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Synthesis of fixed-structure robust controllers using the distributed particle swarm optimizer with cyclic-network topology

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Abstract— **This paper discusses a new meta-heuristic approach with high reliability to the synthesis problem of fixed-structure robust controllers satisfying multiple control specifications. For this purpose, first, the particle swarm optimizer (PSO) with cyclic-network topology is developed. Such a neighborhood topology can ensure a good trade-off between exploration and exploitation ability of the swarm, which results in a significant reduction of the probability of premature convergence to local optima. Second, the proposed distributed PSO algorithm is incorporated with the simple constraint handling method [8] using a virtual objective function to handle multiple control specifications. Then, it is shown how to find optimal parameters of a fixed-structure controller guaranteeing the given specifications based on the developed PSO technique using cyclic-network topology. Third, a typical numerical example to demonstrate its effectiveness is given, which clearly shows that the proposed distributed PSO scheme gives a novel and powerful impetus to the fixed structure robust controller synthesis.**

I. Introduction

This paper describes a new meta-heuristic approach to the synthesis of fixed-structure (order) robust controllers. Applications of the problem also include systematic tuning of proportional-integral-derivative (PID) controller gains.

Practical fixed-structure (such as PID) robust controllers are required in many application fields, due to limitations of available computer resource and necessity of on-site controller tuning. Therefore, a lot of work has been done in recent years such as robust PID tuning (e.g., [1], [2], [3], [4]) and fixed-order H_∞ controller design (e.g., [5], [6], [7]). However, it is difficult to treat both multiple (e.g., H_∞ norm) control specifications and the restricted controller structure simultaneously. More importantly, since many of the existing methods require the knowledge of sophisticated control theory, it is not easy for most engineers in industry to use those methods.

In this line of research, the authors developed an easy-to-use design methodology of fixed-structure controllers based on a meta-heuristic approach, so-called particle swarm optimization (PSO) algorithm [8]. The PSO algorithm was proposed by Kennedy and Eberhart [9], which is a swarm intelligence technique and is one of the evolutionary computation algorithms. The PSO has attracted a lot of attention in recent years, and then, within little more than a decade, hundreds of papers have reported the successful applications of PSO [10]. Also, the empirical evidence of its superiority in solving a variety of non-convex problems has been accumulated [11], [12]. The main achievement of Maruta et al. [8] was to develop a method for handling the optimization problems subject to inequality constraints within the framework of PSO, and then give a novel impetus to the PSO-based design scheme for fixed-structure controllers satisfying multiple H_∞ norm specifications. They also have shown its effectiveness and easy-of-use property via several numerical examples. The method works well in most cases . However, there are cases that their PSO scheme may not provide a sufficient reliability in the sense that the probability of obtaining a desired solution is not high enough (e.g., less than 0.5). One such examples is given in the numerical example 3 in Maruta et al. [8]. Also, if the domain or number of design parameters is too large, the PSO may not work well. Therefore, the enhancement of performance precision of PSO is inevitable to improve its practicability in fixed-structure controller designs.

The above shortcomings of PSO, in fact, result from the neighborhood structure that plays an important role in the evolution law of particles. In a general PSO scheme, a particle swarm tries to find an optimum through an iterative process where particles sample a given search space and then adjust their search directions to sample near to their fitter neighbors. Here, the neighbors mean those particles which share information on individual fitness values. Therefore, the set of neighbor-connections (the swarm's topology) between all of the particles has a significant impact on the exploration and exploitation ability of the swarm (i.e., its ability to perform global search of the given search space, and converge faster to the most promising region, respectively.) [13].

One of the most common topologies is the star topology (see Fig. 2(a)) where the neighborhood of an individual is the entire swarm, which was also adopted in Maruta et al. [8]. Such a fully connected neighborhood topology may have many opportunities of containing a relatively good solution, and thus often exhibits fast convergence of swarm to optima than other topology. However, because of particles' poor exploration ability due to the fast convergence rate, it may lead to the problem of premature convergence to local optima. We can easily infer from the above observation that the low success rate appeared in Maruta et al. [8] results from the particles' poor exploration behavior due to a star neighborhood topology. In order to overcome the drawback of star topology, the ring topology, which has a few

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connected neighbor particles (see Fig. 2(b)), has been studied [14]. It has a tendency to reduce the convergence rate in PSO. Such a characteristic improves the exploration ability and thus reduces the possibility of particles to be trapped into local optima. Nevertheless, its serious drawback is an increased amount of computation time (i.e., convergence speed of particles). It follows from the above observation that one of the extremely important research issues in PSO is, and continues to be, finding a best neighborhood topology ensuring a good trade-off between exploration and exploitation ability of the swarm [14]. This is also connected directly with the improvement of practicability of fixed-structure robust controller designs via the PSO scheme.

The purpose of this paper is to develop an easy-to-use design scheme with superior reliability and validity for fixed-structure controllers satisfying multiple control specifications. The main tool is the distributed particle swarm optimizer with cyclic-network topology. In order to find optimal controller parameters satisfying multiple control specifications, we incorporate this distributed cyclic-network topology based PSO algorithm with the simple constraint handling method [8], which does not require any problem-dependent or user-defined parameters such as penalty factors or Lagrange multipliers. Next, it is shown how to obtain a fixed-structure controller guaranteeing multiple H_∞ specifications based on the developed distributed constrained PSO technique. We also thoroughly study an example of fixed-structure robust controller synthesis with a single H_∞ norm specification, since it is a typical one for explicitly showing the remarkable reliability of the proposed PSO method. It clearly verifies that the proposed distributed constrained PSO methodology with cyclic-network topology gives a novel and powerful impetus to the fixed-structure robust controller syntheses. Note that, although it is omitted here due to the page limitation, various types of fixed-structure robust controllers (e.g., the numerical example 1 with multiple H_∞ specifications presented in Maruta et al. [8]) can be directly designed via our methodology with more promising results.

II. Distributed particle swarm optimizer with cyclic-network topology

In this section, we present a novel distributed PSO algorithm using a cyclic-network topology for fixed-structure robust controller design problems. It features not only superior reliability, but also high practicality, simplicity and implementability . Some of the advantages of the proposed method over conventional PSO methods are discussed in detail.

A. Optimization problem description

In this study, we are interested in general constrained optimization problems that are mathematically formulated as follows:

$$\min_{\boldsymbol{x}\in\mathbb{F}} f(\boldsymbol{x}), \quad f(\boldsymbol{x}) : \mathbb{R}^n \to \mathbb{R} \tag{1}$$

with

$$\mathbb{F} := \{\boldsymbol{x} \in \mathbb{R}^n | h_1(\boldsymbol{x}) < 0, h_2(\boldsymbol{x}) < 0, \cdots, h_m(\boldsymbol{x}) < 0\} \tag{2}$$

where $f(\boldsymbol{x})$ is the linear/nonlinear objective function, which is to be optimized with respect to the design variable vector $\boldsymbol{x} \in \mathbb{R}^n$ where n is the number of independent design variables. In (2), $h_\ell(\boldsymbol{x}) : \mathbb{R}^n \to \mathbb{R}$ is the linear/nonlinear constraint function, and m is the number of constraint conditions. Thus, $\mathbb{F}$ denotes the feasible region, and is assumed to be not empty. Let $\mathbb{D}$ denote the initial search space of $\boldsymbol{x} \in \mathbb{R}^n$, which is supposed to be given by the designer in advance. Note that in order to guarantee particle evolution within the given search space, it is sometimes required that the search space information be incorporated into a form of boundary constraints such that $\underline{\boldsymbol{x}} \le \boldsymbol{x} \le \overline{\boldsymbol{x}}$ where $\underline{\boldsymbol{x}} \in \mathbb{R}^{n_p}$ and $\overline{\boldsymbol{x}} \in \mathbb{R}^n$ denote, respectively, the vectors of the lower and upper bounds of design variables, and $\le$ denotes the element-wise inequality.

In the following subsection, a novel distributed PSO algorithm using a cyclic-network topology to find an optimal solution of the constrained optimization problem defined in (1)-(2) is developed.

B. Distributed PSO with a cyclic-network topology

The PSO algorithm uses a swarm consisting of n_p particles (i.e., $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n_p}$) to search an optimal solution $\boldsymbol{x}^* \in \mathbb{R}^n$ of (1)-(2). In this case, it is crucial to take the given constraint conditions into account in any form to calculate the fitness value of each individual $\boldsymbol{x}_i$, and ultimately to find $\boldsymbol{x}^*$. To achieve this aim within the PSO framework, we incorporate the simple constraint handling method [8] using a virtual objective function $f_v(\boldsymbol{x})$ with the distributed particle evolution law given later.

A virtual objective function $f_v(\boldsymbol{x}) : \mathbb{R}^n \to \mathbb{R}$ could be a function that simultaneously satisfies the following two properties: (i) $f_v(\boldsymbol{x}) < 0$ holds for any $\boldsymbol{x}$ satisfying $h_\ell(\boldsymbol{x}) < 0$, and (ii) $f_v(\boldsymbol{x}_a) < f_v(\boldsymbol{x}_b)$ holds whenever $f(\boldsymbol{x}_a) < f(\boldsymbol{x}_b)$ is satisfied. One possible candidate for $f_v(\boldsymbol{x})$ is $f_v(\boldsymbol{x}) := \arctan\{f(\boldsymbol{x})\} - \pi/2$, which can be used to solve various types of optimization problems (see [8] for details and its distinctive features). Then, the original constrained optimization problem in (1)-(2) can be modified into an unconstrained problem as follows:

$$\min_{\boldsymbol{x}\in\mathbb{R}^n} \mathcal{L}(\boldsymbol{x}) \tag{3}$$

with

$$\mathcal{L}(\boldsymbol{x}) := \begin{cases} h_{\max}(\boldsymbol{x}) & \text{if } h_{\max}(\boldsymbol{x}) \ge 0, \\ f_v(\boldsymbol{x}) & \text{otherwise.} \end{cases} \tag{4}$$

where $h_{\max}(\boldsymbol{x}) := \max[h_1(\boldsymbol{x}), h_2(\boldsymbol{x}), \cdots, h_m(\boldsymbol{x})]$.

It is important to note that, thanks to the flexibility of PSO, $\mathcal{L}(\boldsymbol{x})$ can be used as a new objective function without any problem whatever the objective/constraints functions are [8].

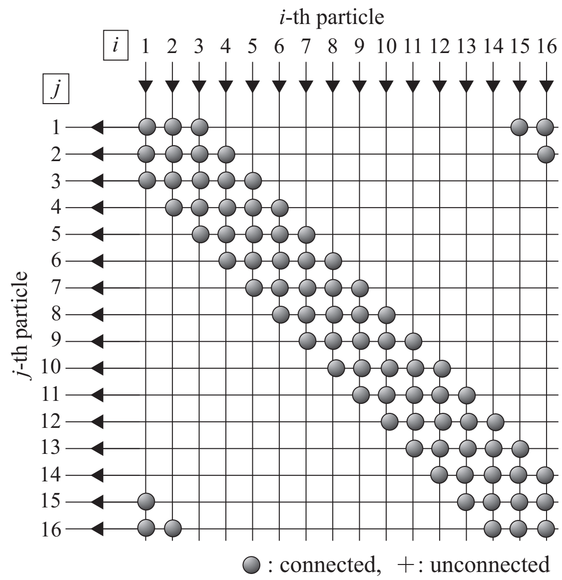


Fig. 1. Information flow in cyclic-network topology for the case of $n_p = 16$ and $n_s = 4$

We next show the way to obtain a solution of the optimization problem (3) with (4) using the developed distributed PSO with a cyclic-network topology. Consider a swarm consisting of n_p particles: $\boldsymbol{x}_1, \boldsymbol{x}_2, \cdots, \boldsymbol{x}_{n_p}$. Each particle $\boldsymbol{x}_i$ is, in effect, an n-dimensional vector. The position of the ith particle and its velocity are denoted, respectively, as $\boldsymbol{x}_i := (x_{i,1}, x_{i,2}, \cdots, x_{i,n})^T \in \mathbb{R}^n$ and $\boldsymbol{v}_i := (v_{i,1}, v_{i,2}, \cdots, v_{i,n})^T \in \mathbb{R}^n$ where $i \in \{1, 2, \cdots, n_p\}$. Then, the position of the ith particle, $\boldsymbol{x}_i \in \mathbb{R}^n$, evolves based on the following update law: For $k = 1, 2, \cdots$, which indicates the iteration number,

$$\boldsymbol{x}_i^{k+1} = \boldsymbol{x}_i^k + \boldsymbol{v}_i^{k+1}, \tag{5}$$

$$\boldsymbol{v}_i^{k+1} = c_0 \boldsymbol{v}_i^k + c_1 r_{1,i}^k (\boldsymbol{x}_{\text{pbest},i}^k - \boldsymbol{x}_i^k) + c_2 r_{2,i}^k (\boldsymbol{x}_{\text{sbest},i}^k - \boldsymbol{x}_i^k), \tag{6}$$

where the inertia factor c_0, the cognitive scaling factor c_1 and the social scaling factor c_2, which are given by the designer, influence on the particle trajectories and thus the convergence and search diversity properties. The random numbers $r_{1,i}^k$ and $r_{2,i}^k$ are uniformly distributed in $[0, 1]$ and represent the stochastic behaviors of PSO.

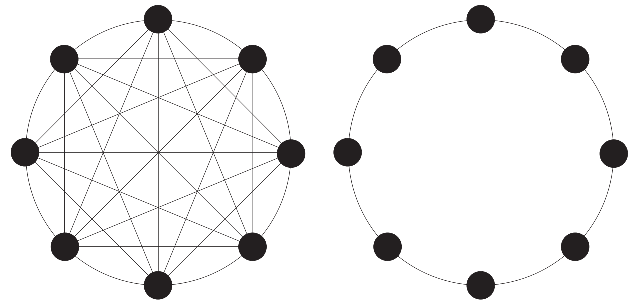
In (6), $\boldsymbol{x}_{\text{pbest},i}^k$ denotes the best previously obtained position of the ith particle; i.e.,

$$\boldsymbol{x}_{\text{pbest},i}^k := \arg \min_{\boldsymbol{x} \in \{\boldsymbol{x}_i^j | j=1,2,\cdots,k\}} \mathcal{L}(\boldsymbol{x}), \tag{7}$$

whereas $\boldsymbol{x}_{\text{sbest},i}^k$ denotes the best position in the social neighborhoods of the ith particle at the current iteration k, whose mathematical formulation is given as

$$\boldsymbol{x}_{\text{sbest},i}^k := \arg \min_{\boldsymbol{x} \in \{\boldsymbol{x}_\ell^k | \ell=i-\frac{n_s}{2},\cdots,i+\frac{n_s}{2}\}} \mathcal{L}(\boldsymbol{x}) \tag{8}$$

where an even-numbered $n_s (\le n_p)$ is the number of neighbors ith particle has, and $\boldsymbol{x}_i^j := \boldsymbol{x}_{(i-1 \bmod n_p)+1}^j$ for $i < 1$ or $n_p + 1 \le i$. Note that the definition of the ith particle's social neighborhoods in (8) means, in fact, the cyclic-network topology (neighborhood structure) among n_s particles (see e.g., Fig. 1). It is here assumed to be a static topology, i.e., the initially determined topology remains fixed during the PSO run.



(a) Star (gbest) topology: $n_s = 8$ (b) Ring (lbest) topology: $n_s = 2$

Fig. 2. A representation of the conventional static social network topologies

It follows from the above observation that the first part of (6) denotes the inertia effect of previous velocity; the second part denotes the cognition by the ith individual itself; the third part denotes the social cooperation among neighborhoods of the ith individual. Note that if $n_s = n_p$, $\boldsymbol{x}_{\text{sbest},i}^k$ in (8) corresponds to one of the most common topologies; i.e., the star (gbest) topology where every individual is connected to every other one (see e.g., Fig. 2(a)). On the other hand, if $n_s = 2$, it denotes the ring (lbest) topology which connects each individual to its two immediate neighbors (see e.g., Fig. 2(b)) [13], [14], [17]. As mentioned in Section I, the neighborhood structure exerts significant influence on the PSO's exploration and exploitation ability (i.e., its ability to perform global search of the given search space and converge faster to the most promising region, respectively.).

When the neighborhood of an individual is the entire swarm (i.e., star topology), it may have many opportunities of containing a relatively good solution. In fact, this fully connected neighborhood topology often exhibits fast convergence of swarm to optima than ring topology. However, because of their poor exploration ability due to the fast convergence rate of particles, it may lead to the problem of premature convergence to local optima. On the other hand, the ring topology, which has few connected neighbor particles, has a tendency to reduce the convergence rate in PSO. Such a characteristic improves the exploration ability and thus reduce the possibility of particles to be trapped into local optima. Nevertheless, its fatal drawback is an increased amount of computation time (i.e., convergence speed of particles). It follows from the above observation that one of the extremely important research issues in PSO is, and continues to be, finding a number of neighbors, n_s, ensuring a good balancing between exploration and exploitation ability of the swarm [14].

In this line of researches, the authors' recent study [18] thoroughly investigated a suitable number of n_s for the related problems in this field, and obtained the following result:

$n_s \approx 2n$ (Two times of the number of design variables).

Note that the details of how the neighborhood structure (i.e., the number of n_s) affects the performance and convergence of the swarm can be found in Maruta et al. [18].

The proposed distributed constrained PSO algorithm using a cyclic-network topology consists of the following steps:

[Step 0] Set $k = 0$. Initialize n_p particles with randomly chosen positions $\boldsymbol{x}_i^0 \in \mathbb{D}$, and $\boldsymbol{v}_i^0 = \boldsymbol{0}$. For $i = 1, 2, \cdots, n_p$, set $\boldsymbol{x}_{\text{pbest},i}^0 = \boldsymbol{x}_i^0$, and $\boldsymbol{x}_{\text{sbest},i}^0$ as

$$\boldsymbol{x}_{\text{sbest},i}^0 \leftarrow \arg \min_{\boldsymbol{x} \in \{\boldsymbol{x}_\ell^0 | \ell = i - \frac{n_s}{2}, \cdots, i + \frac{n_s}{2}\}} \mathcal{L}(\boldsymbol{x})$$

[Step 1] If the termination criterion is satisfied (i.e., $k > k_{\max}$), the algorithm terminates with the solution

$$\boldsymbol{x}^* := \arg \min_{\boldsymbol{x} \in \{\boldsymbol{x}_i^j | i = 1, 2, \cdots, n_p;\ j = 1, 2, \cdots, k\}} \mathcal{L}(\boldsymbol{x}). \tag{9}$$

Otherwise, go to Step 2.

[Step 2] Apply the following evolutionary update law to all particles: For $i = 1, 2, \cdots, n_p$,

$$\boldsymbol{v}_i^{k+1} \leftarrow c_0 \boldsymbol{v}_i^k + c_1 r_{1,i}^k (\boldsymbol{x}_{\text{pbest},i}^k - \boldsymbol{x}_i^k) + c_2 r_{2,i}^k (\boldsymbol{x}_{\text{sbest},i}^k - \boldsymbol{x}_i^k),$$
$$\boldsymbol{x}_i^{k+1} \leftarrow \boldsymbol{x}_i^k + \boldsymbol{v}_i^{k+1}.$$

Set $k = k + 1$, and then determine $\boldsymbol{x}_{\text{pbest},i}^k$ and $\boldsymbol{x}_{\text{sbest},i}^k$ as

$$\boldsymbol{x}_{\text{pbest},i}^k \leftarrow \arg \min_{\boldsymbol{x} \in \{\boldsymbol{x}_i^j | j = 1, 2, \cdots, k\}} \mathcal{L}(\boldsymbol{x}),$$

$$\boldsymbol{x}_{\text{sbest},i}^k \leftarrow \arg \min_{\boldsymbol{x} \in \{\boldsymbol{x}_\ell^k | \ell = i - \frac{n_s}{2}, \cdots, i + \frac{n_s}{2}\}} \mathcal{L}(\boldsymbol{x}).$$

Go to Step 1.

Note that the termination criterion in Step 1 is set as a maximum number of iterations for simplicity; i.e., the algorithm terminates after a user-determined number of iterations $k_{\max}$. For various types of termination criteria, refer to Kwok et al. [19].

In the following section, we present a concrete synthesis procedure of fixed-structure robust controllers based on the aforementioned distributed constrained particle swarm optimizer with a cyclic-network topology.

III. Synthesis of Fixed-Structure Robust Controllers

In this section, we present a design procedure of fixed-structure robust controllers satisfying the given multiple H_∞ performance specifications, which is based on the distributed constrained PSO with a cyclic-network topology presented in Section II-B. We also provide an example verifying the applicability of the proposed methodology to the synthesis of fixed-structure robust controllers.

A. Controller synthesis procedure [8]

Consider the linear time-invariant closed-loop system $\Sigma[\boldsymbol{x}]$:

$$\begin{bmatrix} \boldsymbol{z} \\ \boldsymbol{y} \end{bmatrix} = G(s) \begin{bmatrix} \boldsymbol{w} \\ \boldsymbol{u} \end{bmatrix}, \ \boldsymbol{u} = K(s; \boldsymbol{x}) \boldsymbol{y}, \tag{10}$$

where $G(s)$ denotes the generalized plant, $K(s; \boldsymbol{x})$ denotes the fixed-structure controller which depends on the design parameter $\boldsymbol{x} \in \mathbb{R}^n$. The vectors $\boldsymbol{z}$ and $\boldsymbol{w}$ are, respectively, defined as $\boldsymbol{z} := (\boldsymbol{z}_1^T, \boldsymbol{z}_2^T, \cdots, \boldsymbol{z}_m^T)^T$ where $\boldsymbol{z}_i \in \mathbb{R}^{p_i}$ and $\boldsymbol{w} := (\boldsymbol{w}_1^T, \boldsymbol{w}_2^T, \cdots, \boldsymbol{w}_m^T)^T$ where $\boldsymbol{w} \in \mathbb{R}^{q_i}$. The signals $\boldsymbol{z}_i \in \mathbb{R}^{p_i}$, $\boldsymbol{w}_i \in \mathbb{R}^{q_i}$, $\boldsymbol{y} \in \mathbb{R}^{p_0}$ and $\boldsymbol{u} \in \mathbb{R}^{q_0}$ are the controlled output vector, the external input vector, the measurement vector and the control input vector, respectively. Let $\lambda_i(\Sigma[\boldsymbol{x}])$ denote the ith pole of the system $\Sigma[\boldsymbol{x}]$ and $\lambda_{\max}(\Sigma[\boldsymbol{x}])$ be the pole whose real part is greater than that of any other pole; i.e., $\text{Re}[\lambda_{\max}(\Sigma[\boldsymbol{x}])] = \max_i \{\text{Re}[\lambda_i(\Sigma[\boldsymbol{x}])],\ \forall i\}$. Further, let $T_{z_i w_i}(s; \boldsymbol{x})$ denote the transfer matrix from $\boldsymbol{w}_i$ to $\boldsymbol{z}_i$ for $i = 1, 2, \cdots, m$.

Now the optimization-based controller synthesis problem considered in this paper is stated as follows: Given the objective function

$$J(\boldsymbol{x}) := \|T_{z_1 w_1}(s; \boldsymbol{x})\|_\infty \quad (\text{or } \text{Re}[\lambda_{\max}(\Sigma[\boldsymbol{x}])]) \tag{11}$$

and the admissible level $\gamma_i > 0$, find the design parameter $\boldsymbol{x} \in \mathbb{R}^n$ which minimizes $J(\boldsymbol{x})$ while satisfying the following multiple stability/performance constraint conditions simultaneously:

(C1) $\text{Re}[\lambda_{\max}(\Sigma[\boldsymbol{x}])] < 0$,
(C2) $\|T_{z_i w_i}(s; \boldsymbol{x})\|_\infty < \gamma_i$ for $i = 2, 3, \cdots, m$.

In order to design a controller by minimizing $J(\boldsymbol{x})$ in (11) subject to (C1)-(C2), it is enough to solve the optimization problem (3) with (4). Here, $f_v(\boldsymbol{x})$ and $h_\ell(\boldsymbol{x})$ could be set in the following manners:

If $J(\boldsymbol{x}) := \|T_{z_1 w_1}(s; \boldsymbol{x})\|_\infty$ is given, an example of $f_v(\boldsymbol{x})$ satisfying the required properties mentioned in Section II-B is given by

$$f_v(\boldsymbol{x}) = -\|T_{z_1 w_1}(s; \boldsymbol{x})\|_\infty^{-1}. \tag{12}$$

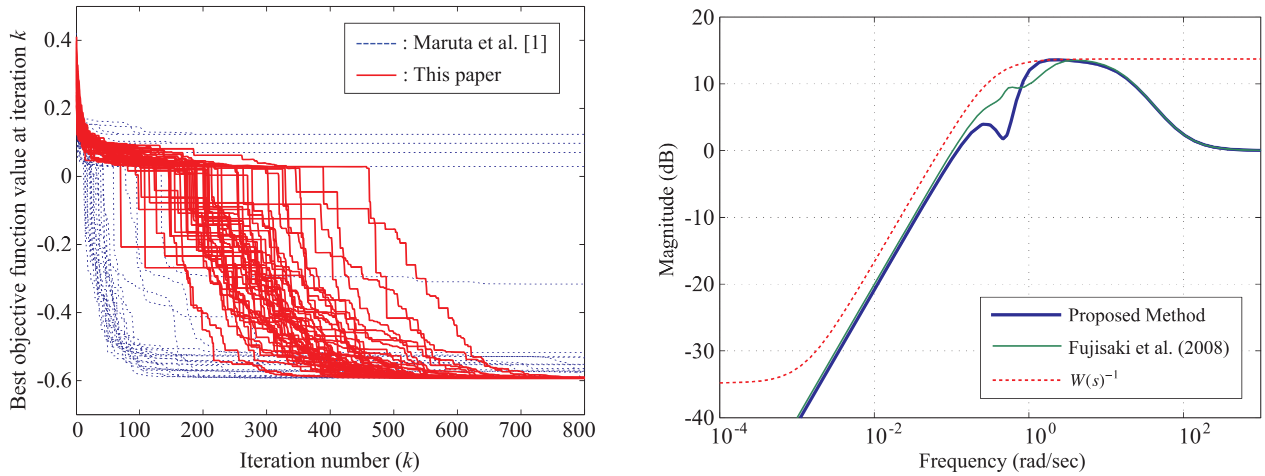
On the other hand, if $J(\boldsymbol{x}) := \text{Re}[\lambda_{\max}(\Sigma[\boldsymbol{x}])]$ is given, it is enough to choose

$$f_v(\boldsymbol{x}) = \text{Re}[\lambda_{\max}(\Sigma[\boldsymbol{x}])]. \tag{13}$$

Then, the feasible region $\mathbb{F}$ of $\boldsymbol{x}$ in (2) could be set as

$$\mathbb{F} := \left\{ \boldsymbol{x} \in \mathbb{R}^n \;\middle|\; \begin{array}{c} \text{Re}[\lambda_{\max}(\Sigma[\boldsymbol{x}])] < 0, \\ \|T_{z_2 w_2}(s; \boldsymbol{x})\|_\infty - \gamma_2 < 0, \\ \vdots \\ \|T_{z_m w_m}(s; \boldsymbol{x})\|_\infty - \gamma_m < 0 \end{array} \right\} \tag{14}$$

Therefore, all we have to do is to solve (3) with (4) using the above-defined $f_v(\boldsymbol{x})$ and the constraint conditions in (14)



(a) Convergence property of objective function values

(b) Bode plot of $S(s;\boldsymbol{x}^*)$

Fig. 3. Simulation results: Convergence property and bode plot

via the distributed constrained PSO algorithm using a cyclic-network topology presented in Section II-B.

In the following subsection, its effectiveness is evaluated through a simulation study, because it is difficult to guarantee the performance of the proposed method theoretically due to probabilistic nature of PSO. Also, the performance superiority of the proposed distributed constrained PSO scheme over our previous PSO method in Maruta et al. [8] in controller design problems is clearly demonstrated.

B. Numerical example

In order to show the effectiveness of our distributed PSO-based controller design methodology over the mixed probabilistic/deterministic approach by Fujisaki et al. [20], the example presented in their paper is handled. Note that, although it is omitted here due to the page limitation, various types of fixed-structure robust controllers (e.g., the numerical example 1 with multiple H_∞ norm specifications presented in Maruta et al. [8]) can be directly designed via our methodology with more promising results. In the following example, all computations are performed with the MATLAB Version 7.10.0.499 (R2010a) 64-bit.

Consider the unity feedback system $\Sigma[\boldsymbol{x}]$ consisting of

$$P(s) = \frac{17\,(1+s)\,(1+16s)\,\left(1-s+s^2\right)}{s\,(1-s)\,(90-s)\,(1+s+4s^2)}, \tag{15}$$

$$K(s) = \frac{\theta_0+\alpha_0 s+\theta_2 s^2}{1+\mu_0 s+\beta_2 s^2}. \tag{16}$$

Let $\boldsymbol{x} := (\theta_0, \alpha_0, \theta_2, \mu_0, \beta_2)^T$ denote the design parameter vector. Its initial search space is supposed to be given by $\mathbb{D} := \{\boldsymbol{x} \in \mathbb{R}^5 |\ -5 \leq x_i \leq 5,\ i = 1, 2, \cdots, 5\}$ based on the problem setting in Fujisaki et al. [20]. Note that the search spaces of θ_0 and θ_2 are not specified in their method, since these are not determined in a probabilistic way. On the other hand, in Fujisaki et al. [20], two types of fixed-structure controllers are designed so that the following two performance specifications are *separately* satisfied:

(i) The first one is designed to satisfy the following pole placement condition:

$$\mathrm{Re}\left[\lambda_{\max}(\Sigma[s;\boldsymbol{x}])\right] < -0.2, \tag{17}$$

(ii) The second one is designed to satisfy the following H_∞ performance condition:

$$\|W(s)S(s;\boldsymbol{x})\|_\infty < 1,\ W(s) := \frac{55\,(1+3s)}{1+800s}. \tag{18}$$

Here, in order to find the optimal design parameter vector $\boldsymbol{x}^* \in \mathbb{R}^5$ satisfying both of these conditions in (17) and (18) *simultaneously*, we solve (3) with (4) using $f_v(\boldsymbol{x}) = \mathrm{Re}\left[\lambda_{\max}(\Sigma[s;\boldsymbol{x}])\right]$ with $\mathbb{F} := \left\{\boldsymbol{x} \in \mathbb{R}^5 \middle| \substack{\mathrm{Re}\left[\lambda_{\max}(\Sigma[s;\boldsymbol{x}])\right] < 0,\\ \|W(s)S(s;\boldsymbol{x})\|_\infty - 1 < 0} \right\}$. The number of particles is set as $n_p = 300$, and the maximum PSO iteration number is $k_{\max} = 4000$. The parameters of particle evolution law in (6) are set as $c_0 = 0.75$ and $c_1 = c_2 = 1.6$. The number of neighbors of the ith particle is set as $n_s = 10$, which is two times of $n = 5$ as mentioned in Section II-B. Then, we run the proposed distributed constrained PSO algorithm 50 times.

The changes of the best objective function values at the first 800 iterations of 50 trials are illustrated in Fig. 3(a). This figure shows that the proposed PSO method can find the best solution with superior reliability as compared with the method adopted in Maruta et al. [8]. The above fact is also confirmed from the statistical comparison result given in Table I. It verifies that the proposed method provides the feasible solution, which achieves $\mathrm{Re}\left[\lambda_{\max}(\Sigma[s;\boldsymbol{x}])\right] < -0.2$, in all trials (100% success rate), while the PSO technique in Maruta et al. [8] achieves the same performance with 44% chance in whole trials. Furthermore, the standard deviation of the obtained feasible solutions explicitly verifies a remarkable reliability of the proposed distributed constrained PSO scheme in this type of fixed-structure robust controller synthesis problems.

TABLE I

STATISTICAL RESULT OF EXAMPLE

	Success rate	Objective function value: $\mathrm{Re}\,[\lambda_{\max}(\Sigma[s;\boldsymbol{x}])]$			
		Best	Worst	Mean	Standard deviation
Maruta et al. [8]	44% (22/50)	-0.593041	-0.560853	-0.590354	0.007106
This paper	100% (50/50)	-0.593041	-0.593041	-0.593041	0

The best controller parameters $\boldsymbol{x}^* := (\theta_0^*, \alpha_0^*, \theta_2^*, \mu_0^*, \beta_2^*)^T$ obtained from the above procedure are as follows:

$$\theta_0^* = -0.584077176726232,\ \alpha_0^* = -0.719319610966632,$$
$$\theta_2^* = -2.57431670869684,\ \mu_0^* = -0.552299376960132,$$
$$\beta_2^* = -1.5498348855402,$$

and the best value of $\mathrm{Re}\,[\lambda_{\max}(\Sigma[s;\boldsymbol{x}^*])]$ is -0.593041. The poles of the corresponding closed-loop system are -18.9979, $-0.5930 \pm 0.2947j$, $-0.5930 \pm 0.2946j$ and $-0.5930 \pm 0.2944j$, which verifies that the pole placement specification is guaranteed with a considerable margin. Fig. 3(b) shows the gain plot of the sensitivity function $S(s;\boldsymbol{x}^*)$, which verifies the given constraint condition is guaranteed.

IV. CONCLUSION

In this paper, we have proposed a new meta-heuristic approach with superior reliability and validity to the synthesis of fixed-structure controllers satisfying multiple control specifications. The main tool is the distributed particle swarm optimizer (PSO) with cyclic-network topology, which achieves significant reduction of the possibility of particles to be trapped into local optima (i.e., a premature convergence phenomenon of particles). In order to handle multiple specifications in controller design, the distributed PSO algorithm has been incorporated with the constraint handling method [8] using a virtual objective function. Since the proposed controller design method does not require strong back ground on sophisticated control theory and it does not depend on types of controller structure nor specifications, it is easy to use for most practical engineers. Its effectiveness has been shown through the detailed numerical example, which clearly demonstrates that the proposed distributed constrained PSO scheme gives a novel and powerful impetus to the fixed-structure robust controller syntheses.

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