Duty Cycle Scheduling in Dynamic Sensor Networks for Controlling Event Detection Probabilities

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Abstract—A sensor network comprising of RF or radarbased sensors has a deteriorating performance in that the effective sensor footprint shrinks as the power level decreases. Power is typically only drawn from the sensor nodes when they are turned on, and as a consequence, the power consumption can be controlled by controlling the duty cycle of the sensors. In this paper, we provide a probabilistic scheduling of the duty cycles in a sensor network deployed in an area of interest based on a Poisson distribution which ensures that a performance measure, e.g., the probability of event detection, is achieved throughout the lifetime of the network. Upper bounds on the performance of the network are given in terms of the decay rates, the spatial distribution intensity, and the desired performance of the network.

I. INTRODUCTION

A sensor network consists of a large number of sensor nodes. Each node is typically a low cost, low power device with limited sensing, processing and communication capabilities. Despite their individual limitations, with proper deployment, sensing and processing algorithms, these sensor nodes can form a highly reliable, efficient and robust network. Sensor networks have a wide range of real life military and civilian applications [1]. Surveillance and reconnaissance of large areas, disaster relief operations, and monitoring parameters of interest in inaccessible areas or extremely large systems are examples of such applications.

Normally, the sensor nodes are battery powered. Despite the low power consumption, the lifetime of each sensor is limited as it is practically impossible to replace the batteries of a large number of sensors deployed in an inaccessible or potentially hostile environment [5]. Hence, power management is a critical problem and subject of an active research effort in the wireless sensor networks community. One obvious way to conserve power is to turn the sensors *off* when they are not needed. However, this must be done intelligently because critical events can be missed and information can be lost while a sensor is *off*.

Cruz and Sarkar form an optimization problem to minimize the power consumption with respect to a QoS constraint. They solve this problem for a system comprising of two wireless receivers and a single transmitter over static channel using dynamic programming (DP). Using these results as a guideline, they develop an adaptive scheme that finds an optimal sleep duration for multiple wireless nodes with respect to average packet delay as the performance constraint [6]. Similarly, Shuman and Liu optimize the sleep duration of the sensor nodes by forming a cost function that includes energy consumption and the cost of delay in packet forwarding [16].

Alfieri *et al.* use the redundancy in the sensor deployment to reduce power consumption by turning only a subset of sensors *on* at each time, thus increasing the lifetime of the network [2]. In [17] and [9], the authors devise distributed protocols for extending the lifetime of the network by turning *on* only a sufficient number of sensors. Each node decides whether it should remain *on* based on its own observation of the surrounding environment. Potkonjak and Slijepcevic propose a heuristic that selects a set of sensor nodes which are mutually exclusive such that members of each set when turned *on* completely cover the area of interest [14]. Since only one set is active at each time, power is conserved by eliminating the redundancy.

In [8], Fekri and Subramanian obtained limits on the sleep duty cycling for energy efficient operation of the system. Dietrich and Dressler has shown how a number of the sensor network performance parameters, such as coverage and connectivity, essentially reduce to network lifetime and thus formulate a concise definition of network lifetime [7]. Hou and Zhang found the upper bounds on the lifetime of a sensor network that can be achieved by various switching algorithms when α portion of the total region is covered by the sensors [11].

Cassandras and Ning proposed a scheme in which using the available statistical information about event times, the sleep time of the receiver is controlled dynamically such that it samples the channel more frequently when an event is more likely to happen, and less frequently when it is not. Moreover they also derived the optimality conditions for minimum energy consumption [3]. Hsin and Liu have investigated the relation between redundancy in sensor deployment and the probability of an event being undetected [12].

In this paper, we extend the work of Hsin and Liu by taking into account the decrease in the sensor footprint due to power decay. This behavior is observed in RF type sensors where size of the sensor footprint is proportional to the available power to the sensor. We propose a scheduling scheme which can provide a constant performance, namely the event detection probability, throughout the lifetime of the network. We also derive an expression for the lifetime of network. Here we only consider the sensing capability of the network; we are not concerned with the communication among the sensors.

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In Section II, we lay out the basics of the problem under consideration. The event detection probability for both nondecaying and decaying networks are discussed in Section III. Section IV contains the main results of the paper and provides a scheduling strategy to maintain the desired performance throughout the lifetime of the sensor network. Monte-Carlo simulations are given in Section V that illustrates the validity and operation of our proposed scheme. We conclude the paper by the remarks in Section VI.

II. SYSTEM DESCRIPTION

Consider a domain $\mathcal{D} \subset \mathbb{R}^2$ in which sensors are randomly deployed such that the location of each sensor is independent of all the other sensors' locations. For example, such a scenario can arise when sensors are dropped in the region of interest from air. From [10] we know that *n* points which are distributed independently with uniform distribution within a large region $\mathcal{D} \subset \mathbb{R}^2$ are those of a spatial Poisson point process.

As such, this sensor deployment can be modeled as a stationary spatial Poisson point process with constant intensity λ (expected number of sensors in unit area). Given a set in \mathcal{D} with area A, the probability of having n sensors in this area is given by

$$P_n = \frac{(\lambda A)^n e^{-\lambda A}}{n!}.$$
(1)

All the sensors are considered to have identical battery power and sensing capabilities when deployed. The sensors are all RF or radar based where the footprint of each sensor *i* is a closed ball, $\mathcal{B}_{r(t)}(x_i)$, of radius r(t), centered at x_i , position of the sensor. The radius of the footprint depends on the available power level for each sensor. We consider a Boolean sensing model, i.e., events are detected only if they are within the footprint of the sensor.

To conserve power, we let the sensors be on with probability q. Each sensor can switch its state from on to off or vice versa, only at discrete time instances $k\Delta t$ (or simply at instance k), where Δt is the sample time. The state of a sensor at instance k is maintained throughout the interval [k, k + 1) of length Δt . A sensor can sense only when it is on and for an event to be detected it should be within the footprint of at least one on sensor.

When a sensor is *on* it consumes power and as a consequence its footprint shrinks. Using the discrete time version of the battery dynamics in [15], we model the power of each sensor in the *on* state using the following dynamics

$$\eta(k+1) = \eta(k) - \Delta t \gamma \eta(k),$$

where γ is the decay constant and $\eta(k)$ represents the remaining battery power. We define a switching signal $\sigma(k)$ as

$$\sigma(k) = \begin{cases} 1 & \text{if the sensor is } on \text{ at time } k. \\ 0 & \text{if the sensor is } off \text{ at time } k. \end{cases}$$

Since the sensor is *on* with probability q, the expected value of $\sigma(k)$ is $E\{\sigma(k)\} = \hat{\sigma}(k) = q(k)$. We know that power is

only consumed when the sensor is *on* so we can modify the power model as

$$\eta(k+1) = \eta(k) - \Delta t \gamma \sigma(k) \eta(k), \qquad (2)$$

Since $\hat{\sigma}(k) = q(k)$ and $\sigma(i)$ is independent of $\sigma(j)$ for all $i \neq j$, the expected power level of each sensor is

$$\hat{\eta}(k+1) = \left[\prod_{i=0}^{k} \left(1 - \Delta t \gamma q(i)\right)\right] \eta(0).$$
(3)

Moreover, for all $t \in [k, k+1)$, we assume that $\eta(t) = \eta(k)$.¹

III. PROBABILITY OF EVENT DETECTION

Consider a non-persistent event that happens at location $x_e \in \mathcal{D}$ at some arbitrary time $t \in [k, k+1)$. A non-persistent event does not leave a mark in the environment and can only be detected when it occurs. Hence, this event is detected if it is within the footprint of at least one sensor which is in *on* state at time k. We first consider a non-decaying sensor network, i.e. network of sensors whose footprints does not change with time.

Lemma 3.1: [12] The probability of an event going undetected by a non-decaying sensor network deployed randomly with an intensity λ is given by $P_u = e^{-\lambda \pi r^2 q}$.

Proof: The probability of an event going undetected is equal to the sum of probabilities of the event being in range of $n \in [0, \infty]$ sensors and all of them being *off*; The proof of this Lemma can be found in [12].

Now, consider a decaying network where the power of the sensors is consumed when they are *on*, resulting in a decrease in the area of the sensor footprints which is proportional to the decay in power [4]. Martin *et al.* have shown that if the sensor range model is based on the RF power density function for an isotropic antenna, the sensor footprint is proportional to the available power of the sensor node [13], i.e.

$$r^2(t) \propto \eta(t). \tag{4}$$

where r(t) is the radius of the sensor footprint at time $t \in [k, k+1)$. Hence, the area of the sensor footprint at time t is

$$A(t) = \pi r(t)^2 = \alpha \eta(t).$$
(5)

where $\alpha = \zeta \pi$ is a constant with ζ being the constant of proportionality in Equation (4).

Theorem 3.2: The probability of an event being detected by a decaying sensor network is given by $P_d(k) = 1 - e^{-\lambda c \left[\prod_{i=0}^{k-1} (1 - \Delta t \gamma q(i))\right]q(k)},$ as the number of sensors goes to infinity.

¹We should note that in this analysis we are not considering the potential power consumption due to switching between the *on* and *off* states.

Proof: From Lemma 3.1, we know that an event at $x_e \in \mathcal{D}$ is detected in a non-decaying sensor network if there is at least one on sensor in $\mathcal{B}_r(x_e)$, where r is the radius of the sensors footprint. For a decaying network this reasoning can not be used directly. Although all the sensors are initially identical, we have no reason to believe that the battery powers and sensors footprint areas are the same throughout the network at any time $k \neq 0$ due to sensor switching and power decay. Instead, at each time k, there are a finite number of possible footprints, $N(k) \in [1, M]$, corresponding to all possible onoff combinations, where M is the total number of sensors in the network. At each time k, associate $A_i(k)$ with the i^{th} possible footprint, $i = 1, 2, ..., \min(M, 2^k)$, according to some indexing of the possible footprints.² Considering that the sensor deployment follows a Poisson process model, the probability of n sensors being in a given set with area $A_i(k)$ is given by (1) as

$$P_n^i(k) = \frac{(\delta_i(k)\lambda A_i(k))^n e^{-\delta_i(k)\lambda A_i(k)}}{n!},$$

Thus, the probability of an event going undetected by all the sensors of footprint area $A_i(k)$ is

$$P_u^i(k) = \sum_{n=0}^{\infty} (1-q)^n \frac{(\delta_i(k)\lambda A_i(k))^n e^{\delta_i(k)\lambda A_i(k)}}{n!}$$
$$= e^{-\delta_i(k)\lambda A_i(k)q},$$

where

$$\delta_i(k) = \frac{\text{number of sensors with footprint area } A_i(k)}{\text{total number of sensors}}$$

and $\sum_{i=1}^{N(k)} \delta_i(k) = 1$. Hence $\delta_i(k)\lambda$ is the intensity of sensors with footprint area $A_i(k)$. The total probability of an event going undetected by all the sensors in the network is

$$P_u(k) = \prod_{i=1}^{N(k)} P_u^i(k) = e^{-\left(\sum_{i=1}^{N(k)} \delta_i(k) A_i(k)\right)\lambda q}.$$
 (6)

It is important to note that $\sum_{i=1}^{N(k)} \delta_i(k) A_i(k)$ is the weighted average of footprint area of all the sensors in the network. So the above expression can be written as

$$\sum_{i=1}^{N(k)} \delta_i(k) A_i(k) = \frac{1}{M} \sum_{j=1}^M A_j(k).$$

From the Law of Large Numbers we know that for a large number of sensors this mean of sensor footprints will approach $\hat{A}(k)$, the exprected area of the sensors' footprints, or formally

$$\frac{1}{M}\sum_{j=1}^M A_j(k) \to \hat{A}(k) \quad as \ M \to \infty.$$

Replacing this in Equation (6), we get

$$P_u(k) = e^{-\lambda \hat{A}(k)q}.$$
(7)

²Strictly speaking, i is really a funtion of k, but we suppress this for notational convinience.

The expected footprint area can easily be computed by replacing the expected power from Equation (3) into Equation (5) which yields

$$\hat{A}(k) = c \left[\prod_{i=0}^{k-1} (1 - \Delta k \gamma q(i)) \right], \tag{8}$$

Where $c = \alpha \eta(0)$ is a constant. Noting that $P_d = 1 - P_u$ and substituting the value of $\hat{A}(t)$ in Equation (7) concludes the proof.

Note that if the probability of sensors being *on*, q, is constant then the chance of an event being detected, P_d , clearly decreases with time.

IV. DUTY CYCLE SCHEDULING FOR CONSTANT EVENT DETECTION PROBABILITY

In many practical applications of sensor networks it is desired to maintain a minimum satisfactory probability of detecting events.

Definition 4.1: As the number of sensors goes to infinity, the *desired network performance*, P_{des} , is the minimum satisfactory probability of an event being detected.

Consider the case where the desired event detection probability is a given performance parameter P_{des} . Hence $\beta = 1 - P_{des}$, where β is the probability of the event going undetected. Duty cycle scheduling of the sensors can help maintain such performance in the face of the decreasing sensor power.

Using the result of Theorem 3.2, we have

$$\left[\prod_{i=0}^{k-1} \left(1 - \Delta t \gamma q(i)\right)\right] q(k) = \frac{\ln(\frac{1}{\beta})}{\lambda c}.$$
(9)

Putting k = 0 in the above equation, we can compute the initial value of q as

$$q(0) = \frac{\ln(\frac{1}{\beta})}{\lambda c}.$$
(10)

Theorem 4.1: As the number of sensors goes to infinity, the maximum achievable event detection probability in a sensor network with given spatial distribution intensity λ is $1 - e^{(-\lambda c)}$.

Proof: Consider Equation (10) which gives the initial probability of a sensor to be in *on* state. This probability should always remain in the interval [0, 1]. Since $\beta \in [0, 1]$, it is guaranteed that

$$q(0) = \frac{\ln(\frac{1}{\beta})}{\lambda c} \ge 0,$$

for all given β , λ and c.

To ensure that $q(0) \leq 1$, we have

$$q(0) = \frac{\ln(\frac{1}{\beta})}{\lambda c} \le 1$$

or

$$\beta \ge e^{-\lambda c}.$$

Hence,
$$P_{des} \leq 1 - e^{-\lambda c}$$
.

Our goal is to control q(k), the probability of the sensors being *on* at time instant k, such that a desired performance is achieved. In other words, we are looking to find $u(k) \in \mathbb{R}$ such that

$$q(k+1) = u(k),$$
 (11)

gives a scheduling scheme for the sensors' duty cycle which ensures that the overall network event detection performance will be maintained at the desired level.

Rearranging the terms of Equation (9), we get an expression for the evolution of q(k) as

$$q(k+1) = \frac{1}{1 - \Delta t \gamma q(k)} q(k).$$
 (12)

In other words, our proposed duty cycle control law is of the form

$$u(k) = \frac{1}{1 - \Delta t \gamma q(k)} q(k).$$
(13)

Solving the resulting controlled dynamical Equation (12) with the initial condition (10) gives an expression for q(k) as

$$q(k) = \frac{-1}{\gamma k \Delta t + \frac{\lambda c}{\ln(\beta)}}.$$
 (14)

Therefore, Equation (14) gives a scheduling strategy, for the duty cycle of the sensors, which ensures a constant event detection probability. However, this can be achieved for a limited time, beyond which it is impossible to maintain the desired event detection probability.

Definition 4.2: As the number of sensors goes to infinity, the *lifetime of the sensor network* is the maximal time beyond which the desired network performance cannot be achieved.

Characterizing this lifetime is essential to the design of the sensor network.

Theorem 4.2: As the number of sensors goes to infinity, the lifetime of the sensor network with desired event detection probability of $1 - \beta$ is given by $\frac{1}{\gamma} (\frac{\lambda c}{\ln(1/\beta)} - 1)$.

Proof: At the end of the lifetime of the sensor network all sensors should be on and contributing, i.e., $q(k_f) = 1$, where k_f denotes the final time instance. Otherwise, if a sensor is off, turning that on will increase the detection probability by the probability of the event being in the footprint of that sensor. This in turn increases the lifetime of the sensor network which is contradictory to the assumption of being at the end of lifetime of the network. Using Equation (14) sensors are all on when $\gamma k_f \Delta t + \frac{\lambda c}{\ln(\beta)} = -1$, i.e., $k_f \Delta t = \frac{1}{\gamma} (\frac{\lambda c}{\ln(1/\beta)} - 1)$.



Fig. 1. Evolution of probability of a sensor being *on* for a given desired performance P_{des} where $\lambda = 10$, c = 1 and r = 2. In each case, lifetime of the network is achieved when q = 1.

Figure (1) depicts how the duty cycle of sensors (probability of sensors being *on*) needed to maintain a constant event detection probability, varies with time. For a constant event detection probability P_{des} , lifetime of the network is achieved when all sensors are turned *on*, as is shown in the proof of the Theorem 4.2. Also, it can be seen that as P_{des} increases, the lifetime of the network decreases.

Corollary 4.3: As the number of sensors goes to infinity, given a desired lifetime of the sensor network, t_f , the maximum probability of event detection that can be maintained in the time interval $[0, t_f]$ is $P_d = 1 - e^{\frac{-\lambda c}{1 + \gamma k_f \Delta t}}$.

Proof: As mentioned in the proof of the Theorem 4.2, at the end of the lifetime of the sensor network all nodes are *on* and contributing to maintain the desired network performance, i.e., $q(k_f) = 1$. Substituting this value in Equation (14), we have

$$\gamma k_f \Delta t + \frac{\lambda c}{\ln(\beta)} = -1$$

Solving the above equation for β and noting that $P_d = 1 - \beta$ concludes the proof.

Figure (2) shows how the lifetime of a sensor network is related to the desired event detection probability.

V. SIMULATIONS

To put the viability of the proposed duty cycle scheduling strategy to the test we implemented a Monte-Carlo simulation of a sensor network deployed randomly. We consider a 10 by 10 unit rectangular area with $A_{total} = 100$. Sensors are deployed in this field according to a spatial stationary Poisson point process with constant intensity per unit area of $\lambda = 10$. The initial footprint of each sensor is set to be the closed ball of unit radius centered at the position of the sensor. Events are generated randomly at each time instant throughout the area of interest. To increase the accuracy of the results, each value of P_d is averaged over 100 runs of simulation.



Fig. 2. Lifetime t_f of network vs desired network performance P_{des} . The parameters are $\lambda = 10$, c = 1 and r(0) = 2.



Fig. 3. Event detection probability P_d vs time t for non-decaying networks with q = 0.1.

We begin by simulating a non-decaying network with q(t) = 0.1. Figure (3) shows the resulting event detection probability. The resulting detection probability $P_d = 0.61$ is close to that computed analytically from Lemma 3.1, $P_d = 1 - P_u = 0.63$.

Figure (4) depict the simulation result for a decaying network where each sensor is *on* with a constant probability of q(t) = 0.1 and power of the *on* sensors decay with the rate $\gamma = 1$. We keep track of each individual sensor's decaying footprint (power is only drawn when sensor is *on*). The expected footprint remains close to that computed analytically from Equation (8) and the network performance, i.e. event detection probability, as anticipated decreases with time.

To ensure that we maintain the desired performance throughout the lifetime of the network, we need to vary q according to Equation (14) as is shown in figure (5).

Figure (6) illustrates the simulation results for decaying network with scheduling scheme (solid line). We set the desired network performance at $P_{des} = 0.63$. In the simulation, after the initial settling time (probabilities are compute at real time), $P_d \approx 0.62$ which is very close to required



Fig. 4. Event detection probability P_d vs time t for decaying networks with $q = 0.1, \gamma = 1, \lambda = 10$ and A = 1.



Fig. 5. Probability of sensor being on q vs time t for $P_{des} = 0.63$.

performance indicating the validity of our scheme. Moreover, a comparison is done with the case if no scheduling scheme is applied (dashed line). The improvement in the result due to our proposed scheme is obvious from this plot.

VI. CONCLUSIONS

Sensor networks that are deployed in an area of interest are usually battery powered and have a limited lifetime. Radio Frequency based sensors typically have a shrinking footprint whose size is proportional to the available power level. In this work, we proposed a probabilistic duty cycle scheduling strategy that maximizes the lifetime of decaying networks while guaranteeing a minimum level of performance. We used the event detection probability as a measure for the desired performance of the network. A relationship between the lifetime of the network and desired level of performance of the network is also derived. Finally, we validate our analytical results using Monte-Carlo simulations of the proposed strategies.



Fig. 6. Event detection probability P_d vs time t for decaying networks with given $P_{des} = 0.63$ (constant dashed line); with scheduling scheme (solid line) and without scheduling scheme (decaying dashed line). Here $\lambda = 10$, A(0) = 1, $\gamma = 1$ and c = 1.

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