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An Integral Sliding Mode Augmentation Scheme for Fault Tolerant Control

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Abstract— **In this paper an integral sliding mode control allocation scheme is used to enhance the functional capability and reliability of an existing commercial-aircraft controller by introducing fault tolerance. The design philosophy is to distribute the control signals to primary control surfaces in the nominal fault free scenario, whilst the secondary control surfaces are only activated when the effectiveness levels of the primary control surfaces are not sufficient to handle the situation in the presence of faults/failures. The key advantage of the proposed scheme over other sliding mode control allocation schemes is that the one proposed in this paper can be retro-fitted to an existing state feedback controller designed using only the primary actuators. Simulation results using the FTLAB747 software, show good results, and validate the effectiveness of the proposed scheme.**

I. Introduction

Faults in safety critical systems, if not mitigated can result in catastrophic consequences. Fault tolerant control (FTC) attempts to maintain closed loop performance in safety critical systems in the face of multiple faults and failures. Redundant actuators are key to the design of FTC systems, because in emergency situations when faults or failures occur in the primary control surfaces, this redundancy can be exploited to retain acceptable performance [7]. Many different approaches have been suggested: see for example [22] for an overview of the various methods.

Control Allocation (CA) is one technique to manage redundancy in overactuated systems, and can be used in combination with other control design methods to distribute the control signals to functioning actuators during fault free as well as in fault/failure scenarios, without reconfiguring the underlying controller. In [17], a comparison of different control allocation methods is made, whereas in [12] optimal control and CA are compared in terms of redistributing a virtual control signal among redundant actuators. The benefits of using CA in terms of FTC are exploited in [3], [6] for high performance aircraft. The combination of CA with sliding mode control approaches is used in [1], [14], [18] and [2] to achieve fault tolerance. The use of sliding mode ideas to achieve tolerance to actuator faults and failures is a natural extension of the well-known robustness properties to matched uncertainty exhibited by sliding mode control schemes [20], [8]. The idea behind the CA scheme in [2] is to automatically stop sending control signals to actuators which are faulty, and to redistribute the control effort to healthy ones to maintain the desired performance objectives. This approach has been extensively tested on the aircraft benchmark problem associated with the GARTEUR AG16 programme [7]. To enforce a sliding mode throughout the entire system response, the idea of an integral sliding mode was proposed in [21], [20]. More recently in [5], [4] integral sliding modes were investigated for systems with unmatched uncertainty. In [11], an integral sliding mode approach was considered as a candidate for FTC, where robustness against faults and certain total actuator failures is guaranteed throughout the entire system response.

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In this paper a novel fault tolerant CA scheme incorporating integral modes is proposed. The approach proposed in this paper is quite different from [11], since it represents an a-posteri design approach building on a given existing state feedback control law designed using the primary control surfaces. The idea is that if there are no faults in the primary actuators, the integral sliding mode control allocation scheme behaves exactly as the given baseline controller. Only in the case of faults and failures to the primary control surfaces, are the FTC aspects of the integral sliding mode scheme invoked. To accommodate this philosophy, a completely different development and design methodology compared to [11] must be adopted. In [11] all the design parameters of the integral sliding mode CA scheme are synthesized simultaneously. Here a given baseline controller is used as a given starting point, and the integral sliding mode design is interlaced with the existing controller. In this way the integral sliding mode CA scheme proposed in this paper can be retro-fitted to any existing control scheme to introduce fault tolerance. It is different to the integral sliding mode schemes originally proposed in [21], [20], [5] because of the partitioned structure of the actuators into two sets: the primary ones and the secondary ones. This results in a very specific design formulation – which is solved in this paper. The efficacy of the proposed scheme is tested in simulation, on the high-fidelity nonlinear model, which forms the basis of the GARTEUR AG16 benchmark from [7].

II. Problem Formulation

Consider an uncertain linear system subject to actuator faults or failures written as

$$\dot{x}(t) = Ax(t) + BWu(t) + f(t) \tag{1}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$ and $W \in \mathbb{R}^{m\times m}$ is a diagonal semi-positive definite weighting matrix. The function $f(t)$ is a disturbance which is unknown but is assumed to be bounded. Suppose the input distribution can be partitioned as

$$B = \begin{bmatrix} B_o & B_s \end{bmatrix} \tag{2}$$

where $B_o \in \mathbb{R}^{n \times l}$ and $B_s \in \mathbb{R}^{n \times m-l}$. Here B_o is the input distribution matrix associated with the primary actuators, whilst B_s is associated with secondary actuators which impart redundancy into the system. Partition the weighting matrix as $W = diag[W_1, W_2]$ where $W_1 = diag[w_1, .., w_l]$ and $W_2 = diag[w_{l+1}, .., w_m]$. These matrices model the effectiveness level of the primary and secondary actuators respectively. If $w_i = 1$, it means that the corresponding ith actuator has no fault and is working perfectly, whereas if $1 > w_i > 0$, an actuator fault is present. If $w_i = 0$, the actuator has completely failed.

Assume B_o has full column rank and therefore there exists an orthogonal matrix $T_o \in \mathbb{R}^{n \times n}$ such that

$$T_o B_o = \begin{bmatrix} 0 \\ B_{21} \end{bmatrix} \tag{3}$$

where $B_{21} \in \mathbb{R}^{l \times l}$ (and B_{21} is nonsingular). This represents so-called QR decomposition. It is assumed that the function $f(t)$ satisfies the matching condition [8], [21]

$$f(t) = B_o \xi(t) \tag{4}$$

where $\xi(t)$ is some bounded unknown disturbance. By a suitable change of coordinates $x \mapsto T_o x$ it can be assumed, the input distribution matrix B_o has the form on the right hand side of (3) and therefore the original matrix B in (1), in suitable coordinates, has the form

$$B = \begin{bmatrix} 0 & B_{12} \\ B_{21} & B_{22} \end{bmatrix} \tag{5}$$

where $B_{22} \in \mathbb{R}^{l \times (m-l)}$. Also scale the states to ensure that $B_{21}^T B_{21} = I_l$. This can be achieved easily without any loss of generality. Assume for the system (A, B_o) a controller, based only on the primary actuators, has been designed. Specifically assume a state feedback gain matrix $F \in \mathbb{R}^{l \times n}$ has been designed so that the closed loop system

$$\dot{x}(t) = (A + B_o F)x(t) \tag{6}$$

is stable and has a suitable dynamic response which satisfies the requirements imposed on the designer. In the remainder of the paper it will be assumed without loss of generality that states associated with (1) have been chosen so that the input distribution matrix B has the form given in (5). A control allocation scheme will now be retro-fitted to the existing controller for the primary surfaces given by

$$u_o(t) = Fx(t) \tag{7}$$

The physical control signals which are sent to all the actuators are assumed to be given by

$$u(t) = N\nu(t) \tag{8}$$

where $\nu(t) \in \mathbb{R}^l$ is the virtual control effort which will be discussed later. The control allocation matrix is given by

$$N = \begin{bmatrix} I_l \\ N_2(I_l - W_1) \end{bmatrix} \tag{9}$$

where

$$N_2 = B_{22}^T B_{21} (B_{21}^T B_{22} W_2 B_{22}^T B_{21})^{-1} \tag{10}$$

In order that $\det(B_{21}^T B_{22} W_2 B_{22}^T B_{21})^{-1} \neq 0$, because by construction $\det(B_{21}) \neq 0$, this is equivalent to $\det(B_{22} W_2 B_{22}^T) \neq 0$. This imposes a limitation on the number of elements of W_2 that can become zero and therefore limits the number of total failures in the secondary actuators that can be accommodated.

Substituting (8) and (4) into (1) gives

$$\dot{x}(t) = Ax(t) + \begin{bmatrix} B_{12} W_2 N_2 (I_l - W_1) \\ B_{21} W_1 + B_{22} W_2 N_2 (I_l - W_1) \end{bmatrix} \nu(t) + B_o \xi(t) \tag{11}$$

Notice that since by construction $B_{21}^T B_{21} = I_l$ and B_{21} is square, then $B_{21} B_{21}^T = I_l$ and it follows that

$$\begin{aligned} B_{22} W_2 N_2 &= B_{21} B_{21}^T B_{22} W_2 N_2 \\ &= B_{21} \end{aligned} \tag{12}$$

since $B_{21}^T B_{22} W_2 N_2 = I$ from the definition of N_2 in (10). Substituting from (12) in (11) yields

$$\dot{x}(t) = Ax(t) + \underbrace{\begin{bmatrix} B_{12} W_2 N_2 (I_l - W_1) \\ B_{21} \end{bmatrix}}_{B_w} \nu(t) + B_o \xi(t) \tag{13}$$

It is easy to see that during a fault-free situation (i.e. when $W = I$), equation (13) becomes

$$\dot{x}(t) = Ax(t) + \underbrace{\begin{bmatrix} 0 \\ B_{21} \end{bmatrix}}_{B_o} \nu(t) + B_o \xi(t) \tag{14}$$

Consequently if $\nu(t) = u_o(t)$, then nominal baseline performance is achieved. Furthermore when $W = I$, substituting in (8) and (9), it follows if $\nu(t) = Fx(t)$ then

$$u(t) = \begin{bmatrix} Fx(t) \\ 0 \end{bmatrix}$$

and only the primary actuators are used. However to introduce robustness and tolerance faults, rather than using $\nu(t) = Fx(t)$, an integral sliding mode scheme will be introduced for the synthesis of the virtual control signal $\nu(t)$.

III. Integral sliding mode controller design

A. *Integral switching function:*

The integral switching function suggested in [5] and [11], aims to eliminate the reaching phase present in traditional sliding mode control – i.e. the sliding mode will exist from the beginning. Choose as the sliding surface $\mathcal{S} = \{x \in \mathbb{R}^n : \sigma(x,t) = 0\}$, where

$$\sigma(x,t) := Gx(t) - Gx(t_0) - G \int_{t_0}^{t} \big(A + B_o F\big) x(\tau) d\tau \tag{15}$$

and $G \in \mathbb{R}^{l \times n}$ is the design freedom. In this paper

$$G := B_o^T \tag{16}$$

is suggested. With this choice of G it follows

$$GB_o = B_{21}^T B_{21} = I_l$$

and

$$
\begin{aligned}
GB_w &= \begin{bmatrix} 0 & B_{21}^T \end{bmatrix} \begin{bmatrix} B_{12}W_2N_2(I_l - W_1) \\ B_{21} \end{bmatrix} \\
&= B_{21}^T B_{21} = I_l
\end{aligned}
$$

It is necessary to analyze the sliding motion associated with the surface in (15) and the choice of G in (16), in the presence of faults or failures. Taking the time derivative of (15) yields

$$\dot{\sigma}(t) = G\dot{x}(t) - GAx(t) - GB_oFx(t) \tag{17}$$

Substituting the value of (13) into (17) and after simplifying gives

$$\dot{\sigma}(t) = GB_w\nu(t) + GB_o\xi(t) - GB_oFx(t) \tag{18}$$

An expression for the equivalent control [20], [8] can be obtained by setting $\dot{\sigma}(t) = 0$ in (18) and by solving for $\nu_{eq}(t)$. By taking into account the fact that $GB_w := I_l$ and $GB_o := I_l$ it follows

$$\nu_{eq}(t) = Fx(t) - \xi(t) \tag{19}$$

The equations of motion during sliding can be obtained by substituting (19) into (13) to obtain

$$\dot{x}(t) = Ax(t) + \begin{bmatrix} B_{12}W_2N_2(I_l - W_1) \\ B_{21} \end{bmatrix} (Fx(t) - \xi(t)) + B_o\xi(t) \tag{20}$$

Using the fact that

$$B_o = \begin{bmatrix} 0 \\ B_{21} \end{bmatrix}$$

equation (20) can be written as

$$\dot{x}(t) = (A + B_oF)x(t) + \begin{bmatrix} B_{12}W_2N_2(I_l - W_1) \\ 0_l \end{bmatrix} \big(Fx(t) - \xi(t)\big) \tag{21}$$

which can be rewritten as

$$\dot{x}(t) = (A + B_oF)x(t) + \tilde{B}\big[\Phi(t)\big(Fx(t) - \xi(t)\big)\big] \tag{22}$$

where

$$\tilde{B} := \begin{bmatrix} B_{12} \\ 0 \end{bmatrix} \tag{23}$$

and

$$\Phi(t) := W_2N_2(I_l - W_1) \tag{24}$$

The representation in (22) will be used as the basis for the closed-loop analysis.

B. Closed-loop Stability Analysis:

It is clear that during fault-free conditions (i.e. when $W = I$), $\Phi(t) = 0$, and equation (22) becomes

$$\dot{x}(t) = (A + B_oF)x(t) \tag{25}$$

which is stable by design of the original a-priori baseline controller $u_o = Fx(t)$. Also the effect of disturbance signal $\xi(t)$ during sliding in the nominal case (when $W = I$) is completely rejected. However the sliding motion in (22) depends on the matrix W, and a stability analysis needs to be carried out to ensure closed-loop stability for different faults and failures. To this end write the equation (22) as

$$\dot{x}(t) = \underbrace{(A + B_oF)}_{\tilde{A}} x(t) + \tilde{B}\overbrace{\Phi(t)\underbrace{Fx(t)}_{\tilde{y}}}^{\tilde{u}} - \tilde{B}\Phi(t)\xi(t) \tag{26}$$

and define

$$\gamma_2 = \|\tilde{G}(s)\|_\infty \tag{27}$$

where

$$\tilde{G}(s) := F(sI - \tilde{A})^{-1}\tilde{B} \tag{28}$$

As argued earlier in (12), it is easy to verify that W_2N_2 is a pseudo inverse for $B_{21}^TB_{22}$, and so using arguments similar to those in [1], the boundedness of properties of the pseudo inverse proved in [19] ensures $\|W_2N_2\| < \gamma_1$ for some γ_1 provided $\det(B_{22}W_2B_{22}^T) \neq 0$. Define a scalar γ_1^* to be the smallest number satisfying

$$\|\Phi(t)\| < \gamma_1^* \tag{29}$$

Since $\|\Phi\| \leq \|I_l - W_1\|\|W_2N_2\| < \|W_2N_2\|$, the existence of γ_1^* is guaranteed.

Proposition 1: During fault or failure conditions, for any combination of $0 < w_i \leq 1$, the closed loop system will be stable if:

$$\gamma_2\gamma_1^* < 1 \tag{30}$$

Proof: The closed-loop system in the presence of faults and failures in (26) can be written as

$$\dot{x}(t) = \tilde{A}x(t) + \tilde{B}\tilde{u}(t) \tag{31}$$

$$\tilde{y}(t) = Fx(t) \tag{32}$$

where

$$\tilde{u}(t) = \Phi(t)\tilde{y}(t) \tag{33}$$

(Note the term $\tilde{B}\Phi(t)\xi(t)$ constitutes an external disturbance and does not affect closed-loop stability). This form of equation (26) can be effectively thought of as the feedback interconnection of the feedforward linear system $\tilde{G}(s)$ with an uncertain feedback term $\Phi(t)$. According to the small gain theorem [15], this interconnection, and hence equation (26), will be stable if

$$\gamma_2\gamma_1^* < 1 \tag{34}$$

■

C. Integral Sliding Mode Control laws:

The proposed integral sliding mode control law, which depends on the nominal system (14) is defined as

$$\nu(t) = \nu_l(t) + \nu_n(t) \tag{35}$$

where the linear part of the control law is

$$\nu_l(t) = Fx(t) \tag{36}$$

and the nonlinear part, which is responsible for inducing sliding despite faults/failures, is defined as

$$\nu_n(t) = -\rho\frac{\sigma(x,t)}{\|\sigma(x,t)\|} \qquad \text{for } \sigma(x,t) \neq 0 \tag{37}$$

where ρ is a scalar gain to enforce the sliding motion.
Now it will be shown that the control law in (35) satisfies the reachability condition [20], which guarantees that the designed control law drives the system trajectories to the switching surface and maintains it on the surface. Substituting equation (35)-(37) into (18) yields

$$\begin{aligned}\dot{\sigma}(t) &= Fx(t) - \rho\frac{\sigma(t)}{\|\sigma(t)\|} + \xi(t) - Fx(t) \\ &= -\rho\frac{\sigma(t)}{\|\sigma(t)\|} + \xi(t) \end{aligned} \quad (38)$$

Consider the positive definite, candidate Lyapunov function

$$V(t) = \frac{1}{2}\sigma^T\sigma \quad (39)$$

Taking the time derivative of the Lyapunov function in (39) and substituting from (38) results in

$$\dot{V} = -\rho\|\sigma\| + \sigma^T\xi(t) \quad (40)$$

Since it is assumed that the upper bound of the disturbance term $\xi(t)$ is known, to overcome $\xi(t)$ in (40), the scalar gain ρ should have the value

$$\rho \geq \max_t \|\xi(t)\| + \eta \quad (41)$$

where η is a positive design scalar. Substituting (41) into (40) gives

$$\dot{V} \leq -\eta\|\sigma\| = -\sqrt{2}\eta V^{1/2} \quad (42)$$

This is sufficient to guarantee that the sliding motion is attained in finite time and maintained for all subsequent time.

Finally to get the physical control law, substituting (35)-(37) into (8) yields

$$u(t) = \begin{bmatrix} I_l \\ N_2(I_l - W_1) \end{bmatrix} \left(Fx(t) - \rho\frac{\sigma(x,t)}{\|\sigma(x,t)\|}\right) \quad (43)$$

where N_2 is from (10).

IV. Fault Tolerant Controller design

The objective here is to design an FTC based on an existing baseline control law by using the proposed integral sliding mode CA scheme described earlier. The design process will be demonstrated by means of the lateral dynamics of a large transport aircraft.

A. Baseline Control Law for Yaw Damper

A yaw damper is a stability-augmentation system for the lateral dynamics of an aircraft [10]. Here the baseline control law F for the nominal system (14) is obtained using eigenstructure assignment [16]. The desired closed-loop eigenvalues have been obtained from chapter 10 in [10] and the best possible eigenvectors for this situation are documented in [9]. As in [10], a linearization of the benchmark model from [7] is obtained around an operating condition of straight and level flight at $40,000$ ft and a forward speed of 774 ft/sec (Mach 0.8) as given in [13]. Together with a washout filter (high-pass) on the yaw rate, the augmented state space representation is

$$A = \begin{bmatrix} -0.3330 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0.0816 & 1 \\ 0 & 0.0413 & -0.0537 & -0.9944 & 0.0823 \\ 0 & -0.0012 & 0.6090 & -0.0869 & -0.0335 \\ 0 & 0.0002 & -2.9236 & 0.3681 & -0.4514 \end{bmatrix}$$

$$B = \left[\begin{array}{cc|ccccccccc} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0.0070 & 0 & 0.0008 & 0.0003 & -0.0003 & -0.0008 & 0.0001 & 0.0001 & -0.0001 & -0.0001 \\ -0.4438 & -0.0082 & -0.0145 & -0.0046 & 0.0046 & 0.0145 & 0.0314 & 0.0179 & -0.0179 & -0.0314 \\ 0.1451 & -0.1329 & -0.2033 & -0.0625 & 0.0625 & 0.2033 & 0.0054 & 0.0031 & -0.0031 & -0.0054 \end{array}\right]$$

The states are $x = \{x_{wo}, \phi, \beta, r, p\}^T$, where x_{wo} is the washout filter state, ϕ is roll angle (rad), β is side slip angle (rad), r is yaw rate (rad/sec) and p is roll rate (rad/sec). The available control inputs are $\delta_{lat} = \{\delta_r, \delta_a, \delta_{sp1-4}, \delta_{sp5}, \delta_{sp8}, \delta_{sp9-12}, th_1, th_2, th_3, th_4\}$ which represent aileron deflections (rad), spoiler deflections (left: 1-4, 5 and right: 8, 9-12)(rad), rudder deflection (rad), and individual engine thrusts (N) scaled by 10^5. For this design it is assumed that the left aileron moves in an anti-symmetric fashion to the right one (the outer ailerons on each wing are not active during cruise flight). Note further transformations are required to the model above to enforce the structure in (5) and to ensure $B_{21}^T B_{21} = I_2$. The ideal closed-loop eigenvalues suggested in [10] are,

$$\{-0.0051, -0.468, -0.279 \pm 0.628j, -1.106\}$$

where the natural motion corresponding to the complex eigenvalues is referred to as dutch roll, and the motion corresponding to the stable real eigenvalues is referred to as spiral mode (-0.0051), washout filter (-0.468) and roll mode (-1.106). The ideal selection of eigenvectors for decoupling the modes suggested in [9] are

$$\underbrace{\begin{bmatrix} * \\ 0 \\ 1 \\ * \\ * \end{bmatrix} \begin{bmatrix} * \\ 0 \\ 1 \\ * \\ * \end{bmatrix}}_{\text{Dutch roll mode}} \quad \underbrace{\begin{bmatrix} * \\ 1 \\ 0 \\ * \\ * \end{bmatrix}}_{\text{roll mode}} \quad \underbrace{\begin{bmatrix} * \\ 1 \\ 0 \\ * \\ * \end{bmatrix}}_{\text{spiral mode}} \quad \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}}_{\text{washout filter}}$$

where $*$ denotes that the magnitude of the element is unimportant. To achieve the ideal eigenstructure suggested in [9] for the system (A, B_o), where B_o is the input control distribution matrix associated with the primary control surfaces (rudder and aileron in this case), the ideal baseline control law F is obtained as

$$F = \begin{bmatrix} -0.4086 & -0.0101 & 0.2824 & 1.1705 & -0.1476 \\ -0.3259 & 0.0340 & -21.2779 & 4.3306 & 4.7983 \end{bmatrix} \quad (44)$$

This controller is assumed to exist a-priori.

B. Fault tolerant yaw damper controller design

The baseline control law associated with (44), which is responsible for the nominal performance, considers only the primary control surfaces. For fault tolerance, the controller suggested in (43) will be employed. The proposed CA scheme ensures that during a primary control surface failure, the control signals can be redistributed to the available secondary control surfaces automatically without reconfiguring the controller to maintain the nominal performance. During fault/failure scenarios the closed-loop stability condition in (30) must be satisfied. From the particular choice of matrix F in (44), using equation (27), $\|\tilde{G}(s)\|_\infty = \gamma_2 = 0.0295$. During normal operation, the ailerons are the primary control surfaces for ϕ regulation, and the spoilers are the redundancy; whereas the rudder is the primary control surface for β, and differential engine thrust is the redundancy. From a numerical search, it can be verified that $\gamma_1 = 12.7667$. Hence simple calculations show that the stability condition of *Proposition 1* in (30) is satisfied since $\gamma_2\gamma_1 = 0.3768 < 1$. This shows the system is stable for all $0 < w_i \leq 1$ provided that suitable redundancy is available: i.e. $\det(B_{22}W_2B_{22}^T)^{-1} \neq 0$.

V. Fault Tolerant Simulation Results

A. FTLAB747 $V6.5/7.1/2006b$

This software runs under MATLAB/Simulink, and represents a 'real world' model of B747-100/200 aircraft. This high-fidelity nonlinear model contains 77 states, incorporating rigid body variables, actuators, sensors and aero-engine dynamics. It has been used as the basis for the GARTEUR AG16 benchmark [7].

B. Nonlinear Simulation Results

The simulations in this paper are all based on the nonlinear benchmark model of a large transport aircraft from [7]. In the simulations the discontinuity in the nonlinear control term in (37) is smoothed by using the sigmoidal approximation $\frac{\sigma}{\|\sigma\|+\delta}$ [8], where the value of the scalar is chosen as $\delta = 0.001$. The simulation objective here is to investigate the lateral dynamics response due to an initial $\beta(0) = 1^o$ disturbance.

In Figures 1 and 2, the closed-loop system response of the nonlinear benchmark together with the ideal response of the (A, B_o) system with only the baseline control law $u_o = Fx(t)$ with an initial $\beta(0) = 1^o$ perturbation is shown in the case of fault free primary actuators. The states show the decoupled response of the roll and yaw motions. Furthermore note that the secondary actuators are not active during the closed-loop response.

Figures 3 and 4, show the system states and actuators deflections when both the primary actuators become stuck at some offset positions. It can be seen in Figure 3 that even in this extreme failure case the nominal performance is still maintained. The spoilers and the engine thrusts (left and right) work together to counteract the effect of the primary actuator failures.

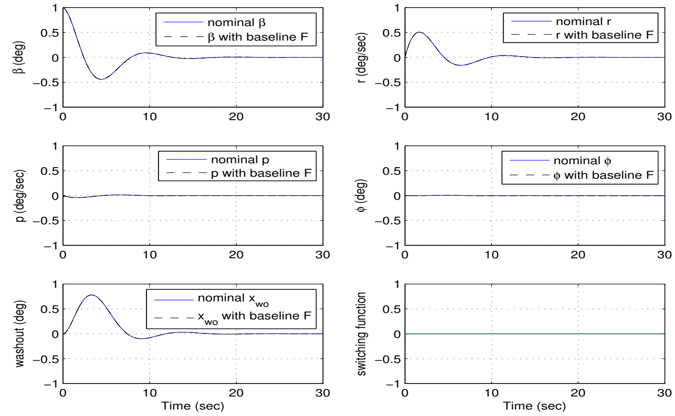

Fig. 1. Nominal scenario: System States Vs ideal states with baseline F

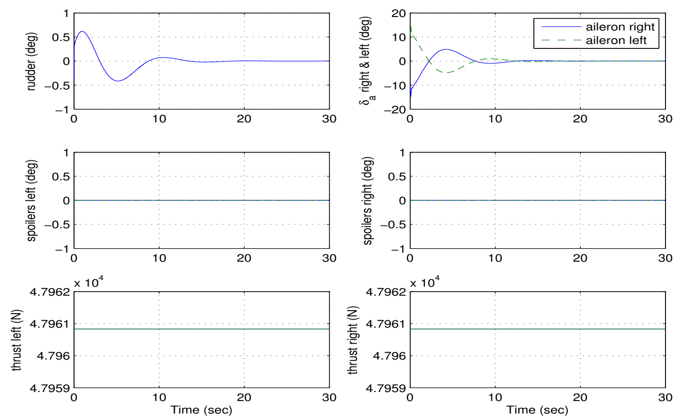

Fig. 2. Nominal scenario: actuators deflections

Figures 5 and 6, show another severe failure case, where the rudder becomes stuck (at some offset position) and the engines develop faults (Engines 1 and 2 on the left wing, and engine 3 on the right wing). To compensate for the failure, engine 4 works actively, together with contributions from the left spoilers (sp1-4 and 5) and the ailerons. Figure 5 shows that due to the availability of redundant actuators near nominal performance is still maintained.

VI. Conclusion

In this paper a novel fault tolerant control scheme to augment an existing baseline control law is proposed. The objective is to maintain closed loop performance in the face of faults and failures to the primary actuators. The integral sliding mode control allocation method which is proposed can be retro-fitted to an existing baseline controller designed for the primary actuators, and in the fault free case, the controller exactly reproduces the original baseline control action. An advantage of the proposed scheme is that the baseline controller structure does not need to be changed, and one controller can be used in nominal as well as in fault/failure scenarios. When the primary control surfaces become faulty, the proposed scheme redistributes the control signals to the functioning secondary actuators, to maintain the nominal performance, via the integral sliding mode scheme coupled with the control allocation.

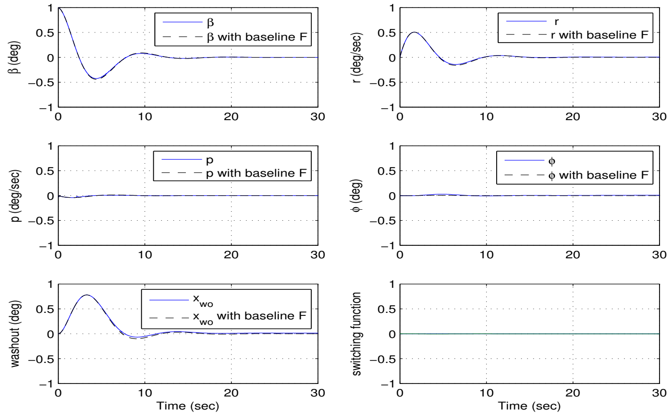

Fig. 3. Primary failure: System States Vs ideal states with baseline F

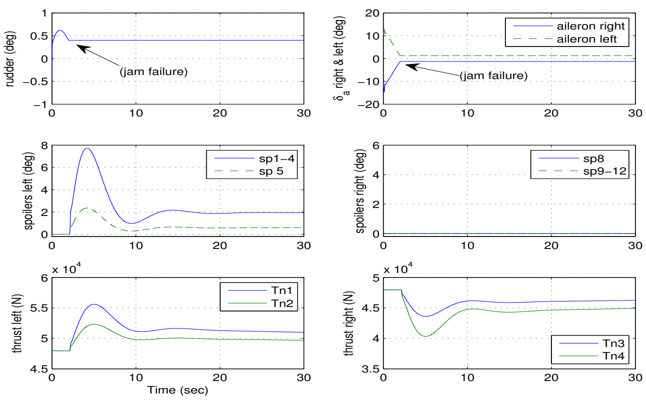

Fig. 4. Primary failure: actuators deflections

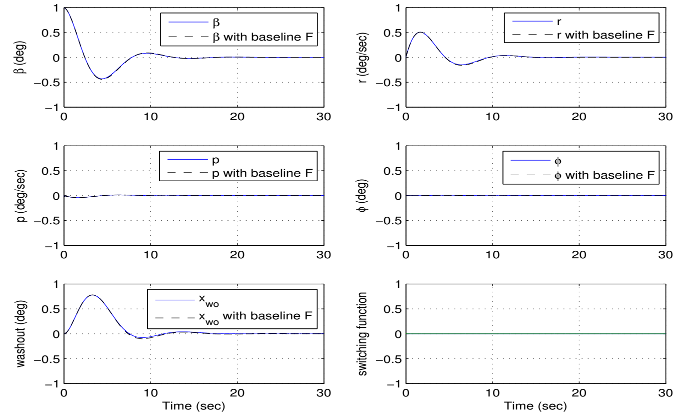

Fig. 5. Rudder and three engines failure: System States Vs ideal states with baseline F

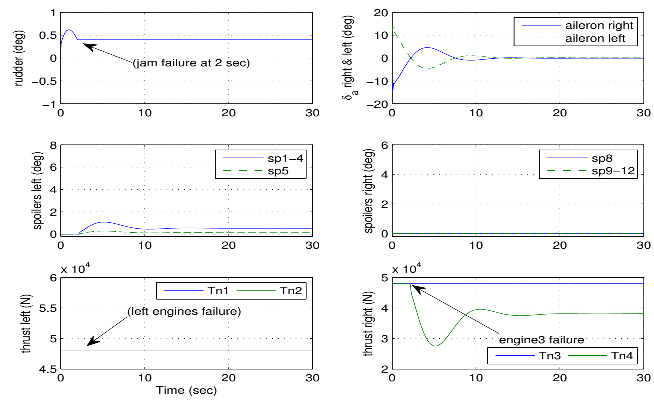

Fig. 6. Rudder and three engines failure: actuators deflections

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