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Stable Grasping Control Method of Dual-fingered Robot Hands for Force Angle Optimization and Position Regulation

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Abstract— **This paper proposes a method for controlling an object with arbitrarily smooth surfaces in a horizontal plane by a dual-fingered robots. The proposed control method achieves both (a) the stable grasping with the optimal force angles, in order to lower the probability of the object slipping out of the finger-tips, and (b) the position regulation without visual sensing. The shape of an object is not limited as long as the contact point is positioned in the vicinity of the smooth curvatures since the controller is allowed to use the tactile sensor. We analyze the dynamic stability of the proposed control method via Lyapunov stability theory. Finally, simulation results are presented to validate the proposed control method.**

I. Introduction

Dexterous manipulation by multi-fingered robot hands is one of the challenging problems in robotics. In order to mimic a human hand, various research has been conducted. In the early research on hand robots, the multi-joint-fingered models which are similar to the human hand in appearance were primarily presented [1]-[3]. By increasing the interest in the dexterous manipulation, the researches on the various sensors and control methods have been required. Furthermore, the control strategies have been subdivided depending on the grasp taxonomies such as power grasp [4], pinch [5], pen spinning [6], and rope knotting [7], etc.

Among them, pinching motion is one of the challenging areas because it requires tiny contact points between the finger-tips and an object with the rolling contact constraints. There have been many research efforts in multi-fingered robot hands based on rolling contact constraints. Maekawa *et al.* [8]-[10] proposed a grasping-force compensation algorithm based on rolling manipulation using tactile sensing. However this control method did not guarantee the dynamic stability of the system. Doulgeri *et al.* [11] designed a force position controller using force/tactile sensors. This control method is adapted to cope with the kinematic uncertainties in a dynamic sense. Ozawa *et al.* [12], [13] proposed a control method that can realize stable grasp without force/tactile sensors. However, it requires a priori information about the object shape.

This paper deals with a problem of the blind grasp and the optimal force angle control with dynamic stability converging to the form of force/torque balance for an arbitrarily shape object with smooth surfaces. It is assumed that dual-fingered robot hands pinch an object with hemispherical finger-tips in a horizontal plane. The proposed control method improves the grasping stability by optimizing the force angle and provides the position regulation in x- and y-coordinates, concurrently. It is assumed that all the kinematic parameters of the fingers and the measured data of joint angles, joint angular velocities, and contact angles are known but any kinematic and dynamic data such as a shape and a center of mass, are not given. In Section II, we present the kinematic constraints and dynamics of dual fingers with an arbitrarily shaped object. Then, we define the stable grasping conditions and the optimal force angle. In Section III, a control method for the optimal force angle and regulation is proposed and its stability is proved via Lyapunov direct method. In Section IV, computer simulation results are shown to verify the effectiveness of the proposed control method. Finally, Section V gives some conclusions.

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II. Preliminary and Problem Formulation

A. Kinematics of a Dual Fingered Robot

For the sake of generality of a grasped object, we only assume that the object is surrounded with smooth curvatures in the vicinity of the contact points without any knowledge of an object. It is the marginal assumption for applying the rolling contact constraints between the finger-tips and the object surface. A model of a dual-fingered robot grasping an arbitrarily shaped object with smooth curvatures is shown in Fig. 1. There exists a cross point Q_1 of two lines which are extended from the tangential components of the grasping points if those are not parallel. Then, it is possible to find a line $\overline{Q_1Q_2}$ which divides an angle $\angle P_1Q_1P_2$ into two equal angles ϕ_0, where P_1 and P_2 are the contact points. Since it is supposed that the finger-tips are not deformable and the object is rigid, we obtain ϕ_0 from the geometrical relation as follows:

$$\phi_0 = \pi - \frac{\psi_1 + \psi_2}{2}, \tag{1}$$

where

$$\psi_1 = \sum_{j=1}^{n_1} q_{1j} + \phi_{t1}, \quad \psi_2 = \sum_{j=1}^{n_2} q_{2j} + \phi_{t2}.$$

In these expressions, n_1 and n_2 are the degrees of the fingers, q_{1i} and q_{2j} are the finger joint angles and ϕ_{t1} and ϕ_{t2} are the contact angles of each finger. If the tangential components of

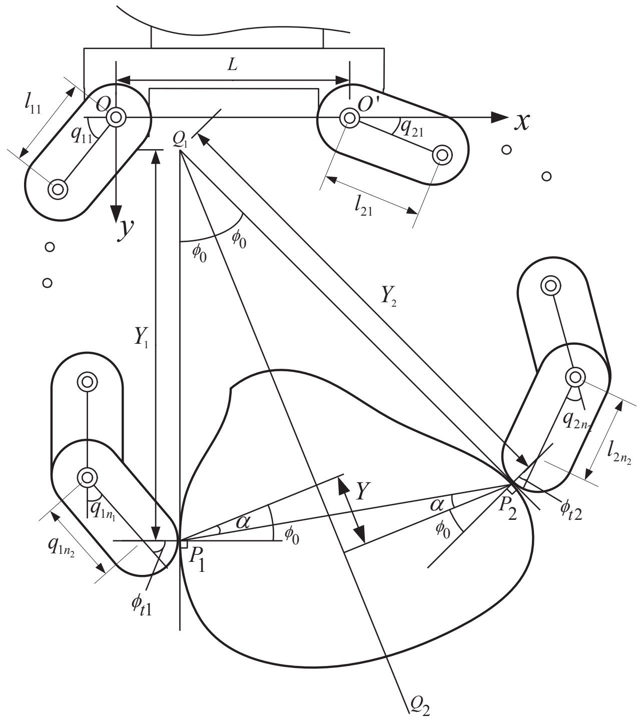

Fig. 1. Dual-fingered robot grasping an arbitrarily shaped object.

the grasping points are parallel, ϕ_0 must be $\frac{\pi}{2}$. From the non-slip condition, it is easy to derive the following geometrical relations:

$$
\begin{aligned}
(Y_1 + Y_2)\sin\phi_0 = {} & (x_{02} - x_{01})\cos\phi + (y_{01} - y_{02})\sin\phi \\
& - (r_1 + r_2)\cos\phi_0, \qquad (2)\\
(Y_1 - Y_2)\cos\phi_0 = {} & -(x_{02} - x_{01})\sin\phi + (y_{01} - y_{02})\cos\phi \\
& + (r_1 - r_2)\sin\phi_0, \qquad (3)\\
(Y_1 + Y_2)\sin\phi_0 = {} & l\cos\alpha, \qquad (4)\\
(Y_2 - Y_1)\cos\phi_0 = {} & l\sin\alpha, \qquad (5)
\end{aligned}
$$

where

$$Y_1 = c_{10} + r_1\left(-\phi + \phi_0 + \sum_{i=1}^{n_1} q_{1i}\right), \tag{6}$$

$$Y_2 = c_{20} + r_2\left(\phi + \phi_0 + \sum_{i=1}^{n_2} q_{2i}\right), \tag{7}$$

$$\phi = \frac{\psi_1 - \psi_2}{2}. \tag{8}$$

In these expressions, x_{0i} and y_{0i}, for $i = 1,\ 2$, are the finger-tip positions, r_1 and r_2 are the radii of finger-tips, and c_{10} and c_{20} are some constants. α denotes the angle between $\overline{P_1P_2}$ and a perpendicular line from P_1(or P_2) to $\overline{Q_1Q_2}$ and l denotes the length of $\overline{P_1P_2}$. When $\phi_0 \neq \frac{\pi}{2}$, Y_1 and Y_2 are lengths of $\overline{Q_1P_1}$ and $\overline{Q_2P_2}$, respectively.

Based on the constraints, the kinematic constraint between the angular velocities of the finger joints and the velocities of the center of mass of an object is derived as follows[15]:

$$J_o\dot{z}_o = J_f\dot{q} \tag{9}$$

where $J_o \in \mathbb{R}^{4\times 3}$ and $J_f \in \mathbb{R}^{4\times(n_1+n_2)}$ denote the Jacobian matrices of an object and fingers, respectively. $q = [q_{11}, .., q_{1n_1}, q_{21}, .., q_{2n_2}]^T \in \mathbb{R}^{n_1+n_2}$ and $z_o = [x_o, y_o, \psi_o]^T \in \mathbb{R}^3$ indicate the joint angles of fingers and the center of mass of an object, respectively.

Remark 1: J_o is not observable because the center of mass of an object is unknown. That is, we cannot use J_o to design the controller.

Remark 2: J_f can be obtained from the rolling contact constraints [16]. Using the finger kinematics, the joint angles q and the contact angles ψ_1 and ψ_2, J_f is only expressed as follows:

$$J_f = \begin{bmatrix} J_{11}^T & 0_{n_1\times 1} & J_{13}^T & 0_{n_1\times 1} \\ 0_{n_2\times 1} & J_{22}^T & 0_{n_2\times 1} & J_{24}^T \end{bmatrix}^T, \tag{10}$$

where

$$J_{11}^T = J_{01}^T\begin{bmatrix}\cos\psi_1 \\ -\sin\psi_1\end{bmatrix}, \quad J_{13}^T = -J_{01}^T\begin{bmatrix}\sin\psi_1 \\ \cos\psi_1\end{bmatrix} - r_1\mathbf{e}_1,$$

$$J_{22}^T = J_{02}^T\begin{bmatrix}-\cos\psi_2 \\ \sin\psi_2\end{bmatrix}, \quad J_{24}^T = -J_{02}^T\begin{bmatrix}\sin\psi_2 \\ \cos\psi_2\end{bmatrix} - r_2\mathbf{e}_2,$$

$$
\begin{aligned}
J_{0i}^T &= \left(\left(\frac{\partial x_{0i}}{\partial q_i}\right)^T, \left(\frac{\partial y_{0i}}{\partial q_i}\right)^T\right) \in \mathbb{R}^{n_i\times 2},\\
\psi_i &= \sum_{j=1}^{n_i} q_{ij} + \phi_{ti},\\
x_{0i} &= -\sum_{j=1}^{n_i} l_{ij}\cos\left(\sum_{k=1}^{j} q_{ik}\right),\\
y_{0i} &= \sum_{j=1}^{n_i} l_{ij}\sin\left(\sum_{k=1}^{j} q_{ik}\right) \qquad \text{for } i = 1, 2.
\end{aligned}
$$

In these expressions, $0_{n_1\times 1}$ and $0_{n_2\times 1}$ are $n_1 \times 1$ and $n_2 \times 1$ zero matrices, respectively, L is the length between O and O', $\mathbf{e}_1 = (1, 1, ...1)^T \in \mathbb{R}^{n_1}$, $\mathbf{e}_2 = (1, 1, ...1)^T \in \mathbb{R}^{n_2}$, l_{1j} and l_{2j} are the lengths of the jth link of the fingers 1 and 2, respectively.

B. Dynamics of a Dual Fingered Robot

The dynamics of the dual fingers and an object can be expressed as [14]:

$$H_f(q)\ddot{q} + C_f(q, \dot{q})\dot{q} + J_f^T(q)F_f = u, \tag{11}$$

$$H_o\ddot{z}_o = J_o^T F_f, \tag{12}$$

where $H_f(q) = \text{diag}\,[H_{f1}(q_1), H_{f2}(q_2)]$; $H_{f1}(q_1) \in \mathbb{R}^{n_1\times n_1}$ and $H_{f2}(q_2) \in \mathbb{R}^{n_2\times n_2}$ are symmetric positive definite inertia matrices of the left and right fingers. $H_o = \text{diag}[M_o, M_o, I_o]$; M_0 and I_0 denote the mass and the inertia of an object, respectively. $C_f(q, \dot{q}) = \text{diag}[C_{f1}(q_1, \dot{q}_1), C_{f2}(q_2, \dot{q}_2)]$; $C_{f1}(q_1, \dot{q}_1) \in \mathbb{R}^{n_1\times n_1}$ and $C_{f2}(q_2, \dot{q}_2) \in \mathbb{R}^{n_2\times n_2}$ are the Coriolis-centripetal matrices of the left and right fingers, and $F_f = [f_1, f_2, \lambda_1, \lambda_2]^T \in \mathbb{R}^{4\times 1}$ is the vector of grasping force with the normal forces f_1 and f_2 and the tangential forces λ_1 and λ_2. The grasping force F_f can be expressed as [17]:

$$F_f = \left(J_o^T\right)^+ F_f + \left(I - \left(J_o^T\right)^+ J_o^T\right) F_f, \tag{13}$$

where $\left(J_o^T\right)^+$ is the generalized inverse of J_o^T, which is identical to $(J_0 J_0^T)^{-1} J_0$. The first term denotes the manipulation force that causes the motion of an object which is equivalent to $H_o \ddot{z}_o$, and the second term represents the internal force which does not affect on the movement of an object. Substituting (13) into (11) and (12) yields

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} + \left\{\left(I - J_o J_o^+\right) J_f\right\} F_f = u, \tag{14}$$

where

$$H(q) = H_f(q) + \left(J_o^+ J_f\right)^T H_o \left(J_o^+ J_f\right),$$
$$C(q,\dot{q}) = C_f(q,\dot{q}) + J_f^T \left(J_o^+\right)^T H_o \left(J_o^+ \dot{J}_f - J_o^+ \dot{J}_o J_o^+ J_f\right).$$

Note that $H(q)$ and $C(q,\dot{q})$ are the symmetric and positive definite inertia matrix and the Coriolis-centripetal matrix of the overall system, respectively. From (9), Jacobian matrix in the third term of (14) is constrained by the following equation:

$$\left(I - J_o J_o^+\right) J_f \dot{q} = \left(I - J_o J_o^+\right) J_o \dot{z}_o = 0, \tag{15}$$

which means that the internal force term $\{(I - J_o J_o^+) J_f\}^T F_f$ does not affect on the energy variation of the overall system. That is, even if the internal force term is omitted in (14), the state q in the overall system is not affected as long as it can be derived via Lagrangian without constraints. Hence, we will use the following overall dynamic equation to prove the stability of the control method:

$$H(q)\ddot{q} + C(q,\dot{q})\dot{q} = u. \tag{16}$$

Property 1: The matrix $C(q,\dot{q})$ and the time derivative $\dot{H}(q)$ of the inertia matrix satisfy [18]:

1) $\dot{q}^T \left[\frac{1}{2}\dot{H}(q) - C(q,\dot{q})\right] \dot{q} = 0 \quad \forall q, \dot{q} \in \mathbb{R}^{n_1+n_2}$.
2) $\dot{H}(q) = C(q,\dot{q}) + C(q,\dot{q})^T \quad \forall q, \dot{q} \in \mathbb{R}^{n_1+n_2}$.

C. Stable Grasp with an Optimal Angle

The finger-tip force vector F_f can be decomposed into the manipulation force vector and the internal force vector as (13). The internal force vector must satisfy the following definition:

Definition 1: $F_f = [f_1,\ f_2,\ \lambda_1,\ \lambda_2]^T$ is called the stable internal grasping force for a dual-fingered robot if the following two conditions are satisfied [17]:

Condition 1: The grasping force must be equilibrium with the positive normal forces.

$$J_o^T F_f = 0 \ \text{ with } f_1,\ f_2 > 0.$$

Condition 2: The angles of the grasping forces must exist in the friction cone.

$$\frac{f_i}{\sqrt{f_i^2 + \lambda_i^2}} > \frac{1}{\sqrt{1+\mu_i^2}} \quad \text{for} \quad i = 1, 2.$$

where μ_i is the maximum static friction coefficient at the contact point.

the above conditions are necessary to satisfy the force/torque balance and prevent the slippage of an object between the finger-tips and the object surfaces, respectively. Unless slippage happens, Condition 2 is ignorable. However, to reduce the grasping force angle as much as possible is effective in decreasing the probability of the object slipping out. We present the additional definition of the stable grasp with the optimal force angle.

Definition 2: For an arbitrarily shaped object, a dual-fingered robot can realize the stable grasp with the optimal force angle as ϕ_0 when $\overline{Q_1 Q_2}$ is orthogonal to $\overline{P_1 P_2}$ or $\alpha = 0$ under Definition 1.

Unless satisfying Definition 2, the force angles are not equivalent and the probability of the slippage at the contact point with a higher force angle is higher than another points. The greater the difference between the grasping force angle, the greater the probability of slippage is increased. When the stable grasp with the optimal force angles is realized, the force angles at the left and right contact points are equivalent to each other and it has the lowest slippage probability.

III. Controlling Optimal Force Angles and Regulating Position of the Object in a Blind Manner

A. Definition of Target Point in a Blind Manner

Humans can manipulate an object using tactile information in a blind manner, which is less precise than a method using visual feedback. For the sake of the position regulation of an object in a blind manner, it is necessary to define a specific point which can be defined without visual feedback. Hence, we choose the target point $z_t := (x_t,\ y_t)^T$ as the center of finger-tips as follows:

$$x_t = \frac{x_{01} + x_{02}}{2} \quad \text{and} \quad y_t = \frac{y_{01} + y_{02}}{2}. \tag{17}$$

Accordingly, the error $e_t := (e_x,\ e_y)^T$ is defined as follows:

$$e_x = x_t - x_d = \frac{x_{01} + x_{02}}{2} - x_d,$$
$$e_y = y_t - y_d = \frac{y_{01} + y_{02}}{2} - y_d,$$

where x_d and y_d are the desired x- and y- coordinates, respectively.

B. Controller Design

We propose a control law for the realization of stable grasp with optimal force angles and the regulation of the object position in x- and y-coordinates, simultaneously as follows:

$$u = -K_v \dot{q} - J_p K_p e_t - f_d J_f^T(q) \left[\cos\phi_0\ \ \cos\phi_0\ \ \sin\phi_0\ \ \sin\phi_0\right]^T, \tag{18}$$

where $K_v \in \mathbb{R}^{(n_1+n_2)\times(n_1+n_2)}$ and $K_p \in \mathbb{R}^{2\times 2}$ are positive definite diagonal matrices and f_d is a positive constant. The first term in the right hand side of (18) is introduced for damping, the second term is done for regulating the position in a blind manner, and the third term is done for establishing the desired grasping force f_d and cancelling the rotational moment of an object. The Jacobian matrix J_p is defined as follows:

$$J_p^T = 2\frac{\partial e_t^T}{\partial q} = \left[J_{01},\ J_{02}\right] \in \mathbb{R}^{2\times(n_1+n_2)}.$$

Remark 3: This control law only requires the measured data of joint angles, joint angular velocities, contact angles, and kinematic parameters of fingers, but no object information, no preplanning, and no force sensors.

Property 2: The third term in (18) has the equivalent equation as follows:

$$J_f^T(q)\left[\cos\phi_0\ \ \cos\phi_0\ \ \sin\phi_0\ \ \sin\phi_0\right]^T = J_\alpha f_d l \sin\alpha, \quad (19)$$

where J_α denotes the Jacobian matrix of α with respect to q.

Proof: See Appendix I. ■

C. Stability Analysis

In this subsection, we analyze the stability of the stable grasp control method with the position regulation for dual fingered robot. Let us define a Lyapunov function candidate as

$$\begin{aligned} V(\dot{q},\alpha,e_t) =& \frac{1}{2}\dot{q}^T H(q)\dot{q} + \int_0^\alpha f_d l \sin\alpha d\alpha + \frac{1}{2}e_t^T K_p e_t \\ & + \epsilon\dot{q}^T H(q)\left\{J_\alpha\sin\alpha + \rho J_p e_t\right\}, \end{aligned} \quad (20)$$

where ϵ and ρ are some positive constant values. To show that (20) is positive definite function, we can rewrite (20) as follows:

$$\begin{aligned} V \geq\ & \frac{1}{2}\dot{q}^T H(q)\dot{q} + 2 f_d l_{\min}\sin^2\left(\frac{\alpha}{2}\right) + \frac{1}{2}e_t^T K_p e_t \\ & + \epsilon\dot{q}^T H(q)\left\{J_\alpha\sin\alpha + \rho J_p e_t\right\}, \end{aligned} \quad (21)$$

where $l_{\min}$ is the minimum length of $\overline{P_1P_2}$. Since $\sin^2\left(\frac{\alpha}{2}\right) < \sin^2\alpha$ in $-\frac{\pi}{2} < \alpha < \frac{\pi}{2}$, (21) becomes

$$\begin{aligned} V \geq & \frac{3}{8}\dot{q}^T H(q)\dot{q} + \left(\frac{1}{4}\dot{q} + 2\epsilon\rho J_\alpha\sin\alpha\right)^T H(q) \\ & \times\left(\frac{1}{4}\dot{q} + 2\epsilon\rho J_\alpha\sin\alpha\right) \\ & + \left(\frac{1}{4}\dot{q} + 2\epsilon J_p e_t\right)^T H(q)\left(\frac{1}{4}\dot{q} + 2\epsilon J_p e_t\right) \\ & + 2\left(f_d l_{\min} - 2\epsilon^2 J_\alpha^T H(q) J_\alpha\right)\sin^2\left(\frac{\alpha}{2}\right) \\ & + \frac{1}{2}e_t^T\left(K_p - 8\epsilon^2\rho^2 J_p^T H(q) J_p\right)e_t. \end{aligned} \quad (22)$$

As seen from (22), V is a positive definite in $(\dot{q},\alpha,e_t)$ when ϵ is chosen so that

$$\epsilon < \min\left[\sqrt{\frac{f_d l_{\min}}{2(J_\alpha^T H(q) J_\alpha)_M}},\ \sqrt{\frac{(K_p)_m}{8\rho^2(J_p^T H(q) J_p)_M}}\ \right], \quad (23)$$

where $(\cdot)_M$ and $(\cdot)_m$ indicate the largest and smallest eigenvalues of the matrix, respectively. From (23), we can determine any positive constant of f_d and any positive diagonal matrix of K_p by setting ϵ to a sufficiently small constant. We use the following lemma to prove the stability of the proposed controller:

Lemma 1: $a\|x\|^2 - b\|x\|\|y\| + c\|y\|^2 > (\sqrt{a}\|x\| - \sqrt{c}\|y\|)^2$ is provided that $b^2 < 4ac$.

Theorem 1: Consider the dual-fingered robot system (11) and (12) that is grasping an arbitrarily shaped object in a horizontal plane illustrated in Fig. 1. If the control law (18) is applied to the system with positive constants k_v and f_d, the desired grasping force f_d can be guaranteed with the dynamic force/torque balance and the asymptotic convergence of α and e_t to zeros. That is, (18) realizes the stable grasp and the position regulation in x- and y-coordinates with the optimal force angles.

Proof: See Appendix II. ■

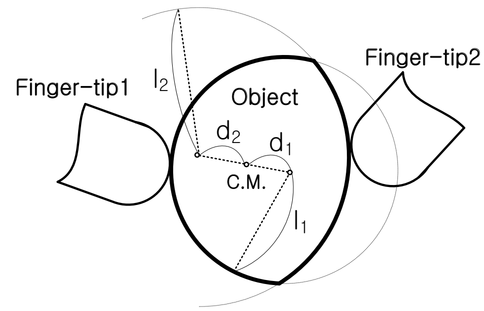

Fig. 2. Arbitrarily shaped object.

IV. Simulation Results

In this section, simulation results are presented to illustrate the effectiveness of the proposed controller for dual-fingered robots. We consider two planar fingers with three degrees of freedom grasping an object moving in a horizontal plane. The object has two curvatures with difference radii and centers as shown in Fig. 2. The centers from the center of mass are $d_1 = 2$ [cm] and $d_2 = 100$ [cm], and the radii of curvatures are $l_1 = 2$ [cm] and $l_2 = 1.03$ [m], respectively. The initial position vector and the desired position of the x- and y- coordinates are $(x_i,\ y_i) = (4,\ 8)$[cm] and $(x_d,\ y_d) = (6,\ 4)$[cm], respectively. The mass and the moment of inertia of all links are set to 0.03 [kg] and $3\times10^{-}5$ [kg $\cdot$ m^2] and the lengths of the links l_{1i}, l_{2i}, and l_{3i} for i =1, 2 are set to 4 [cm], 3 [cm], and 3 [cm], respectively. The radii of the finger-tips are set to 1 [cm]. The damping gain and the desired grasping force are chosen as $k_v = 0.008$ and $f_d = 0.8$ [N], respectively. The initial joint and the force angles are set to $q = (1,\ 0.96,\ 1.02,\ 0.9,\ 1.4,\ 0.39)^T$[rad] and $(\phi_{f1},\phi_{f2}) = (-0.014,\ \ -0.089)$[rad], respectively. We applied the Constraint Stabilization Method(CSM) [19] because this simulation should be conducted under the geometric constraints and the rolling constraints (2)-(7).

Figs. 3 and 4 display the initial and final postures of the dual-fingered robot grasping an arbitrarily shaped object having the different surface curvatures. Since the magnitude of the grasping forces are equal and the directions are toward each other as shown in Fig. 5, it is observed that the dynamic

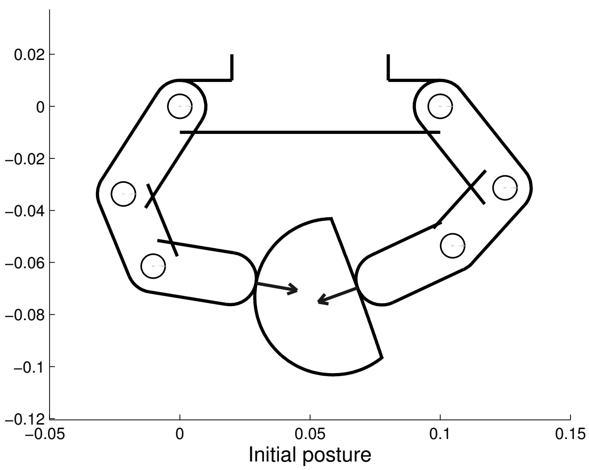

Fig. 3. The initial posture of dual-fingered robot.

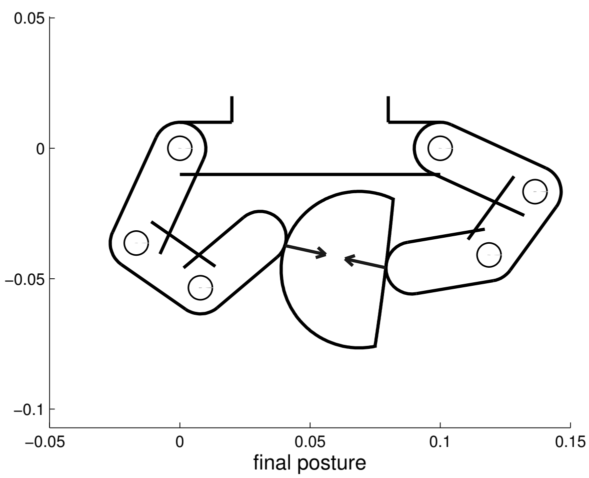

Fig. 4. The final posture of dual-fingered robot.

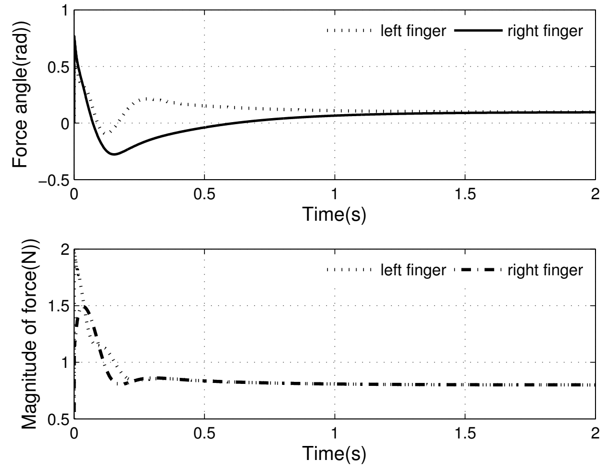

Fig. 5. The magnitude and the angle of grasping forces.

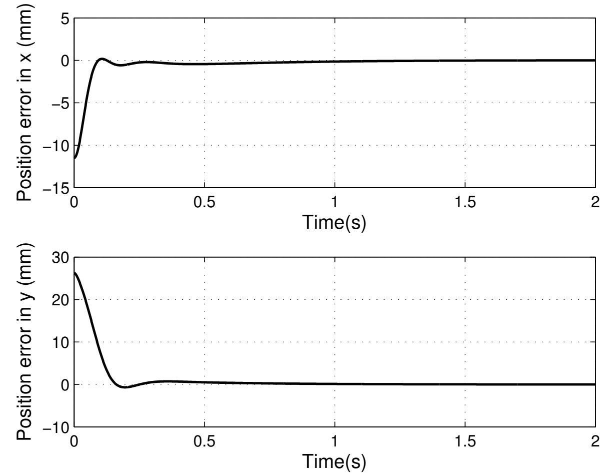

Fig. 6. The position errors.

force/torque closure is realized with the positive internal forces. Fig. 5 also shows that the force angles converge asymptotically to the equivalent value as $0.97[\mathrm{rad}]$. The simulation results of the object position errors are plotted in Fig. 6. Accordingly, we can draw a conclusion that the proposed control method can realize the stable grasp with the optimal force angle and the position regulation in a blind manner for an arbitrarily shaped object.

V. Conclusion

In this paper, a control method for both the stable grasping of an object with optimal force angles and the position regulation of the object in a blind manner has been proposed. First, we have introduced the kinematics and dynamics of the dual-fingered robot grasping an arbitrarily shaped object. Second, the controller has been designed for the force/torque balance with the optimal force angle which can be applied in an object with smooth surfaces. Third, the dynamic stability has been proved via Lyapunov stability analysis. Finally, the numerical simulations for the two three-link fingered robot grasping an object with two different curvatures have been performed to demonstrate the effectiveness of the proposed controller using CSM.

Appendix I

Proof the property 2

Proof: Substituting (10) into J_f^T in (18), we obtain

$$J_f^T(q)\begin{bmatrix} \cos\phi_0 & \cos\phi_0 & \sin\phi_0 & \sin\phi_0 \end{bmatrix}^T = \begin{bmatrix} J_{01}, & -J_{02} \end{bmatrix}^T \begin{bmatrix} -\cos\phi \\ \sin\phi \end{bmatrix} - \begin{bmatrix} r_1\sin\phi_0\mathbf{e}_1 \\ r_2\sin\phi_0\mathbf{e}_2 \end{bmatrix}. \quad (24)$$

The above equation can be rewritten as

$$J_f^T(q)\begin{bmatrix} \cos\phi_0, & \cos\phi_0, & \sin\phi_0, & \sin\phi_0 \end{bmatrix}^T = -\frac{d(x_{01}-x_{02})}{dq}\cos\phi + \frac{d(y_{01}-dy_{02})}{dq}\sin\phi - \begin{bmatrix} r_1\sin\phi_0\mathbf{e}_1 \\ r_2\sin\phi_0\mathbf{e}_2 \end{bmatrix}. \quad (25)$$

Then, substituting the derivative of (2) in q into (25) yields

$$\begin{aligned} & J_f^T(q)\begin{bmatrix} \cos\phi_0, & \cos\phi_0, & \sin\phi_0, & \sin\phi_0 \end{bmatrix}^T \\ &= \frac{d(Y_1+Y_2)}{dq}\sin\phi_0 + (Y_1+Y_2)\cos\phi_0\frac{d\phi_0}{dq} \\ &\quad - \{-(x_{02}-x_{01})\sin\phi + (y_{01}-y_{02})\cos\phi\}\frac{d\phi}{dq} \\ &\quad - \begin{bmatrix} r_1\sin\phi_0\mathbf{e}_1 \\ r_2\sin\phi_0\mathbf{e}_2 \end{bmatrix} - (r_1+r_2)\sin\phi_0\frac{d\phi_0}{dq}. \end{aligned} \tag{26}$$

Substituting (3), (6) and (7) into (26), we obtain

$$\begin{aligned} & J_f^T(q)\begin{bmatrix} \cos\phi_0, & \cos\phi_0, & \sin\phi_0, & \sin\phi_0 \end{bmatrix}^T \\ &= (Y_1+Y_2)\cos\phi_0\frac{d\phi_0}{dq} + (Y_2-Y_1)\cos\phi_0\frac{d\phi}{dq}. \end{aligned} \tag{27}$$

Using (4) and (5), we obtain

$$J_f^T(q)\left[\cos\phi_0,\ \cos\phi_0,\ \sin\phi_0,\ \sin\phi_0\right]^T = J_\alpha l\sin\alpha. \tag{28}$$

This completes the proof of Property 2. ■

Appendix II
Proof of Theorem 1

Proof: The time derivation of (20) yields

$$\begin{aligned} \dot{V} =& \{\dot{q} + \epsilon(J_\alpha\sin\alpha + \rho J_p e_t)\}^T\left\{H(q)\ddot{q} + \frac{1}{2}\dot{H}(q)\dot{q}\right\} \\ &+ \dot{q}^T(J_\alpha f_d l\sin\alpha + J_p K_p e_t) \\ &+ \frac{1}{2}\epsilon(J_\alpha\sin\alpha + \rho J_p e_t)^T\dot{H}(q)\dot{q} \\ &+ \epsilon\dot{q}^T G_1(q,\alpha,e_t)\dot{q} + \epsilon\dot{q}^T G_2(q,\alpha,e_t)\dot{q}, \end{aligned} \tag{29}$$

where

$$\begin{aligned} G_1(q,\alpha,e_t) &= \frac{\partial J_\alpha}{\partial q}\sin\alpha + \rho\frac{\partial J_{p1}}{\partial q}e_x + \rho\frac{\partial J_{p2}}{\partial q}e_y, \\ G_2(q,\alpha,e_t) &= J_\alpha J_\alpha^T\cos\alpha + \frac{\rho}{2}J_p J_p^T. \end{aligned}$$

Substituting (18) into (29) and using Properties 1 and 2, it is obtained that

$$\begin{aligned} \dot{V} =& -\dot{q}\{K_v - \epsilon G_1(q,\alpha,e_t) - G_2(q,\alpha,e_t)\}\dot{q} \\ &- \frac{1}{2}\epsilon J_\alpha^T J_\alpha f_d l\sin^2\alpha - \frac{1}{2}\epsilon\rho e_t^T J_p^T J_p K_p e_t \\ &+ \epsilon(J_\alpha\sin\alpha + J_p e_t)^T\left(C^T - K_v\right)\dot{q} \\ &- \epsilon G_3(q,\alpha,e_t), \end{aligned} \tag{30}$$

where

$$\begin{aligned} G_3(q,\alpha,e_t) =& \frac{1}{2}J_\alpha^T J_\alpha f_d l\sin^2\alpha + \frac{1}{2}\rho e_t^T J_p^T J_p K_p e_t \\ &+ J_\alpha^T J_p K_p e_t\sin\alpha + \rho J_\alpha^T J_p e_t\sin\alpha f_d l. \end{aligned} \tag{31}$$

From the existence of lower and upper bounds of the Jacobian matrices, (31) is lower bounded as follows:

$$\begin{aligned} & G_3(q,\alpha,e_t) \geq \\ &\quad \frac{1}{2}\left(J_\alpha^T J_\alpha\right)_m^2 f_d l_{min}\sin^2\alpha + \frac{\rho}{2}(J_p^T J_p)_m(K_p)_m\|e_t\|^2 \\ &\quad - \left\{(J_\alpha^T J_p)_M(K_p)_M + \rho(J_\alpha^T J_p)_M f_d l_{max}\right\} \\ &\quad \times \|e_t\|\,\|\sin\alpha\|. \end{aligned} \tag{32}$$

Using Lemma 1, $G_3(q,\alpha,e_t)$ is a positive definite function when there exists a positive constant ρ satisfying the following inequality:

$$\begin{aligned} & (f_d l_{max})^2\rho^2 + (K_p)_M^2 \\ &+ f_d\left\{2(K_p)_M l_{max} - \frac{(J_\alpha^T J_p)_m^2}{(J_\alpha^T J_p)_M^2}(K_p)_m l_{min}\right\}\rho < 0. \end{aligned} \tag{33}$$

(33) has a real constant ρ provided that

$$4\{(K_p)_M l_{max}\}^2 > \left\{2(K_p)_M l_{max} - \frac{(J_\alpha^T J_p)_m^2}{(J_\alpha^T J_p)_M^2}(K_p)_m l_{min}\right\}^2.$$

Since $G_3(q,\alpha,e_t)$ is positive definite and all Jacobian matrices are lower and upper bounded, we can obtain the upper bound of (30) as follows:

$$\begin{aligned} \dot{V} \leq& -\left\{(K_v)_m - \epsilon(g_1 + g_2 + h_2^2 + h_4^2)\right\}\|\dot{q}\|^2 \\ &- \epsilon\left(h_1|\sin\alpha| - h_2\|\dot{q}\|\right)^2 - \epsilon\left(h_3\|e_t\| - h_4\|\dot{q}\|\right)^2 \end{aligned} \tag{34}$$

where

$$\begin{aligned} g_1 &= (G_1(q,\alpha,e_t))_M, \\ g_2 &= (G_2(q,\alpha,e_t))_M, \\ h_1 &= \sqrt{\frac{1}{2}(J_\alpha^T J_\alpha)_m f_d l_{min}}, \\ h_2 &= \frac{1}{2h_1}\{C(q,\dot{q})_M + (K_v)_M\}(J_\alpha)_M, \\ h_3 &= \sqrt{\frac{\rho}{2}(J_p^T J_p)_m(K_p)_m}, \\ h_4 &= \frac{1}{2h_3}\{C(q,\dot{q})_M + (K_v)_M\}(J_p)_M. \end{aligned}$$

(34) is a negative definite function in $\dot{q}$, $(h_1|\sin\alpha| - h_2\|\dot{q}\|)$, and $(h_3\|e_t\| - h_4\|\dot{q}\|)$, when ϵ provides the following inequality:

$$\epsilon \leq \frac{(K_v)_m - \eta}{g_1 + g_2 + h_2^2 + h_4^2}, \tag{35}$$

where $0 < \eta < (K_v)_m$.

Corollary 1: From (23) and (35), the condition of ϵ is given as

$$\epsilon < \min\left[\begin{array}{c} \sqrt{\frac{f_d l_{min}}{2(J_\alpha^T H(q) J_\alpha)_M}},\ \sqrt{\frac{(K_p)_m}{8\rho^2(J_p^T H(q) J_p)_M}}, \\ \frac{(K_v)_m - \eta}{g_1 + g_2 + h_2^2 + h_4^2} \end{array}\right].$$

(34) becomes zero when $\dot{q} = 0$, $h_1|\sin\alpha| = h_2\|\dot{q}\|$, and $h_3\|e_t\| = h_4\|\dot{q}\|$. It denotes that (34) becomes zero when $(\dot{q},\sin\alpha,e_t) = (0,0,[0,0]^T)$ and always has a negative value when $(\dot{q},\ \sin\alpha,\ e_t) \neq (0,0,[0,0]^T)$. Hence, we can obtain that $\dot{V}$ is a negative definite function in $(\dot{q},\alpha,e_t)$. By invoking the Lyapunov's direct method, we have proven the uniformly asymptotic stability of the equilibrium point $(\dot{q},\alpha,e_t) = 0$. In order to verify that the grasping forces are positive as f_d, we substitute (18) and $(\dot{q},\alpha,e_t) = 0$ into (11). Then, we have $F_f = f_d\begin{bmatrix} \cos\phi_0, & \cos\phi_0, & \sin\phi_0, & \sin\phi_0 \end{bmatrix}^T$, which indicates

the direction of the grasping force toward each other and the magnitude equivalent to f_d. This completes the proof of Theorem 1. ■

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