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Robust Partially Mode Delay-Dependent $\mathcal{H}_\infty$ Output Feedback Control of Discrete-Time Networked Control Systems

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***Abstract*— This paper examines robust partially mode delay dependent $\mathcal{H}_\infty$ output feedback controller design for discrete-time systems with random communication delays. A finite state Markov chain with partially known transition probabilities is used to model random communication delays between sensors and controller. Based on Lyapunov-Krasovskii functional, a novel methodology for designing a partially mode delay-dependent output feedback controller is proposed. Using cone complementarity linearization algorithm bilinear matrix inequalities (BMIs) are solved to obtain the controller gains. We also show that the results for completely known transition probabilities and completely unknown transition probabilities can be derived as special cases of our result. The effectiveness of the proposed design methodology is demonstrated by a numerical example. To the best of authors' knowledge, the problem of designing an output feedback controller for a partially known transition probability has not been fully investigated.**

I. Introduction

Networked control systems (NCSs), where the spatially distributed system components are connected via a network, are a branch in control systems that has been receiving significant research interest in recent years. Its main feature, lack of wires, provide several advantages such as modularity, quick and easy maintenance and low cost, most of which have been very difficult to achieve in traditional point-to-point architecture [1]-[5]. Introduction of network in the control loop and the limited bandwidth of the network creates numerous challenges, mainly network induced delays. Since these delays are usually time-varying and nondeterministic, the traditional control methodologies for time delay systems (TDSs) may not apply to NCSs [6]-[10]. Other issues such as packet dropout, packet reordering, quantization error and variable sampling/transmission intervals also contributes to the need for a specific control scheme.

In many literatures, Markov chain is used to model the network delays, making the overall system Markov jump systems (MJSs)[11]-[16]. However in complex systems, it is very difficult to obtain fully known transition probability matrix. This has fueled recent researches where the transition probably matrix is partially known [15], [16]. In [15], a mode-dependent state feedback controller is designed where the controller gain is not only dependent on the delay upper bound but also the delay range. However, this approach is still conservative since it is not completely mode-dependent. The paper in [15] derives the conditions for the existence of the controller by completely separating the known and unknown parts, which makes it even more conservative, that is the unknown part does not contain any transition probabilities. In [16] $\mathcal{H}_\infty$ filtering problem is investigated where the unknown elements are estimated. This particular filtering design method bridges two extreme cases, mode-independent and mode-dependent. However this method does not contain transition probabilities in the filtering design. In [18] $\mathcal{H}_\infty$ output feedback control of NCSs, with completely known transition probabilities, is considered. In this paper we adapt the approaches in [18] and [19], where $\mathcal{H}_\infty$ state feedback control of NCSs with partially known transition probability matrix is considered, to investigate output feedback control of NCSs with partially known transition probability matrix. To the best of authors' knowledge this has not yet been investigated in NCSs with partially known transition probability matrix.

The aim of this paper is to consider a class of uncertain discrete-time linear systems with random communication delays. A Markov chain with a partially known transition probability matrix is used to model the network induced delay. The number of nodes in the Markov chain depends on the number of possible delays in the network. A partially mode delay-dependent output feedback controller is proposed based on the Lyapunov-Krasovskii functional. The partially mode delay dependent controller is obtained by solving Bilinear matrix inequalities (BMIs) using the cone complementarity linearization algorithm.

The main contributions of the paper can be summarized as follows:

- The existing method in [15] handles the unknown part without using any transition probability information. However we know that the sum of unknown transition probabilities is equal to one minus the sum of known transition probabilities. Therefore, in this paper, we incorporate this information into the unknown part to yield less conservative results.
- It is shown that the proposed method is a generalization of completely known [20] and completely unknown

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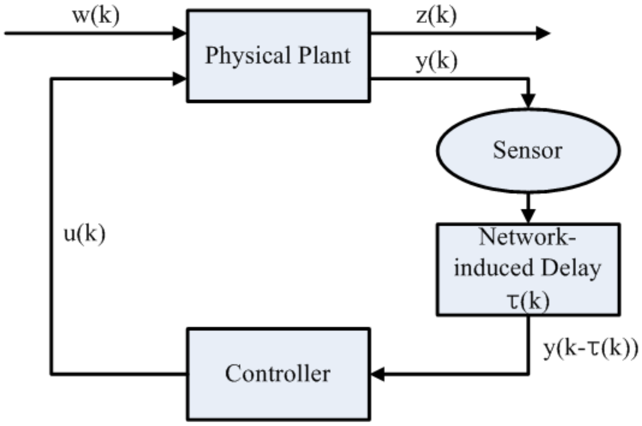

Fig. 1. Networked control systems with sensor-to-controller delay

transition probabilities [21]-[24].

The rest of the note is organized as follows. Section II presents system description and definitions including modeling of the delays using a finite state Markov chain with partially known transition probabilities. Section III proposes an algorithm to solve BMIs in order to obtain partially mode-dependent controller gains for systems with partially known transition probability matrix. Section IV illustrates the effectiveness of the proposed design methodology using a servo motor system. Conclusions are presented in Section V.

II. System Description and Definitions

Consider the simple system setup in Figure 1. A class of uncertain discrete-time linear systems under consideration is described by the following model:

$$\begin{aligned} x(k+1) &= [A+\Delta A(k)]x(k)+[B_1+\Delta B_1(k)]w(k) \\ &\quad +[B_2+\Delta B_2(k)]u(k), \quad x(0)=0 \\ z(k) &= [C_1+\Delta C_1(k)]x(k) \\ &\quad +[D_{11}+\Delta D_{11}(k)]w(k) \\ &\quad +[D_{12}+\Delta D_{12}(k)]u(k) \\ y(k) &= C_2 x(k) \end{aligned} \tag{1}$$

where $x(k) \in \Re^n, u(k) \in \Re^m, z(k) \in \Re^{m_1}, y(k) \in \Re^{m_2}$ are the state, input, controlled output and measured output, respectively and $w(k) \in \Re^{m_3}$ is the disturbance which belongs to $\mathcal{L}_2[0,\infty)$, the space of square summable vector sequence over $[0,\infty]$. The matrices A, B_1, B_2, C_1, D_{11}, D_{12} and C_2 are of known dimensions. The matrix functions $\Delta A(k)$, $\Delta B_1(k)$, $\Delta B_2(k)$, $\Delta C_1(k)$, $\Delta D_{11}(k)$ and $\Delta D_{12}(k)$ represent the time-varying uncertainties in the system which satisfy the following assumption.

Assumption 2.1:

$$\begin{bmatrix} \Delta A(k) & \Delta B_1(k) & \Delta B_2(k) \\ \Delta C_1(k) & \Delta D_{11}(k) & \Delta D_{12}(k) \end{bmatrix} = \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} F(k) \begin{bmatrix} H_1 & H_2 & H_3 \end{bmatrix}$$

where H_i and E_i are known matrices which characterize the structure of the uncertainties. Furthermore, there exists a positive-definite matrix $\mathcal{W}$ such that the following inequality holds:

$$F^T(k)\mathcal{W}F(k) \leq \mathcal{W} \tag{2}$$

Let $\{r_k, k\}$ be a discrete homogeneous Markov chain taking values in a finite set $\mathcal{S} = \{1,2,\cdots,s\}$, with the following transition probability from mode i at k to mode j at time $k+1$

$$p_{ij} := \textbf{Prob}\{r_{k+1}=j|r_k=i\}$$

where $i,j \in \mathcal{S}$.

In this paper, the random delays τ_k is modeled by a finite state Markov process as $\tau_k = \tau(r_k)$ with $0 \leq \tau(1) < \tau(2) < \cdots < \tau(s) \leq \infty$. We assume that the controller will always use the most recent data, that is, if there is no new information coming at step $k+1$ (data could be lost or there is a longer delay), then $x(k-\tau_k)$ will be used for feedback. Thus the delay τ_k can only increase at most by 1 at each step, and we constrain

$$\textbf{Prob}\{\tau_{k+1} > \tau_k + 1\} = 0$$

Note that it is very unlikely that all the elements in the transition probability matrix are known especially when the systems are complex. Along with this assumption, the structured transition probability matrix [25] is restructured as shown below:

$$P_\tau = \begin{bmatrix} p_{11} & p_{12} & 0 & 0 & \dots & 0 \\ ? & ? & p_{23} & 0 & \dots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ \vdots & \vdots & \vdots & \vdots & \vdots & p_{(s-1)s} \\ p_{s1} & ? & ? & p_{s4} & \dots & p_{ss} \end{bmatrix} \tag{3}$$

where "?" represents the unknown, but time-invariant terms. Note $0 \leq p_{ij} \leq 1$ and $\sum_{j=1}^{i+1} p_{ij} = 1$.

In this paper, the controller is in the following form:

$$\begin{aligned} \hat{x}(k+1) &= A_c(i)\hat{x}(k) + B_c(i)y(k-\tau_k) \\ u(k) &= C_c(i)\hat{x}(k) \end{aligned} \tag{4}$$

where $\hat{x}(k)$ is the controller's state, $A_c(i), B_c(i)$ and $C_c(i)$ are the controller matrices. The closed loop system of (1) with (4) is given as follows:

$$\begin{aligned} \zeta(k+1) &= [A_{cl}(i)+\bar{E}_1F(k)\bar{H}_1(i)]\zeta(k) \\ &\quad +B_{cl}(i)\bar{C}_2\zeta(k-\tau_k) \\ &\quad +[\bar{B}_1+\bar{E}_1F(k)H_2]w(k) \\ z(k) &= [C_{cl}(i)+E_2F(k)\bar{H}_1(i)]\zeta(k) \\ &\quad +[D_{11}+E_2F(k)H_2]w(k). \end{aligned} \tag{5}$$

where $\zeta(k) = [x(k)\ \ \hat{x}(k)]^T$,

$$A_{cl}(i) = \begin{bmatrix} A & B_2C_c(i) \\ 0 & A_c(i) \end{bmatrix}, B_{cl}(i) = \begin{bmatrix} 0 \\ B_c(i) \end{bmatrix},$$
$$\bar{B}_1 = \begin{bmatrix} B_1 \\ 0 \end{bmatrix}, \bar{C}_2 = \begin{bmatrix} C_2 & 0 \end{bmatrix},$$
$$\bar{E}_1 = \begin{bmatrix} E_1 \\ 0 \end{bmatrix}, \bar{H}_1(i) = \begin{bmatrix} H_1 & H_3C_c(i) \end{bmatrix},$$
$$C_{cl}(i) = \begin{bmatrix} C_1 & D_{12}C_c(i) \end{bmatrix}.$$

In this paper we denote

$$\begin{aligned} \mathcal{S}^i_{\mathcal{K}} &\triangleq \{j : \text{if } p_{ij} \text{ is known}\}, \\ \mathcal{S}^i_{\mathcal{UK}} &\triangleq \{j : \text{if } p_{ij} \text{ is unknown}\} \end{aligned} \tag{6}$$

where $\forall i \in \mathcal{S}$.

Moreover, for $\mathcal{S}^i_{\mathcal{K}} \neq \emptyset$, the following is defined.

$$\mathcal{S}^i_{\mathcal{K}} = \{\mathcal{K}^i_1, \ldots, \mathcal{K}^i_m\}, \quad 1 \leq m \leq s \tag{7}$$

where $\mathcal{K}^i_m \in \mathbb{N}^+$ represents the mth known element in the ith row of matrix P_τ. Also we denote $p^i_{\mathcal{K}} \triangleq \sum_{j\in\mathcal{S}^i_{\mathcal{K}}}^{i+1} p_{ij}$ and $p^i_{\mathcal{UK}} \triangleq \sum_{j\in\mathcal{S}^i_{\mathcal{UK}}}^{i+1} p_{ij}$ respectively.

Problem Formulation: Given a prescribed $\gamma > 0$ design an output feedback controller of the form (4) such that

1) the system (1) with (4) and $w(k) = 0$ is stochastically stable, i.e., there exists a constant $0 < \alpha < \infty$ such that
$$E\left\{\sum_{\ell=0}^{\infty} x^T(\ell)x(\ell)\right\} < \alpha \tag{8}$$
for all $x(0), r_0$.
2) Under the zero-initial condition, the controlled output $z(k)$ satisfies
$$E\left\{\sum_{k=0}^{\infty} z^T(k)z(k)|r_0\right\} < \gamma \sum_{k=0}^{\infty} w^T(k)w(k) \tag{9}$$
for all nonzero $w(k)$.

The following lemmas which will play a vital role in deriving our main results are shown below.

Lemma 2.1: [19] Let $\bar{x}(k) = x(k+1) - x(k)$ and $\tilde{\zeta}(k) = \Big[\zeta^T(k)\ \ \zeta^T(k-\tau(r_k))\ \ w^T(k)\ \ \zeta^T(k)\bar{H}_1^T(i)F^T(k)$ $\zeta^T(k-\tau(r_k))\bar{C}_2^T H_3^T F^T \quad w^T(k)H_2^T F^T(k)\Big]^T \in \Re^l$, then for any matrices $R \in \Re^{n\times n}$, $M \in \Re^{n\times l}$ and $Z \in \Re^{l\times l}$ satisfying

$$\begin{bmatrix} R & M \\ M^T & Z \end{bmatrix} \geq 0 \tag{10}$$

the following inequality holds

$$-\sum_{i=k-\tau_k}^{k-1} \bar{x}^T(i)R\bar{x}(i) \leq \tilde{\zeta}^T(k)\Big\{\Upsilon_1 + \Upsilon_1^T + \tau_k Z\Big\}\tilde{\zeta}(k) \tag{11}$$

where $\Upsilon_1 = M^T[\text{diag}\{I, 0\}\ \ \text{diag}\{-I, 0\}\ \ 0\ \ 0\ \ 0\ \ 0]$.

Lemma 2.2: [16] For given scalars $a_1 \geq 0$ and $b_i \geq 0$, $i = 1, 2, \ldots, N$, we have

$$\sum_{i=1}^{N} a_i b_i \leq \sum_{i=1}^{N} a_i \sum_{i=1}^{N} b_i \tag{12}$$

Lemma 2.3: [16] For given scalar $\lambda \geq 0$ and matrix $P_i \geq 0$, $i = 1, 2, \ldots, N$, we have

$$\sum_{i=1}^{N} \lambda_i P_i \leq \sum_{i=1}^{N} \lambda_i \sum_{i=1}^{N} P_i \tag{13}$$

III. Robust Delay Dependent $\mathcal{H}_\infty$ Output Feedback Control Design with Partially Known Transition Probabilities

The following theorem proposes stability criteria for the system shown in (1) with partially known transition probabilities.

Theorem 3.1: [19] For given controller gains $K(i)$, $i \in \mathcal{S}$, and $\gamma > 0$, if there exist sets of positive-definite matrices $P(i)$, $R_1(i)$, R_1, $R_2(i)$, R_2, $W_1(i)$, $W_2(i)$, $W_3(i)$, Q, $Z(i)$ and matrices $M(i)$, $\Omega_1(i)$, $\Omega_2(i)$, $\forall i \in \mathcal{S}$, satisfying the following inequalities

$$R_1 > R_1(i), \quad R_2 > R_2(i) \tag{14}$$

$$\begin{aligned} &\Lambda(i) + \Gamma_2^T(i)\tau(s)R_2\Gamma_2(i) + \Upsilon_1(i) + \Upsilon_1^T(i) + \tau(i)Z(i) \\ &\qquad + \Xi^T(i)\Xi(i) + \Omega_1(i) + \Omega_2(i) < 0 \end{aligned} \tag{15}$$

$$\Gamma_1^T(i)\tilde{P_{\mathcal{K}}}(i)\Gamma_1(i) + \Gamma_2^T(i)\tilde{\tau_{\mathcal{K}}}R_1\Gamma_2(i) - \Omega_1(i) < 0, \quad \forall j \in \mathcal{S}^i_{\mathcal{K}} \tag{16}$$

$$\begin{aligned} &\left(1 - p^i_{\mathcal{K}}\right)\left[\Gamma_1^T(i)P(j)\Gamma_1(i) + \Gamma_2^T(i)\tau(j)R_1\Gamma_2(i)\right] \\ &\qquad - \Omega_2(i) < 0, \ \forall j \in \mathcal{S}^i_{\mathcal{UK}} \end{aligned} \tag{17}$$

and

$$\begin{bmatrix} (1 - p_{i(i+1)})R_1(i) + R_2(i) & M(i) \\ M^T(i) & Z(i) \end{bmatrix} \geq 0 \tag{18}$$

$$\begin{bmatrix} p^i_{\mathcal{K}}R_1(i) + R_2(i) & M(i) \\ M^T(i) & Z(i) \end{bmatrix} \geq 0, \quad p_{i(i+1)} \in \mathcal{S}^i_{\mathcal{UK}} \tag{19}$$

where

$$\begin{aligned} \Gamma_1(i) &= \Big[A_{cl}(i)\ \ B_{cl}(i)\bar{C}_2\ \ \bar{B}_1\ \ \bar{E}_1\ \ \bar{E}_1\ \ \bar{E}_1\Big] \\ \Xi(i) &= \Big[C_{cl}\ \ 0\ \ D_{11}\ \ E_2\ \ E_2\ \ E_2\Big] \\ \Gamma_2(i) &= \Big[\bar{A}\ \ 0\ \ \bar{B}_1\ \ \bar{E}_1\ \ \bar{E}_1\ \ \bar{E}_1\Big] \\ \Upsilon_1 &= M^T[\text{diag}\{I, 0\}\ \ \text{diag}\{-I, 0\}\ \ 0\ \ 0\ \ 0\ \ 0] \\ \tilde{P}_{\mathcal{K}}(i) &= \textstyle\sum_{j\in\mathcal{S}^i_{\mathcal{K}}}^{i+1} p_{ij}P(j) \\ \tilde{\tau}_{\mathcal{K}}(i) &= \textstyle\sum_{j\in\mathcal{S}^i_{\mathcal{UK}}}^{i+1} p_{ij}\tau(j) \\ \bar{A} &= \begin{bmatrix} A - I & B_2C_c(i) \\ 0 & 0 \end{bmatrix} \end{aligned}$$

$$\Lambda(i) = \mathrm{diag}\Big\{\Big((\tau(s)-\tau(1)+1)Q + \bar{H}_1^T(i)W_1(i)\bar{H}_1(i) - P(i)\Big), \Big(\bar{C}_2^T H_3^T W_2(i) H_3 \bar{C}_2 - Q\Big), \Big(H_2^T W_3(i) H_2 - \gamma I\Big), -W_1(i), -W_2(i), -W_3(i)\Big\} \quad (20)$$

Then the closed-loop system is stochastically stable with the prescribed $\mathcal{H}_\infty$ performance.

The following theorem provides a robust $\mathcal{H}_\infty$ controller design procedure for the system (1) with partially known transition probabilities.

Theorem 3.2: For a given $\gamma > 0$, if there exist sets of positive-definite matrices $X(i)$, $Y(i)$, $\mathcal{Y}(i)$, $\tilde{R}_1(i)$, $\tilde{R}_1$, $\tilde{R}_2(i)$, $\tilde{R}_2$, $W_1(i)$, $W_2(i)$, $W_3(i)$, Q, $\mathcal{Q}$, $\tilde{W}_1(i)$, $\tilde{W}_2(i)$, N_1, N_2, $\tilde{Z}(i)$, $S(i,j)$ and matrices $\tilde{M}(i)$, $\tilde{\Omega}_1(i)$, $\tilde{\Omega}_2(i)$, $\mathcal{A}(i)$, $\mathcal{B}(i)$, $\mathcal{C}(i)$ and $J(i)$ for $i = 1, 2, \cdots, s$ satisfying the following inequalities

$$\tilde{R}_1 > \tilde{R}_1(i), \quad \tilde{R}_2 > \tilde{R}_2(i) \quad (21)$$

$$\begin{bmatrix} -\Omega_1(i) & \sqrt{p_{\mathcal{K}}^i}\tilde{\Gamma}_1^T(i) & \sqrt{\bar{\tau}_k}\tilde{\Gamma}_2^T(i) \\ * & -\Phi_{\mathcal{K}}(i) & 0 \\ * & * & -N_1 \end{bmatrix} < 0, \quad \forall j \in \mathcal{S}_{\mathcal{K}}^i \quad (23)$$

$$\begin{bmatrix} S(i,j) & J^T(i) \\ * & Y(j) \end{bmatrix} > 0 \quad (25)$$

$$\begin{bmatrix} (1-p_{i(i+1)})\tilde{R}_1(i) + \tilde{R}_2(i) & \tilde{M}(i) \\ * & \tilde{Z}(i) \end{bmatrix} \geq 0 \quad (26)$$

$$\begin{bmatrix} p_{\mathcal{K}}^i\tilde{R}_1(i) + \tilde{R}_2(i) & \tilde{M}(i) \\ * & \tilde{Z}(i) \end{bmatrix} \geq 0, \quad p_{i(i+1)} \in \mathcal{S}_{\mathcal{UK}}^i \quad (27)$$

and

$$N_1\tilde{R}_1 = I, \; N_2\tilde{R}_2 = I, \tilde{W}_1(i)W_1(i) = I \text{ and } \tilde{W}_2(i)W_2(i) = I, \quad (28)$$

where

$$\begin{aligned}
\tilde{\Lambda}(i) &= \mathrm{diag}\Big\{-\begin{bmatrix} Y(i) & I \\ I & X(i) \end{bmatrix}, -Q, \Big(H_2^T W_3(i) H_2 - \gamma I\Big), -W_1(i), -W_2(i), -W_3(i)\Big\} \\
\tilde{\Gamma}_1(i) &= \begin{bmatrix} \check{A}_{cl}(i) & \check{B}_{cl}(i)\bar{C}_2(i) & \check{B}_1 & \check{E}_1 & \check{E}_1 & \check{E}_1 \end{bmatrix} \\
\tilde{\Gamma}_2(i) &= \begin{bmatrix} \check{A}(i) & 0 & \bar{B}_1 & \bar{E}_1 & \bar{E}_1 & \bar{E}_1 \end{bmatrix} \\
\tilde{\Gamma}_3(i) &= \Big[\sqrt{(\tau(s)-\tau(1)+1)}T(i) \; 0 \; 0 \; 0 \; 0 \; 0 \Big] \\
\mathcal{W} &= \mathrm{diag}\Big\{\tilde{W}_1(i), \tilde{W}_2(i)\Big\}
\end{aligned}$$

$$\begin{aligned}
\Phi_{\mathcal{K}} &= \begin{bmatrix} p_{\mathcal{K}}^i\big(\tilde{S}(i) - J(i) - J^T(i)\big) & I \\ I & \sum_{j\in\mathcal{S}_{\mathcal{K}}^i}^{i+1} p_{ij}X(j) \end{bmatrix} \\
\Phi_{\mathcal{UK}} &= \begin{bmatrix} \big(1-p_{\mathcal{K}}^i\big)^{-1}\big(\bar{S}(i) - J(i) - J^T(i)\big) & I \\ I & (1-p_{\mathcal{K}}^i)\sum_{j\in\mathcal{S}_{\mathcal{UK}}^i}^{i+1} X(j) \end{bmatrix} \\
\tilde{\Xi}(i) &= \begin{bmatrix} \check{C}_{cl}(i) & 0 & D_{11} & E_2 & E_2 & E_2 \end{bmatrix} \\
\mathcal{H} &= \begin{bmatrix} \check{H}_1(i) & 0 & 0 & 0 & 0 & 0 \\ 0 & H_3\bar{C}_2 & 0 & 0 & 0 & 0 \end{bmatrix} \\
\tilde{\Upsilon}_1(i) &= \tilde{M}^T(i)[\mathrm{diag}\{I,0\} \; \mathrm{diag}\{-I,0\} \; 0 \; 0 \; 0 \; 0] \\
\check{A}_{cl}(i) &= \begin{bmatrix} AY(i) + B_2\mathcal{C}(i) & A \\ \mathcal{A}(i) & \sum_{j=1}^{i+1} \bar{p_{ij}}X(j)A \end{bmatrix} \\
\check{B}_1(i) &= \begin{bmatrix} B_1 \\ \sum_{j=1}^{i+1} \bar{p_{ij}}X(j)B_1 \end{bmatrix} \\
\check{A}(i) &= \begin{bmatrix} (A-I)Y(i) + B_2\mathcal{C}(i) & A - I \\ 0 & 0 \end{bmatrix} \\
\check{C}_{cl}(i) &= \begin{bmatrix} C_1Y(i) + D_{12}\mathcal{C}(i) & C_1 \end{bmatrix} \\
T(i) &= \begin{bmatrix} Y(i) & I \\ Y(i) & 0 \end{bmatrix}, \; \check{B}_{cl}(i) = \begin{bmatrix} 0 \\ \mathcal{B}(i) \end{bmatrix} \\
\check{H}_1(i) &= \begin{bmatrix} H_1Y(i) + H_3\mathcal{C}(i) & H_1 \end{bmatrix} \\
\check{E}_1(i) &= \begin{bmatrix} E_1 \\ \sum_{j=1}^{i+1} \bar{p_{ij}}X(j)E_1 \end{bmatrix}
\end{aligned}$$

and

$$\begin{aligned}
\sum_{j=1}^{i+1}\bar{p}_{ij}X(i) &= \sum_{j\in\mathcal{S}_{\mathcal{K}}^i}^{i+1} p_{ij}X(j) + (1-p_{\mathcal{K}}^i)\sum_{j\in\mathcal{S}_{\mathcal{UK}}^i}^{i+1} X(j) \\
\sum_{j=1}^{i+1}\bar{p}_{ij}Y^{-1}(j) &= \sum_{j\in\mathcal{S}_{\mathcal{K}}^i}^{i+1} p_{ij}Y^{-1}(j) + (1-p_{\mathcal{K}}^i)\sum_{j\in\mathcal{S}_{\mathcal{UK}}^i}^{i+1} Y^{-1}(j).
\end{aligned}$$

Then the closed-loop system is stochastically stable with the prescribed $\mathcal{H}_\infty$ performance. Furthermore, the controller is given as follows:

$$\begin{aligned}
A_c(i) &= \Big(\sum_{j=1}^{i+1}\bar{p_{ij}}Y^{-1}(j) - \sum_{j=1}^{i+1}\bar{p_{ij}}X(j)\Big)^{-1} \Big(\mathcal{A}(i) - \sum_{j=1}^{i+1}\bar{p_{ij}}X(j)(AY(i) + B_2\mathcal{C}(i))\Big)Y^{-1}(i) \\
B_c(i) &= \Big(\sum_{j=1}^{i+1}\bar{p_{ij}}Y^{-1}(j) - \sum_{j=1}^{i+1}\bar{p_{ij}}X(i)\Big)^{-1}\mathcal{B}(i) \\
C_c(i) &= \mathcal{C}(i)Y^{-1}(i). \quad (29)
\end{aligned}$$

Proof: The proof is omitted due to lack of space.

$$\begin{bmatrix} \tilde{\Lambda}(i)+\tilde{\Upsilon}_1(i)+\tilde{\Upsilon}_1^T(i)+\tau(i)\tilde{Z}(i)+\tilde{\Omega}_1(i)+\tilde{\Omega}_2(i) & \sqrt{\tau(s)}\tilde{\Gamma}_2^T(i) & \tilde{\Gamma}_3^T(i) & \tilde{\Xi}^T(i) & \mathcal{H}^T(i) \\ * & -N_2 & 0 & 0 & 0 \\ * & * & -\mathcal{Q} & 0 & 0 \\ * & * & * & -I & 0 \\ * & * & * & * & -\mathcal{W} \end{bmatrix} < 0 \quad (22)$$

$$\begin{bmatrix} -\Omega_2(i) & \sqrt{(1-p^i_{\mathcal{K}})}\tilde{\Gamma}_1^T(i) & \sqrt{(1-p^i_{\mathcal{K}})\sum_{j\in\mathcal{S}^i_{\mathcal{UK}}}\tau(j)}\tilde{\Gamma}_2^T(i) \\ * & -\Phi_{\mathcal{UK}}(i) & 0 \\ * & * & -N_1 \end{bmatrix} < 0, \ \forall j \in \mathcal{S}^i_{\mathcal{UK}} \quad (24)$$

$\nabla\nabla\nabla$

Remark 3.1: When $\mathcal{S}^i_{\mathcal{UK}} = 0$, we can see that the overall result is reduced to systems with completely known transition probability matrix. When $\mathcal{S}^i_{\mathcal{K}} = 0$ the overall system becomes that with completely unknown transition probability matrix. Furthermore when $\mathcal{S}^i_{\mathcal{K}} = 0$ it is clear that the system is no longer mode-dependent. Therefore it is clear that existing literatures such as [20], where the transition probabilities are completely known, or [21]-[24], where the transition probabilities are completely unknown, are special case of our proposed result.

In accordance with the cone complementary algorithm [26], the nonconvex feasibility problem formulated by (21)-(26) can be converted into the following nonlinear minimisation problem subject to LMIs:

Minimize $Tr\Big(N_1\tilde{R}_1 + N_2\tilde{R}_2 + \tilde{W}_1(i)W_1(i) + \tilde{W}_2(i)W_2(i) + Q\mathcal{Q}\Big)$

Subject to (21)-(26) and

$$\begin{bmatrix} N_1 & I \\ I & \tilde{R}_1 \end{bmatrix} \geq 0, \begin{bmatrix} N_2 & I \\ I & \tilde{R}_2 \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} \tilde{W}_1(i) & I \\ I & W_1(i) \end{bmatrix} \geq 0, \begin{bmatrix} \tilde{W}_2(i) & I \\ I & W_2(i) \end{bmatrix} \geq 0,$$
$$\begin{bmatrix} Q & I \\ I & \mathcal{Q} \end{bmatrix} \geq 0 \quad (30)$$

To solve this optimization problem, the algorithm shown in [19] can be used.

IV. Example

Consider a servo system where the discrete state space representation of the system is given as follows:

$$A = \begin{bmatrix} 0.2802 & -0.0273 \\ 1 & 0 \end{bmatrix} \quad B_1 = \begin{bmatrix} 0.002 \\ 0.003 \end{bmatrix}$$
$$C_1 = \begin{bmatrix} 0.8875 & -0.1404 \end{bmatrix} \quad B_2 = \begin{bmatrix} 0 \\ 0.1 \end{bmatrix}$$
$$D_{11} = 0 \quad D_{12} = 0.005 \quad (31)$$

and the uncertainties are characterized by matrices below:

$$E_1 = \begin{bmatrix} 0.005 \\ 0.002 \end{bmatrix} \quad H_1 = \begin{bmatrix} 0.007 & 0.002 \end{bmatrix}$$
$$E_2 = 0.001 \quad H_2 = 0.004 \quad H_3 = 0.003 \quad (32)$$

The delays are characterized by a Markov chain taking values in a finite set $\mathcal{S} = \{1, 2\}$, which correspond to 2, 3 seconds delays, respectively. The transition probability matrix is given by;

$$P_\tau = \begin{bmatrix} 0.4 & 0.6 \\ ? & ? \end{bmatrix} \quad (33)$$

Using Theorem III, we obtain the controller matrices as follows:

$$A_c(1) = \begin{bmatrix} 0.2842 & -0.0266 \\ -0.0857 & 0.8772 \end{bmatrix}$$
$$B_c(1) = \begin{bmatrix} 0.0007 \\ 0.0059 \end{bmatrix}$$
$$C_c(1) = \begin{bmatrix} -10.9181 & 8.7029 \end{bmatrix}$$
$$A_c(2) = \begin{bmatrix} 0.2820 & -0.0267 \\ -0.0691 & 0.9463 \end{bmatrix}$$
$$B_c(2) = 1 \times 10^{-3} \times \begin{bmatrix} 0.3987 \\ -0.8123 \end{bmatrix}$$
$$C_c(2) = \begin{bmatrix} -10.6811 & 9.3458 \end{bmatrix} \quad (34)$$

Consider the following two cases.

Case 1: Let us say that the transition probability matrix is given by;

$$P_{\tau 1} = \begin{bmatrix} 0.4 & 0.6 \\ (0.3) & (0.7) \end{bmatrix} \quad (35)$$

Case 2: The transition probability is now given by;

$$P_{\tau 2} = \begin{bmatrix} 0.4 & 0.6 \\ (0.9) & (0.1) \end{bmatrix} \quad (36)$$

the results obtained for these cases are shown on Figure 2.

Remark 4.1: Figure 2, shows the ratio of energy of the controlled output to the energy of the disturbance. In Case 1, the attenuation level is approximately equal to $\sqrt{2.1 \times 10^{-6}} = 0.0014$, which is less than the prescribed level $\gamma = 0.5$. Similarly, the ratio is also less than the prescribed level in Case 2. State transition is made according to the transition probability matrices P_τ. The same controller gains in (34) control systems with two different transition probability matrix.

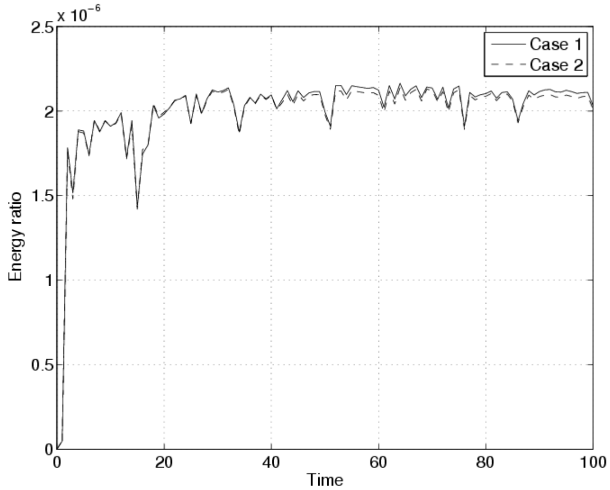

Fig. 2. Ratio of energy of the controlled output to the energy of the disturbance ($\gamma = 0.5$), $P_{\tau 1}$

V. CONCLUSIONS

In this paper, stability criteria and partially mode delay dependent $\mathcal{H}_\infty$ output feedback controller are developed for a class of networked control systems. Random network-induced delays are modeled by Markov processes with partially known transition probability matrix. Conditions for stochastic stability with a given attenuation gain are derived by using Lyapunov-Krasovskii functional. The partially mode delay dependent controller design technique is given in terms of the solvability of bilinear matrix inequalities. An iterative algorithm is proposed to change this non-convex problem into quasi-convex optimization problems, which can be solved effectively by available mathematical tools. Finally, the effectiveness of the proposed design methodology is illustrated by a servo motor numerical example.

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