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A Nonlinear Adaptive Observer Approach for State of Charge Estimation of Lithium-Ion Batteries

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***Abstract*—A state of charge estimation method for lithium-ion batteries is presented. First, the problem is formulated, and existing literature is reviewed. Then an equivalent circuit is used to model the battery, and an indirect nonlinear adaptive observer approach is developed to estimate the state of charge. Parameter identification and state estimation scheme are discussed. Simulation result based on real-world battery test data is shown to support the validity of the proposed method. Issues related to production code implementation are also addressed.**

I. Introduction

For hybrid vehicle technology, one of the key enablers is the energy storage system (ESS). While other types of hybrid vehicles are being developed, the majority of hybrid vehicles are so-called hybrid-electric vehicles, or HEV, which use batteries as ESS. The types of batteries in use include lithium-ion, NiMH, and lead-acid. Currently an increasing number of manufacturers are competing toward developing lithium-ion batteries for HEV, PHEV (plug-in HEV), and BEV (battery electric vehicle) applications due to the facts that lithium-ion batteries have higher power and energy densities, higher operating voltages, lower self-discharge rate, and longer cycle life.

Lithium-ion batteries have limitations: they cannot be over-charged or over-discharged; otherwise there may be permanent damages to the cells, even fire hazard. Lithium-ion cells within a battery pack have to be "balanced" using advanced methods such as cell balancing. Some useful techniques for NiMH cell balancing, for example overcharging the battery, can not be used here. In order to use lithium-ion battery for automotive propulsion purpose, these limitations have to be overcome.

Some fundamental control problems for lithium-ion batteries used for PHEV/BEV include:

- State of Charge (SOC) estimation;
- Power capability estimation;
- Health monitoring;
- Cell balancing

All these control problems are being dealt with in engineering practice by numerous OEMs and suppliers. Still, much work needs to be done in order to provide more accurate estimation of various battery control related variables in order to improve vehicle performance and fuel economy, to enhance system safety, to reduce system cost, and to improve battery life. One of the key issues to be discussed here is the SOC estimation. In this paper a model-based approach is taken to address this problem for lithium-ion batteries.

A. *Problem Statement*

SOC is defined as percentage of available charge as compared with max charge capacity. For a battery with capacity Q, charge/discharge efficiency η, input current I (charge current is considered negative), and sample period T_s, SOC can be calculated as below:

$$SOC(k+1)=SOC(k)-T_s * \frac{\eta * I(k)}{Q} \quad (1)$$

A straightforward method of calculating SOC is to use ampere-hour integration (Equation (1)). Unfortunately, due to the nature of the method, the SOC as calculated in Equation (1) tends to drift from real SOC due to error in initial SOC value estimation, current sensor inaccuracy, error in charge efficiency estimation, and battery capacity change over its life span. Hence, there are numerous model-based SOC estimation methods to overcome these shortcomings. Some of the related publications are reviewed below.

B. *Literature Review*

Chiasson et al [1] used an equivalent circuit model of a class of electrochemical batteries to perform SOC estimation via open circuit voltage (OCV) estimation. Dominico et al [2] used an averaged, first principle, electrochemical lithium-ion battery model for SOC estimation using Extended Kalman Filter (EKF). Laig-Horstebrock et al [3] used a model consisting of a resistor in series with Warburg impedance. Mean voltage across the capacitors of the Warburg impedance is chosen as a measure of the SOC. Tate et al [4] used an EKF for SOC and hysteresis estimation, for NiMH battery with a generic nonlinear system model. Tate et al [5] used an equivalent circuit model consisting of a voltage source (OCV), a resistor, an RC network and another

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voltage source (representing hysteresis related to charge and discharge activities). The EKF based estimator state consists of SOC, ohmic resistance, and double layer voltage. Plett [6] used a generic nonlinear system model as equivalent circuit model, and used EKF for state variable estimations with SOC as one of the states. Plett [7] further developed a Sigma-Point KF approach to combine parameter estimation and SOC estimation. Verbrugge et al [8] used an equivalent circuit model consisting of a resistor, a RC network and an OCV, and used regression method, based on measurement history of terminal voltage, input current and temperature, to determine the resistor value and the SOC value. Similar work can also be found at Verbrugge et al [9] for battery SOC and battery state of health estimation. Ashizawa et al [10] used an equivalent circuit model and an adaptive filter to estimate the model parameters and then the OCV.

C. *Motivation of the Project*

Given that lithium-ion batteries exhibit vastly different characteristics compared with lead-acid and NiMH batteries, it is clear that most existing model-based SOC estimation methods may either be inappropriate, or too complicated in terms of CPU utilization and/or memory demand, for practical use. To this end, an indirect nonlinear adaptive observer approach is developed to estimate lithium-ion battery SOC. This approach is based on the understanding of the fundamental properties of the lithium-ion batteries, the requirement on the accuracy of SOC estimation, and the awareness of the limitations of microcontroller used to program such control algorithms in automotive engineering applications.

D. *Outline of the Paper*

This paper is organized as follows. In section II, some lithium-ion battery properties are discussed. In section III, the model to be used for SOC estimation purpose, as well as the overall architecture are provided. Identification method for parameters of the model is discussed. Then, the SOC estimation scheme, a nonlinear adaptive observer, is presented. In section IV, simulation results are presented to show the validity of the proposed method. In section V, implementation issues are addressed. Conclusion is provided in section VI.

II. PROBLEM FORMULATION

In this section the problems are formulated. Models associated with the SOC estimation problem are described.

A. *SOC Estimation: SOC vs. Open Circuit Voltage*

Due to the nature and history of the electrochemical reactions within the battery, even when no load is attached the measurement of terminal voltage is often not the true OCV. For a given family of batteries, the settling time varies. For the lithium-ion batteries studied in this paper, the time for the transient to settle is in the range of a few hundred seconds.

A sample SOC-OCV curve at given temperature is shown below.

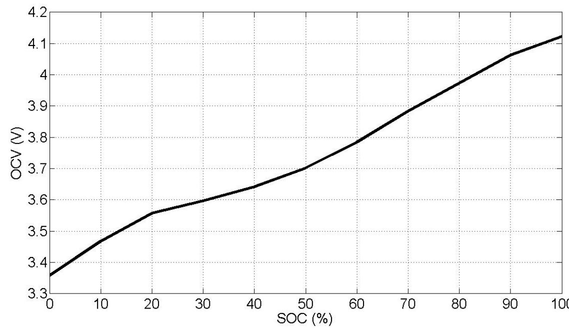

Figure 1 Sample SOC-OCV Curve

A key property of lithium-ion batteries is that OCV is a monotonically increasing, one-to-one function of SOC, and vice versa. The one-to-one mapping between OCV and SOC leads to simplification in the modelling and estimation for the related battery variables. In other words, an estimation of SOC is equivalent to estimation of OCV and vice versa.

Battery charge efficiency is considered a constant value throughout this study. It turns out that the proposed method is robust against this assumption.

B. *Plant Model and Related SOC Estimation Problem*

The lithium-ion battery cell is modelled as a nonlinear dynamic systems with inputs (current, and temperature), and output (cell voltage), and various states, including the OCV. The SOC estimation is based on a so-called indirect nonlinear adaptive observation scheme: The plant model parameters are first identified and the identified parameters are fed to the OCV estimator. The estimated OCV is fed back to the parameter identifier, as it depends on a known value of the OCV.

III. RESULTS

A. *Model*

There are two main approaches to the modeling of lithium-ion batteries, the first principles model approach [2], and the equivalent circuit model approach [1]. While there are advantages in using first principles model for battery controls, due to limited capabilities of microcontrollers used in automotive control modules, an equivalent circuit model is used. This model consists of a voltage source, known as the open circuit voltage of the battery, in series connection with a resistor representing the electrolyte and contactor resistance, and a number of RC networks representing electrochemical kinetics and diffusion relaxation [11].

First, the following notations are used:

Inputs	$I \in R$	Battery current
	$T \in R$	Battery temperature

Outputs	V$\in R^{+}$	Battery cell voltage
Noise:	ε	Gaussian noise, zero mean, covariance matrix R_{ε}
	ω	Gaussian noise, zero mean, covariance matrix R_{ω}
	ζ	Gaussian noise, zero mean, covariance matrix R_{ζ}
States	Xp$\in R^{P,+}$	Vector of model parameters
	OCV$\in R^{+}$	Open circuit voltage
	Xv$\in R^{(N-1)}$	Voltage of the capacitances in the RC networks

P, N: integers

The parameters associated with the battery cell model are considered slowly time-varying. As such, they can be modelled as follows:

$$X_p(k+1) = X_p(k) + \varepsilon(k) \tag{2}$$

The state-space equation of the overall battery cell is:

$$\begin{bmatrix} OCV(k+1) \\ \\ X_v(k+1) \end{bmatrix} = A(Xp(k)) * \begin{bmatrix} OCV(k) \\ \\ X_v(k) \end{bmatrix} + f(OCV(k), X_p(k)) * I(k) + \omega(k)$$

$$V(k) = C(Xp(k)) * \begin{bmatrix} OCV(k) \\ \\ X_v(k) \end{bmatrix} + D(Xp(k)) * I(k) + \varsigma(k) \tag{3}$$

Remark 1: The assumptions about noises ε. ω, ζ are not essential for this study.

Remark 2: For the lithium-ion battery studied in this paper, the nonlinear term f is rather mild. Indeed, the vector f in Equation (3) only has one nonlinear component, as it can be represented as:

$$f = \begin{bmatrix} f_1(OCV\ (k)) \\ b_2(X_p(k)) \\ \dots \\ b_{N-1}(X_p(k)) \\ b_N(X_p(k)) \end{bmatrix} \tag{4}$$

Within Equation (4), only the input matrix entry for state variable OCV has a nonlinear term, the others are linear terms dependent on parameter vector $X_p(k)$.

Based on experimental data, the nonlinear term f_1 can be described as a piecewise linear function:

$$f_1(OCV\ (k)) = \begin{cases} l_1, & if \quad OCV\ (k) \in [OCV_0, OCV_1] \\ l_2, & if \quad OCV\ (k) \in [OCV_1, OCV_2] \\ \cdots & \cdots \qquad \cdots \\ l_{M-1}, & if \quad OCV\ (k) \in [OCV_{M-2}, OCV_{M-1}] \\ l_M, & if \quad OCV\ (k) \in [OCV_{M-1}, OCV_M] \end{cases} \tag{5}$$

Where l_1, l_2, …, l_M are constants, and OCV_0, OCV_1, …, OCV_M are constant values between OCV_{min} and OCV_{max}, the minimum and maximum possible values of OCV. M is some integer value.

From the above description, the SOC estimation problem is actually a state estimation problem for a class of piece-wise linear systems. Another important factor to consider is that the constant values of l_1, l_2, …, l_M, are not too far apart. In other words, the nonlinearity is rather mild.

There are many existing results on constructing state estimators for this class of systems, for example, [5,6,9,12]. The proposed approach is to use an indirect nonlinear adaptive observer to first obtain parameter estimation, and then construct an equivalent nonlinear observation scheme.

B. *Block Diagram*

The overall battery SOC estimation block diagram using discrete time domain variables is shown below:

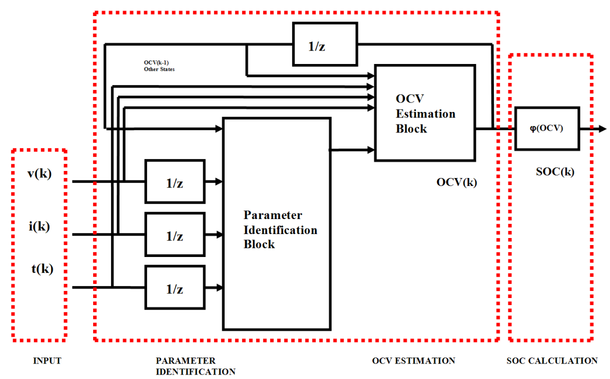

Figure 2 Block Diagram for SOC Estimation

Here, v(k), i(k) and t(k) are the measurements of cell voltage, current and temperature (not used directly) at the k-th time instant. The "Identification Block" provides estimation of plant parameters. These parameters are fed into the OCV estimation block, where the OCV is estimated (and translated into SOC after the output phase). Due to the nature of the nonlinear function φ: OCV → SOC, both OCV estimation and SOC estimation are interchangeably used in this study.

The overall estimation task is allocated in four blocks: measurements, parameter identification, OCV estimation, and SOC estimation. The measurement block is straightforward, while the SOC estimation block is direct algebraic transformation. Below only the parameter identification block and the OCV estimation block are described.

C. *Parameter Identification Block*

For the parameter identification block, the inputs are the current and temperature measurements (the temperature measurement is not directly used; rather, it is used for initialization purpose only), and the last estimated OCV value and other internal state variables; the output is the terminal voltage. The state set is the vector of unknown parameters.

While there are many ways of identifying the parameters based on Equations (2) and (3), a nonlinear algebraic transformation g: $X_p \rightarrow \hat{X}_p$, is used to make the system linear in terms of I/O mapping with regard to the transformed parameter vector $\hat{X}_p$. Such a nonlinear algebraic transformation performed on the model parameter vector, can alleviate the difficulty of dealing with nonlinear terms in the parameter identification block directly using KFs[12]. The drawback of this approach is that the assumption about noise type and covariance may not be valid anymore. Further, as it shall be discussed later, identification based on Equation (6) may involve higher order derivatives of related signals, which may not be easy to obtain. Consider the computational complexity of other methods (for example, EKF [5,12]), it is a trade-off well-worth taken.

Based on the transformed parameter vector and Equations (2) (3), a generalized Input/Output relationship with regard to $\hat{X}_p$ can be established as follows:

$$Y(k) = \Phi^T(k) * \hat{X}\, p(k) \tag{6}$$

Where Y(k) is a vector consisting of terminal voltage, estimated open circuit voltage, and their derivatives, as well as combinations of these variables. $\Phi(k)$ is a matrix with its elements made up from other state variables, input and their combinations. $\hat{X}\, p(k)$ is the transformed parameter vector to be identified.

Once the linear I/O map (6) is established, it is straightforward to use existing method and related software (for example, Matlab/Simulink) to perform the parameter identification task. The approach taken here is the well-known KF parameter identification scheme [12, 13].

D. *OCV Estimation Block*

The inputs of the OCV estimation block are current and the parameters obtained in parameter identification block. The output of this block is the terminal voltage. The state variables are the OCV, and a number of internal states, which will not be used by vehicle system controller; rather, they are used by the parameter identification block. With the parameter vector represented by $\hat{X}_p$, system (3) can be written as:

$$\begin{bmatrix} OCV(k+1) \\ \\ X_v(k+1) \end{bmatrix} = A(\hat{X}\, p(k)) * \begin{bmatrix} OCV(k) \\ \\ X_v(k) \end{bmatrix} + \begin{bmatrix} f_1(OCV(k)) \\ b_2(\hat{X}_p(k)) \\ \vdots \\ b_{n-1}(\hat{X}_p(k)) \\ b_n(\hat{X}_p(k)) \end{bmatrix} * I(k) + \omega(k)$$

$$V(k) = C(\hat{X}\, p(k)) * \begin{bmatrix} OCV(k) \\ \\ X_v(k) \end{bmatrix} + D(\hat{X}\, p(k)) * I(k) + \varsigma(t) \tag{7}$$

A nonlinear observer (NLO) is constructed for OCV estimation purpose. The motivation is as follows. While it is true that KF is the optimal observer for linear time-invariant systems with Gaussian noise in both state and output measurements, the system in this study is neither linear nor necessarily having Gaussian type noise in the state and/or output measurements. Notice that SOC is a slow-varying variable itself, with proper filtering of measurement variables, it is possible to alleviate the impact of measurement noises without using more computationally demanding method such as EKF. Further, in order to use the EKF, deliberate calibrations have to be done. One more important point to make is that even with proper modelling of the nonlinear components in Equation (7), it is not clear whether the closed loop is stable. To this end, it is meaningful to investigate alternative approach. Starting from Equation (7), a Luenberger observer is constructed below. With knowledge from real world battery testing regarding the model parameters, it is possible to construct feedback gain K (see below) such that the closed loop system is stable under all operating conditions:

$$\begin{bmatrix} \tilde{OCV}(k+1) \\ \\ \tilde{X}_v(k+1) \end{bmatrix} = A(\hat{X}\, p(k)) * \begin{bmatrix} \tilde{OCV}(k) \\ \\ \tilde{X}_v(k) \end{bmatrix} + \begin{bmatrix} f_1(\tilde{OCV}(k)) \\ b_2(\hat{X}_p(k)) \\ \vdots \\ b_{n-1}(\hat{X}_p(k)) \\ b_n(\hat{X}_p(k)) \end{bmatrix} * I(k) + K * (V(k) - \tilde{V}(k))$$

$$\tilde{V}(k) = C(\hat{X}\, p(k)) * \begin{bmatrix} \tilde{OCV}(k) \\ \\ \tilde{X}_v(k) \end{bmatrix} + D(\hat{X}\, p(k)) * I(k) \tag{8}$$

K is an Nx1 real vector to be calibrated. When proper model is obtained and parameter identified, the calibration task is to ensure that system (8) is stable, which amounts to a pole-placement problem for systems (3)(4)(5)(7). The noise terms in Equation (7) are omitted in constructing Equation (8).

While a stability result for observer has not been established yet, extensive simulation results have shown that with proper selection of gain matrix K, the observer (Equation (8)) converges nicely to target state values with reasonable convergence speed for numerous test profiles.

Remark 3: One advantage of the NLO (Equation (8)) is that it does not depend on assumptions about the noise types in measurements so the calibration effort needed will be much lighter. It will be very interesting to compare this method with KF type observer when other classes of measurement noises are introduced.

Remark 4: Another advantage of this approach is the modularity of the battery control software: the parameter identification block can be re-used for other battery control tasks such as battery power capability estimation and battery health management.

Remark 5: The proposed indirect nonlinear adaptive observer method is applicable for other types of batteries whose SOC and OCV relationship can be mapped similar to: φ: OCV $\rightarrow$ SOC, where φ is one-to-one and monotonically increasing.

IV. Simulation Study

Extensive simulation studies have been carried out. Here one example is shown. A Ford internal driving cycle related test data from battery lab. is used to test the algorithms. For the drive cycle, the OCV estimation from the NLO is obtained, and compared with OCV obtained by reverse lookup of φ: OCV $\rightarrow$ SOC, where SOC is calculated based on ampere-hour integration method. In order to have a better understanding on how this NLO approach works, an OCV estimation scheme using an EKF for system (7) is also constructed and tested for the same drive cycle. Test results are plotted, respectively, for comparison purpose.

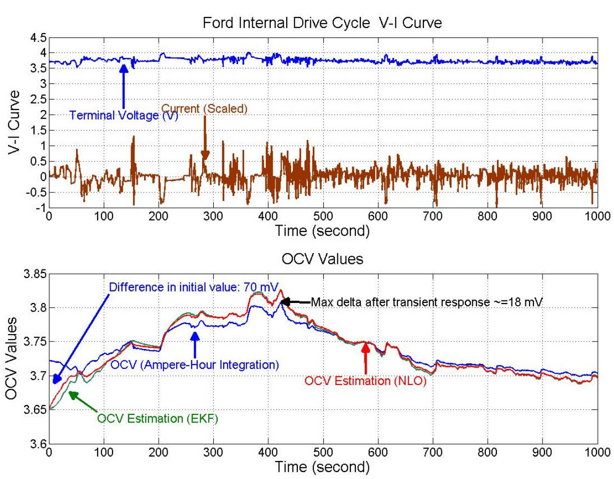

Figure 3: OCV Estimation for a Ford Drive Cycle

In Fig. 3, the first subplot is the V-I curve, with current I being scaled; the second plot shows an OCV obtained by reverse lookup of φ: OCV $\rightarrow$ SOC, where SOC is calculated based on ampere-hour integration method; the OCV estimation via NLO, and the OCV estimation via EKF. For both NLO and EKF, initial OCV values are biased from real values by 70 mV.

From the plots in Figure 3, it is clear that for the Ford internal drive cycle data based test, the NLO based OCV estimation is comparable to the EKF based OCV estimation and follows closely with the ampere-hour integration based OCV value after certain transient period due to parameter learning process and biased initial values in the range of 70 mV. Both methods show differences with ampere-hour integration method based OCV value, which may not be exact due to initial value bias and ampere-hour integration errors. The maximum difference between OCV from NLO estimation and that of the ampere-hour integration method at any give time point except for the initial transient response period for the entire drive cycle is about 18 mV, or about 3% SOC.

V. Implementation Issues

To implement the above methods into production battery control module, a number of issues have to be resolved.

A. *Signal Filtering and Synchronization*

The noises associated with current and voltage measurements in a vehicle operating environment are not necessarily Gaussian. The measurement noise issue is more profound since an indirect nonlinear adaptive observer approach is used, wherein parameters have to be identified utilizing numerical differentiation [15,16,17] of the measured variables. A number of filtering methods is investigated. One approach found to be very effective is the Savitzky-Golay filter [16]. A lower order SG filter is used for all measured signals, and appropriate synchronization step is taken to make sure the applied signals or their combinations are in synchronization with each other (i.e., signals used in Equations (6,8)).

B. *Initial Value Determination*

As it was already discussed, the initial SOC value can be obtained rather accurately after extended period of vehicle shutdown. The initial values for the related model parameters can be difficult to set, however, as most of them depend on ambient temperature, state of charge, and stage of life of the battery. A consequence of this is that the last estimated parameter values are normally not very useful for SOC estimation purpose at next vehicle start up, no matter how accurate they are. One way to provide reasonably accurate initial plant model parameters is to use battery life test data and open loop parameter identification technique to construct look-up tables for initialization purpose.

C. *Stability and Performance Monitoring*

Since the proposed approach to SOC estimation is essentially an indirect nonlinear adaptive observer, it has the inherent characteristics of all (nonlinear) adaptive control methods [14]. For automotive applications, the following questions are of particular interests: Is the closed-loop system stable? Is the performance acceptable? What to do if the answers are "no"?

To improve the robustness of proposed solution, a high-level supervisor is added to monitor the stability and performance of the proposed method. Once it is determined that the performance and/or stability of the closed loop system is compromised, the system may reset on its own and proper control actions are taken so the vehicle does not need to be shut down.

D. *Processor Load and Capacity Consideration*

To implement the proposed algorithms in production battery control module, one has to consider the limitations of the related hardware in terms of processor load (CPU-utilization ratio) and available RAM/ROM[18] [19]. The reason is that SOC is essentially cell level property. For a given battery pack with series connection configuration, one may have to deal with near 100 cells for lithium-ion battery used in PHEV/BEVs. If KFs are used for all these cells for parameter identification, the demand for computing power (CPU-utilization) as well as RAM/ROM usage will be tremendous. In automotive controls design, one has to take cost factor into consideration. To this end, some investigation has been done to make sure accurate SOC estimation is obtained while limitations of battery control module are dealt with properly.

VI. CONCLUSIONS

In this paper a model-based SOC estimation method is presented. The overall approach is an indirect nonlinear adaptive observer. KF is used in the parameter identification block, while an NLO is constructed for the state observation purposes. These algorithms are tested using real world battery test data. Simulation results show that OCV estimations are consistent with existing non-model based algorithms. Issues related to practical implementation of the proposed algorithms in real world battery controls hardware modules are also addressed.

It would be very interesting to see how the OCV estimation compares on a point-to-point basis with true OCV if the latter can be experimentally determined. Also, both the parameter estimation block and OCV estimation block parameters have to be calibrated carefully so better performance can be obtained. The other interesting issue to be studied is the robustness of this method against charge efficiency map inaccuracy and battery capacity change. These topics will be investigated in the near future.

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REFERENCES

[1] J. Chiasson and B. Vairamohan, "Estimating the State of Charge of a Battery," *IEEE Transactions on Control Systems Technology*, vol. 13, no. 3, pp. 465~470, 2005

[2] D. D. Domenico, G. Fiengo, and A. Stefanopoulou, "Lithium-Ion Battery State Of Charge Estimation With A Kalman Filter Based On A Electrochemical Model," in *Proceedings of the 17th IEEE International Conference on Control Applications*, IEEE. San Antonio, Texas, USA

[3] H. Laig-Horstebrock, E. Meissner, and G. Richter, "Method For Determining The State Of Charge And Loading Capacity Of An Electrical Storage Battery," US *Patent and Trademark Office, Patent 6362598*, Mar. 26, 2002.

[4] E. D. Tate, M. W. Verbrugge, and S. Sarbacker, "State Of Charge Prediction Method And Apparatus For A Battery," *US Patent and Trademark Office, Patent 6441586*, Apr. 27, 2002.

[5] E. D. Tate, M. W. Verbrugge, B. Koch, and D. Frisch, "Method For Estimating States And Parameters Of An Electrochemical Cell," *US Patent and Trademark Office, Patent 7109685*, Sept. 19, 2006.

[6] G. L. Plett, "Method and Apparatus for a Battery State of Charge Estimator," *US Patent and Trademark Office, Patent 6534954,* Mar. 18, 2003.

[7] G. L. Plett, "System and Method for Determining both an Estimated Battery State Vector and an Estimated Battery Parameter Vector," *US Patent and Trademark Office, Patent 7521895,* Apr. 21, 2009.

[8] M. W. Verbrugge, E. D. Tate, D. Frisch, R. Ying, B. J. Koch, and S. Sarbacker, "Method and Apparatus for a Battery State of Charge Estimator," *US Patent and Trademark Office, Patent 6639385*, Oct. 28, 2003.

[9] M. W. Verbrugge, E. D. Tate, D. R. Frisch, and B. J. Koch, "Method and Apparatus for Generalized Recursive Least-Square Process for Battery State of Charge and State of Health," *U.S. Patent Application Publication 2004/0162683*, Aug. 19, 2004.

[10] H. Ashizawa, H. Nakamura, and H. Asai, "State-Of-Charge Estimating Device of Secondary Battery," *US Patent and Trademark Office, Patent 7352156*, Apr. 1, 2008.

[11] Hanlei Zhang, and Mo-Yuen Chow, "Comprehensive Dynamic Battery Modeling for PHEV Applications," in Proceedings of IEEE Power and Energy Society General Meeting, Minneapolis, Minnesota, USA, July 25- 29, 2010.

[12] G. Welch and G. Bishop, "An Introduction to the Kalman Filter," http://www.cs.unc.edu/~welch/kalman/kalmanIntro.html

[13] Matlab Help Manual, Mathworks Inc.

[14] Ioannou, P. A., Sun, J., Robust Adaptive Control, Prentice Hall, Inc.,1996

[15] S. Diop, J. W. Grizzle, F. Chaplais, On Numerical Differentiation Algorithms for Nonlinear Estimation. In the *Proceedings of the IEEE Conference on Decision and Control*, Sydney, Australia, 2000.

[16] Abraham Savitzky, Marcel J. E. Golay, Smoothing and Differentiation of Data by Simplified Least Squares Procedures, *Analytical Chemistry*, Vol. 36, No. 8, July 1964

[17] P. Holoborodko, "Numerical Methods," www.holoborodko.com.

[18] G. Plett, Efficient Battery Pack State Estimation using Bar-Delta Filtering, *EVS24 Stavanger*, Norway, May 13 - 16, 2009

[19] Carmelo Speltino, Domenico Di Domenico, Giovanni Fiengo, Anna G. Stefanopoulou, "Cell Equalization In Battery Stacks Through State Of Charge Estimation Polling", *2010 American Control Conference*.