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Adaptive Controller Design for Uncertain Nonlinear Systems with Input Magnitude and Rate Limitations

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***Abstract*— An adaptive controller for a class of multiple-input-multiple-output (MIMO) uncertain nonlinear systems with extern disturbance and control input limitations is presented in this paper. The controller is designed with a priori consideration of input limitation effects, hence it can generate control signals satisfying input limitations. This controller uses adaptive radial basis function (RBF) neural networks to approximate the unknown nonlinearities. To compensate the effects of input limitations, an auxiliary system is constructed and used in neural network parameter update laws. Furthermore, in order to deal with approximation errors for unknown nonlinearities and extern disturbances, a supervisory control is designed, which guarantees that the closed loop system achieves a desired level H^∞ tracking performance. The closed loop system performance is analyzed by Lyapunov method. Steady state and transient tracking performance index are established and can be adjusted by design parameters. Computer simulations are presented to illustrate the efficiency and tracking performance of the proposed controller.**

Index Terms*— Input Saturation, RBF Neural Network, Adaptive Control, Nonlinear System, H^∞ Control Performance

I. INTRODUCTION

Every physical control system contains actuators with amplitude and rate limitations, such as the elevator of an aircraft can only generate a limited force or torques in a limited rate. Magnitude limitation or rate limitation of actuators is one of the main sources of control system performance limitation. In controller design, actuator dynamics should be considered. The controllers that ignore actuator limitations may cause the closed loop system performance to degenerate or even make the closed system unstable, and decrease the lifetime of the actuators, or damage the actuators. Hence how to incorporate the actuator dynamics in controller design is a research subject both having practical interest and theoretical significance.

The design of stabilizing controllers with a priori consideration of the actuator saturation effect for nonlinear systems with unknown nonlinearities and external disturbance is a challenging problem. For uncertain nonlinear single-input-single-output (SISO) systems with input saturation, Zhou [1, Chapter 11] proposed an adaptive backstepping controller which took the input saturation into controller design. An auxiliary system was constructed to compensate the effect of saturation. J. Farrell et.al presented adaptive backstepping approach [2] and online approximation based adaptive backstepping approach [3–6] for unknown nonlinear systems with known magnitude, rate, bandwidth constraints on intermediate states or actuators without disturbance. Those approaches also used constructed auxiliary systems for generating a modified tracking error to guarantee stability during saturation.

In this paper, we will address the problem of controlling a class of multiple-input-multiple-output (MIMO) uncertain nonlinear systems in the presence of disturbances and control input limitations. In the controller design process, adaptive RBF neural networks are used to approximate unknown nonlinearities. In order to deal with actuator limitations, an auxiliary system is constructed and used in parameter update laws of the RBF neural network to compensate the effects of input limitations. A supervisory control is designed to attenuate the effects of approximation errors and external disturbance. The performance of the closed loop system is obtained through Lyapunov analysis. Explicit bounds on the performance of the tracking error in terms of design parameters are given. Hence, the bounds of tracking errors can be adjusted by tuning the design parameters. The proposed controller can generate control signals satisfying actuator magnitude and rate limitations, and guarantee a H^∞ tracking performances of the closed loop system.

The rest of this paper is organized as follows. In Section II, the problem statement is presented. In Section III, the adaptive control scheme is discussed, and the performance is analyzed. A numerical example is shown in Section IV. Section V concludes the paper. Throughout this paper, $|\cdot|$ indicates the absolute value, $\|\cdot\|$ indicates the Euclidean vector norm, and $\|\cdot\|_2$ indicates the L_2 norm.

II. PROBLEM FORMULATION

Consider the following MIMO fully-actuated affine nonlinear system

$$\begin{aligned} \dot{\mathbf{x}} &= \mathbf{f}(\mathbf{x}) + \sum_{i=1}^{m} \mathbf{g}_i(\mathbf{x}) u_i + \mathbf{d} \\ y_j &= h_j(\mathbf{x}) \quad j = 1, \cdots, m \end{aligned} \tag{1}$$

where $\mathbf{x} \in \mathbb{R}^n$ is the state, $\mathbf{f}(\mathbf{x}), \mathbf{g}_i(\mathbf{x}) (i = 1, \cdots, m)$ are unknown but smooth vector fields, $\mathbf{y} = (y_1, \cdots, y_m)^T \in \mathbb{R}^m$ is the system output, $h_1(\mathbf{x}), \cdots, h_m(\mathbf{x})$ are continuous functions, $\mathbf{d} = (d_1, \cdots, d_n)^T$ is extern disturbance. d_i is unknown but bounded. $u_i, i = 1, \cdots, m$ are inputs, which satisfy the following constraints

$$|u_i| \le u_{i\max}, \quad |\dot{u}_i| \le v_{i\max} \tag{2}$$

where $u_{i\max}, v_{i\max}$ are positive constants.

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The control objective is to find control laws which satisfy magnitude and rate constraints that make y_i follow a given bounded reference signal y_{id} with a H^∞ tracking performance in the presence of extern disturbance.

We make the following assumptions on System (1):

Assumption 1: All the states of this system are observable, and the output signals and references signals are continuous differentiable.

Assumption 2: System (1) has a (vector) relative degree $\{r_1, \cdots, r_m\}(\sum_{i=1}^m r_i = n)$ at its state space.

Assumption 3: $L_{\mathbf{d}} L_{\mathbf{f}}^{\sigma_i} h_i(\mathbf{x}) = 0 (i = 1, \cdots, m)$ for any $\sigma_i < r_i - 1$, where $L_{\mathbf{f}} h_i$ represents the Lie derivative of h_i along vector field $\mathbf{f}(\mathbf{x})$, and $L_{\mathbf{f}}^k h_i$ is defined recursively as $L_{\mathbf{f}}^k h_i \stackrel{\text{def}}{=} L_{\mathbf{f}}(L_{\mathbf{f}}^{k-1} h_i)$. σ_i is called the *disturbance characteristic index* [7] of y_i.

According to Assumption 2, there exist r functions $\phi_{i1} = h_i(\mathbf{x}), \phi_{i2}(\mathbf{x}) = L_{\mathbf{f}} h_i(\mathbf{x}), \cdots, \phi_{ir_i}(\mathbf{x}) = L_{\mathbf{f}}^{r_i-1} h_i(x)$ such that under the coordinate transformation $(z_{11}, \cdots, z_{1r_1}, \cdots, z_{m1}, \cdots, z_{mr_m})^T = \mathbf{\Phi}(\mathbf{x}) = (\phi_{11}, \cdots, \phi_{1r_1}, \cdots, \phi_{m1}, \cdots, \phi_{mr_m})^T$, System (1) can be transformed into the following form

$$\begin{aligned} \dot{z}_{i1} &= z_{i2}, \cdots, \dot{z}_{i,r_i-1} = z_{ir_i} \\ \dot{z}_{ir_i} &= b_i(\mathbf{z}) + \sum_{j=1}^{m} a_{ij}(\mathbf{z}) u_j + c_i(\mathbf{z}) \\ y_i &= z_{i1} \qquad i = 1, \cdots, m \end{aligned} \tag{3}$$

where $\mathbf{z} = (\mathbf{z}_1^T, \cdots, \mathbf{z}_m^T)^T, \mathbf{z}_i = (z_{i1}, \cdots, z_{ir_i})^T, i = 1, \cdots, m,\ a_{ij}(\mathbf{z}) = L_{\mathbf{g}_j} L_{\mathbf{f}}^{r_i-1} h_i(\mathbf{\Phi}^{-1}(\mathbf{z})), b_i(\mathbf{z}) = L_{\mathbf{f}}^{r_i} h_i(\mathbf{\Phi}^{-1}(\mathbf{z})), c_i(\mathbf{z}) = L_{\mathbf{d}} L_{\mathbf{f}}^{r_i-1} h_i(\mathbf{\Phi}^{-1}(\mathbf{z}))$.

System (3) characterizes the *norm form* [8] of System (1) with a (vector) relative degree $\{r_1, \cdots, r_m\}$, and has a general form as follows

$$\begin{aligned} \dot{x}_{i1} &= x_{i2}, \cdots, \dot{x}_{i,r_i-1} = x_{ir_i} \\ \dot{x}_{ir_i} &= f_i(\mathbf{x}) + \sum_{j=1}^{m} g_{ij}(\mathbf{x}) u_j + d_i \\ y_i &= x_{i1} \qquad i = 1, \cdots, m \end{aligned} \tag{4}$$

where $\mathbf{x} = (x_{11}, \cdots, x_{1r_1}, x_{21}, \cdots, x_{2r_2}, \cdots, x_{m1}, \cdots, x_{mr_m})^T \in \mathbb{R}^{\sum_{i=1}^m r_i}$; $f_i(\mathbf{x})(i = 1, \cdots, m), g_{ij}(i, j = 1, \cdots, m)$ are smooth functions; d_i still represents bounded extern disturbance, $u_i(i = 1, \cdots, m)$ are control inputs satisfying the constraints (2). System (4) also can be rewritten in the following compact form

$$\mathbf{Y}^{(\mathbf{r})} = \mathbf{F}(\mathbf{x}) + \mathbf{G}(\mathbf{x})\mathbf{u} + \mathbf{d} \tag{5}$$

where

$$\mathbf{F}(x) = \begin{bmatrix} f_1(\mathbf{x}) \\ \vdots \\ f_m(\mathbf{x}) \end{bmatrix}, \mathbf{u} = \begin{bmatrix} u_1 \\ \vdots \\ u_m \end{bmatrix}, \mathbf{d}(\mathbf{x}) = \begin{bmatrix} d_1 \\ \vdots \\ d_m \end{bmatrix}$$

$$\mathbf{G}(\mathbf{x}) = \begin{bmatrix} g_{11}(\mathbf{x}) & \cdots & g_{1m}(\mathbf{x}) \\ \vdots & \ddots & \vdots \\ g_{m1}(\mathbf{x}) & \cdots & g_{mm}(\mathbf{x}) \end{bmatrix}, \mathbf{Y}^{(\mathbf{r})} = \begin{bmatrix} y_1^{(r_1)} \\ \vdots \\ y_m^{(r_m)} \end{bmatrix}$$

In the following, the controller design for System (5) will be considered.

III. Controller Design

To begin, define $\tau_1, \cdots, \tau_m$ as follows

$$\tau_i = y_{1d}^{(r_i)} + \sum_{j=1}^{r_i} \lambda_{ij} e_i^{(j-1)} \quad i = 1, \cdots, m \tag{6}$$

where $y_{id}, i = 1, \cdots, m$ are the reference signals, $e_i = y_{id} - y_i (i = 1, \cdots, m)$ are the tracking errors, $\lambda_{i1}, \cdots, \lambda_{ir_i}$ are parameters to be chosen such that the roots of the equation $s^{r_i} + \lambda_{ir_i} s^{r_i-1} + \cdots + \lambda_{i2} s + \lambda_{i1} = 0$ in the open left-half complex plane. Let $\boldsymbol{\tau} = (\tau_1, \cdots, \tau_m)^T$. If $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are known and the constrains on control inputs are ignored, then based on dynamic inversion algorithm, the control law

$$\mathbf{u}_{c'} = \mathbf{G}^{\#}(\mathbf{x})(-\mathbf{F}(\mathbf{x}) + \boldsymbol{\tau} + \mathbf{u}_d) \tag{7}$$

can be applied to System (5) to achieve the following error dynamic system

$$\begin{bmatrix} e_1^{(r_1)} + \sum_{j=1}^{r_1} \lambda_{1j} e_1^{(j-1)} \\ \vdots \\ e_m^{(r_m)} + \sum_{j=1}^{r_m} \lambda_{mj} e_m^{(j-1)} \end{bmatrix} = -\mathbf{u_d} - \mathbf{d} \tag{8}$$

where $\mathbf{G}^{\#}(\mathbf{x})$ represents the generalized inversion [9] of $\mathbf{G}(\mathbf{x})$, $\mathbf{u_d} \in \mathbb{R}^m$ is a supervisor control used to attenuate the extern disturbance $\mathbf{d}$ and will be decided later.

Because $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are unknown vector and matrix respectively, the above control law (7) can not be implemented. Besides, there is no guarantee that $\mathbf{u}_{c'}$ satisfies the constraints (2). Neural networks [10], [11] or fuzzy logic systems [12–15] can be used as universal approximators to approximate any continuous functions at any arbitrary accuracy as long as the network is big enough or the fuzzy rules are sufficient. In this work, in order to treat this tracking control design problem, radial basis-function (RBF) neural networks are used to approximate the unknown functions, that is, $f_i(\mathbf{x}), i = 1, \cdots, m$, and $g_{ij}(\mathbf{x}), i, j = 1, \cdots, m$ are approximated as follows:

$$f_i(\mathbf{x}) \approx \hat{f}_i(\mathbf{x}|\mathbf{\Theta}_{f_i}) = \mathbf{\Theta}_{f_i}^T \mathbf{\Phi}_{f_i}(\mathbf{x}) \ i = 1, \cdots, m \tag{9}$$

$$g_{ij}(\mathbf{x}) \approx \hat{g}_{ij}(\mathbf{x}|\mathbf{\Theta}_{g_{ij}}) = \mathbf{\Theta}_{g_{ij}}^T \mathbf{\Phi}_{g_{ij}}(\mathbf{x}) \ i, j = 1, \cdots, m \tag{10}$$

where $\mathbf{\Theta}_{f_i} \in \mathbb{R}^{M_{f_i}}, \mathbf{\Theta}_{g_{ij}} \in \mathbb{R}^{M_{g_{ij}}}$ are weight vectors, and $\mathbf{\Phi}_{f_i}(\mathbf{x}) \in \mathbb{R}^{M_{f_i}}(\mathbf{x}), \mathbf{\Phi}_{g_{ij}}(\mathbf{x}) \in \mathbb{R}^{M_{g_{ij}}}(\mathbf{x})$ are radial bases, $M_{f_i}, M_{g_{ij}}$ are the corresponding dimensions of the bases.

Denote

$$\hat{\mathbf{F}}(\mathbf{x}|\mathbf{\Theta_F}) = \begin{bmatrix} \hat{f}_1(\mathbf{x}) \\ \vdots \\ \hat{f}_m(\mathbf{x}) \end{bmatrix} \hat{\mathbf{G}}(\mathbf{x}|\mathbf{\Theta_G}) = \begin{bmatrix} \hat{g}_{11}(\mathbf{x}) & \cdots & \hat{g}_{1m}(\mathbf{x}) \\ \vdots & \ddots & \vdots \\ \hat{g}_{m1}(\mathbf{x}) & \cdots & \hat{g}_{mm}(\mathbf{x}) \end{bmatrix} \tag{11}$$

In the control law design, $\hat{\mathbf{F}}(\mathbf{x}|\mathbf{\Theta_F})$ will be used as an estimation of $\mathbf{F}(\mathbf{x})$, and $\hat{\mathbf{G}}(\mathbf{x}|\mathbf{\Theta_G})$ will be used as an estimation of $\mathbf{G}(\mathbf{x})$.

Using the approximation (11) and considering the constraints imposed on the control inputs, we modify the control law (7) as follows:

$$\mathbf{u}_c = \hat{\mathbf{G}}^{\#}(\mathbf{x}|\mathbf{\Theta_G}) \left(-\hat{\mathbf{F}}(\mathbf{x}|\mathbf{\Theta_F}) + \boldsymbol{\tau} + \boldsymbol{\xi} + \mathbf{u_d}\right) \tag{12}$$

$$\mathbf{u} = \mathbf{sat}(\mathbf{u}_c) \tag{13}$$

where $\mathbf{sat}(\mathbf{u}_c)$ represents the magnitude and rate limitations on $\mathbf{u}_c$ and

$$\boldsymbol{\xi} = \begin{bmatrix} -\xi_{11}^{(r_1)} - \sum_{i=1}^{r_1} \lambda_{1i} \xi_{11}^{(i-1)} - \dot{\xi}_{1r_1} - c_{1r_1} \xi_{1r_1} \\ \vdots \\ -\xi_{m1}^{(r_m)} - \sum_{i=1}^{r_m} \lambda_{mi} \xi_{m1}^{(i-1)} - \dot{\xi}_{mr_m} - c_{mr_m} \xi_{mr_m} \end{bmatrix} \tag{14}$$

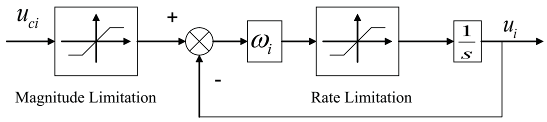

Fig. 1. Schematic for magnitude and rate limitations

$\xi_{11}, \cdots, \xi_{1r_1}, \xi_{21}, \cdots, \xi_{2r_2}, \cdots, \xi_{m1}, \cdots, \xi_{mr_m}$ are the states of the following constructed auxiliary system (15) which uses the difference of $\mathbf{u}_c$ before and after magnitude and rate limitations as input.

$$\begin{aligned} &\dot{\xi}_{i1} = \xi_{i2} - c_{i1}\xi_{i1}, \cdots, \dot{\xi}_{i,r_i-1} = \xi_{ir_i} - c_{i,r_i-1}\xi_{i,r_i-1} \\ &\dot{\xi}_{ir_i} = -c_{ir_i}\xi_{ir_i} + \sum_{j=1}^{m} \hat{g}_{ij}(\mathbf{x})(u_j - u_{cj}) \quad i = 1, \cdots, m \end{aligned} \tag{15}$$

u_{cj} is the j^{th} element of $\mathbf{u}_c$, and u_j is the j^{th} element of $\mathbf{u}$. $c_{ij}(i = 1, \cdots, m, j = 1, \cdots, r_i)$ are positive design parameters. $\boldsymbol{\xi}$ is used to compensate the effect of actuator saturation.

$\mathbf{u}_c$ is obtained from (7) according to *certainty equivalence principle* [16] which is widely used in adaptive control schemes. $\mathbf{u}_c$ also may not satisfy the constraints (2), hence magnitude and rate limitations are imposed to generate signal $\mathbf{u}$ which satisfies the constraints (2). $\mathbf{u}$ will be applied to System (5) as the control input. The difference of $\mathbf{u}_c$ before and after magnitude and rate limitations are used to generate the signal $\boldsymbol{\xi}$ which will be used in parameters update laws. The magnitude and rate limitations on $\mathbf{u}_c$, i.e., $\mathbf{sat}(\mathbf{u}_c)$, can always be implemented by assuming a first-order model for the dynamics of each component of $\mathbf{u}_c$, for example, $\dot{u}_i = \text{sat}_R(\omega_i(\text{sat}_M(u_{ci}) - u_i))$, where ω_i is a positive constant, $\text{sat}_R(\cdot), \text{sat}_M(\cdot)$ represent the rate and magnitude functions respectively. The function

$$\text{sat}_R(x) = \begin{cases} R & \text{if } x \geq R \\ x & \text{if } |x| < R \\ -R & \text{if } x \leq -R \end{cases} \tag{16}$$

and $\text{sat}_M(x)$ is defined similarly. Fig. 1 gives a visual description for this example.

In the following, we will specify the RBF parameter update laws for $\boldsymbol{\Theta}_{f_i}(i = 1, \cdots, m), \boldsymbol{\Theta}_{g_{ij}}(i, j = 1, \cdots, m)$ and supervisor control $\mathbf{u_d}$, so that desired tracking performance can be achieved. Applying the control law (12)-(13) to System (5) yields

$$\boldsymbol{\tau} - \mathbf{Y^{(r)}} + \boldsymbol{\xi}_1 = \hat{\mathbf{F}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{F}) - \mathbf{F}(\mathbf{x}) + \left(\hat{\mathbf{G}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{G}) - \mathbf{G}(\mathbf{x})\right)\mathbf{u} - \mathbf{u}_d - \mathbf{d} \tag{17}$$

where $\boldsymbol{\xi}_1 = (-\xi_{11}^{(r_1)} - \sum_{i=1}^{r_1} \lambda_{1i}\xi_{11}^{(i-1)}, \cdots, -\xi_{m1}^{(r_m)} - \sum_{i=1}^{r_m} \lambda_{mi}\xi_{m1}^{(i-1)})^T$.

Define the optimal approximation weight vectors for $f_i(i = 1, \cdots, m)$, $g_{ij}(i, j = 1, \cdots, m)$ as follows

$$\boldsymbol{\Theta}_{f_i}^* = \arg \min_{\boldsymbol{\Theta}_{f_i} \in \Omega_\mathbf{F}} \left[\sup_{\mathbf{x} \in \mathbf{U}_c} |f_i(\mathbf{x}) - \hat{f}_i(\mathbf{x}|\boldsymbol{\Theta}_{f_i})| \right] \tag{18}$$

$$\boldsymbol{\Theta}_{g_{ij}}^* = \arg \min_{\boldsymbol{\Theta}_{g_{ij}} \in \Omega_\mathbf{G}} \left[\sup_{\mathbf{x} \in \mathbf{U}_c} |g_{ij}(\mathbf{x}) - \hat{g}_{ij}(\mathbf{x}|\boldsymbol{\Theta}_{g_{ij}})| \right] \tag{19}$$

where $\Omega_\mathbf{F}, \Omega_\mathbf{G}, \mathbf{U}_c$ denote the sets of suitable bounds on $\boldsymbol{\Theta}_{f_i}, \boldsymbol{\Theta}_{g_{ij}}$, and $\mathbf{x}$ respectively. $\boldsymbol{\Theta}_{f_i}^*(i = 1, \cdots, m), \boldsymbol{\Theta}_{g_{ij}}^*(i, j = 1, \cdots, m)$ are constant vectors. The optimal approximations for $\mathbf{F}(\mathbf{x})$ and $\mathbf{G}(\mathbf{x})$ are denoted as $\hat{\mathbf{F}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{F}^*), \hat{\mathbf{G}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{G}^*)$ respectively. Define the minimum approximation error as

$$\mathbf{w} \stackrel{\text{def}}{=} \hat{\mathbf{F}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{F}^*) - \mathbf{F}(\mathbf{x}) + \left[\hat{\mathbf{G}}(\mathbf{x}|\boldsymbol{\Theta}_\mathbf{G}^*) - \mathbf{G}(\mathbf{x})\right]\mathbf{u}$$

According to neural network theory, the following assumption is reasonable:

Assumption 4: The minimum approximation error is square integrable, i.e., $\int_0^T \mathbf{w}^T\mathbf{w}dt < \infty$

Define the modified tracking error as

$$\bar{e}_i \stackrel{\text{def}}{=} y_{id} - y_i - \xi_{i1}, \quad i = 1, \cdots, m \tag{20}$$

Using the definition (20) and the optimal approximation for $\mathbf{F}(\mathbf{x})$, $\mathbf{G}(\mathbf{x})$, (17) can be rewritten as

$$\begin{aligned} &\bar{e}_i^{(r_i)} + \sum_{k=1}^{r_i} \lambda_{ik}\bar{e}_i^{(k-1)} = \hat{f}_i(\mathbf{x}|\boldsymbol{\Theta}_{f_i}) - \hat{f}_i\left(\mathbf{x}|\boldsymbol{\Theta}_{f_i}^*\right) - u_{d_i} - d_i \\ &+ \sum_{j=1}^{m} \left(\hat{g}_{ij}(\mathbf{x}|\boldsymbol{\Theta}_{g_{ij}}) - \hat{g}_{ij}\left(\mathbf{x}|\boldsymbol{\Theta}_{g_{ij}}^*\right)\right)u_j + w_i \;\; i = 1, \cdots, m \end{aligned} \tag{21}$$

where u_{d_i}, d_i, w_i are the i^{th} element of $\mathbf{u_d}, \mathbf{d}$, and $\mathbf{w}$, respectively. Defining $\bar{\mathbf{e}}_i = [\bar{e}_i, \cdots, \bar{e}_i^{(r_i-1)}]^T$, $\tilde{\boldsymbol{\Theta}}_{f_i} = \hat{\boldsymbol{\Theta}}_{f_i} - \boldsymbol{\Theta}_{f_i}^*$, $\tilde{\boldsymbol{\Theta}}_{g_{ij}} = \hat{\boldsymbol{\Theta}}_{g_{ij}} - \boldsymbol{\Theta}_{g_{ij}}^*$, then equations (21) can be rewritten in the following form:

$$\begin{aligned} \dot{\bar{\mathbf{e}}}_i =& \mathbf{A}_i\bar{\mathbf{e}}_i + \mathbf{B}_i\Big(\tilde{\boldsymbol{\Theta}}_{f_i}^T\boldsymbol{\Phi}_{f_i}(\mathbf{x}) \\ &+ \sum_{j=1}^{m}\left(\tilde{\boldsymbol{\Theta}}_{g_{ij}}^T\boldsymbol{\Phi}_{g_{ij}}(\mathbf{x})\right)u_j - u_{d_i} - d_i + w_i\Big) \;\; i = 1, \cdots, m \end{aligned} \tag{22}$$

where

$$\mathbf{A}_i = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ -\lambda_{i1} & -\lambda_{i2} & -\lambda_{i3} & \cdots & -\lambda_{ir_i} \end{bmatrix}, \mathbf{B}_i = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix} \tag{23}$$

Finally, for the nonlinear system (5), the following theorem can be obtained

Theorem 1: If we select the control law (13), adopt the following parameters update laws and u_{d_i}

$$\dot{\hat{\boldsymbol{\Theta}}}_{f_i} = -\Gamma_{f_i}\boldsymbol{\Phi}_{f_i}(\mathbf{x})\mathbf{B}_i^T\mathbf{P}_i\bar{\mathbf{e}}_i, i = 1, \cdots, m \tag{24}$$

$$\dot{\hat{\boldsymbol{\Theta}}}_{g_{ij}} = -\Gamma_{g_{ij}}\boldsymbol{\Phi}_{g_{ij}}(\mathbf{x})\mathbf{B}_i^T\mathbf{P}_i\bar{\mathbf{e}}_i u_j, i, j = 1, \cdots, m \tag{25}$$

$$u_{d_i} = \frac{1}{2\rho_i^2}\bar{\mathbf{e}}_i^T\mathbf{P}_i\mathbf{B}_i, i = 1, \cdots, m \tag{26}$$

then the following H^∞ tracking performance can be obtained:

$$\begin{aligned} \int_0^T \bar{\mathbf{e}}^T\mathbf{Q}\bar{\mathbf{e}}dt \leq& \bar{\mathbf{e}}^T(\mathbf{0})\mathbf{P}\bar{\mathbf{e}}(\mathbf{0}) + \sum_{i=1}^{m}\tilde{\boldsymbol{\Theta}}_{f_i}^T(\mathbf{0})\Gamma_{f_i}^{-1}\tilde{\boldsymbol{\Theta}}_{f_i}(\mathbf{0}) \\ &+ \sum_{i=1}^{m}\rho_i^2\int_0^T \varrho_i^2 dt + \sum_{i,j=1}^{m}\tilde{\boldsymbol{\Theta}}_{g_{ij}}^T(\mathbf{0})\Gamma_{g_{ij}}^{-1}\tilde{\boldsymbol{\Theta}}_{g_{ij}}(\mathbf{0}) \end{aligned} \tag{27}$$

where $\Gamma_{f_i}(i = 1, \cdots, m), \Gamma_{g_{ij}}(i, j = 1, \cdots, m)$ are positive definite diagonal matrices to be designed, $\rho_i(i = 1, \cdots, m)$ are positive constant to be designed, $\bar{\mathbf{e}} = [\bar{\mathbf{e}}_1^T, \cdots, \bar{\mathbf{e}}_m^T]^T$, $\varrho_i \stackrel{\text{def}}{=} -d_i + w_i$, $\mathbf{Q} = \text{diag}(\mathbf{Q}_1, \cdots, \mathbf{Q}_m)$ and $\mathbf{Q}_i \in \mathbb{R}^{m \times m}(i = 1, \cdots, m)$ are arbitrary symmetric positive definite matrices, $\mathbf{P} = \text{diag}(\mathbf{P}_1, \cdots, \mathbf{P}_m)$ and $\mathbf{P}_i(i = 1, \cdots, m)$ are the symmetric positive definite solution of the following Lyapunov equations

$$\mathbf{P}_i\mathbf{A}_i + \mathbf{A}_i^T\mathbf{P}_i = -\mathbf{Q}_i \tag{28}$$

Proof: Define the Lyapunov function V_i for the i^{th} subsystem.

$$V_i = \frac{1}{2}\bar{\mathbf{e}}_i^T \mathbf{P}_i \bar{\mathbf{e}}_i + \frac{1}{2}\tilde{\mathbf{\Theta}}_{f_i}^T \Gamma_{f_i}^{-1} \tilde{\mathbf{\Theta}}_{f_i} + \frac{1}{2}\sum_{j=1}^{m} \tilde{\mathbf{\Theta}}_{g_{ij}}^T \Gamma_{g_{ij}}^{-1} \tilde{\mathbf{\Theta}}_{g_{ij}} \tag{29}$$

The time derivative of V_i is

$$\begin{aligned}\dot{V}_i =& \frac{1}{2}\left(\bar{\mathbf{e}}_i^T \mathbf{A}_i^T \mathbf{P}_i \bar{\mathbf{e}}_i + \bar{\mathbf{e}}_i^T \mathbf{P}_i \mathbf{A}_i \bar{\mathbf{e}}_i\right) + \tilde{\mathbf{\Theta}}_{f_i}^T \mathbf{\Phi}_{\mathbf{F}}(\mathbf{x}) \mathbf{B}_i^T \mathbf{P}_i \bar{\mathbf{e}}_i \\ &+ \sum_{i=1}^{m} \tilde{\mathbf{\Theta}}_{g_{ij}}^T \mathbf{\Phi}_{\mathbf{G}}(\mathbf{x}) \mathbf{B}_i^T \mathbf{P}_i \bar{\mathbf{e}}_i u_j + \\ &+ \frac{1}{2}(-u_{d_i} + \varrho_i)(\mathbf{B}_i^T \mathbf{P}_i \bar{\mathbf{e}}_i + \bar{\mathbf{e}}_i^T \mathbf{P}_i \mathbf{B}_i) \\ &+ \tilde{\mathbf{\Theta}}_{f_i}^T \Gamma_{f_i}^{-1} \dot{\tilde{\mathbf{\Theta}}}_{f_i} + \sum_{j=1}^{m} \tilde{\mathbf{\Theta}}_{g_{ij}}^T \Gamma_{g_{ij}}^{-1} \dot{\tilde{\mathbf{\Theta}}}_{g_{ij}} \\ \leq& -\frac{1}{2}\bar{\mathbf{e}}_i^T \mathbf{Q} \bar{\mathbf{e}}_i + \frac{1}{2}\varrho_i(\mathbf{B}_i^T \mathbf{P}_i \bar{\mathbf{e}}_i + \bar{\mathbf{e}}_i^T \mathbf{P}_i \mathbf{B}_i) \\ &- \frac{1}{2\rho^2}\bar{\mathbf{e}}_i^T \mathbf{P}_i \mathbf{B}_i \mathbf{B}_i^T \mathbf{P}_i \bar{\mathbf{e}}_i \\ \leq& -\frac{1}{2}\bar{\mathbf{e}}_i^T \mathbf{Q} \bar{\mathbf{e}}_i + \frac{1}{2}\rho_i^2 \varrho_i^2\end{aligned} \tag{30}$$

The following inequality can be obtained from (30)

$$\frac{1}{2}\bar{\mathbf{e}}_i^T \mathbf{Q} \bar{\mathbf{e}}_i \leq -\dot{V}_i + \frac{1}{2}\rho_i^2 \varrho_i^2 \tag{31}$$

Integrating both sides of the above inequality yields

$$\frac{1}{2}\int_0^T \bar{\mathbf{e}}_i^T \mathbf{Q} \bar{\mathbf{e}}_i dt \leq V_i(0) + \frac{\rho_i^2}{2}\int_0^T \varrho_i^2 dt \tag{32}$$

Giving Lyapunov function $V = \sum_{i=1}^m V_i$ and according to the definition of V_i, (27) is obtained. This completes the proof. □

Corollary 1: For the i^{th} subsystem of (22), it is assumed that $\int_0^T d_i^2 dt < \infty$. If the control law (13) and the parameter update laws (24)-(25) are adopted, then the following statements hold:

i) the closed loop system is stable and the modified steady tracking error satisfies $\lim_{t\to\infty} \bar{e}_i = 0$, i.e., $\lim_{t\to\infty} |y_{id}(t) - y_i(t) - \xi_{i1}(t)| = 0$

ii) A bound of the transient tracking error will be given by

$$\begin{aligned}\|e_i\|_2^2 \leq& \frac{\bar{\mathbf{e}}_i^T(\mathbf{0}) \mathbf{P}_i \bar{\mathbf{e}}_i(\mathbf{0}) + \tilde{\mathbf{\Theta}}_{f_i}^T(\mathbf{0}) \Gamma_{f_i}^{-1} \tilde{\mathbf{\Theta}}_{f_i}(\mathbf{0}) + \rho_i^2 \int_0^T \varrho_i^2 dt}{\lambda_{\min}(\mathbf{Q}_i)} \\ &+ \frac{\sum_{j=1}^m \tilde{\mathbf{\Theta}}_{g_{ij}}^T(\mathbf{0}) \Gamma_{g_{ij}}^{-1} \tilde{\mathbf{\Theta}}_{g_{ij}}(\mathbf{0})}{\lambda_{\min}(\mathbf{Q}_i)} + \frac{1}{2\kappa_i}\|\sum_{j=1}^m \hat{g}_{ij}(\mathbf{x}) \Delta u_j\|_2^2\end{aligned} \tag{33}$$

where κ_i is a positive constant to be defined later.

Proof: From (30), it can be obtained that $\dot{V}_i \leq -\frac{1}{2}\lambda_{\min}(\mathbf{Q}_i)\|\bar{\mathbf{e}}_i\|^2 + \frac{1}{2}\rho_i^2 \varrho_i^2$, where $\lambda_{\min}(\mathbf{Q}_i)$ represents the minimum eigenvalue of matrix $\mathbf{Q}_i$. $\dot{V}_i$ is negative whenever $\|\bar{\mathbf{e}}_i\| \geq \frac{\rho_i|\varrho_i|}{\sqrt{\lambda_{\min}(\mathbf{Q}_i)}}$. Hence the modified tracking error (20) will stay in the region $\|\bar{\mathbf{e}}_i\| \leq \frac{\rho_i|\varrho_i|}{\sqrt{\lambda_{\min}(\mathbf{Q}_i)}}$. Obviously $\bar{e}_i^2 \leq \|\bar{\mathbf{e}}_i\|^2 \leq \frac{-2\dot{V}_i + \rho_i^2 \varrho_i^2}{\lambda_{\min}(\mathbf{Q}_i)}$, hence

$$\int_0^T \bar{e}_i^2 dt \leq \frac{2V_i(0) + \rho_i^2 \int_0^T \varrho_i^2 dt}{\lambda_{\min}(\mathbf{Q}_i)} \tag{34}$$

Assumption 4 implies $\int_0^T w_i^2 dt < \infty$, then $\int_0^T \varrho_i^2 dt = \int_0^T (w_i - d_i)^2 dt < \infty$. (34) means $\bar{e}_i \in L_2$. According to Barbalat lemma, $\lim_{t\to\infty} \bar{e}_i = 0$. This proves the statement i).

For the proof of the statement ii), let $\boldsymbol{\xi}_i \stackrel{\text{def}}{=} (\xi_{i1}, \cdots, \xi_{ir_i})^T$, and $\Delta u_j = u_j - u_{cj}$. Defining $V_{\boldsymbol{\xi}_i} = \frac{1}{2}\boldsymbol{\xi}_i^T \boldsymbol{\xi}_i$, then we have

$$\begin{aligned}\dot{V}_{\boldsymbol{\xi}_i} =& \sum_{j=1}^{r_i - 1} \xi_{ij}(\xi_{i,j+1} - c_{ij}\xi_{ij}) - c_{ir_i}\xi_{ir_i}^2 + \xi_{ir_i}\sum_{j=1}^m \hat{g}_{ij}(\mathbf{x})\Delta u_j \\ \leq& -\sum_{j=1}^{r_i - 1} \bar{c}_{ij}\xi_{ij}^2 + (1 - c_{ir_i})\xi_{ir_i}^2 + \frac{\left(\sum_{j=1}^m \hat{g}_{ij}(\mathbf{x})\Delta u_j\right)^2}{2} \\ \leq& -\kappa_i \sum_{j=1}^{r_i} \xi_{ij}^2 + \frac{\left(\sum_{j=1}^m \hat{g}_{ij}(\mathbf{x})\Delta u_j\right)^2}{2}\end{aligned} \tag{35}$$

where $\bar{c}_{i1} = c_{i1} - \frac{1}{2}, \bar{c}_{ij} = c_{ij} - 1 (j = 2, \cdots, r_i - 1), \kappa_i = \min\{\bar{c}_{ij}(j = 1, \cdots, r_i - 1), c_{ir_i} - 1\}$. Parameters $c_{ij}(j = 1, \cdots, r_i)$ are chosen to make $\kappa_i > 0$.

Integrating both side of the inequality (35) yields

$$\begin{aligned}\|\boldsymbol{\xi}_i\|_2^2 =& \int_0^\infty {\boldsymbol{\xi}_i}^T(t)\boldsymbol{\xi}_i(t)dt \leq \frac{(V_{\boldsymbol{\xi}_i}(0) - V_{\boldsymbol{\xi}_i}(\infty))}{\kappa_i} \\ &+ \frac{1}{2\kappa_i}\|\sum_{j=1}^m \hat{g}_{ij}(\mathbf{x})\Delta u_j\|_2^2\end{aligned} \tag{36}$$

Setting $\boldsymbol{\xi}_i(0) = \mathbf{0}$, then $V_{\boldsymbol{\xi}_i}(0) = 0$, and

$$\|\boldsymbol{\xi}_i\|_2 \leq \frac{1}{\sqrt{2\kappa_i}}\|\sum_{j=1}^m \hat{g}_{ij}(\mathbf{x})\Delta u_j\|_2 \tag{37}$$

It is straightforward that

$$\|e_i\|_2^2 = \|y_{id}(t) - y_i(t)\|_2^2 \leq \|y_{id}(t) - y_i(t) - \xi_{i1}(t)\|_2^2 + \|\boldsymbol{\xi}_i(t)\|_2^2 \tag{38}$$

Substituting (34) and (37) into (38) yields

$$\|e_i\|_2^2 \leq \frac{2V_i(0)}{\lambda_{\min}(\mathbf{Q}_i)} + \frac{\rho_i^2}{\lambda_{\min}(\mathbf{Q}_i)}\int_0^\infty \varrho_i^2(t)dt + \frac{1}{2\kappa_i}\|\sum_{j=1}^m \hat{g}_{ij}(\mathbf{x})\Delta u_j\|_2^2 \tag{39}$$

According to the definition of V_i, (33) is obtained. The statement ii) holds. □

Remark 1: According to Theorem 1, the i^{th} subsystem achieves a H^∞ tracking performance with a prescribed disturbance attenuation level ρ_i, i.e., the L_2 gain from w_i to the extended tracking error $\bar{e}_i$ must be equal or less than ρ_i.

Remark 2: Because design parameters are chosen to make $\mathbf{A}_i$ a Hurwitz stable matrix, there exists unique symmetric positive definite matrix $\mathbf{P}_i$ satisfying Lyapunov equation (28).

Remark 3: If the reference signals $y_{id}, i = 1, \cdots, m$, are chosen small, then there may be no actuator saturation on the signal $\mathbf{u}_c$ obtained by *certainty equivalence principle*, i.e. $\Delta\mathbf{u} = \mathbf{u} - \mathbf{u}_c = \mathbf{0}$, and the obtained controller becomes an approximate nonlinear dynamic inversion controller. In this situation, $\xi_{i1} = 0$. $\lim_{t\to\infty} |y_{id}(t) - y_i(t) - \xi_{i1}(t)| = \lim_{t\to\infty} |y_{id}(t) - y_i(t)| = 0$, that means every output will track their reference signal asymptotically. If $\mathbf{\Delta u} \neq \mathbf{0}$ but $\|\mathbf{\Delta u}\| \to 0$ as $t \to \infty$, then $\lim_{t\to\infty} \dot{V}_{\boldsymbol{\xi}_i} \leq \lim_{t\to\infty} \big(-\kappa_i \sum_{j=1}^{r_i} \xi_{ij}^2 +$

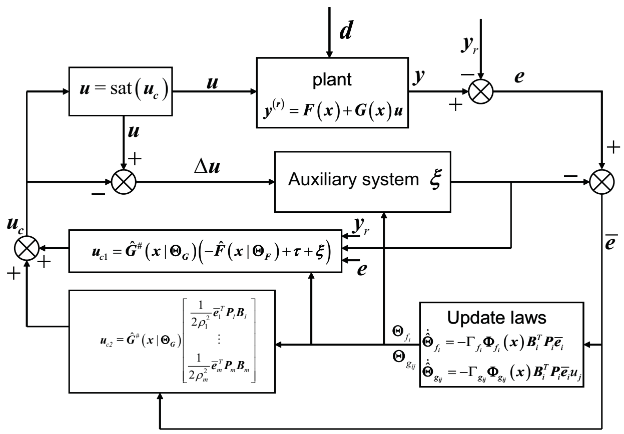

Fig. 2. The overall structure of adaptive H^{∞} tracking control scheme

$\frac{\left(\sum_{j=1}^{m}\hat{g}_{ij}(\mathbf{x})\Delta u_j\right)^2}{2}) = \lim_{t\to\infty} -\kappa_i \sum_{j=1}^{r_i}\xi_{ij}^2 \le 0$. Therefore $\xi_{i1} \to 0$ as $t \to \infty$, and $\lim_{t\to\infty}|y_{id}(t) - y_i(t) - \xi_{i1}(t)| = \lim_{t\to\infty}|y_{id}(t) - y_i(t)| = 0$. This implies that if the signal $\mathbf{u}_c$ has no saturation or $\mathbf{u}_c$ is not saturated as $t \to \infty$, then the desired H^{∞} tracking performance is ensured.

Remark 4: The bound for $\|y_{id}(t) - y_i(t)\|_2$ is an explicit function of the design parameters. According to the statement ii), this bound depends on the initial estimate errors $\hat{\mathbf{\Theta}}_{f_i}(0), \hat{\mathbf{\Theta}}_{g_{ij}}(0)(j = 1, \cdots, m)$. The effects of initial estimate errors on this bound can be decreased by increasing the values of the diagonal adaptation gain matrices $\Gamma_{f_i}, \Gamma_{g_{ij}}(j = 1, \cdots, m)$ and by choosing positive definite symmetric matrix $\mathbf{Q}_i$ with larger minimum eigenvalue. On the other hand, the effects of extern disturbances and $\Delta\mathbf{u}$ on the transient performance can be reduced by decreasing ρ_i and increasing κ_i.

Remark 5: Parameters $\lambda_{ij}, i = 1, \cdots, m, j = 1, \cdots, r_i$ should be selected such that $\mathbf{A}_i(i = 1, \cdots, m)$ are Hurwitz stable. Parameters $c_{ij}, i = 1, \cdots, m, j = 1, \cdots, r_i$ should be selected such that $\kappa_i > 0(i = 1, \cdots, m)$.

Based on the previous analysis, the design procedure for the adaptive H^{∞} tracking control scheme in Fig.2 is given as follows:

Step 1: Select the radial bases $\mathbf{\Phi}_{f_i}(\mathbf{x})(i = 1, \cdots, m)$, $\mathbf{\Phi}_{g_{ij}}(\mathbf{x})(i, j = 1, \cdots, m)$.

Step 2: Select the parameters $\lambda_{ij}(j = 1, \cdots, r_i, i = 1, \cdots, m)$, $c_{ij}(j = 1, \cdots, r_i, i = 1, \cdots, m)$, and $\rho_i(i = 1, \cdots, m)$. Select the parameter update gain matrices $\Gamma_{f_i}(i = 1, \cdots, m), \Gamma_{g_{ij}}(i, j = 1, \cdots, m)$.

Step 3: Set $\xi_{ij}(0) = 0$, and construct the auxiliary system (15).

Step 4: Select $\mathbf{Q}_i(i = 1, \cdots, m)$ and solve Lyapunov equations (28) to get $\mathbf{P}_i$.

Step 5: Obtain the control law (12)-(13) and the parameter update laws (24) and (25).

Remark 6: Although the true value of $\mathbf{G}(\mathbf{x})$ may be invertible, the estimate matrix $\hat{\mathbf{G}}(\mathbf{x}|\mathbf{\Theta}_{\mathbf{G}})$ may become singular during the adaptive process, so Moore-Penrose generalized matrix inverse [9] of $\hat{\mathbf{G}}(\mathbf{x}|\mathbf{\Theta}_{\mathbf{G}})$ is used in Step 5.

IV. Numerical Example

In this section, we apply the controller to control a nonlinear system with input magnitude and rate limitations. The dynamic model of this nonlinear system is as follows

$$\begin{aligned} \dot{x}_1 &= -(x_1 + x_2^2) + 10u_1 + \sin^2(x_2)u_2 + 0.2d_1(t) \\ \dot{x}_2 &= -x_1^2 + x_1^2 u_1 + u_2 + 0.2d_2(t) \\ y_1 &= x_1 \quad y_2 = x_2 \end{aligned} \tag{40}$$

where u_1, u_2 are control inputs and have the limitations $|u_i| \le 5, |\dot{u}_i| \le 10, i = 1, 2$, and $d_1(t), d_2(t)$ are uniformly distributed random noise in $[0, 1]$. $x_1(0) = 1, x_2(0) = 0$. The reference trajectories $y_{1d} = \sin(t), y_{2d} = \cos(t)$ are used in this computer simulation.

Obviously, the relative degree of the plant (40) is $r_1 = r_2 = 1$. Rewrite the plant (40) as

$$\begin{bmatrix} \dot{y}_1 \\ \dot{y}_2 \end{bmatrix} = \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} + \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \end{bmatrix} + 0.2 \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

where $f_1 = -(x_1 + x_2^2)$, $f_2 = -x_1^2$, $g_{11} = 10$, $g_{12} = \sin^2(x_2)$, $g_{21} = x_1^2, g_{22} = 1$.

According to the design procedure, the H^{∞} tracking design is given as follows

Step 1: We choose a 11-dimensional Gauss radial base for approximating f_1, i.e.,

$$\mathbf{\Phi}_{f_1}(\mathbf{x}) = \left[\exp\left(-\frac{\|\mathbf{x} - \mathbf{c}_1\|^2}{b_1^2}\right), \cdots, \exp\left(-\frac{\|\mathbf{x} - \mathbf{c}_{11}\|^2}{b_{11}^2}\right)\right]^T$$

where $\mathbf{x} = (x_1, x_2)^T$, $\mathbf{c}_i(i = 1, \cdots, 11)$ are the center of the radial base, and are chosen as $\mathbf{c}_1 = [-2, -2]^T$, $\mathbf{c}_2 = [-1.6, -1.6]^T$, $\mathbf{c}_3 = [-1.2, -1.2]^T$, $\mathbf{c}_4 = [-0.8, -0.8]^T$, $\mathbf{c}_5 = [-0.4, -0.4]^T$, $\mathbf{c}_6 = [0, 0]^T$, $\mathbf{c}_7 = [0.4, 0.4]^T$, $\mathbf{c}_8 = [0.8, 0.8]^T$, $\mathbf{c}_9 = [1.2, 1.2]^T$, $\mathbf{c}_{10} = [1.6, 1.6]^T$, $\mathbf{c}_{11} = [2, 2]^T$, $b_i = 1, i = 1, \cdots, 11$. The radial bases for $f_2, g_{11}, g_{12}, g_{21}, g_{22}$ are chosen the same as f_1.

Step 2: Select the coefficients $\lambda_{11} = 5$, $\lambda_{21} = 5$. Now $\mathbf{A}_1 = [-5]_{1\times1}$, $\mathbf{A}_2 = [-5]_{1\times1}$, $\mathbf{B}_1 = \mathbf{B}_2 = [1]_{1\times1}$. Select the coefficients $c_{11} = 5$, $c_{21} = 5$. Choose $\rho_1 = \rho_2 = 0.5$ and the parameter update gain matrices $\Gamma_{f_1} = \Gamma_{f_2} = \text{diag}(10, 10, 10, 10, 10, 10, 10, 10, 10, 10, 10)$, $\Gamma_{g_{11}} = \Gamma_{g_{12}} = \Gamma_{g_{21}} = \Gamma_{g_{22}} = \text{diag}(5, 5, 5, 5, 5, 5, 5, 5, 5, 5, 5)$

Step 3: Set $\xi_{11}(0) = 0, \xi_{21}(0) = 0$, and construct following auxiliary system

$$\begin{aligned} \dot{\xi}_{11} &= -c_{11}\xi_{11} + \hat{g}_{11}(u_1 - u_{c1}) + \hat{g}_{12}(u_2 - u_{c2}) \\ \dot{\xi}_{21} &= -c_{21}\xi_{21} + \hat{g}_{21}(u_1 - u_{c1}) + \hat{g}_{22}(u_2 - u_{c2}) \end{aligned}$$

Step 4: Select $\mathbf{Q}_1 = [10]_{1\times1}$ and $\mathbf{Q}_2 = [10]_{1\times1}$. Solving Lyapunov equation (28), we obtain $\mathbf{P}_1 = \mathbf{P}_2 = [1]_{1\times1}$.

Step 5: Set the parameters update laws as

$$\begin{aligned} &\dot{\hat{\mathbf{\Theta}}}_{f_1} = -\Gamma_{f_1}\mathbf{\Phi}_{f_1}(\mathbf{x})\mathbf{B}_1^T\mathbf{P}_1\bar{\mathbf{e}}_1, \dot{\hat{\mathbf{\Theta}}}_{f_2} = -\Gamma_{f_2}\mathbf{\Phi}_{f_2}(\mathbf{x})\mathbf{B}_2^T\mathbf{P}_2\bar{\mathbf{e}}_2, \\ &\dot{\hat{\mathbf{\Theta}}}_{g_{11}} = -\Gamma_{g_{11}}\mathbf{\Phi}_{g_{11}}(\mathbf{x})\mathbf{B}_1^T\mathbf{P}_1\bar{\mathbf{e}}_1 u_1, \\ &\dot{\hat{\mathbf{\Theta}}}_{g_{12}} = -\Gamma_{g_{12}}\mathbf{\Phi}_{g_{12}}(\mathbf{x})\mathbf{B}_1^T\mathbf{P}_1\bar{\mathbf{e}}_1 u_2, \\ &\dot{\hat{\mathbf{\Theta}}}_{g_{21}} = -\Gamma_{g_{21}}\mathbf{\Phi}_{g_{21}}(\mathbf{x})\mathbf{B}_2^T\mathbf{P}_2\bar{\mathbf{e}}_2 u_1, \\ &\dot{\hat{\mathbf{\Theta}}}_{g_{22}} = -\Gamma_{g_{22}}\mathbf{\Phi}_{g_{22}}(\mathbf{x})\mathbf{B}_2^T\mathbf{P}_2\bar{\mathbf{e}}_2 u_2 \end{aligned}$$

The supervisory control are chosen as $u_{d_1} = \frac{1}{2\rho_1^2}\bar{\mathbf{e}}_1^T\mathbf{P}_1\mathbf{B}_1$, $u_{d_2} = \frac{1}{2\rho_2^2}\bar{\mathbf{e}}_2^T\mathbf{P}_2\mathbf{B}_2$. The initial values for $\mathbf{\Theta}_{f_i}(0)(i = 1, 2)$ and $\mathbf{\Theta}_{g_{ij}}(0)(i, j = 1, 2)$ are chosen as uniformly distributed pseudorandom numbers in interval $[0, 1]$.

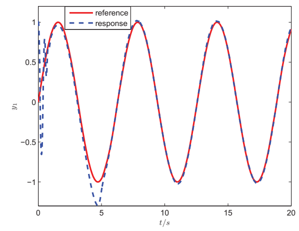

Fig. 3. The trajectory of y_1

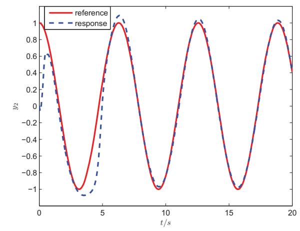

Fig. 4. The trajectory of y_2

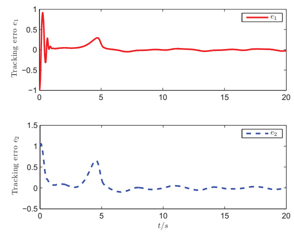

Fig. 5. The output tracking errors e_1, e_2

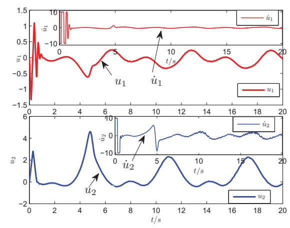

Fig. 6. u_1, u_2 and $\dot{u}_1, \dot{u}_2$

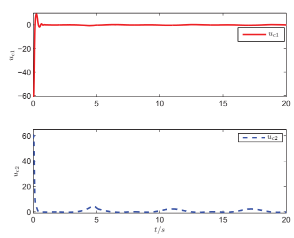

Fig. 7. u_{c1}, u_{c2}

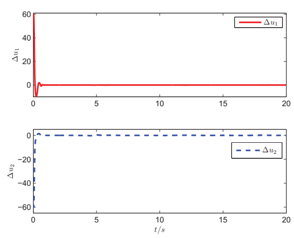

Fig. 8. $\Delta u_1, \Delta u_2$

According to (12), u_{c1} and u_{c2} will be obtained. u_1, u_2 will be calculated as $\dot{u}_1 = \text{sat}_{10}(20.5\text{sat}_5(u_{c1}) - u_1), \dot{u}_2 = \text{sat}_{10}(20.5\text{sat}_5(u_{c2}) - u_2)$.

Simulation results are presented in Fig. 3 to Fig. 8. Fig. 3 shows the curves of output $y_1(t)$ and its reference trajectory. Fig. 4 shows the curves of output $y_2(t)$ and its reference trajectory. Curves in Fig. 5 describe the tracking errors. These simulation curves indicate that the outputs track their reference values well, and the effects of approximation error and extern disturbance on tracking errors are effectively attenuated. The control signals $u_1(t), u_2(t)$ and their derivatives $\dot{u}_1(t), \dot{u}_2(t)$ are given in Fig. 6. It is observed that the control signals $u_1(t)$ and $u_2(t)$ satisfy their limitations. Fig. 7 shows the signals $u_{c1}(t), u_{c2}(t)$ obtained by *certainty equivalence principle*. Obviously they do not satisfy the control input limitations. Fig. 8 shows the signals $\Delta u_1(t), \Delta u_2(t)$. These curves in Fig. 8 tell us that $\Delta u_1(t), \Delta u_2(t)$ tend to zero soon as time goes. Hence $y_i(t), i = 1, 2$ asymptotically tend to $y_{id}(t), i = 1, 2$. These results indicate that the proposed adaptive controller is effective.

V. Conclusions

In this work, an adaptive controller based on adaptive radial basis neural network is proposed for a class of nonlinear MIMO systems with control input magnitude and rate limitations to achieve the desired disturbance attenuation in the presence of extern disturbance. An auxiliary system is constructed to compensate the effects of actuator magnitude and rate limitations. Supervisory controls are used to attenuate the effects of extern disturbance and approximation so as to achieve a desired level disturbance attenuation tracking performance. The closed loop tracking performance is analyzed. The bound of tracking error is given in terms of design parameters. The proposed controller can generate control signals satisfying their constraints and guarantee a desired closed loop performance. Simulation results illustrate the effectiveness of the proposed controller.

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