$$

$$\boldsymbol{\theta}(k)=\left[K_0(k),K_1(k),K_2(k),M(k),R_2(k),A(k),B(k)\right]^T \tag{14}$$

and λ $(0<\lambda<1)$ is the forgetting factor. According to (6) and (7), $R_1(k)$ and $C_1(k)$ can be calculated using $\boldsymbol{\theta}(k)$.

The schematic diagram for the online parameter identification of the battery is shown in Fig. 2. The initial values of the parameter estimate $\boldsymbol{\theta}(0)$ and its error covariance matrix $\boldsymbol{P}(0)$ are firstly provided. Then, the parameter vector $\boldsymbol{\theta}(k)$ can be updated based on the online collected regressor.

III. EXPERIMENTAL RESULTS

A. Battery test bench

The experimental setup is shown in Fig. 3, which consists of a Digatron Battery Testing System (BTS-600), a battery management module, a controller area network (CAN) communication unit and a Labview-based virtual measurement unit. The Digatron Battery Testing System is responsible for loading the battery based on the designed program with maximum voltage of 500 V and maximum charging/discharging current of 500 A. The recorded signals include load current, terminal voltage, temperature, accumulative Amp-hour (Ah) and Watt-hour (Wh). The battery management module also collects the voltage and temperature of each cell in the battery. The errors of the Hall current and voltage sensors are less than 0.2% and 0.5%, respectively. The measured load current is transmitted to the battery management module through CAN bus driven by the Labview program and CAN communication unit. Both the Labview-based measurement unit and the management module have a low-pass filtering function incorporated. The current is integrated to obtain "true" battery SOC based on the nominal battery capacity. The measured voltage, temperature and computed SOC are then transmitted through CAN bus to the Labview for real-time display. A Lithium-Ion battery module composed of sixteen cells in series was tested. Each healthy cell has a nominal output voltage of 3.6 V and a nominal capacity of 105 Ah. The actual capacity of the module was around 100 Ah, due to deviant behaviors of cells in the module.

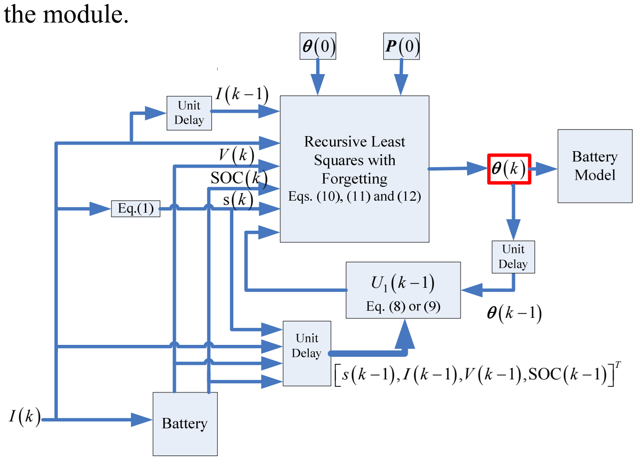

Fig. 2. Schematic of the recursive least square method for battery model update

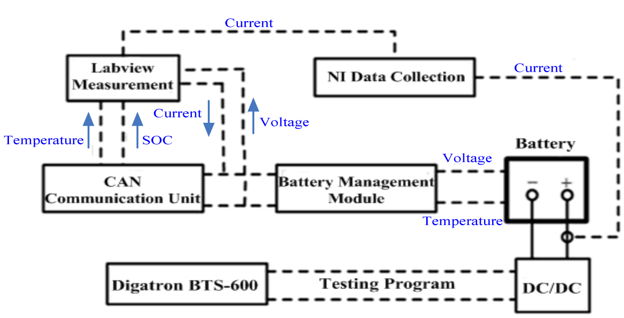

Fig. 3. Schematic diagram of the battery test bench

B. Hybrid pulse test

A hybrid pulse test which was comprised of a sequence of Hybrid Pulse Power Characterization (HPPC) profiles, constant-current discharge pulses and rests was conducted. Data points including current, voltage and SOC were collected once per second. The HPPC current profile is shown in Fig. 4. The voltage, current and SOC profiles for the hybrid pulse test are shown in Fig. 5. The forgetting factor λ for the parameter identification algorithm was set to 0.9996 in the hybrid pulse test to balance between parameter tracking and plant uncertainties. Due to the long duration of the hybrid pulse test, a relatively large value of λ was needed to satisfy a precondition that the RC parameters are positive. A quite inaccurate $\boldsymbol{\theta}(0)$ and a large error covariance matrix $\boldsymbol{P}(0)$ were used. The estimated results are shown in Figures 6-9. It can be seen from the results that the initial parameter errors can be quickly compensated by the recursive least squares algorithm with forgetting. Apart from a few of abrupt changes caused by noise, these estimated parameters after correcting the initial errors seem to be slowly time-varying. The model response starting from the first second in the hybrid pulse test is shown in Fig. 10. It is obvious that the online identified model can accurately describe the dynamical voltage behavior of the Lithium-Ion battery in the hybrid pulse test. The relative error of the model response is shown in Fig. 11. The maximum and mean relative errors are 1.664% and 0.043%, respectively.

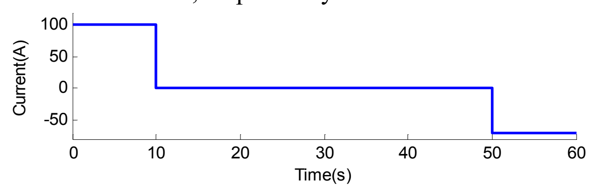

Fig. 4. HPPC current profile

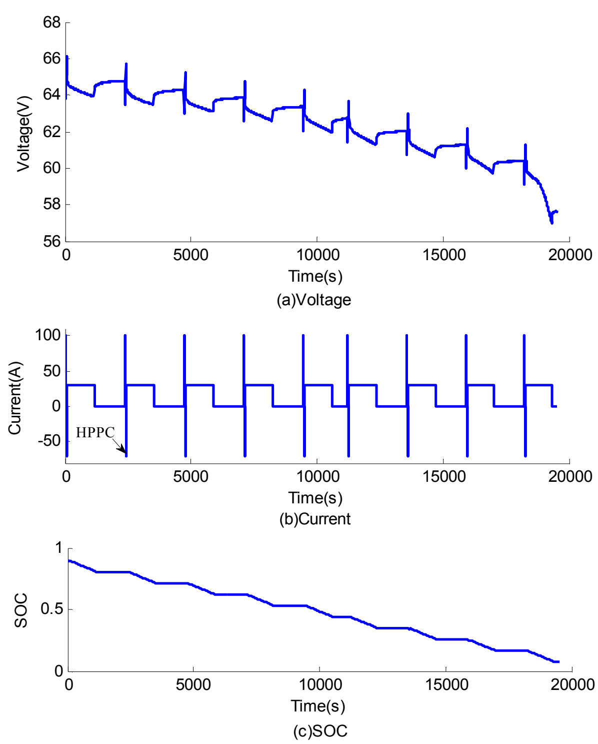

Fig. 5. Measured and calculated battery response in the hybrid pulse test

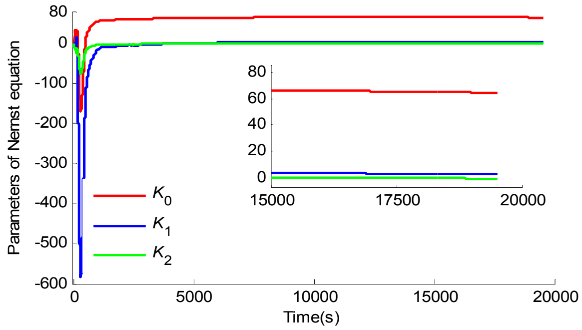

Fig. 6 Estimated K_0, K_1 and K_2 in the hybrid pulse test

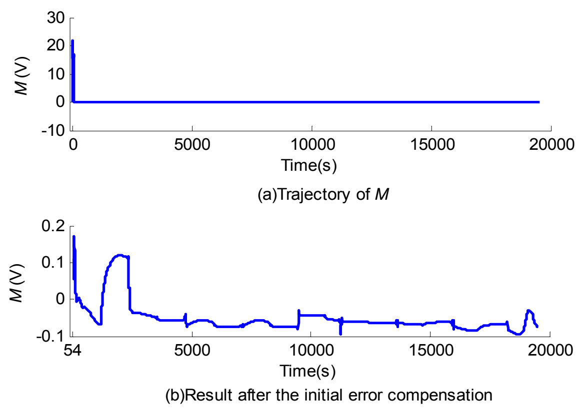

Fig. 7. Estimated M in the hybrid pulse test

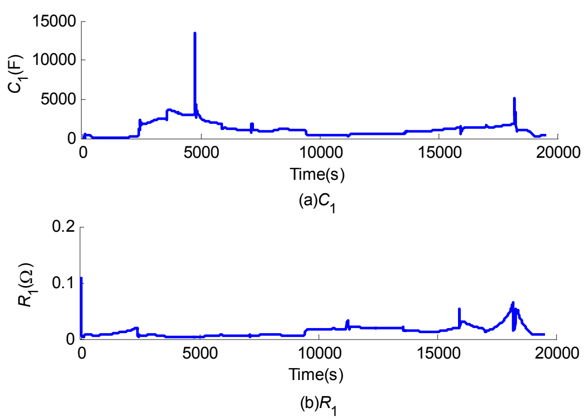

Fig. 8. Estimated C_1 and R_1 in the hybrid pulse test

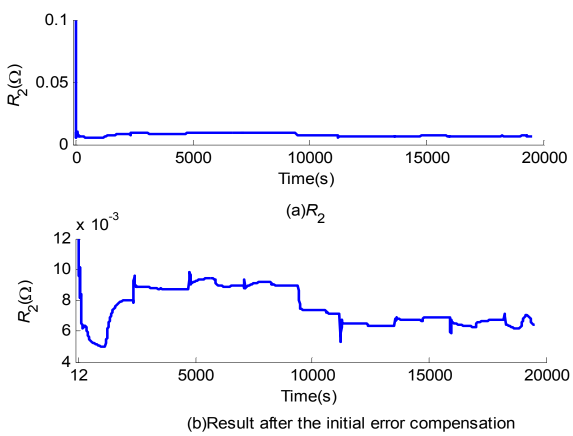

Fig. 9. Estimated R_2 in the hybrid pulse test

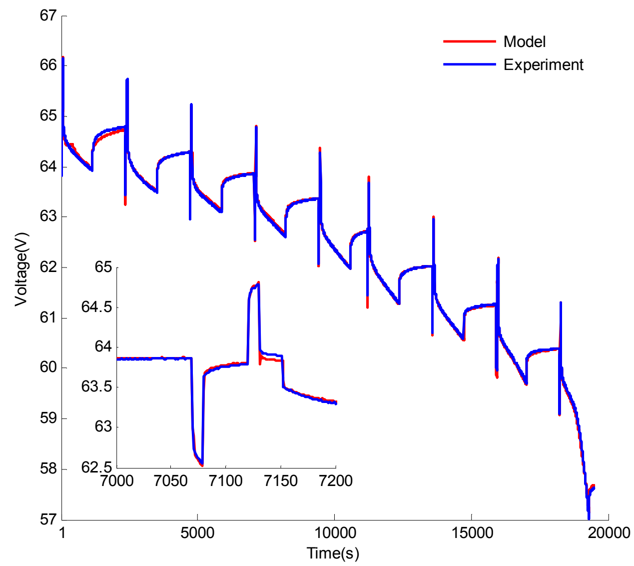

Fig. 10. Comparison of the measured and estimated battery voltage in the hybrid pulse test

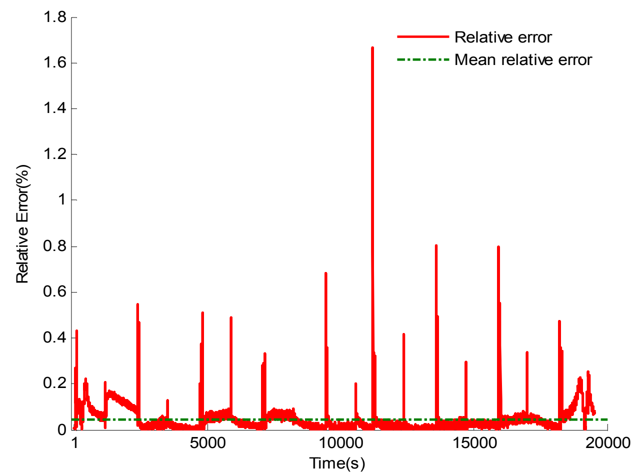

Fig.11. Relative error of the model prediction in the hybrid pulse test

C. Transient battery power test

Another experimental data set collected to further evaluate the performance of the proposed battery model. The test was comprised of twelve cycles in series. The cycle was a variant of the standard Dynamic Stress Test (DST) cycle, which is used to simulate the actual driving cycles of electric vehicles [16]. The voltage, current and SOC profiles for the variable power test are shown in Fig. 12. It can be seen that the maximum discharging and charging current rates are 5C and 2C, respectively, which is much higher than the HPPC test, and closer to what a battery might experience on a PHEV or BEV. Therefore, this transient test can better evaluate the performance of the battery model in real applications. The forgetting factor λ for the parameter identification algorithm was set to 0.995. The same $\boldsymbol{\theta}(0)$ and error covariance matrix $\boldsymbol{P}(0)$ as those in the hybrid pulse test were used. The estimation results of K_0, K_1 and K_2 are shown in Fig. 13.

Fig. 14 shows the estimation results of M, C_1, R_1 and R_2 converge after a long initial transient, perhaps due to the fact the initial parameter values are very different from their true values. The parameter trajectories in the transient power test are quite different from those in the hybrid pulse test. The model response throughout the transient power test is shown in Fig. 15. The model prediction error is shown in Fig. 16. The maximum and mean relative errors are 2.121% and 0.115%, respectively.

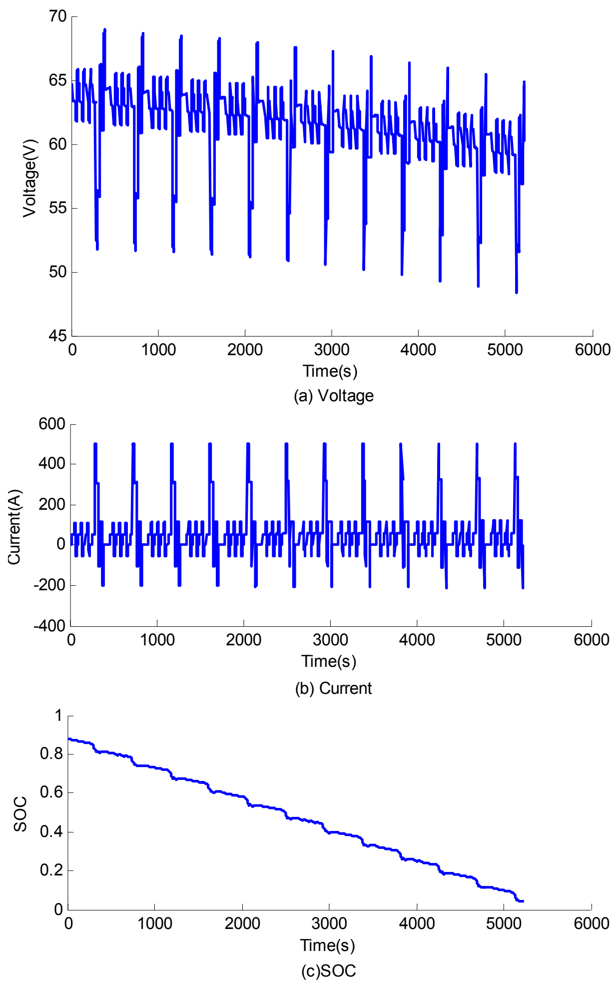

Fig. 12. Measured and calculated battery response in the transient power test

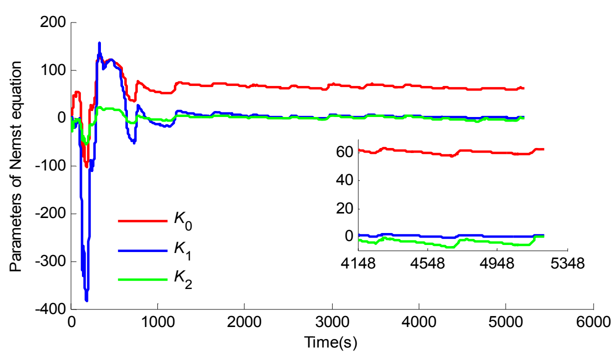

Fig. 13. Trajectories of K_0, K_1 and K_2 in the transient power test

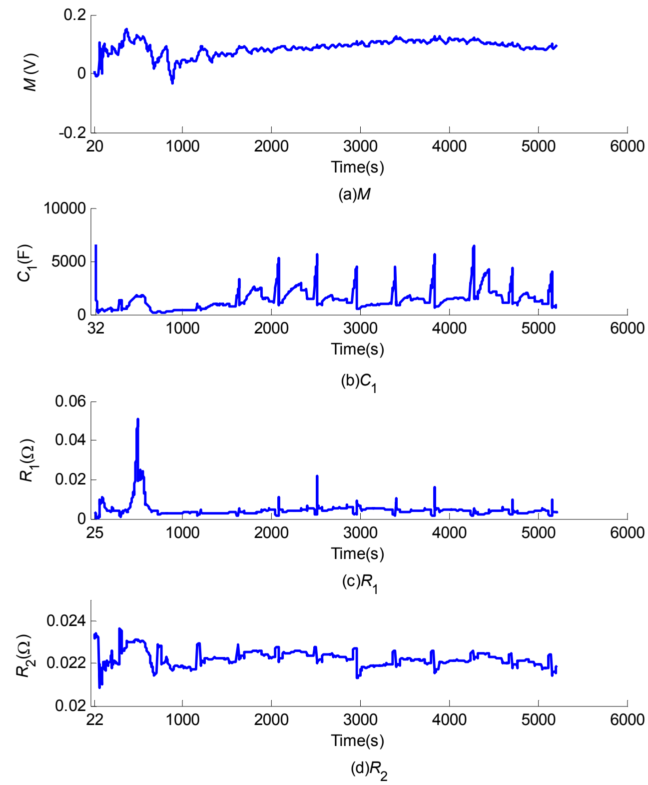

Fig. 14. Results of M, C_1, R_1 and R_2 in the transient power test

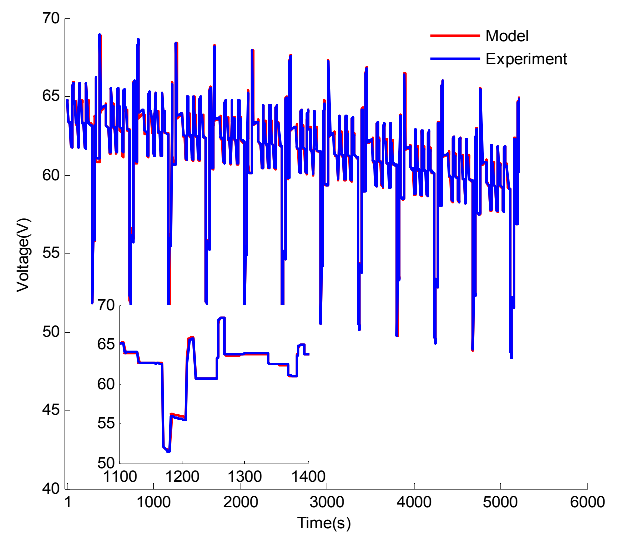

Fig. 15. Comparison of the measured and estimated battery voltage in the transient power test

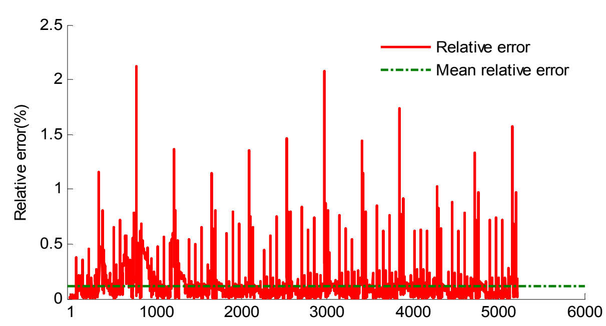

Fig. 16. Relative error of the model response in the transient power test

IV. CONCLUSION

In this paper, a battery model was established to simulate the dynamical voltage behavior of a Lithium-Ion battery. The relationship between battery open-circuit voltage (OCV) and the battery State-of-Charge (SOC) can be represented by an adaptation of the Nernst equation. A zero-state hysteresis correction term is used to depict the hysteresis effect of the battery. A parallel RC network is used to describe the relaxation effect of the battery. A recursive least squares algorithm with forgetting was used to estimate the model parameters. Two different experimental data sets were used to validate the battery model. Results show that the model predicts the voltage of the battery with small error (mean error ~ 0.1%) in both data sets. We are currently working on the SOC estimation algorithm using the developed model. The model update and SOC estimation are interactive. The SOC estimates are used to update the battery model; the updated model is relied upon to estimate SOC. As the recursion times increase, good estimates of the battery SOC can be expected.

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