2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Balance Maintenance of the Synthetic-Wheel Biped in the Presence of Impulsive Disturbances

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***Abstract*— A recent addition to the biped literature is the Synthetic-Wheel Biped. This platform has arc-shaped feet and is prone to rolling on its feet when disturbed. A disturbance rejection algorithm is proposed for the Synthetic-Wheel Biped such that it can stop moving and regain its upright posture. The algorithm imposes a symmetric gait and uses impulsive control inputs to keep the states of the system bounded and reject the effect of the disturbance. It is assumed that the external disturbances are applied when the biped is in its upright posture and the disturbances are modeled by initial velocity conditions. The biped takes a few steps to recover its upright configuration. Numerical simulations show the effectiveness of the proposed algorithm in rejecting the external disturbances applied on the Synthetic-Wheel Biped.**

NOMENCLATURE

For the nomenclature below, subscript $k \in \{st, sw, t\}$ where st, sw, t denote the stance leg, swing leg and torso and $i, j \in \{1, 2, 3\}$.

d_k	distance of center of mass of the k-th link, (m)
l_t	length of the torso, (m)
g	acceleration due to gravity, ($9.81\ m.s^{-2}$)
R	radius of curvature of the feet and length of the stance and swing legs, (m)
α	angle of the torso with respect to the vertical axis, measured positive in the ccw direction, (rad)
α_d	constant desired value of α, (rad)
θ_j	angular displacement of the j-th link, (rad)
$\dot{\theta}_j$	angular velocity of the j-th link, (rad/s)
$\dot{\theta}_j^+$	angular velocity of the j-th link, immediately after the application of impulsive torque, (rad/s)
$\dot{\theta}_j^-$	angular velocity of the j-th link, immediately before the application of impulsive torque, (rad/s)
τ_1	generalized force corresponding to θ_2, $(N.m)$
τ_2	generalized force corresponding to θ_3, $(N.m)$
θ	vector of generalized coordinates: θ_1, θ_2 and θ_3
$\bar{\theta}$	vector of generalized coordinates: θ_1, v_1 and v_2
$\hat{\theta}$	vector of control variables v_1 and v_2
P	coordinate transformation matrix describing stance and swing-leg interchange
θ_{new}	the value of θ after interchange of stance and swing legs
θ_{old}	the value of θ before interchange of stance and swing legs
I_1	impulse corresponding to θ_2 coordinate, $(N.m.s)$
I_2	impulse corresponding to θ_3 coordinate, $(N.m.s)$
S_j	$\sin\theta_j$
C_j	$\cos\theta_j$
S_{ij}	$\sin(\theta_i + \theta_j)$
C_{ij}	$\cos(\theta_i + \theta_j)$

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I. INTRODUCTION

Bipeds can provide greater maneuverability over a wide variety of terrains as compared to wheeled mobile robots and their anthropomorphic characteristics make them well suited to sharing the workspace with humans. The design and control of bipeds has been a subject of considerable research for the past four decades and has resulted in the development of a number of different biped platforms, [1], [2], [3], [4], for example, and walking strategies [5], [6], [7], [8].

The dynamics of bipeds are complicated, nonlinear, and often unstable due to impact from footground interaction and discrete events, such as switching support between legs. This makes the control problem difficult, and it is not surprising that stability and robustness of human locomotion has been difficult to emulate in bipeds. The concept of Zero-Moment-Point (ZMP) developed by Vukobratovic and Juricic [5] has been widely used to design robust controllers to handle external disturbances [9], [10], [11], [12]. Abdallah and Goswami [13] developed an algorithm based on ZMP approach for balance maintenance of a fully actuated biped standing upright and subjected to an external disturbance. Their two-phase algorithm consists of reflex and recovery phases in which the external disturbance is absorbed and the biped home position is recovered. Prahlad et al. [14] proposed a ZMP compensation method in which the joint trajectories, instead of joint torques, are changed to keep the ZMP inside the support polygon. Zheng [15] developed an acceleration compensation approach, which is based on choosing the biped joint accelerations to reduce the angle deviation of the supporting foot due to the external disturbance. Hobbelen and Wisse [16] investigated the effect of leg retraction on the disturbance rejection of limit cycle walkers[1] and used the Gait Sensivity Norm [17], to find the optimal swing-leg retraction. Other nonlinear control methods such as sliding mode control and passivity-based control have been proposed for a 5-DOF under-actuated biped [18] and a force-controllable humanoid [19] for disturbance rejection.

We consider the problem of control design to reject impulsive disturbances which cause discrete jumps in the system velocities. To this end we enlarge the set of admissible controls to include impulsive inputs. An impulsive input

[1] limit cycle walkers are the bipeds that show stable limit cycle motion

can produce large changes in system velocities over a short duration of time. There has been a fair amount of theoretical research on impulsive control [20], [21], [22], [23] and in recent years, researchers have studied diverse application problems that include under-actuated systems [24], [25], [26].

In this paper, an algorithm is proposed to cancel the effect of impulsive disturbances applied to the Synthetic-Wheel Biped [27] in its upright configuration. The Synthetic-Wheel Biped, which appeared in the literature recently, has curved feet and will roll on its feet when disturbed. Our algorithm allows the biped to take a few steps in the direction of the applied disturbance and recover its upright posture.

II. Biped Dynamics and Control

A. Equations of Motion

A schematic of the Synthetic-Wheel Biped [27] is shown in Fig.1. It is an under-actuated three-DOF system described by the generalized coordinates θ_1, θ_2 and θ_3. The two actuators that correspond to θ_2 and θ_3 coordinates can actively control the torso and the swing leg motion while the stance leg motion is completely passive. Assuming no friction in the joints, the equations of motion for the biped can be obtained using the Lagrangian formulation as follows,

$$A(\theta)\ddot{\theta} + B(\theta,\dot{\theta})\dot{\theta} + G(\theta) = T \tag{1}$$

where

$$\theta = \begin{pmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \end{pmatrix}, \qquad T = \begin{pmatrix} 0 \\ \tau_1 \\ \tau_2 \end{pmatrix} \tag{2}$$

In (1), $A(\theta)$ represents the symmetric inertia matrix, $B(\theta,\dot{\theta})$ is the matrix containing the Coriolis and centrifugal forces, and $G(\theta)$ is the vector of gravitational forces. The coordinate θ_1 denotes the passive DOF and hence its corresponding generalized force is zero. The terms $A(\theta)$, $B(\theta,\dot{\theta})$ and $G(\theta)$ are defined as follows:

$$A = [A_{ij}]_{3\times 3}\,, \quad B = [B_{ij}]_{3\times 3}\,, \quad G = [G_i]_{3\times 1} \tag{3}$$

and can be obtained following the Lagrangian Formulation.

B. Symmetric Gait

A symmetric gait for the Synthetic-Wheel Biped is generated by imposing the following constraints on the motion of the torso and the swing leg [27]:

$$\begin{aligned} \mathcal{C}_1 &: \alpha = \alpha_d \\ \mathcal{C}_2 &: \theta_3 = -2\theta_1 \end{aligned} \tag{4}$$

where α, depicted in Fig.2, is defined by the relation

$$\alpha = \theta_1 + \theta_2 - \pi \tag{5}$$

The constraint $\mathcal{C}_1$ ensures that the torso maintains angle α_d with respect to the vertical axis while constraint $\mathcal{C}_2$ ensures that the swing leg is symmetric with respect to the stance leg about the vertical axis at all times. The constrained system

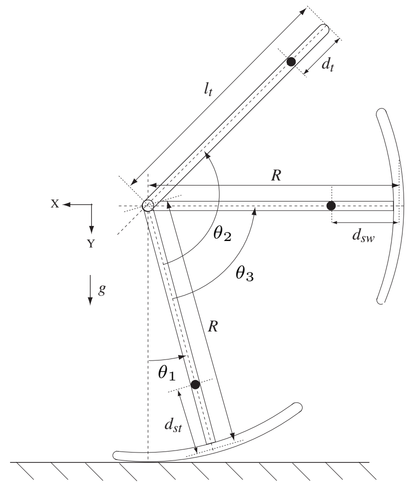

Fig. 1. A schematic of the Synthetic-Wheel Biped. the link angles θ_1, θ_2 and θ_3 are measured counter-clockwise with respect to the horizontal axis.

has one passive DOF with the following dynamics, which is derived by substituting (4) into (1):

$$A_c(\theta_1)\,\ddot{\theta}_1 + B_c(\theta_1,\dot{\theta}_1)\,\dot{\theta}_1 + G_c = 0 \tag{6}$$

where the expressions for $A_c(\theta_1)$, $B_c(\theta_1,\dot{\theta}_1)$ and G_c can be found in [27]. For any set of reasonable parameter values, it can be verified that $\ddot{\theta}_1$ will be positive for positive angle α_d and vice versa. This implies that we can slow down the biped velocity $\dot{\theta}_1$ by applying the constraints in (4) and choosing α_d to have the opposite sign of $\dot{\theta}_1$.

C. Interchange of Stance and Swing Legs

Assuming that the biped has a positive velocity, it will roll on its stance leg and the point of contact with the ground will move from the heel to the toe. The stance and swing legs can be interchanged at any time; but to have the maximum step size, the swing leg should touch down when the heel of the swing leg is right in front of the toe of the stance leg, as seen in Fig.2.

Without assuming maximum step size, an interchange of stance and swing legs will result in a transformation of the generalized coordinates and their velocities given by the following relations:

$$\begin{aligned} \theta_{new} &= P\,\theta_{old} \\ \dot{\theta}_{new} &= P\,\dot{\theta}_{old} \end{aligned} \tag{7}$$

where the entries of the transformation matrix P are

$$P = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{bmatrix} \tag{8}$$

and can be obtained from Fig.2.

D. Control Design for Symmetric Gait

In this section, we present the controller for imposing the symmetric gait constraints in (4). First, the new set of generalized coordinates is defined as

$$\bar{\theta} = \begin{bmatrix} \theta_1 & v_1 & v_2 \end{bmatrix}^T \tag{9}$$

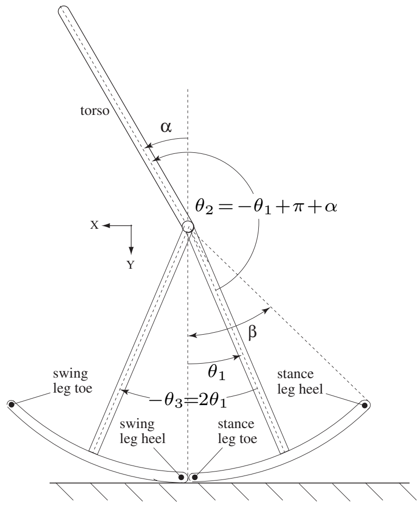

Fig. 2. Biped configuration at the time of interchange of stance and swing legs.

where

$$\begin{aligned} v_1 &= \alpha - \alpha_d \\ v_2 &= \theta_3 + 2\theta_1 \end{aligned} \tag{10}$$

The original generalized coordinates θ_2 and θ_3 are related to the new coordinates v_1 and v_2 according to the following relations, which can be derived from (5) and (10):

$$\begin{aligned} \theta_2 =& v_1 - \theta_1 + \pi + \alpha_d \\ \theta_3 =& v_2 - 2\theta_1 \end{aligned} \tag{11}$$

Substituting (11) into (1), we obtain the dynamics of the system in terms of the new generalized coordinates as follows:

$$\bar{A}(\bar{\theta})\,\ddot{\bar{\theta}} + \bar{B}(\bar{\theta},\dot{\bar{\theta}})\,\dot{\bar{\theta}} + \bar{G}(\bar{\theta}) = T \tag{12}$$

where $\bar{A}$, $\bar{B}$, and $\bar{G}$ have the same dimensions as A, B, and G respectively. The generalized force corresponding to θ_1 in (12) is zero and this allows us to eliminate $\ddot{\theta}_1$ from the two equations corresponding to the generalized coordinates v_1 and v_2. The reduced-order equations have the form

$$\hat{A}(\bar{\theta})\,\ddot{\hat{\theta}} + \hat{B}(\bar{\theta},\dot{\bar{\theta}})\,\dot{\hat{\theta}} + \hat{G}(\bar{\theta}) = \hat{T} \tag{13}$$

where $\hat{A} \in R^{2\times 2}$, $\hat{B} \in R^{2\times 2}$ and $\hat{G} \in R^{2\times 1}$, and

$$\hat{\theta} = [v_1 \quad v_2]^T, \qquad \hat{T} = [\tau_1 \quad \tau_2]^T \tag{14}$$

Equation (13) represents a completely actuated system and we use feedback linearization to design our controller as follows:

$$\hat{T} = \hat{B}(\bar{\theta},\dot{\bar{\theta}})\,\dot{\hat{\theta}} + \hat{G}(\bar{\theta}) - \hat{A}(\bar{\theta})(K_d\,\dot{\hat{\theta}} + K_p\,\hat{\theta}) \tag{15}$$

where K_d and K_p are diagonal, positive-definite matrices of dimension two. Indeed, substitution of (15) into (13) yields

$$\ddot{\hat{\theta}} + K_d\,\dot{\hat{\theta}} + K_p\,\hat{\theta} = 0 \tag{16}$$

which implies $\hat{\theta} \to 0$ as $t \to \infty$. This simply follows that $v_1, v_2 \to 0$ as $t \to \infty$, *i.e.*, the constraints in (4) are satisfied.

The controller in (15) has been implemented and experimentally verified in our laboratory biped [27]. The experimental results show that v_1 and v_2 do not converge to zero but oscillate around zero due to impulsive disturbances from the ground at the time of swing-leg touchdown. The disturbance rejection algorithm presented in this paper provides a way to avoid these impulsive disturbances from the ground while taking steps that are initiated to maintain balance after the biped has been subjected to an external disturbance.

III. Impulsive Torques and Effects

A. Braking Torque for the Swing Leg

Consider the action that results in exponential convergence of the swing leg velocity $\dot{\theta}_3$ to zero while keeping $\dot{\theta}_1$ unchanged. To this end, the following dynamics are assumed,

$$\begin{aligned} \ddot{\theta}_1 =& 0 \\ \ddot{\theta}_3 =& -k_1\dot{\theta}_3 \end{aligned} \tag{17}$$

where k_1 is a positive constant determining the rate of convergence of $\dot{\theta}_3$ to zero. To compute the torque required for this action, we multiply (1) with the inverse of the inertia matrix to obtain

$$\begin{pmatrix} \ddot{\theta}_1 \\ \ddot{\theta}_2 \\ \ddot{\theta}_3 \end{pmatrix} = \frac{1}{A_{33}(A_{11}A_{22} - A_{12}^2) - A_{22}A_{13}^2} \begin{bmatrix} h_1 - A_{12}A_{33}\tau_1 - A_{13}A_{22}\tau_2 \\ h_2 + (A_{11}A_{33} - A_{13}^2)\tau_1 + A_{12}A_{13}\tau_2 \\ h_3 + A_{12}A_{13}\tau_1 + (A_{11}A_{22} - A_{12}^2)\tau_2 \end{bmatrix} \tag{18}$$

where h_1, h_2 and h_3 are given by the following expressions,

$$\begin{aligned} h_1 =& -A_{22}A_{33}(B_{11}\dot{\theta}_1 + B_{12}\dot{\theta}_2 + B_{13}\dot{\theta}_3) \\ & - A_{22}A_{33}G_1 + A_{12}A_{33}G_2 + A_{22}A_{13}G_3 \\ h_2 =& A_{12}A_{13}(B_{11}\dot{\theta}_1 + B_{12}\dot{\theta}_2 + B_{13}\dot{\theta}_3) \\ & + A_{12}A_{33}G_1 + (A_{13}^2 - A_{11}A_{33})G_2 - A_{12}A_{13}G_3 \\ h_3 =& A_{22}A_{13}(B_{11}\dot{\theta}_1 + B_{12}\dot{\theta}_2 + B_{13}\dot{\theta}_3) \\ & + A_{13}A_{22}G_1 - A_{12}A_{13}G_2 + (A_{12}^2 - A_{11}A_{22})G_3 \end{aligned} \tag{19}$$

Substituting (17) into the first and third equations in (18) results in

$$\begin{aligned} \tau_1 =& \frac{1}{A_{12}}\left[\frac{A_{12}^2h_1 - A_{22}(A_{11}h_1 + A_{13}h_3)}{A_{13}^2A_{22} + (A_{12}^2 - A_{11}A_{22})A_{33}} + k_1A_{13}A_{22}\dot{\theta}_3\right] \\ \tau_2 =& \frac{A_{13}h_1 + A_{33}h_3}{A_{13}^2A_{22} + (A_{12}^2 - A_{11}A_{22})A_{33}} - k_1A_{33}\dot{\theta}_3 \end{aligned} \tag{20}$$

If the constant k_1 has a very large value, the torque expressions in (20) will be impulsive in nature and will stop the swing leg relative velocity, $\dot{\theta}_3$, in a very short period of time without changing the stance leg velocity.

B. Braking Torque for the Torso

Consider an action which results in exponential convergence of the torso absolute velocity, $\dot{\alpha}$, to zero while maintaining equal and opposite velocities for the stance and swing legs, *i.e.* $\dot{\theta}_3 = -2\,\dot{\theta}_1$, in conformity with the symmetric gait.

Using (5), we consider the following dynamics to achieve the goal,

$$\begin{aligned}\ddot{\theta}_1 + \ddot{\theta}_2 =& - k_2(\dot{\theta}_1 + \dot{\theta}_2)\\ \ddot{\theta}_1 + 2\ddot{\theta}_3 =& - k_3(\dot{\theta}_1 + 2\dot{\theta}_3)\end{aligned} \tag{21}$$

where k_2 and k_3 are some positive constants determining the rate of convergence of the velocities to their desired values. The control inputs that result in the dynamics in (21) can be obtained by substituting (18) into (21) and solving for τ_1 and τ_2. The complicated torque expressions are not explicitly mentioned here for the sake of simplicity. If we choose very large values for k_2 and k_3 in (21), the resulted torque expressions will be impulsive in nature and stop the motion of the torso in a very short period of time without affecting the symmetric velocity condition associated with the symmetric gait.

C. Effect of Impulse on the System Velocities

The application of impulsive torques for θ_2 and θ_3 generalized coordinates results in velocity jumps in all three coordinates θ_1, θ_2 and θ_3. The relationship between the jumps in velocities can be derived from Lagrange's equations as discussed next. Consider the integral of the equations of motion in (1) over the short interval of time Δt during which the impulsive forces and moments act, *i.e.*

$$\int_0^{\Delta t} \Big[A(\theta)\ddot{\theta} + B(\theta,\dot{\theta})\dot{\theta} + G(\theta)\Big] \; dt = \int_0^{\Delta t} T \; dt \tag{22}$$

The above equation can be rewritten as:

$$A(\theta)\Delta\dot{\theta} + B(\theta,\dot{\theta})\Delta\theta + \int_0^{\Delta t} G(\theta) \; dt = \int_0^{\Delta t} T \; dt \tag{23}$$

Since Δt is very short time interval and the configuration of the biped does not change during this time, *i.e.* $\Delta\theta = 0$, (23) will be simplified to

$$A(\theta)\Delta\dot{\theta} = T_{imp} \tag{24}$$

where $T_{imp} = [0 \;\; I_1 \;\; I_2]^T$ represents the vector of impulses applied to the biped. The above equation can be decomposed into the following three equations:

$$A_{11}(\dot{\theta}_1^+ - \dot{\theta}_1^-) + A_{12}(\dot{\theta}_2^+ - \dot{\theta}_2^-) + A_{13}(\dot{\theta}_3^+ - \dot{\theta}_3^-) = 0 \tag{25}$$

$$A_{21}(\dot{\theta}_1^+ - \dot{\theta}_1^-) + A_{22}(\dot{\theta}_2^+ - \dot{\theta}_2^-) + A_{23}(\dot{\theta}_3^+ - \dot{\theta}_3^-) = I_1 \tag{26}$$

$$A_{31}(\dot{\theta}_1^+ - \dot{\theta}_1^-) + A_{32}(\dot{\theta}_2^+ - \dot{\theta}_2^-) + A_{33}(\dot{\theta}_3^+ - \dot{\theta}_3^-) = I_2 \tag{27}$$

As the first special case, consider the impulsive action described in section III-A in which $\dot{\theta}_3^+ = 0$ and $\dot{\theta}_1^+ = \dot{\theta}_1^-$. Using (25), the relative velocity of torso after the impulsive action can be obtained as

$$\dot{\theta}_2^+ = \dot{\theta}_2^- + \frac{A_{13}}{A_{12}} \; \dot{\theta}_3^- \tag{28}$$

As the second case, consider the impulsive action in section III-B which results in $\dot{\theta}_2^+ = -\dot{\theta}_1^+$ and $\dot{\theta}_3^+ = -2\,\dot{\theta}_1^+$. Then the velocity of the swing leg after the impulse can be obtained from (25) as follows:

$$\dot{\theta}_1^+ = \frac{A_{11}\dot{\theta}_1^- + A_{12}\dot{\theta}_2^- + A_{13}\dot{\theta}_3^-}{A_{11} - A_{12} - 2A_{13}} \tag{29}$$

It should be noted that the impulses I_1 and I_2 can be computed from (26) and (27). These impulses can also be approximated by the time integral of the torque expressions obtained in sections III-A for large gain k_1 and the torque expressions in section III-B for large gains k_2 and k_3.

IV. Disturbance Rejection Algorithm

We propose an algorithm to reject the external disturbances applied to the Synthetic-Wheel Biped. The disturbances are assumed to be of the form of impulsive forces that are applied to the torso while the biped is standing upright and their effects are therefore modeled by jumps in the generalized velocities. The disturbances are assumed to be large enough that precludes the possibility of stabilizing the upright posture without taking a step. The problem at hand is therefore to reject the disturbances and stabilize the upright posture by taking a few steps in the forward or backward direction.

The disturbance rejection algorithm is based on slowing down the biped using the symmetric gait while imposing constraints on the legs and torso. These constraints are:

$$\mathcal{C}_3 \;\; : \;\; -\beta/2 \leq \theta_1 \leq \beta/2 \tag{30}$$

$$\mathcal{C}_4 \;\; : \;\; -\gamma \leq \alpha \leq \gamma \tag{31}$$

where β is the foot arc angle (see Fig.2) and γ is some small positive angle. The constraint in (30) guarantees that the stance leg does not go beyond the foot arc angle and the biped does not fall. The constraint in (31) ensures that the torso deviation from the upright configuration is too large since a large deviation may be irrecoverable.

We propose the following three-step algorithm to reject external impulsive disturbances and stabilize the upright posture of the Synthetic-Wheel Biped:

1. *Initialization*:
   (a) Linearize the dynamic equations in (1) about the desired equilibrium point $(\theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, \theta_3, \dot{\theta}_3) = (0, 0, \pi, 0, 0, 0)$.
   (b) Design a linear controller to render the desired equilibrium locally asymptotically stable [2]. Let R_A be the region of attraction of the desired equilibrium.
   (c) Choose a small positive angle γ such that any configuration of the biped satisfying $|\theta_1| \leq \beta/2$, $\dot{\theta}_1 \approx 0$, $|\alpha| \leq \gamma$, $\dot{\alpha} \approx 0$, $|\theta_3| \leq \beta$ and $\dot{\theta}_3 \approx 0$ lies inside R_A.
2. *Disturbance Rejection*:
   (a) Following the external disturbance, apply the torque expression in (15) to implement the symmetric gait. The torque expression in (15) depends on the value of the torso desired angle, α_d, which is chosen to have the opposite sign of the stance leg velocity, $\dot{\theta}_1$, so the biped slows down. This follows from our discussion in section II-B.

[2]This is always possible because the linearized system is controllable

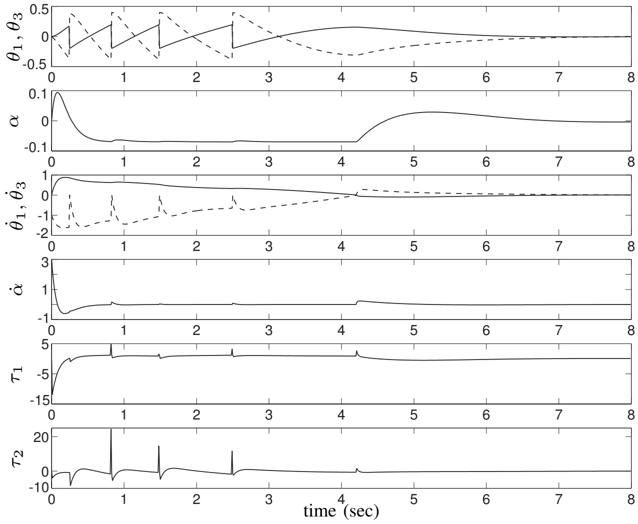

Fig. 3. Plot of joint angles (rad), joint angular velocities (rad/s), and control inputs (N.m) for the first simulation. The dashed lines correspond to the plots of θ_3 and $\dot{\theta}_3$.

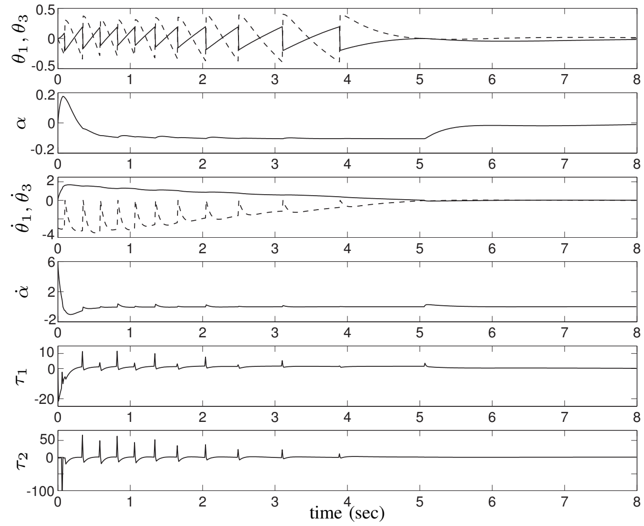

Fig. 4. Plot of joint angles (rad), joint angular velocities (rad/s), and control inputs (N.m) for the second simulation. The dashed lines correspond to the plots of θ_3 and $\dot{\theta}_3$.

(b) If the stance leg angle θ_1 reaches the boundary of the interval $[-\beta/2, \beta/2]$, apply the torque expressions in (20) to quickly enforce $\dot{\theta}_3 = 0$ while keeping the stance leg velocity unchanged. The stance leg and swing leg are immediately interchanged using (7) and the control torques in step 2(a) are applied thereafter. By enforcing $\dot{\theta}_3 = 0$, we avoid impulsive disturbance from the ground to the biped (swing leg) at the time of leg interchange. It can be seen from (7) that this also eliminates jumps in $\dot{\theta}_1$ at the time of leg interchange, which is necessary for smooth walking.

(c) If the torso angle α reaches the boundary of the interval $[-\gamma, \gamma]$, apply the torque expressions obtained in section III-B to quickly stop the torso while keeping the velocity of the legs symmetric. Then continue to apply the control torques in step 2(a).

e) If the stance leg velocity is close to zero, *i.e.* $\dot{\theta}_1 \approx 0$, terminate this step and go to step 3. If the constraints in (30) and (31) are not violated, $\dot{\theta}_1$ will go to zero since α_d was chosen to have a sign opposite to that of $\dot{\theta}_1$.

3. *Stabilization*:
With the biped satisfying the constraints in (30) and (31) while maintaining the symmetric gait, $\dot{\theta}_1 \approx 0$ ensures that the biped configuration will be inside the region of attraction R_A. Invoke the linear controller designed in step 1 to stabilize the desired equilibrium.

V. Numerical Simulations

We present the simulation results for the Synthetic-Wheel Biped with kinematic and dynamic parameters shown in Table I. Since $\beta = 22.5°$, the constraint in (30) is given by $-11.25° \leq \theta_1 \leq 11.25°$. The value of γ for the constraint in (31) was chosen as $\gamma = 10°$.

In the first simulation, the impulsive disturbance on the biped is modeled by the following initial conditions:

$$(\theta_{10}, \dot{\theta}_{10}, \theta_{20}, \dot{\theta}_{20}, \theta_{30}, \dot{\theta}_{30}) = (0, 0, \pi, 3, 0, -1) \qquad (32)$$

Figure 3 shows the simulation results which includes the plots of the joint angles and angular velocities and the control inputs. To illustrate the symmetric gait condition in (4), the angular positions of the stance and swing legs are shown on the same subplot using solid and dashed lines, respectively. The angular velocity plots of the stance and swing legs (solid and dashed lines) also satisfy the symmetric gait condition except at times when impulsive torques are applied. The desired angle of torso in (4) was chosen to be $\alpha_d = -4°$. A negative value of α_d was chosen since the biped moves in the positive direction due to the external disturbance, *i.e.*, $\dot{\theta}_1 > 0$. It can be seen form the plots in Fig.3 that the biped is able to reject the initial disturbance and stabilize its upright posture in about 8 seconds. The biped takes four steps to reject the initial disturbance before the linear controller is invoked at $t = 4.22\ s$. The peaks in the torque plots correspond to impulsive torques in (20) and are applied when the stance leg gets to the boundary of the interval $[-11.25°, -11.25°]$.

In the second simulation, the initial disturbance is larger and is modeled by the following initial condition:

$$(\theta_{10}, \dot{\theta}_{10}, \theta_{20}, \dot{\theta}_{20}, \theta_{30}, \dot{\theta}_{30}) = (0, 0.1, \pi, 5.5, 0, -3) \qquad (33)$$

TABLE I
PARAMETERS OF THE SYNTHETIC-WHEEL BIPED [27]

Kinematic parameters			
	Length (m)	Foot radius, R (m)	Foot arc, β (deg)
Inner leg	0.635	0.635	22.5
Outer leg	0.635	0.635	22.5
Torso	0.457	-	-
Dynamic parameters			
	Mass (kg)	Inertia (kgm^2)	d in Fig.2 (m)
Inner leg	1.64	0.094	0.285
Outer leg	3.64	0.128	0.355
Torso	11.87	0.198	0.307

The simulation results are shown in Fig.4. As in the first simulation, dashed lines are used to plot the swing-leg angle and angular velocity. The desired angle of torso is chosen to be $\alpha_d = -6^\circ$. This is larger in magnitude than the value of α_d in the first simulation since the magnitude of the initial disturbance is larger. Nevertheless, the biped has to take a greater number of steps to reject the initial disturbance. The linear controller is invoked at $t = 5.05\ s$ to stabilize the its upright posture. The first peaks in the torque plots correspond to the impulsive torques in section III-B which are applied to stop the torso from violating the constraint in (31). The subsequent peaks correspond to the impulsive torques in (20) which are applied to brake the swing leg immediately prior to swing-leg touchdown.

Remark 1: The continuous torques in Figs.3 and 4 are well below the maximum continuous torques of the motors used in the Synthetic-Wheel Biped prototype. The impulsive torques were modeled as Dirac delta functions but the numerical simulations show that our algorithm is effective even when the input is bounded and its time support is not infinitesimal. The large value of the impulsive torques is not a problem since electric motors can apply substantially larger torques[3] than their maximum continuous torque over small intervals of time.

VI. Conclusions

We proposed an algorithm to reject the effect of impulsive disturbances applied to the Synthetic-Wheel Biped in its upright configuration. The impulsive disturbance is modeled by initial velocity conditions of the biped. The algorithm uses a combination of continuous and impulsive control inputs to generate a symmetric walking gait and bound the states of the system. The symmetric gait is designed to allow the biped to take a few steps to reject the effect of the disturbance and the states are bounded to avoid the biped from falling down during the gait cycle. The algorithm was implemented on the Synthetic-Wheel Biped using kinematic and dynamic parameters provided in [27]. Simulation results show the effectiveness of the algorithm in rejecting the effect of the disturbance and stabilization of the upright posture. Our future work will focus on experimental verification of the proposed algorithm.

VII. Acknowledgments

The funding provided by the National Science Foundation, NSF Grant CMMI 0925055, is appreciated.

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