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Trajectory tracking control of a small unmanned helicopter using MPC and Backstepping

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***Abstract*— A hierarchical inner-outer loop-based controller is proposed to solve the trajectory tracking problem of a small unmanned helicopter. The outer loop employs model predictive controller to track the reference trajectory, while the inner loop controller is designed by means of backstepping techniques that allow the stabilization of the attitude. The obtained control method takes advantage of both controllers and it is simple and easy to implement and tune in future real flight test. Finally, the computer simulations are conducted to illustrate the tracking performance of the proposed control method.**

I. INTRODUCTION

Since the last decades, there has been significant interest in using unmanned helicopters for applications in dealing with emergency situations, such as an earthquake, flood, an active volcano, or a nuclear disaster. However, helicopters are well known to be highly nonlinear systems with significant dynamic coupling and inherently unstable characteristic, therefore design of flying control systems has now become a very challenging area of research[1].

In recent years, a wide set of control methodologies, from classical PID control to LQR and H-Infinity control, have been reported [2,3,4]. These controllers have achieved quite modest performance: the flight models are limited to hover and low-speed straight flight, or they lose tracking accuracy considerably as the speed is increased and maneuvering flight is attempted. In [5], gain scheduling control is used to obtain acceptable performance for the full flight envelope. The main drawback of this approach is the severe trade-off between control performance and the number of the required trim points.

In order to overcome the drawbacks of the previous approaches, a variety of nonlinear flight control techniques have been developed. Such as feedback linearization [6], dynamic inversion [7,8], neural networks [9], etc. However, these controllers are very difficult to implement in real flight.

Model predictive control (MPC) is a form of optimization algorithm, which computes a future control sequence in a finite horizon in such a way that the prediction of the plant output is driven close to the reference [10]. This is accomplished by minimizing a cost function which is constructed based not only on actual state error but also on thefuture behavior of the system. To avoid model mismatch and external disturbance, only the first control in this sequence is applied to the plant. Compared with the conventional control which uses a pre-computed control law, model predictive control action is obtained on-line. Therefore, the controller will be more robust and could guide the system more smoothly.

Backstepping (BS) is mainly applicable to systems having a cascaded or triangular structure. The central idea of the approach is to recursively design controllers for subsystems in the structure and "step back" the feedback signals towards the contro input [11]. This differs from the conventional feedback linearization in that it can avoid cancellation of useful nonlinearities in pursuing the objectives of stabilization and tracking. In addition, by ultilizing the control Lyapunov function, it also has the flexibility in introducing appropriate dynamics to make the system behave in a desired manner. Especially in the flight control problem, in which case, unlike the traditional control philosophy, it could guarantee the stability and tracking performance in three channels simultaneously.

This paper presents a hierarchical inner-outer loop based flight controller for an unmanned helicopter, which takes advantage of the decoupling of the nonlinear translational and rotation dynamics of the rigid body. The outer loop makes use of model predictive controller to tracks the reference position, and the inner loop uses backstepping controller to track the attitude commands. This new method combines the advantages of both MPC and backstepping, particularly it is simple, easy to implement and to tune in future flight test.

The organization of this paper is as follows: In Section 2, an overview of the nonlinear unmanned helicopter model and the hierarchical control strategy are presented. The predictive controller for the translational movement is designed in Section 3. In Section 4, the attitude controller based on backstepping is developed. The simulation results are given in Section 5, and Section 6 presents conclusion of this paper.

II. HELICOPTER MODEL AND CONTROL STRATEGY

A. Helicopter dynamics model

The dynamics of small-scale helicopters can be adequately described by a set of the rigid body equations. The external forces f^b and torques τ^b are applied at the center of mass with respect to the body frame. m and $I \in R^{3\times 3}$ respectively denote the mass and the moment inertial matrix of the model helicopter. The position and velocity of the helicopter are given by $P^i = [x\ y\ z]^T$ and $V^i = [u\ v\ w]^T$ respectively in the inertial frame. The helicopter dynamics are written as

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follows[6]:

$$\dot{P}^i = V^i \tag{1}$$
$$m\dot{V}^i = R(\eta) f^b \tag{2}$$
$$\dot{\eta} = H(\eta)\omega^b \tag{3}$$
$$I\dot{\omega}^b = -\omega^b \times I\omega^b + \tau^b \tag{4}$$

where $\omega^b = [p\ q\ r]^T$ represents the helicopter angular rate vector with respect to its body axes. The Euler angle vector $\eta = [\varphi\ \theta\ \psi]^T$ is defined in the roll-pitch-yaw sequence. Therefore, the helicopter's rotation matrix from the body axes to the inertial axes is expressed by

$$R(\eta) = \begin{bmatrix} c_\theta c_\psi & s_\varphi s_\theta c_\psi - c_\varphi s_\psi & c_\varphi s_\theta c_\psi + s_\varphi s_\psi \\ c_\theta s_\psi & s_\varphi s_\theta s_\psi + c_\varphi c_\psi & c_\varphi s_\theta s_\psi - s_\varphi c_\psi \\ -s_\theta & s_\varphi c_\theta & c_\varphi c_\theta \end{bmatrix} \tag{5}$$

The transformation matrix $H(\eta)$ is shown as follows:

$$H(\eta) = \begin{bmatrix} 1 & s_\varphi t_\theta & c_\varphi t_\theta \\ 0 & c_\varphi & -s_\varphi \\ 0 & s_\varphi / c_\theta & c_\varphi / c_\theta \end{bmatrix} \tag{6}$$

where $c_{(\cdot)}$, $s_{(\cdot)}$, $t_{(\cdot)}$ are abbreviations for $\cos(\cdot)$, $\sin(\cdot)$ and $\tan(\cdot)$.

Following the modeling approach of [6], [12], [13], there are four control inputs associated with helicopter. The control inputs are defined as $U = [T_M\ T_T\ a\ b]^T$. The components T_M and T_T are the magnitudes of the generated thrusts by the main and tail rotor, respectively. a and b are longitudinal and lateral tilt angles of the tip path plane of the main rotor with respect to the shaft. Denote the thrust vector of the main and tail rotor by $\vec{T}_M$ and $\vec{T}_T$ respectively, then

$$\vec{T}_M = \begin{bmatrix} X_M \\ Y_M \\ Z_M \end{bmatrix} = \begin{bmatrix} -s_a c_b \\ c_a s_b \\ -c_a c_b \end{bmatrix} T_M \approx \begin{bmatrix} -a \\ b \\ -1 \end{bmatrix} T_M \tag{7}$$

The above equation is simplified by assuming small angle approximation since the tilt angles a and b are small.

$$\vec{T}_T = [0\ Y_T\ 0]^T = [0\ -1\ 0]^T T_T \tag{8}$$

Therefore, the complete force vector is

$$f^b = \begin{bmatrix} X_M \\ Y_M + Y_T \\ Z_M \end{bmatrix} + R^T \begin{bmatrix} 0 \\ 0 \\ mg \end{bmatrix} \tag{9}$$

We neglect the contribution of T_M along the x direction and we assume that the contribution of T_T and the contribution of T_M along the y direction is matched, thus obtained the following model for f^b

$$f^b = [0\ 0\ -T_M]^T + R^T [0\ 0\ mg]^T \tag{10}$$

Denote $\vec{h}_M = [x_m\ y_m\ z_m]^T$ and $\vec{h}_T = [x_t\ y_t\ z_t]^T$ as the position vectors of the main and tail rotor shafts respectively, with respect to the body frame. Then the torques generated by $\vec{T}_M$ and $\vec{T}_T$ are $\tau_M^b = \vec{h}_M \times \vec{T}_M$ and $\tau_T^b = \vec{h}_T \times \vec{T}_T$, respectively, and the total torques are

$$\tau^b = \tau_M + \tau_M^b + \tau_T^b \tag{11}$$

where

$$\begin{aligned} \tau_M &= [R_M\ M_M\ N_M]^T \\ R_M &= c_m b - Q_M s_a c_b \\ M_M &= c_m a + Q_M s_b c_a \\ N_M &= -Q_M c_a c_b \\ Q_M &= C^M T_M^{1.5} + D^M \end{aligned} \tag{12}$$

In the above equations, c_m is a positive constant associated with the main rotor's stiffness and Q_M is the main rotor's anti-torque. The positive constants C^M and D^M are associated with the generation of the reaction torque Q_M. The equation (11) can be rewritten as

$$\tau^b = A(T_M) v_c + B(T_M) \tag{13}$$

where $v_c = (a\ b\ T_T)^T$, $A(T_M) \in R^{3\times 3}$, $B(T_M) \in R^{3\times 1}$.

B. Control Strategy

This paper presents a hierarchical inner-outer loop based flight controller for a unmanned helicopter. The outer loop employs model predictive controller to track the reference position, and the inner loop uses backstepping controller to track the attitude commands. The block diagram of the overall controller is shown in Fig.1.

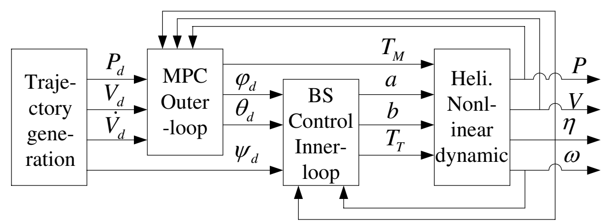

Fig. 1. Structure of the inner-outer loop controller

III. TRAJECTORY TRACKING CONTROLLER DESIGN

From the equations (1), (2), (5) and (10), we could get

$$\begin{cases} \ddot{x} = \frac{1}{m}(c_\psi s_\theta c_\varphi + s_\psi s_\varphi) T_M \\ \ddot{y} = \frac{1}{m}(s_\psi s_\theta c_\varphi - c_\psi s_\varphi) T_M \\ \ddot{z} = g + \frac{1}{m}(c_\theta c_\varphi) T_M \end{cases} \tag{14}$$

We transform the above equations into the following state space form

$$\dot{\xi}(t) = f(\xi(t), F_\xi(t)) = \begin{bmatrix} u \\ F_x \frac{T_M}{m} \\ v \\ F_y \frac{T_M}{m} \\ w \\ g + (c_\theta c_\varphi) \frac{T_M}{m} \end{bmatrix} \tag{15}$$

where,

$$\begin{aligned} F_x(t) &\triangleq c_\psi s_\theta c_\varphi + s_\psi s_\varphi \\ F_y(t) &\triangleq s_\psi s_\theta c_\varphi - c_\psi s_\varphi \end{aligned} \tag{16}$$

$\xi(t) = [x(t)\ \ u(t)\ \ y(t)\ \ v(t)\ \ z(t)\ \ w(t)]^T$ is the state vector of the system, $F_\xi(t)$ is the control input. The objective of this paper is to guarantee the helicopter follows a previously defined reference trajectory with minimum error. However, due to the fact that the destination coordinates vary in time, we assume a reference virtual helicopter having the same mathematical model is placed on the track, i.e.,

$$\dot{\xi}_r(t) = f(\xi_r(t), F_{\xi r}(t)) \tag{17}$$

where $\xi_r(t) = [x_r(t) \quad u_r(t) \quad y_r(t) \quad v_r(t) \quad z_r(t) \quad w_r(t)]^T$ and $F_{\xi r}(t) = [F_{xr} \quad F_{yr} \quad T_{M\,r}]^T$ are the reference states and control inputs, respectively. Under the assumption that the helicopter height has been stabilized, we could get the reference control inputs for the translation movements:

$$T_{M\,r} = m \cdot (\ddot{z}_r - g), F_{xr} = \frac{\ddot{x}_r \cdot m}{T_{M\,r}}, F_{yr} = \frac{\ddot{y}_r \cdot m}{T_{M\,r}}$$

By subtracting the reference model (17) from (15), we get the translation error model

$$\dot{\tilde{\xi}}(t) = A(t) \cdot \tilde{\xi}(t) + B(t) \cdot \tilde{F}_\xi(t) \tag{18}$$

Where $\tilde{\xi}(t) = \xi(t) - \xi_r(t)$ is state error, and $\tilde{F}_\xi(t) = F_\xi(t) - F_{\xi r}(t)$ is control error.

Using Euler's method, a time-variant discrete linear model is obtained

$$\tilde{\xi}(k+1) = \bar{A} \cdot \tilde{\xi}(k) + \bar{B}(k) \cdot \tilde{F}_\xi(k) \tag{19}$$

Following [12], the error model (19) can be split up into two subsystems: height error and x and y motions error. Matrices for each subsystem are the following:

$$\bar{A}_z = \begin{bmatrix} 1 & \Delta t \\ 0 & 1 \end{bmatrix} \tag{20}$$

$$\bar{B}_z = \begin{bmatrix} 0 \\ \frac{\Delta t}{m}\cos(\theta(k))\cos(\varphi(k)) \end{bmatrix} \tag{21}$$

$$\bar{A}_{xy} = \begin{bmatrix} 1 & \Delta t & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & \Delta t \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{22}$$

$$\bar{B}_{xy} = \begin{bmatrix} 0 & 0 \\ \frac{\Delta t}{m} T_M(k) & 0 \\ 0 & 0 \\ 0 & \frac{\Delta t}{m} T_M(k) \end{bmatrix} \tag{23}$$

where Δt is the sampling time. Following the above model, we consider the height control problem first. Define a cost function as

$$\begin{aligned} J_z &= [\hat{\tilde{\xi}}_z - \hat{\tilde{\xi}}_{zr}]' Q_z [\hat{\tilde{\xi}}_z - \hat{\tilde{\xi}}_{zr}] \\ &+ [\hat{\tilde{F}}_{\xi z} - \hat{\tilde{F}}_{\xi zr}]' R_z [\hat{\tilde{F}}_{\xi z} - \hat{\tilde{F}}_{\xi zr}] \end{aligned} \tag{24}$$

where Q_z and R_z are positive weighting matrices, the predictions of plant output $\hat{\tilde{\xi}}_z$ are computed using (19), (20), (21)[14]:

$$\hat{\tilde{\xi}}_z = P_z(k\,|k) \cdot \tilde{\xi}_z(k\,|k) + H_z(k\,|k) \cdot \hat{\tilde{F}}_{\xi z} \tag{25}$$

where

$$\hat{\tilde{\xi}}_z \triangleq \begin{bmatrix} \tilde{\xi}_z(k+1\,|k) \\ \vdots \\ \tilde{\xi}_z(k+N_{2z}\,|k) \end{bmatrix}$$

$$\hat{\tilde{F}}_{\xi z} \triangleq \begin{bmatrix} \tilde{F}_{\xi z}(k\,|k) \\ \vdots \\ \tilde{F}_{\xi z}(k+N_{Fz}-1\,|k) \end{bmatrix}$$

$$\hat{\tilde{\xi}}_{zr} \triangleq \begin{bmatrix} \tilde{\xi}_{zr}(k+1\,|k) - \tilde{\xi}_{zr}(k\,|k) \\ \vdots \\ \tilde{\xi}_{zr}(k+N_{2z}\,|k) - \tilde{\xi}_{zr}(k\,|k) \end{bmatrix}$$

$$\hat{\tilde{F}}_{\xi zr} \triangleq \begin{bmatrix} \tilde{F}_{\xi zr}(k\,|k) - \tilde{F}_{\xi zr}(k-1\,|k) \\ \vdots \\ \tilde{F}_{\xi zr}(k+N_{Fz}-1\,|k) - \tilde{F}_{\xi zr}(k-1\,|k) \end{bmatrix}$$

$$P_z(k\,|k) \triangleq \begin{bmatrix} \bar{A}_z(k\,|k) \\ \bar{A}_z(k\,|k)\bar{A}_z(k+1\,|k) \\ \vdots \\ \alpha(k,0,2) \\ \alpha(k,0,1) \end{bmatrix}$$

$$H_z(k\,|k) \triangleq \begin{bmatrix} \bar{B}_z(k\,|k) & 0 & \cdots & 0 \\ \bar{A}_z(k+1\,|k)\bar{B}_z(k\,|k) & \bar{B}_z(k+1\,|k) & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ \alpha(k,1,2)\bar{B}_z(k\,|k) & \alpha(k,2,2)\bar{B}_z(k+1\,|k) & \cdots & 0 \\ \alpha(k,1,1)\bar{B}_z(k\,|k) & \alpha(k,2,1)\bar{B}_z(k+1\,|k) & \cdots & \bar{B}_z(k+N_{Fz}-1\,|k) \end{bmatrix}$$

$\alpha(k,j,l) \triangleq \prod_{i=j}^{N_{2z}-l} \bar{A}_z(k+i\,|k)$, N_{2z} is the cost horizon and N_{Fz} is the control horizon.

Let $\partial J_z \big/ \partial \hat{\tilde{F}}_{\xi z} = 0$, we get the optimal control law in this horizon as

$$\begin{aligned} \hat{\tilde{F}}_{\xi z} &= [H_z' Q_z H_z + R_z]^{-1} \cdot [H_z' Q_z (\hat{\tilde{\xi}}_{zr} \\ &- P_z \tilde{\xi}_z(k)) + R_z \hat{\tilde{F}}_{\xi zr}] \end{aligned} \tag{26}$$

At each sampling time k, only $\hat{\tilde{F}}_{\xi z}(k\,|k)$ is needed. Then we obtain the height control signal as

$$T_M(k) = \hat{\tilde{F}}_{\xi z}(k\,|k) + T_{M\,r}(k) \tag{27}$$

The x and y motion control signal is computed using the same procedure as the height control, we get

$$\begin{aligned} \hat{\tilde{F}}_{\xi xy} &= [H_{xy}' \cdot Q_{xy} \cdot H_{xy} + R_{xy}]^{-1} \cdot [H_{xy}' \\ &\cdot Q_{xy} \cdot (\hat{\tilde{\xi}}_{xyr} - P_{xy} \cdot \tilde{\xi}_{xy}(k)) + R_{xy} \cdot \hat{\tilde{F}}_{\xi xyr}] \end{aligned} \tag{28}$$

where $\hat{\tilde{F}}_{\xi xy}(k|k) = [\tilde{F}_{\xi x}(k) \;\; \tilde{F}_{\xi y}(k)]^T$, then

$$\begin{bmatrix} F_x(k) \\ F_y(k) \end{bmatrix} = \begin{bmatrix} \tilde{F}_{\xi x}(k) \\ \tilde{F}_{\xi y}(k) \end{bmatrix} + \begin{bmatrix} F_{xr}(k) \\ F_{yr}(k) \end{bmatrix} \tag{29}$$

Using equations (16) and (29), the reference roll and pitch angles are derived as following:

$$\begin{cases} \varphi_d = \sin^{-1}(\sin(\psi_d)F_x - \cos(\psi_d)F_y) \\ \theta_d = \frac{F_x - \sin(\psi_d)\sin(\varphi_d)}{\cos(\psi_d)\cos(\varphi_d)} \end{cases}$$

The reference yaw angle ψ_d is set as desired. These reference angles are needed by the attitude loop.

IV. ATTITUDE CONTROLLER DESIGN

The objective in this section is to design a backstepping control law $v_c = (a \;\; b \;\; T_T)^T$ for the rotational dynamics (3) and (4) to track the desired attitude angles $\eta_d = [\varphi_d \;\; \theta_d \;\; \psi_d]^T$. First, define the following control Lyapunov function:

$$W_1 = \tfrac{1}{2}(\eta - \eta_d)^T K_\eta (\eta - \eta_d)$$

where K_η is a positive definite matrix. The reference attitude angles $\eta_d = [\varphi_d \; \theta_d \; \psi_d]^T$ is generated by the outer loop. This gives

$$\dot{W}_1 = (H\omega^b)^T K_\eta \tilde{\eta}$$

If $\omega^b = \omega_d^b \triangleq -\alpha H^{-1}\tilde{\eta}$, where α is a positive scalar, $\tilde{\eta} = \eta - \eta_d$, then $\dot{W}_1 \leq 0$. For notational simplicity, we denote $H^{-1} = \gamma$.

Next, define an error $z_1 \triangleq \omega^b - \omega_d^b$ and have another control Lyapunov function as follows:

$$W_2 = \tfrac{1}{2}(\eta - \eta_d)^T K_\eta (\eta - \eta_d) + \tfrac{1}{2} z_1^T I \, z_1$$

then,

$$\begin{aligned} \dot{W}_2 = {} & z_1^T (H^T K_\eta \tilde{\eta} - \omega^b \times I\omega^b + \tau^b \\ & + \alpha I \gamma \dot{\eta} + \alpha I \dot{\gamma} \tilde{\eta}) + (H\omega_d^b)^T K_\eta \tilde{\eta} \end{aligned}$$

If we make the torque τ^b as

$$\tau^b = \omega^b \times I\omega^b - \alpha I \gamma \dot{\eta} - \alpha I \dot{\gamma} \tilde{\eta} - H^T K_\eta \tilde{\eta}$$

then $\dot{W}_2 \leq 0$. Substituting the above equation into (13), we get the attitude control law as

$$v_c = A^{-1}(T_M)(\tau^b - B(T_M))$$

V. SIMULATION RESULTS

We define the following ascendant helix curve as the reference trajectory:

$$P_d = \begin{pmatrix} 2\cos(t/3) \\ 2\sin(t/3) \\ -1 - t \end{pmatrix}, \psi_d = 0$$

The initial conditions of the helicopter are $(x, y, z) = (1.5, 0, -1)$ m, $(\varphi, \theta, \psi) = (0, 0, 0.5)$ rad, the helicopter parameters follows [15] as:

$$\begin{cases} m = 8.2kg, g = 9.81m/s^2, x_t = -0.91m \\ I = diag(0.18, 0.34, 0.28)kg \cdot m^2, y_t = 0 \\ z_t = -0.08m, z_m = -0.235m, x_m = y_m = 0 \\ c_m = 52N \cdot m/rad, D^M = 0.6304N \cdot m \\ C^M = 0.004452m\big/\sqrt{N} \end{cases}$$

The model predictive controller parameters are adjusted as:

$$\begin{cases} N_{2z} = 5, \quad N_{Fz} = 3 \\ Q_z = diag(20, 15, \cdots, 20, 15) \in R^{10\times10} \\ R_z = diag(0.05, \cdots, 0.05) \in R^{3\times3} \\ N_{2xy} = 10, \quad N_{Fxy} = 6 \\ Q_{xy} = diag(5, 15, \cdots, 5, 15) \in R^{40\times40} \\ R_{xy} = diag(85, 85, \cdots, 85, 85) \in R^{12\times12} \end{cases}$$

In order to evaluate the robustness feature of the controller, we assume the persistent wind gusts of magnitudes $1m/s, 1m/s, 0.5m/s$ are added to the x, y, z directions, respectively.

The simulation results of the reference tracking under the perturbations illustrate in Figs 2-6. Figs 2-5 show the helicopter can follow the reference trajectory under the proposed control method. Figs 3 and 5 show that at the beginning of the disturbances are added to the helicopter, there are a considerable large deviations in the tracking trajectory, but they are converged to the desired values quickly. Fig 6 shows the control input signals, they have not exceeded the constraints of the helicopter.

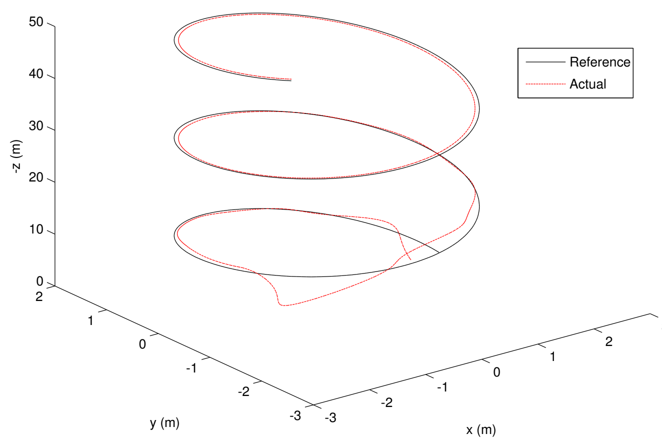

Fig. 2. 3D position trajectory

VI. CONCLUSIONS

This paper presents a hierarchical inner-outer loop based flight controller for an unmanned helicopter, which takes advantage of the decoupling of the nonlinear translational and rotation dynamics of the rigid body. The outer loop employs model predictive controller to track the reference position, and the inner loop uses backstepping controller to track the attitude commands. This is the main idea of the

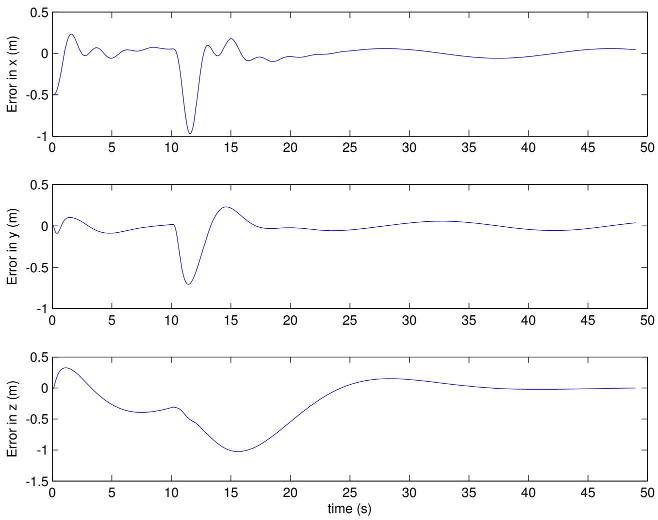

Fig. 3. Position errors in x,y,z directions

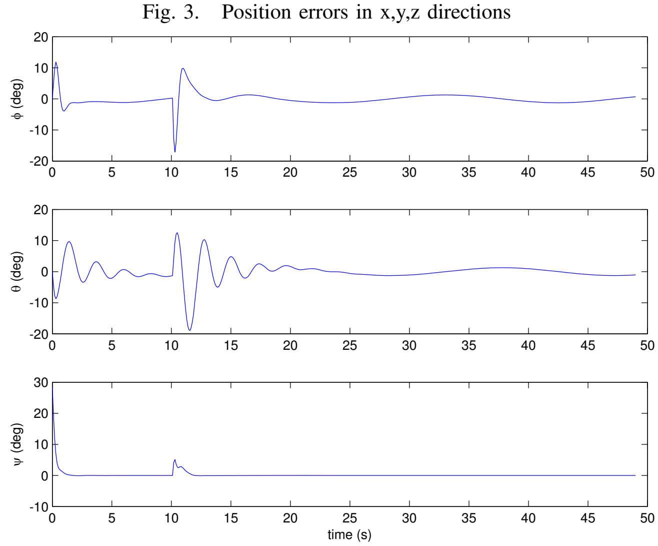

Fig. 4. Actual attitude angles

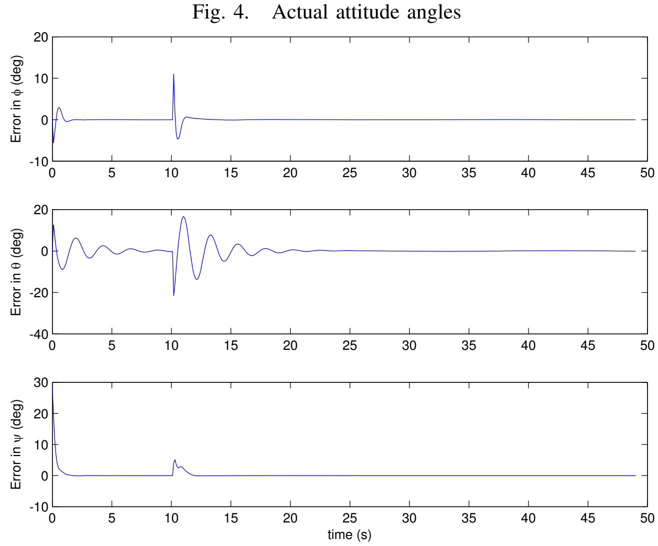

Fig. 5. Attitude angles error between actual and reference

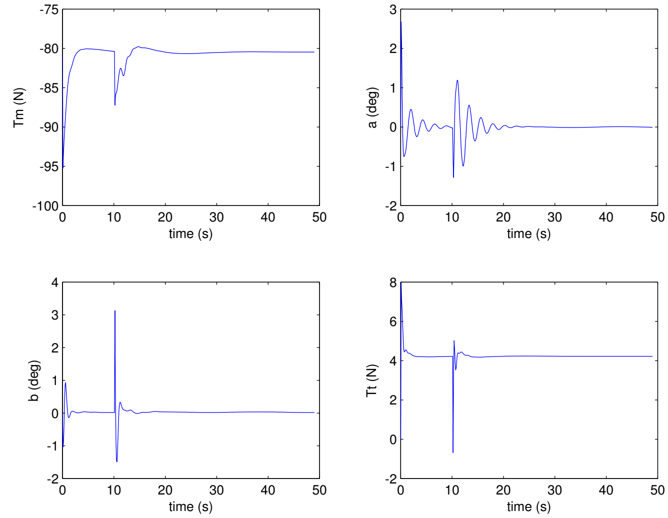

Fig. 6. Control inputs

design. The new method combines the advantages of both MPC and backstepping. Particularly the hierarchical structure is simple, easy to implement and to tune in future real flight test.

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