

# Robust Demand-side Plug-in Electric Vehicle Load Control for Renewable Energy Management

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**Abstract**—Plug-in electric vehicles can provide the power grid with some degree of control authority over fluctuations in electric load, thanks to their charging flexibility. The magnitude of this control authority depends on a variety of factors including the number of vehicles plugged into the grid, their instantaneous power demands, and the degree of flexibility in these demands. This paper addresses the problem of using a universally broadcast control signal to directly control the charge rate of a fleet of plug-in electric vehicles connected to the grid. The paper specifically seeks a control algorithm that is robust to uncertainties in renewable energy generation and the number of grid-connected vehicles. We adopt the sliding mode control strategy to achieve stability and robustness with respect to the collective effects of system uncertainties. The control law and robustness conditions are derived using the Lyapunov stability criterion. The paper shows that using only the real-time imbalance between the electricity supply and demand as a measured system output, the controller is able to precisely attenuate this imbalance, achieving reliable demand-side load management. Numerical simulations are provided to evaluate the performance of this controller.

## I. INTRODUCTION

This paper proposes a robust feedback strategy for controlling the aggregate grid power demanded by a large set of plug-in electric vehicles (PEVs). This problem is challenging due to uncertainty in the total number of PEVs plugged into the grid, the total grid load, and the amount of available grid power at any given time. The paper's overarching goal is to address these uncertainties using a direct load control strategy based on sliding mode control principles.

In a broad sense, this paper is part of a growing literature on the interplay between PEVs and the power grid [1-5]. This literature highlights both positive and negative potential impacts of PEVs on the grid. On the negative side, PEVs represent an additional grid load that may overstretch the power grid, especially in localities with high levels of PEV adoption. On the positive side, the ability to potentially control PEVs as a dispatchable load may enable the grid to both reduce strain during peak hours and accommodate renewable generation to a greater extent [6-8]. To achieve these benefits, the grid needs (i) the ability to communicate

with its PEV loads, and (ii) the ability to control them in a stable and robust manner based on this communication.

Demand-side power management or direct load control is a key enabler for achieving reliable grid operation in the presence of large renewable power penetration [8-10]. Since renewable energy is intermittent, it cannot be fully dispatched upon generation. Thus, commensurate storage capacity needs to be dedicated, either on the generation side or demand side, to mitigate the intermittency of renewable energy sources. This storage capacity can be built specifically for the grid (e.g., pumped hydro units, grid battery energy storage, etc.). Alternatively, the grid can make use of the inherent storage capacities of certain deferrable loads, such as PEVs, air conditioning systems, water heating systems, etc. [10].

Direct load control of PEVs can be challenging due to the uncertainties on both the demand and generation sides. Quantities such as the number of active (i.e., grid connected) PEVs, the overall grid load, and the total available renewable power may be highly uncertain, difficult to measure, or both. This complicates the problem of controlling PEV load to minimize the mismatch between the grid's total power supply and demand. The literature on direct PEV load control recognizes the uncertainties inherent in this problem, but has yet to address them fully. This motivates this paper's development of a robust strategy which can effectively and conveniently control PEV load in the presence of uncertainty.

The remainder of this paper begins by formulating the direct grid load control problem for PEVs under uncertain operating conditions. We assume that the controller is only provided with the imbalance between the total available power and the load, and attempt to suppress this imbalance. We also assume that the exact number of PEVs connected to the grid at a given time is not measurable, but the grid is able to forecast a lower bound for this number at any given time. We adopt the sliding mode control method to derive a high-performance trajectory tracking controller that adapts the load trajectory to the available power trajectory in real time. More specifically, the controller receives the real-time power supply/demand error signal, computes the appropriate scaling of PEV charge rates, then broadcasts this scaling as a universal control signal to the entire PEV population. The stability and robustness properties of this controller are guaranteed using the Lyapunov stability criterion. Finally, the paper provides several numerical simulations to illustrate the performance of this control scheme.

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The remainder of the paper is organized as follows: Section II introduces and formulates the problem of PEV charge rate control. In Section III, a robust demand-side PEV load management strategy is developed based on sliding mode control theory. Section IV presents the numerical simulations of the resulting closed-loop system, and Section V provides the paper's concluding remarks.

## II. PROBLEM STATEMENT AND SYSTEM FORMULATION

Our goal in this paper is to develop a control scheme that adapts the grid demand trajectory, consisting of both PEV and non-PEV loads, to the real-time available power trajectory generated through both renewable and non-renewable resources. The selected control input is a normalized signal,  $u(t)$ ,  $0 \leq u(t) \leq 1$ , broadcast by the grid operator to all active PEVs, scaling their instantaneous charging rates (See Fig. 1). In the extreme case when  $u(t)$  becomes zero, PEVs are stopped from charging, whereas when  $u(t)$  becomes 1, they charge at their maximum power. Therefore, the grid has full control authority over the PEV portion of total power demand.

We envision PEVs receiving the above control signal through a uni-directional (wired or wireless) communication network, and adapting their charging power accordingly. Since this control strategy only requires the uni-directional broadcast of a single signal, we neglect communication delays in this paper's analyses. Future work will examine communication delays in more depth, focusing on their potential impact on controller stability and bandwidth.

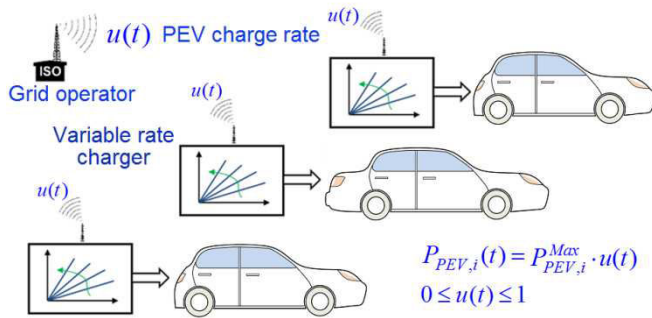


Fig 1. Demand-side PEV load control strategy: PEVs receive a universal control signal  $u(t)$  broadcast by the grid operator through a one-way communication network, and adapt their charging power immediately.

To develop the proposed controller, we begin by expressing total grid power demand as follows:

$$\begin{aligned} P_{demand}(t) &= P_{nonPEV}(t) + P_{PEV}(t) \\ &= P_{nonPEV}(t) + \sum_{i=1}^{N(t)} P_{PEV,i}(t) \\ &= P_{nonPEV}(t) + \sum_{i=1}^{N(t)} P_{PEV,i}^{Max} \cdot u(t) \end{aligned} \quad (1)$$

where  $P_{nonPEV}$  and  $P_{PEV}$  represent the non-PEV and PEV portions of the power demand, respectively,  $P_{PEV,i}^{Max}$  is the  $i^{th}$

PEV's maximum charging power, and  $N(t)$  is the number of active PEVs. The index  $i$  may rotate among the PEVs when one or more of them finish charging.

If we assume that the average value of the PEVs' maximum charging power is independent from the number of PEVs (a legitimate assumption when  $N$  is sufficiently large) the total grid demand can be approximated by:

$$P_{demand}(t) = P_{nonPEV}(t) + \bar{P}_{PEV}^{Max} \cdot N(t) \cdot u(t) \quad (2)$$

where  $\bar{P}_{PEV}^{Max}$  is the average value of the PEVs' maximum charging power.

Equation (2) represents a simplified model for the aggregate grid demand in the presence of controllable PEV load. This aggregate demand depends on the number of active PEVs,  $N(t)$ . For control design purposes, this paper treats  $N(t)$  as a continuous signal with a bounded derivative. The only information one needs to possess about  $N(t)$  to implement the proposed controller is the set of bounds on its derivative. To validate this controller, however, one needs to simulate  $N(t)$ , ideally in a manner representing real-world PEV trips. This paper's controller validation study uses a sinusoidal pattern for  $N(t)$  for simplicity. Ongoing research by the authors is currently validating the proposed controller against more sophisticated real-world driving patterns similar to those used in Ref. [5].

## III. ROBUST DEMAND-SIDE PEV LOAD CONTROL

This section develops a robust load control strategy for the grid demand model in Equation (2). The objective is to force the total power demand  $P_{demand}(t)$  to track a power supply trajectory  $P_{supply}(t)$ , representing the total grid power supplied from both renewable and non-renewable resources:

$$P_{supply}(t) = P_{nonRnew}(t) + P_{Rnew}(t) \quad (3)$$

where  $P_{nonRnew}(t)$  is the non-renewable power generation mainly dedicated to the non-PEV load, and  $P_{Rnew}(t)$  represents the renewable energy supply intended for PEVs.

We seek a control law for the PEV charge rate  $u(t)$  that stabilizes the imbalance between the power supply and demand, represented by a measurable error signal defined as:

$$\begin{aligned} e(t) &= P_{supply}(t) - P_{demand}(t) = \\ &= \{P_{nonRnew}(t) + P_{Rnew}(t) - P_{nonPEV}(t)\} - \bar{P}_{PEV}^{Max} \cdot N(t) \cdot u(t) \end{aligned} \quad (4)$$

The first term inside the brackets in Eq. (4) is the net available power for PEV charging, representing the desired power trajectory to be tracked by the PEVs. Thus, the tracking error can be modified to:

$$e(t) = P_{des}(t) - P_{PEV}(t) = P_{des}(t) - \bar{P}_{PEV}^{Max} \cdot N(t) \cdot u(t) \quad (5)$$

where  $P_{des}(t) = P_{nonRnew}(t) + P_{Rnew}(t) - P_{nonPEV}(t)$ .

Now the control objective is to suppress the tracking error,  $e(t)$ , under uncertainties in the desired power trajectory,  $P_{des}(t)$ , and the number of active PEVs,  $N(t)$ . In the next section we adopt the sliding mode control strategy to achieve this goal.

### A. Control Design

The goal of sliding mode control is to design asymptotically stable manifolds such that the system states converge to these manifolds and slide along them toward the origin [11, 12]. The sliding manifold we consider in the PEV load control problem in this paper is the tracking error itself. Therefore, if we achieve stability of the sliding manifold, the control objective is directly met. We start the control design process by defining a positive-definite Lyapunov candidate function:

$$V(t) = \frac{1}{2} e^2(t) \quad (6)$$

Taking the time derivative of the above Lyapunov function and using Eq. (5) yields:

$$\begin{aligned} \dot{V}(t) &= e(t)\dot{e}(t) = \\ e(t) &\left\{ \dot{P}_{des}(t) - \bar{P}_{PEV}^{Max} \cdot \dot{N}(t) \cdot u(t) - \bar{P}_{PEV}^{Max} \cdot N(t) \cdot \dot{u}(t) \right\} \end{aligned} \quad (7)$$

Now our goal is to choose a control law for  $u(t)$  such that the time derivative of the Lyapunov candidate function becomes strictly negative for nonzero  $e(t)$ . This way  $V(t)$  becomes a strictly decreasing function, leading to the convergence of the tracking error to zero. Before proceeding to the next step, we make a few assumptions:

**Assumption 1:** We assume that the number of PEVs available for charging,  $N(t)$ , never equals zero during the PEV load control process. In other words, at least one PEV is always available for charging.

**Assumption 2:** We assume that the rate of variation of the number of PEVs on the grid is bounded, i.e.:

$$\left| \dot{N}(t) \right| < N_{\max}^{dot} \quad (8)$$

**Assumption 3:** We assume that the desired PEV power trajectory is one time continuously differentiable:

$$\left| \dot{P}_{des}(t) \right| < P_{des,\max}^{dot} \quad (9)$$

**Assumption 4:** We also assume that the desired PEV power trajectory satisfies a trackability condition given by:

$$0 \leq P_{des}(t) \leq \bar{P}_{PEV}^{Max} \cdot N(t) \quad (10)$$

Now we present the following theorem for the charge rate control of PEVs:

**Theorem:** For the system described by Eq. (5), satisfying Assumptions 1-4, the following control law results in the convergence of the tracking error to the origin:

$$u(t) = u(0) + \int_0^t \eta(\tau) \cdot \text{sgn}\{e(\tau)\} d\tau \quad (11)$$

where  $\eta(t)$  is, in general, a user-defined continuous time-varying control gain satisfying a robustness condition given by:

$$\left| \frac{\dot{P}_{des}(t) - \bar{P}_{PEV}^{Max} \cdot \dot{N}(t) \cdot u(t)}{\bar{P}_{PEV}^{Max} \cdot N(t)} \right| < \eta(t) \quad (12)$$

**Proof of the Theorem:** Taking the time derivative of the control law in Eq. (11), and replacing it in Eq. (7) yields:

$$\begin{aligned} \dot{V}(t) &= \left\{ \dot{P}_{des}(t) - \bar{P}_{PEV}^{Max} \cdot \dot{N}(t) \cdot u(t) \right\} e(t) \\ &\quad - \eta(t) \bar{P}_{PEV}^{Max} \cdot N(t) \cdot \text{sgn}\{e(t)\} \cdot e(t) \\ &= \left\{ \dot{P}_{des}(t) - \bar{P}_{PEV}^{Max} \cdot \dot{N}(t) \cdot u(t) \right\} e(t) - \eta(t) \bar{P}_{PEV}^{Max} \cdot N(t) \cdot |e(t)| \end{aligned} \quad (13)$$

We can show that if the robustness condition given in Eq. (12) holds, the second term in Eq. (13) dominates the first term. For this, we recast Eq. (12) as:

$$\left| \dot{P}_{des}(t) - \bar{P}_{PEV}^{Max} \cdot \dot{N}(t) \cdot u(t) \right| < \eta(t) \cdot \bar{P}_{PEV}^{Max} \cdot N(t) \quad (14)$$

It follows that:

$$\left\{ \dot{P}_{des}(t) - \bar{P}_{PEV}^{Max} \cdot \dot{N}(t) \cdot u(t) \right\} e(t) < \eta \bar{P}_{PEV}^{Max} \cdot N(t) \cdot |e(t)|, \quad e(t) \neq 0 \quad (15)$$

We can conclude from Eq. (13) and (15) that the derivative of the Lyapunov candidate function becomes negative for nonzero  $e(t)$ . This also concludes the proof of the theorem. That is:

$$\begin{aligned} \dot{V}(t) &< 0 \text{ for } e(t) \neq 0 \\ \Rightarrow V(t) &\rightarrow 0 \text{ as } t \rightarrow \infty \\ \Rightarrow e(t) &\rightarrow 0 \text{ as } t \rightarrow \infty \end{aligned} \quad (16)$$

**Remark 1:** The proposed controller, in addition to guaranteeing robust and stable tracking, also guarantees that the control input  $u(t)$  remains bounded between 0 and 1, provided  $u(0)$  is chosen between 0 and 1. To see that, note from Eq. (11) that the continuity of  $\eta(t)$  forces  $u(t)$  to also be continuous. Furthermore, note from Eq. (5) and Eq. (10) that when  $u(t) = 0$ , the error  $e(t)$  will be non-negative. Therefore, based on Eq. (11) and Eq. (12), when  $u(t) = 0$ , one can guarantee that it will not decrease further. Conversely, one can show through a similar sequence of steps than when  $u(t) = 1$ , it will not increase further.

**Remark 2:** Because  $u(t)$  is bounded between 0 and 1 (see Remark 1), the bound on  $\eta(t)$  from Eq. (12) is guaranteed to be finite (under Assumptions 1-4). It may be convenient in practice to set  $\eta$  to a constant predefined value rather than a time trajectory, and tune it such that a desirable control performance is achieved.

**Remark 3:** Sliding mode control uses a signum switching function to achieve the attractivity condition of the sliding manifold despite perturbations. Due to high-frequency switching of the control input the chattering problem may arise in practice. In our control law, Eq. (11), the signum function appears inside an integrator which acts as a perfect filter for high-frequency chattering. However, in practice, the controller is implemented in discrete time, degrading its perfect filtering property. To reduce the chattering effect, we can replace the signum function in the control law with a saturation function defined as:

$$\text{sat}\left\{\frac{e(t)}{\varepsilon}\right\} = \begin{cases} 1 & e(t) > \varepsilon \\ \frac{e(t)}{\varepsilon} & -\varepsilon \leq e(t) \leq \varepsilon \\ -1 & e(t) < -\varepsilon \end{cases} \quad (17)$$

where  $\varepsilon$  is a small control parameter adjusting the slope of the saturation function. This replacement removes the chattering effect from the system. As a trade-off, the controller will only guarantee a zonal convergence of the tracking error, i.e.:  $|e(t)| \leq \varepsilon$ . Outside this zone the saturation function is identical to the signum function. Thus, according to the sliding mode control law, the error trajectory has to converge to this zone.

**Remark 4:** In order to avoid integrator windup in the case of control input saturation, we include an integrator anti-windup projection operator in the control law as follows:

$$u(t) = u(0) + \int_0^t \text{Proj}_u \left[ \eta(\tau) \cdot \text{sat}\left\{\frac{e(\tau)}{\varepsilon}\right\}\right] d\tau \quad (18)$$

where

$$\text{Proj}_u[\theta] = \begin{cases} 0 & \text{if } u(t) = u_{\max} = 1 \text{ and } \theta > 0 \\ 0 & \text{if } u(t) = u_{\min} = 0 \text{ and } \theta < 0 \\ \theta & \text{otherwise} \end{cases} \quad (19)$$

This method is equivalent to the variable structure anti-windup or “integrator clamping” technique in the PID control literature [13, 14].

Equation (18) represents the final form of the control law proposed in this paper for the robust PEV load control problem. In the next section, we present a set of numerical simulations to evaluate the performance of the control law derived in this section.

#### IV. NUMERICAL SIMULATIONS

To examine the performance of the derived control law, we carry out a numerical simulation in this section. We evaluate a population of about 1000 PEVs, smoothly varying in number, within a time interval of 5 hours. The average value of the maximum PEV charging power is set to 2 kW, representing a predominantly residential charging scheme.

A multiple-frequency desired PEV power trajectory is generated within the trackable domain of PEVs, representing the renewable energy flow into the grid. The control gain  $\eta$  is set to a constant value, and tuned to yield desirable control performance. The control parameter  $\varepsilon$  is chosen to provide a tolerance level of  $\pm 10$  kW for the tracking error, nearly corresponding to  $\pm 1\%$  error level, to prevent chattering.

Figure 2 depicts the simulation results. Initially, for the first third of the simulation time, the charge rate of PEVs is set to the constant level of half-power charging,  $u(t) = 0.5$ , without activating the controller. During this period the PEV load varies independently from the desired power trajectory, leading to a large supply/demand imbalance. However, when we switch the controller on, the PEV load trajectory converges to the desired power trajectory and stays on it for

the remaining two-third of the simulation time. The tracking error stays close to zero (Fig. 2(b)), and the control input remains within its specified bounds (Fig. 2(c)). Moreover, Fig. 2(d) illustrates the effectiveness of the saturation function in eliminating the chattering effect. The remaining spikes in the error are due to the PEVs plugging in and out.

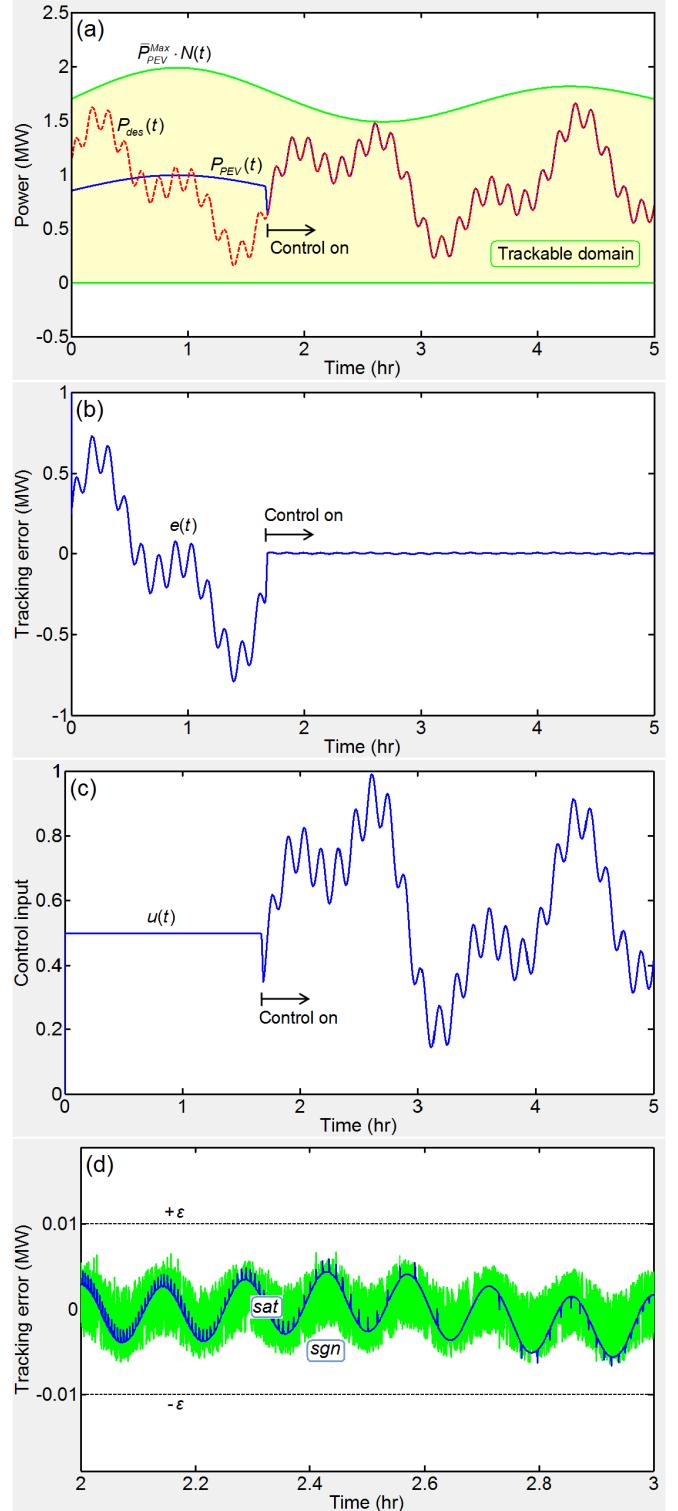


Fig. 2. Demand-side PEV load control simulation: (a) PEV power tracking, (b) tracking error, (c) control input, and (d) comparison of tracking error for two different sliding mode switching schemes.

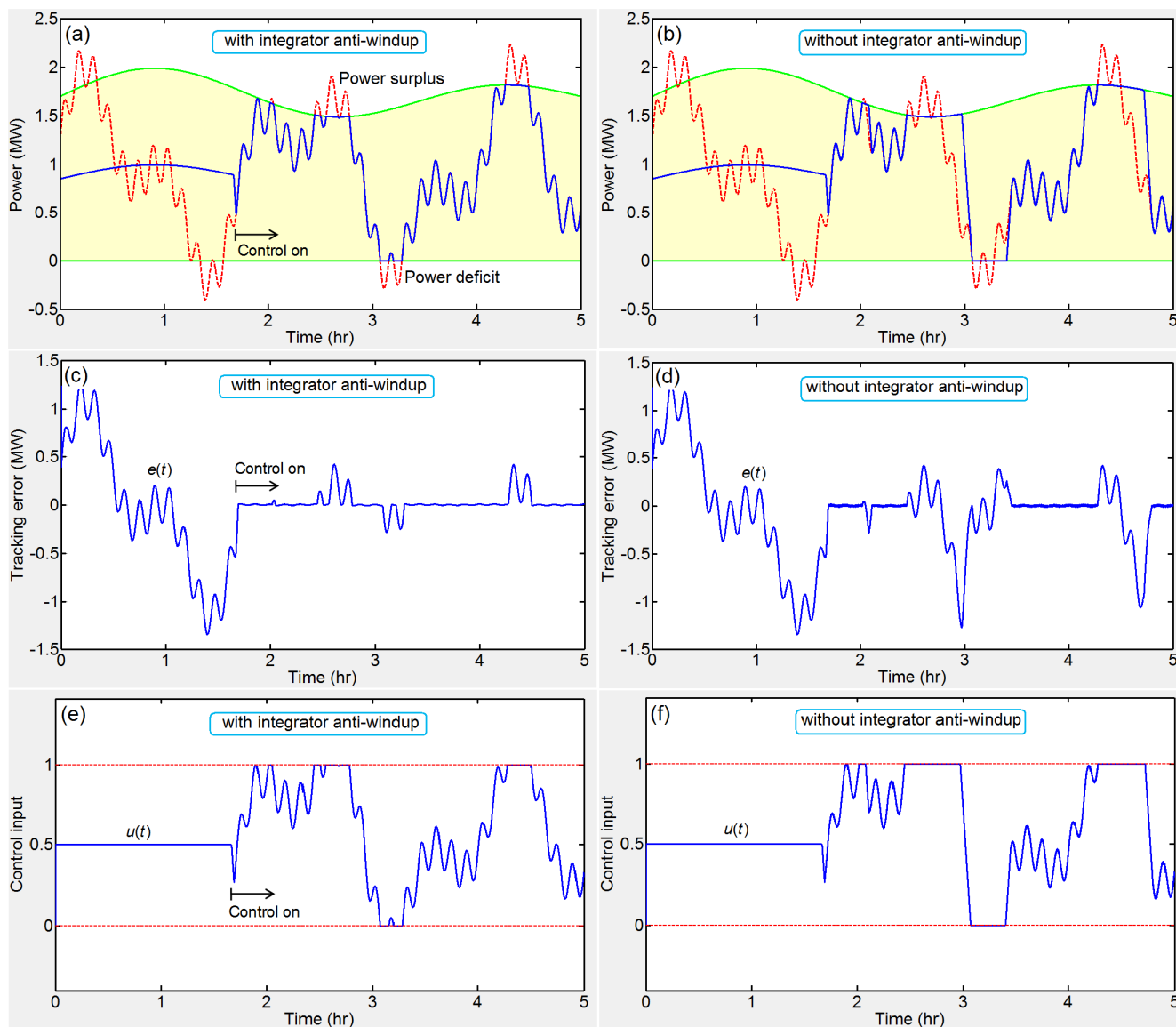


Fig. 3. Comparison between controllers with and without integrator anti-windup scheme.

To demonstrate the advantage of using the projection operator for integrator anti-windup in the case of control saturation, we modify the desired power trajectory such that temporarily escapes the trackable domain of the PHEV load. Whenever there is a power surplus, the control input saturates at 1 and the PEVs charge at their maximum rate, whereas in the case of power deficit the control input saturates at zero, preventing PEVs from charging. The control implementation results are shown in Fig. 3, where a clear difference in performance is seen between the controllers with and without integrator anti-windup. In the case where integrator anti-windup is exploited, the PEV load trajectory and the desired power trajectory merge immediately after the desired power trajectory returns to the trackable domain. In contrast, if no integrator anti-windup scheme is implemented, the controller takes longer to recover. However, we should remark that control saturation during the demand-side management of PEV load can be

problematic for the stability of the power grid system, and hence, must be avoided in practice. Moreover, providing theoretical guarantee for the closed-loop system stability under control saturation can be very challenging.

## V. CONCLUSIONS

This paper uses sliding mode control to achieve robust demand-side management of PEV loads on a grid system. The controller measures the real-time error between the total power supply and demand in a grid serving both PEVs and other loads. The controller suppresses this error by broadcasting a universal signal scaling the power demanded by the PEVs relative to their maximum power capacities. We showed that this controller is able to match power supply and demand in a robust manner despite uncertainties on both the grid side and PEV side, as long as certain trackability assumptions are met. When the desired power

trajectory escapes the trackable domain of the PEV load, the control input saturates and integrator windup ensues. To remedy this, we proposed using an integrator anti-windup strategy, and showed its efficacy in the control recovery period after input saturation. The main advantages of the proposed control scheme in this paper are: (i) robustness with respect to uncertainties and perturbations, and (ii) simplicity of implementation.

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