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On Performance Limits of Feedback Control-Based Stock Trading Strategies

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Abstract— **The starting point for this paper is the control theoretic paradigm for stock trading developed in [1]. Within this framework, a so-called idealized market is characterized by continuous trading and smooth stock price variations. Subsequently, a feedback controller processes the stock price history $p(t)$ to determine the current level of investment $I(t)$. In this idealized setting, we show that that feedback control laws exist which guarantee a profit for all admissible price variations. This first result is only viewed as a benchmark because the controller which achieves this trading profit relies on price signal differentiation which is undesirable. Subsequently, the paper concentrates on more practical differentiator-free controller dynamics. For the simple case of a static linear feedback on the cumulative trading profit or loss $g(t)$, surprisingly, it turns out that a profit is still guaranteed. The final part of the paper involves numerical simulation using historical price; we study the extent to which the idealized market results carry over to real markets.**

I. Introduction

This paper is part of a relatively new line of research involving the application of classical control theoretic concepts to stock trading; e.g., see [1]-[8]. A key idea in the theory underlying the results in this paper is quite simple to explain: As the price of a stock $p(t)$ varies over time, a feedback control trading strategy is used to modulate the amount invested $I(t)$.

To provide further context for this paper, it is also important to take note of the existing literature in finance on technical analysis; e.g., see [9] for an introduction and [10]-[14] illustrating the type of technical analysis issues addressed. In view of this literature, the obvious question to ask is: What might classical control theory have to offer to the area of stock trading? Unlike classical approaches in finance, in this paper, we do not rely on any type of stochastic model for the stock price $p(t)$; e.g., see [15] where geometric Brownian motion is the starting point. Instead, rather than making price predictions, we view $p(t)$ as an uncertain external input against which we seek robust performance.

A second important distinction between our control theoretic approach and the finance literature involves the notion of an *idealized market*; see section below. Given the lack of predictability of stock prices, we take the point of view that a feedback control trading strategy must provide theoretical certifications of performance in an idealized market, before it should be considered for implementation in a real market.

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That is, we view the idealized market as the "proving ground" for theoretical ideas. The trader/researcher first proves theorems in this context as a way to gain credibility regarding the efficacy of an approach in real markets where price is much less predictable and back-testing is costly.

The remainder of this paper is organized as follows: Sections 2 and 3 describe the idealized market paradigm of [1] and associated state equation dynamics. The first set of results, on limits of performance, are given in Section 4. Under the technical conditions associated with idealized markets, we prove that "winning" can be guaranteed in the sense that the account shows a positive trading gain. The controller obtained, however, is not "practical" because it involves price differentiation. Hence, Section 5 is devoted to attainable levels of performance via static output feedback. To this end, we introduce the so-called *Simultaneous Long-Short* (SLS) feedback control and proceed to the main result: *In an idealized market with the SLS static output feedback controller, the trading gain is still positive*. Section 6 is devoted to practical implementation of this controller in real-world discrete-time markets and back-testing using historical data. Finally, Section 7 provides conclusions and describes a future direction for research.

II. The Idealized Market

In the sequel, at time $t \geq 0$, we use notation $p(t)$ for the *stock price*, $I(t)$ for the *amount invested* with $I(t) < 0$ being a short sale, $g(t)$ and $V(t)$ for the *cumulative trading profit or loss* on $[0, t]$ and *account value* respectively. When speaking of a "short sale" above, we mean the following: When the trader has negative investment $I(t) < 0$, this means stock is borrowed from the broker and is immediately sold in the market in the hope that the price will decline. When such a decline occurs, this short seller can realize a profit by buying back the stock and returning the borrowed shares to the broker. Alternatively, if the stock price increases, the short seller may choose to "cover" the trade, return the borrowed stock to the broker and take a loss. The *idealized market* is characterized by a number of assumptions which we now provide:

Continuous and Costless Trading: It is assumed that the trader can react instantaneously to observed price variations with zero transaction cost; i.e., no brokerage commissions or fees. That is, the amount invested $I(t)$ can be continuously updated as price changes occur. Motivation for this assumption is derived in part from the world of high-frequency trading; e.g., with the help of programmed

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trading algorithms, flash traders working for hedge funds can execute literally thousand of trades per second with minimal brokerage costs. In fact, even the small trader using a high-speed internet connection can easily execute many trades per minute. We also note that this assumption is present in the celebrated Black-Scholes model; e.g., see [15].

Continuously Differentiable Prices: We assume that the stock price $p(t)$ is continuously differentiable on $[0, T]$, the time interval of interest. It should be noted that this is the most serious assumption differentiating the idealized market from a real market. That is, this assumption rules out the possibility that price gaps may occur following various real market events such as earnings announcements or major news. As previously mentioned in the introduction, in contrast to most of the finance literature, instead of making predictions based on a geometric Brownian motion model for price, we treat $p(t)$ as an uncertain external input against which we want to robustify the trading gain $g(t)$.

Perfect Liquidity: We assume that the trader faces no gap between a stock's bid and ask prices. That is, orders are filled instantaneously at the market price $p(t)$. We do not view this assumption as serious in the sense that stocks trading large volume on major exchanges typically have bid-ask spreads which are small fractions of a percent.

Trader as a Price-Taker: We assume that the trader is not trading sufficiently large blocks of stock so as to have an influence on the price. Note that this assumption would be faulty in the case of a large hedge or mutual fund. For example, when a hedge fund dumps millions of shares onto the market, the stock price typically declines during the course of the transaction.

Interest Rate and Margin: We assume that the trader accrues interest on any uninvested account funds at the risk-free rate of return r. However, when "extra" funds are brought into the account via a short sale, consistent with the standard practice of brokers, if these funds are "held aside" as cash, no interest is accrued. If the trader opts to use this cash to obtain leverage via purchase of additional stock, margin charges accrue at interest rate m. Another way that margin charges result is when a trader has $I(t) > V(t)$. That is, the trader is essentially being given a loan by the broker and charged margin interest rate m for the use of the funds.

To avoid distracting technical details regarding the way brokers "mark to market" to calculate margin charges, we employ the following model which is a simplification on the way margin accounts are typically handled: When $|I(t)| > V(t)$, margin charges a compounded at interest rate rate m. Note that the absolute value used for $I(t)$ takes care of $I(t) < 0$ when a short is involved. Finally, for simplicity of exposition, in the sequel, we assume that both interest and margin rates are the same. That is, $m = r$. This is a type of efficient market assumption. In practice, it would usually be the case that $m > r$ with the difference $m - r$ being a function of the size of the trader. For example, a large brokerage house trading its own portfolio would have virtually no spread between these two interest rates. With this assumption, we have a very simple equation summarizing interest accruals and margin charges *accruals*. That is, over time interval $[0, t]$, we calculate

$$i(t) \doteq r \int_0^t (V(\tau) - |I(\tau)|) d\tau$$

with the understanding that $i(t) < 0$ represents margin interest owed.

Simplified Collateral Requirement: In a brokerage account, associated with the granting of margin is a collateral requirement on securities. For example, if the account value is V, some clients are allowed to carry $2V$ in equities before forced liquidation of assets occur, larger clients may have larger upper bounds, etc. More generally, our model assumes $\gamma \geq 1$ is specified as part of the trading scenario. Then, we only allow instantaneous investments

$$|I(t)| \leq \gamma V(t)$$

for satisfaction of collateral requirements.

III. Dynamics and Resulting State Equations

Following [1], we consider an infinitesimal time increment dt over which we update both the trading gain g and the account value V. Letting dp be the corresponding stock price increment, the corresponding incremental trading gain is simply the percentage change in price multiplied by the amount invested. Hence, $dg = \frac{dp}{p} I$. During this same time period, the incremental change in the account value is the sum of the contributions from both stock and idle or borrowed cash. That is, $dV = dg + r(V - |I|)dt$. With the starting point above, for the trading profit or loss, we obtain the differential equation

$$\frac{dg}{dt} = \frac{1}{p} \frac{dp}{dt} I(t)$$

and the correspond account value equation is

$$\frac{dV}{dt} = \frac{dg}{dt} + r(V - |I(t)|)$$

with these equations having initial conditions

$$V_0 \doteq V(0) \geq I(0) \doteq I_0; \;\; g(0) = 0.$$

As previously noted, for the differential equations above, we view the price variation $p(t)$ as an external input.

Remarks: We recall that the investment $I(t)$ plays the role of the controller and is yet to be specified. To obtain our first result on performance limits, we first allow $I(t)$ to be rather arbitrary subject to account limits on collateral. To this end, we will allow processing of of the derivative

$$\rho(t) \doteq \frac{1}{p(t)} \frac{dp}{dt}$$

in the maximization of account value. However, recognizing that real-world stock prices are not smooth, it is understood that this first set of results serves only as a benchmark. Accordingly, the second set of results, in Section 5, will address the extent to which trading profits can be guaranteed via a differentiator-free static output linear feedback control.

IV. Performance Limits

Per discussion above, in this section, we seek to maximize the account value without restricting the control dynamics to be differentiator-free. Indeed, we begin with the idealized market with admissible controllers consisting of measurable functions $I(t)$ satisfying the account collateral constraint $|I(t)| \leq \gamma V(t)$. Now, substitution of dg/dt and $\rho(t)$ above into the differential equation for dV/dt yields

$$\frac{dV}{dt} = \rho(t)I(t) + r(V(t) - |I(t)|).$$

For this equation, we consider time intervals of two types.

Type 1 Intervals: On such an interval $[t_1, t_2]$, we have $|\rho(t)| \geq r$. Taking the controller to be of the form

$$I(t) = \gamma^* V(t) \mathrm{sgn}\, \rho(t)$$

with $0 \leq \gamma^* \leq \gamma$, the resulting account value equation is

$$\frac{dV}{dt} = [\gamma^* |\rho(t)| + r(1 - \gamma)]V(t)$$

and the associated endpoint solution is readily calculated to be

$$V(t_2) = e^{r(1-\gamma^*)(t_2-t_1)+\gamma^* \int_{t_1}^{t_2} |\rho(\tau)| d\tau} V(t_1).$$

Type 2 Intervals: On such an interval, $[t_1, t_2]$, we have $|\rho(t)| < r$. In this case, taking the controller to be $I(t) \equiv 0$, the account value equation degenerates to

$$\frac{dV}{dt} = rV(t)$$

with endpoint solution given by $V(t_2) = e^{r(t_2-t_1)}V(t_1)$.

A. Guaranteed Trading Profit and Use of Leverage

From the analysis for the two types of intervals above, it follows that if the idealized market trader uses $0 \leq \gamma^* \leq 1$, the inequality

$$r(1 - \gamma^*)(t_2 - t_1) + \gamma^* \int_{t_1}^{t_2} |\rho(\tau)| d\tau \geq 0$$

is satisfied and it follows that $V(t_2) \geq V(t_1)$.

Remarks: Note that this trader, in opting for $0 < \gamma^* < 1$, has locked in a profit by rejecting the use of available margin. However, observe that this choice of γ^* may be suboptimal in the sense that $\gamma^* = 0$ would yield a larger value of $V(t_2)$ when

$$||\rho||_1 \doteq \int_{t_1}^{t_2} |\rho(\tau)| d\tau$$

is suitably small. That is, with inadequate price volatility, the trader who remains entirely in cash by opting for $\gamma^* = 0$ obtains the risk-free rate of return. On the other hand, suppose the trader chooses to assume risk by using the leverage associated with $\gamma^* > 1$. Notice that enforcement of $V(t_2) \geq V(t_1)$ reduces to

$$||\rho||_1 \geq \frac{(\gamma^* - 1)}{\gamma^*} r(t_2 - t_1).$$

Roughly speaking, this inequality basically tells us what type of percentage variation in the stock price it takes to make it worthwhile to lever the investment. In the limiting case, when a trader is highly confident that this inequality will be satisfied, the optimum is to be maximally levered and use $\gamma^* = \gamma$.

At the other extreme, for a highly non-volatile stock with $||\rho||_1$ suitably small, $\gamma^* = 0$ is optimal even though the inequality $0 < \gamma^* < 1$ guarantees a profit. That is, exploiting the variations in $\rho(t)$ is less rewarding than simply accepting the risk-free rate of return.

V. Trading Via Static Output Feedback

We view the cumulative trading profit or loss $g(t)$ as the system output and consider a static feedback control law of the form $I = f(g)$ with f being a continuous function. That is, the amount invested $I(t)$ in the stock is modulated as a function of the trading profits or losses $g(t)$ accrued over $[0, t]$. In the sequel, we focus on time-invariant linear feedback controls $f(g) = I_0 + Kg$ with $I_0 = I(0)$ being the initial investment.

A. Simultaneous Long-Short (SLS) Linear Feedback Control

To establish the main result, we first construct a controller which is a superposition of two linear feedbacks as described above, one being a a long trade with $I_0, K > 0$ and the other being a short trade with $I_0, K < 0$. These trades can be viewed as running simultaneously in parallel.

SLS Controller Construction: The amount invested in the long trade is $I_L(t)$ and the amount invested in the short trade is $I_S(t)$. Hence, the net overall investment is

$$I(t) = I_L(t) + I_S(t)$$

and, as time evolves, the relative amounts in each of these trades will change. It may well be the case that one of these two trades will become "dominant" as time evolves. For example, in a raging bull market, one would expect to see $I_L(t)$ get large and $I_S(t)$ tending to zero.

With $K > 0$ and $I_0 > 0$ fixed, we define the two feedback controllers by

$$I_L(t) = I_0 + Kg_L(t); \;\; I_S(t) = -I_0 - Kg_S(t)$$

where g_L and g_S are the trading gains or losses for the long and short trades respectively. Hence, the overall investment and trading gains for the combined trade are

$$I(t) = K(g_L(t) - g_S(t)); \;\; g(t) = g_L(t) + g_S(t)$$

which begins at $I(0) = 0$ and $g(0) = 0$. In accordance with Section 3, the individual trades satisfy the differential equation

$$\frac{dg_L}{dt} = \rho(t)(I_0 + Kg_L);$$

$$\frac{dg_S}{dt} = -\rho(t)(I_0 + Kg_S)$$

with initial conditions $g_L(0) = g_S(0) = 0$.

The setup above leads to many results which are consistent with common sense. For example, with $K > 0$ and price $p(t)$ increasing, $I_L(t)$ will increase and $|I_S(t)|$ will decrease. That is, the trader becomes "net long." The question then arises whether the trading gains from the long position will be sufficient to offset losses from the short leading to a net profit. The Arbitrage Theorem to follow answers this question and others in the affirmative provided the the so-called "adequate resource condition" below is satisfied. Given that an idealized market is assumed, this framework should be viewed as one which shows us the limits of state feedback control.

Adequate Resource Condition: In the theorem follow, it is assumed that the combination of initial account value $V(0) = V_0$, feedback gain K and constant γ and prices $p(t)$ are such that the collateral requirement $|I(t)| \leq \gamma V(t)$ is assured over the time interval of interest. Equivalently, if the investment $I(t)$ demands more resources than are currently available in the account, the trader has the ability to respond to a margin call by bringing in more funds.

B. Arbitrage Theorem

At all times $t \geq 0$, assume the adequate resource condition $V(t) \geq \gamma|I(t)|$ is satisfied. Then, the Simultaneous Long-Short static linear feedback controller leads to trading profit

$$g(t) = \frac{I_0}{K}\left[\left(\frac{p(t)}{p(0)}\right)^K + \left(\frac{p(t)}{p(0)}\right)^{-K} - 2\right]$$

satisfying

$$g(t) > 0$$

for all non-zero price variations.

Proof: The differential equations for g_L and g_S above can be readily integrated to obtain solutions

$$g_L(t) = \frac{I_0}{K}\left[\left(\frac{p(t)}{p(0)}\right)^K - 1\right];$$

$$g_S(t) = \frac{I_0}{K}\left[\left(\frac{p(t)}{p(0)}\right)^{-K} - 1\right];$$

with corresponding investments

$$I_L(t) = I_0\left(\frac{p(t)}{p(0)}\right)^K$$

$$I_S(t) = -I_0\left(\frac{p(t)}{p(0)}\right)^{-K}.$$

Summing the two solutions above, the formula for $g(t)$ follows. To prove $g(t) > 0$, without loss of generality, for notational convenience, we assume $p(0) = 1$ and $I_0 = 1$. Now, beginning with

$$g(t) = \frac{1}{K}(p^K(t) + p^{-K}(t) - 2).$$

non-negativity of the profit follows from the fact that the strictly convex function

$$F(p) = \frac{1}{K}(p^K + p^{-K} - 2)$$

has a global minimum at $p = 1$. That is, it is simple to verify that $F(1) = F'(1) = 0;\ \ F''(1) > 0$.

Remarks: In is of interest to understand how the trading profit g depends on the feedback gain K. Using the formula for $g(t)$ above, it is easy to verify that the profit increases monotonically with respect to K. While this makes "high gain" attractive in an idealized market, it may not be the case in a real market. On an intuitive level, in a real market, K should be set as a function of the underlying volatility; this topic is relegated to future research.

C. Improvement Via Controller Reset

Over a time interval $[0, T]$, it is important to note that the theorem above leads to break-even, $g(T) = 0$ when a roundtrip on the stock price occurs; i.e., say $p(T) = p(0)$. The motivation for the discussion to follow is that it is possible to do better than $g(T) = 0$ on such a roundtrip. The key idea is as follows: At some strategically chosen time $t_* > 0$, we re-initialize the controller by setting $I_L(t_*) = I_0$ and $I_S(t_*) = -I_0$. This *reset option* is triggered by stipulating a minimum investment $I_{min} > 0$ and modifying the controller as follows: If at some time $t_* > 0$, either $I_L(t_*) < I_{min}$ or $I_S(t_*) > -I_{min}$, the controller is re-initialized to its starting value $I_L(t_*) = I_0, I_S(t_*) = -I_0$ and the trade continues. We can view this reset process as simply decomposing the initial trade into two separate trades. The first trade goes from zero to t_* and the second trade goes from t_* to T. In accordance with the Arbitrage Theorem, for non-trivial price variations, each of these two trades has a positive trading gain.

Example of Reset: To understand why such a reset procedure may be beneficial, we consider the following simple intuitive example: Suppose the idealized price trajectory is given by $p(t) = 1 + 0.5\sin\frac{\pi t}{100}$ for $0 \leq t \leq 100$ with controller gain $K = 4$ and initial investment and account value $I(0) = V(0) = 10,000$. Then, for $0 \leq t \leq 50$, in accordance with the analysis above, as the price $p(t)$ increases from $p = 1$ to $p = 1.5$, g_L and I_L also increase as the long part of the trade prospers. At the same time, in accordance with the formulae, g_S goes negative and I_S heads towards zero as the short side of the trade is losing and the controller is reducing the short side exposure to cut losses. By the time $t = 50$ arrives, the formulae above result in $g(50) \approx 26,000$, $I_L(50) \approx 22,800$ and $I_S(50) \approx -330$.

Over the second part of the roundtrip, the stock price decreases from $p(50) = 1.5$ to $p(100) = 1$ and all the trading profits are given back with final result $g(T) = 0$. It is apparent from the above that the small short position at time $t = 50$ makes it very difficult to recoup losses as the stock price falls from $t = 50$ to $t = 100$. Hence, over this time period, the long part of the trade is losing a lot of money while the short side is producing minuscule profits.

The value of I_S is not building up quickly enough to exploit the falling price. To remedy this problem, we incorporate the reset option with trigger $I_{min} = 2000$ representing 20% of the initial investment. That is, once either the long or short side of the trade is attenuated by 80%, a reset occurs. Equivalently, we deem the initial trade as complete and begin a "brand new trade." The results obtained are summarized by Figures 1 and 2.

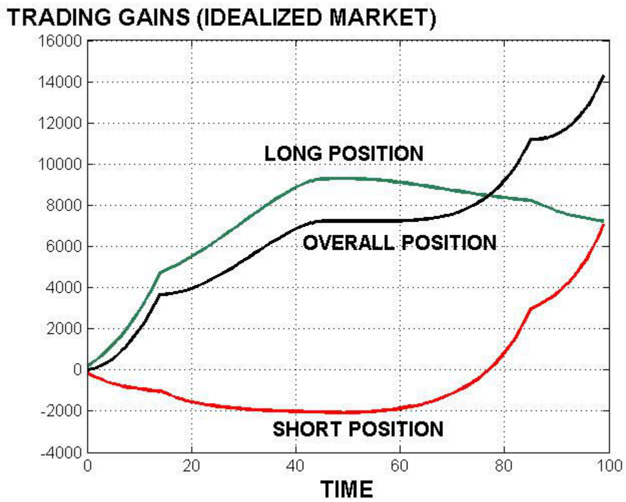

Figure 1: Trading Profit for Sinusoid with Reset

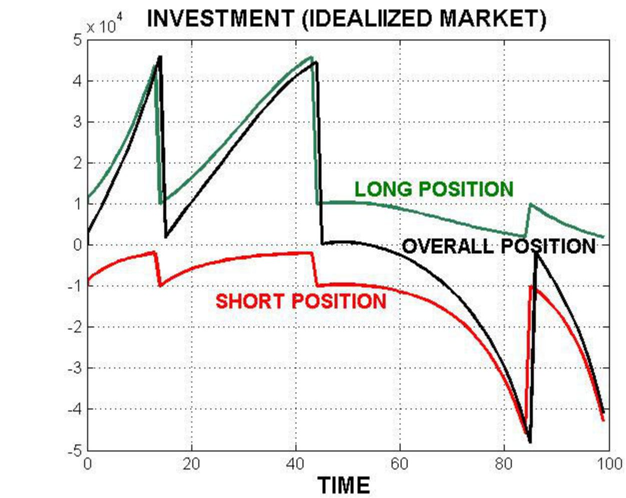

Figure 2: Investment for Sinusoid with Reset

VI. Practical Implementation of SLS Controller

As emphasized in this paper, the use of prices $p(t)$ which are continuously differentiable is an idealization. In real markets, charts of prices at discrete times can appear highly non-differentiable and discontinuous. This raises questions about the efficacy of the static feedback SLS controller in real-world markets. Motivated by the fact that the SLS controller performs well in idealized markets, it becomes a candidate for implementation and back-testing in real markets. Hence, we now assume trading occurs at discrete times t_i and note that the inter-sample time can be either small such as one minute for a high-frequency trader or large such a one day for a mutual fund. Indeed, we let $p(k), V(k), I(k)$ and $g(k)$ denote the discrete-time counterparts of $p(t), V(t), I(t)$ and $g(t)$ respectively. Now, introducing the one-period percentage change in stock price

$$\rho(k) \doteq \frac{p(k+1) - p(k)}{p(k)},$$

we consider various cases for the discrete-time model.

Simplest Case: The simplest scenario occurs when we assume no controller reset and that long and short investments $I_L(k)$ and $I_(k)$ maintain their proper sign; i.e., whereas $I_L(t) \geq 0, I_S(t) \leq 0$ is assured by the dynamics in continuous time, in the discrete-time case, a large value of $\rho(k)$ might lead to an undesirable sign reversal. If, in addition we assume no collateral requirements (say γ is large), it follows from the continuous-time analysis that suitable dynamic update equations are

$$\begin{aligned}
I_L(k+1) &= (1 + K\rho(k))I_L(k); \\
I_S(k+1) &= (1 - K\rho(k))I_S(k); \\
I(k+1) &= I_L(k+1) + I_S(k+1); \\
g_L(k+1) &= g_L(k) + \rho(k)I_L(k); \\
g_S(k+1) &= g_S(k) + \rho(k)I_S(k); \\
g(k+1) &= g_L(k+1) + g_S(k+1); \\
V(k+1) &= V(k) + g(k) + r(V(k) - |I(k)|).
\end{aligned}$$

with r now denoting the one-period risk-free rate of return.

More General Case: To handle the sign restriction conditions on I_L and I_S we modify their update equations to be

$$\begin{aligned}
I_L(k+1) &= \max\{(1 + K\rho(k))I_L(k), 0\}; \\
I_S(k+1) &= \min\{(1 - K\rho(k))I_S(k), 0\}
\end{aligned}$$

and then build in the account collateral requirement by modifying the total investment to be

$$I(k+1) = \min\{I_L(k+1) + |I_S(k+1)|, \gamma V(k)\}.$$

Finally, we enforce $\min\{I_L(k), |I_S(k)|\} < I_{min}$ to trigger controller reset.

A. Example: Trading the Quadruple Q's

In this example, we consider trading the Nasdaq index during the period September 1, 2006 until August 31, 2008. This is the two year period before the precipitous crash of 2008-2009 began. To carry out this trade, we buy and sell shares of the exchange-traded fund QQQQ which tracks the index. As seen in Figure 3, this period includes two round trips of the Nasdaq with price variations between \$37.75 and \$54.18 and the largest daily price change being about four percent. The back-testing was carried out using the daily closing prices, feedback gain $K = 8$, leverage constraint $\gamma = 2$ and

Figure 3: Price of QQQQ Over Trading Period

initial conditions $V_0 = I_0 = 10,000$. For controller reset purposes, we used $I_{min} = 2000$. Over the time period, a five percent interest and margin rate was assumed with daily compounding. As seen in Figures 4 and 5, the inclusion of reset in the controller turns out to be efficacious. The long or short position avoids becoming so small that the controller cannot react to changes in market direction.

Figure 4: Trading Gains/Losses in QQQQ

VII. Conclusion and Further Research

In this paper, we provided a demonstration of the potential for use of control theoretic methods in the stock market. The so-called idealized market is the vehicle through which certifications of performance can be given. Our view is that a necessary but not sufficient condition for use of a trading algorithm in a real market is that it has demonstrable performance properties in an idealized market. This view is based on the idea that variability of prices in real markets makes it possible to easily find historical data which defeats any algorithm in a back-test. In terms of future research, we mention two problems of immediate interest. The first problem might appropriately be called *control gain selection*. How should the feedback gain K be chosen? One possibility would be to address this problem via a data-based adaptive

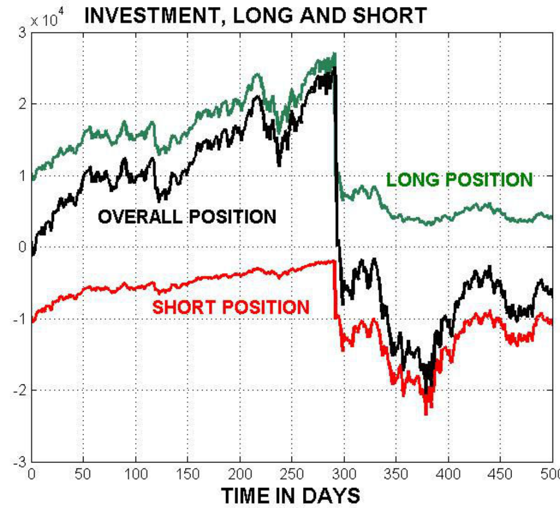

Figure 5: Investment in QQQQ Over Trading Period

method which uses a training set to optimize K. The second problem of interest would be to study other classes of idealized markets. For example, in an idealized market having prices generated via geometric Brownian motion, it would be of interest to see what type of performance results are possible.

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