2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Gaussian Mixture PHD Smoother for Jump Markov Models in Multiple Maneuvering Targets Tracking

Wenling Li, Yingmin Jia, Junping Du and Fashan Yu

Abstract— **This paper presents a Gaussian mixture probability hypothesis density (GM-PHD) smoother for tracking multiple maneuvering targets that follow jump Markov models. Unlike the generalization of the multiple model GM-PHD filters, our aim is to approximate the dynamics of the linear Gaussian jump Markov system (LGJMS) by a best-fitting Gaussian (BFG) distribution so that the GM-PHD smoother can be carried out with respect to an approximated linear Gaussian system. Our approach is inspired by the recognition that the BFG approximation provides an accurate performance measure for the LGJMS. Furthermore, the multiple model estimation is avoided and less computational cost is required. The effectiveness of the proposed smoother is verified with a numerical simulation.**

I. INTRODUCTION

Tracking of multiple targets in the random finite set (RFS) formulation has received much attention in the literature, and multi-target tracking based on RFS have found increasing applications such as radar tracking, sonar tracking, speaker tracking and multi-target tracking from image observations. Based on the finite set statistics (FISST) theory [1], the problem of tracking an unknown and time-varying number of targets in the presence of uncertain data association can be formulated in a rigorous Bayesian framework by constructing the multi-target transition density and multi-target likelihood function. However, the optimal multi-target Bayes filter is generally intractable due to the existence of multiple set integrals and the combinatorial nature of the multi-target densities. To alleviate this intractability, the probability hypothesis density (PHD) filter has been proposed as a first order moment approximation to the multi-target posterior density [2]. It should be pointed out that the PHD filter still requires solving multi-dimensional integrals and does not have closed-form solutions in most practical applications.

This work was supported by the NSFC (60727002, 60774003, 60921001, 90916024), the MOE (20030006003), the COSTIND (A2120061303) and the National 973 Program (2005CB321902).

Wenling Li is with the Seventh Research Division and the Department of Systems and Control, Beihang University (BUAA), Beijing 100191, China (e-mail: lwlmath@gmail.com).

Yingmin Jia is with the Seventh Research Division and the Department of Systems and Control, Beihang University (BUAA), Beijing 100191, China. He is also with the Key Laboratory of Mathematics, Informatics and Behavioral Semantics (LMIB), Ministry of Education, SMSS, Beihang University (BUAA), Beijing 100191, China (e-mail: ymjia@buaa.edu.cn).

Junping Du is with the Beijing Key Laboratory of Intelligent Telecommunications Software and Multimedia, School of Computer Science and Technology, Beijing University of Posts and Telecommunications, Beijing 100876, China (e-mail: junpingdu@126.com).

Fashan Yu is with the School of Electrical Engineering and Automation, Henan Polytechnic University, Jiaozuo 454000, Henan, China (e-mail: yufs@hpu.edu.cn).

The sequential Monte Carlo (SMC) or particle technique has been used to implement the PHD filter in [3] and [4]. The main drawbacks of this approach are the large number of particles and the unreliability of the clustering techniques for extracting state estimates. To overcome these disadvantages, the Gaussian mixture PHD (GM-PHD) filter was developed for linear Gaussian target dynamics and Gaussian birth model [5], in which the weights, means and covariances are propagated analytically by the Kalman filter (KF). Similar idea has been extended to accommodate nonlinear target dynamic and measurement models using the extended Kalman filter (EKF) and the unscented Kalman filter (UKF). The convergence properties of two implementations were analyzed in [6] and [7]. In [6], the convergence results for the mean-square error as well as weak convergence of the empirical particle measure to the true PHD measure have been obtained. Specially, these results show how order of the mean-square error is reduced as the number of particles increases. As shown in [7], the GM-PHD filter can approximate the true PHD filter to any desired degree of accuracy under the linear Gaussian assumption of the dynamic model. The generalizations to the jump Makov models have also been proposed for tracking multiple maneuvering targets in [8]–[11]. As mentioned in [8], the existing multiple model GM-PHD filters are not interacting and how can the PHD filter be combined with the interacting multiple model (IMM) approach [12] remains an interesting and challenging problem in both theory and practice. The main difficulty encountered in combining the IMM approach with the PHD filter is that the mode probabilities in the IMM cannot be derived in the PHD filter since the random finite sets are used in the PHD filter. Recently, the backward smoothing PHD recursion has been derived to improve the tracking performance by employing the physical-space approach [13]. The particle implementation and the Gaussian mixture implementation have been carried out in [14] and [15], respectively. Furthermore, the authors in [16] extended the particle implementation to the multiple model PHD recursion. To our knowledge, the GM-PHD smoother has not been developed for tracking multiple maneuvering targets.

In this paper, we attempt to propose a fixed-lag GM-PHD smoother to address the problem of tracking an unknown and time-varying number of maneuvering targets with Markovian switching dynamics. We do not adopt the multiple model scheme since the existing multiple model GM-PHD filters are not interacting. Instead, our objective is to approximate the dynamics of the linear Gaussian jump Markov system (LGJMS) by a best-fitting Gaussian (BFG) distribution. Based on the BFG approximation, the multiple model estimation for LGJMS is reverted to the single model estimation for an approximated linear Gaussian system. Then the fixed-lag GM-PHD smoother can be carried out with respect to the approximated linear Gaussian system. This is inspired by the recognition that the BFG approximation provides an accurate predictor of the IMM performance [17] and therefore more accurate tracking performance can be expected. In addition, much less computational expense is needed as the multiple model estimation is avoided. Similar ideas have also been used to develop a GM-PHD filter for LGJMS in our previous work [18]. Simulation results are presented to illustrate the performance of the proposed smoother in terms of tracking accuracy and computational cost.

The rest of this paper is organized as follows. The multi-target tracking problem is formulated in the RFS framework and the smoothed PHD recursion is presented in Section II. The BFG approximation for LGJMS is reviewed in Section III along with the Gaussian mixture implementation to the smoothed PHD recursion. In Section IV, the performance of the proposed smoother is evaluated by a numerical example. Conclusion is drawn in Section V.

II. RFS FORMULATION FOR MULTI-TARGET TRACKING

An RFS X is a finite set valued random variable, which can be described by a discrete probability distribution and a family of joint probability densities. Moreover, the discrete distribution characterizes the cardinality of X whereas an appropriate density characterizes the joint distribution of the elements in X. Note that there is a nature way to model the collections of individual targets and measurements with RFSs to capture the time-varying feature of the number of targets present and the number of measurements received at each time step. Suppose that there are n_k targets with states $x_{k,1}, \cdots, x_{k,n_k}$ and m_k measurements $z_{k,1}, \cdots, z_{k,m_k}$ at time step k, we can represent the multi-target state and multi-target measurement as two RFSs [1]

$$X_k \triangleq \{x_{k,1}, \cdots, x_{k,n_k}\} \subset \mathcal{X} \tag{1}$$

$$Z_k \triangleq \{z_{k,1}, \cdots, z_{k,m_k}\} \subset \mathcal{Z} \tag{2}$$

where $\mathcal{X} \subset \mathbb{R}^n$ and $\mathcal{Z} \subset \mathbb{R}^p$ denote the state and observation space, respectively. Then, the multi-target tracking can be formulated as a filtering process with state space $\mathcal{X}$ and observation space $\mathcal{Z}$.

The FISST theory provides a rigorous Bayesian formulation to deal with multi-target tracking problems in the RFS framework. Using the FISST approach, the optimal multi-target Bayes recursion can be derived by propagating the multi-target posterior density in time. However, this recursion involves multiple integrals and the multi-target densities are combinatorial, which makes it computationally intractable. Inspired by the single-target tracking theory, this problem is alleviated by approximating the multi-target posterior density with its statistical moments and propagating the moments instead.

The PHD recursion, which propagates the first order moment or the intensity function of multi-target RFSs, provides a computationally cheaper alternative to the optimal multi-target Bayes recursion under the assumption that the clutter RFS and the predicted multi-target RFSs are Poisson [5]. Specifically, given the posterior intensity $\nu_{k-1|k-1}(x_{k-1}|Z_{1:k-1})$ at time $k-1$, the predicted intensity $\nu_{k|k-1}(x_k|Z_{1:k-1})$, the posterior intensity $\nu_{k|k}(x_k|Z_{1:k})$ and the smoothed intensity $\nu_{t|k}(x_t|Z_{1:k})$ are calculated as follows. For simplicity, $\nu_{a|b}(x_a|Z_{1:b})$ is shortly denoted by $\nu_{a|b}$.

PHD Prediction:

$$\nu_{k|k-1} = \int [p_s f(x_k|x_{k-1}) + \beta_{k|k-1}(x_k|x_{k-1})] \times \nu_{k-1|k-1} dx_{k-1} + \gamma_k(x_k) \tag{3}$$

where p_s is the surviving probability. $f(\cdot|\cdot)$ is the single-target transition density. $\beta_{k|k-1}(\cdot|\cdot)$ and $\gamma_k(\cdot)$ denote the intensity of the spawned target RFS and the intensity of the spontaneously birth target RFS, respectively.

PHD Update:

$$\nu_{k|k} = (1-p_d)\nu_{k|k-1} + \sum_{z \in Z_k} \frac{p_d h(z|x_k)\nu_{k|k-1}}{\kappa_k(z) + \int p_d h(z|x_k)\nu_{k|k-1} dx_k} \tag{4}$$

where p_d is the detection probability. $h(\cdot|\cdot)$ is the single-target measurement likelihood. $\kappa_k(\cdot)$ denotes the intensity of the clutter RFS.

PHD Smoothing:

$$\nu_{t|k} = \nu_{t|t}\Big[(1-p_s) + p_s \int \frac{\nu_{t+1|k} f(x_{t+1}|x_t)}{\nu_{t+1|t}} dx_{t+1}\Big] \tag{5}$$

where $t = k - L$. L is the time lag of the smoothing algorithm. $\nu_{t|k}$ and $\nu_{t+1|k}$ are the smoothed PHD at time t and $t+1$, respectively. $\nu_{t|t}$ is the filtered PHD at time t, and $\nu_{t+1|t}$ is the predicted PHD at time $t+1$. Note that this recursion is initialized with the filtering results at the present time k and stopped at time $k-L$, which is called as the fixed-lag smoothing.

Although the PHD recursion given by (3)-(5) operates on the single-target state space and avoids the explicit problem of data association, the PHD recursion still does not admit closed-form solutions in general due to the multi-dimensional integrals and hence numerical integration methods are required. In the following, the Gaussian mixture implementation is used to derive the fixed-lag PHD smoother.

III. GM-PHD SMOOTHER FOR MULTIPLE MANEUVERING TARGETS TRACKING

In this section, we first review the BFG approximation for LGJMS and derive the recursive formulas for calculating the mean and covariance of the Gaussian distribution. Then, we develop the fixed-lag GM-PHD smoother by applying the Rauch-Tung-Striebel type smoothing algorithm to the approximated linear Gaussian model.

A. BFG approximation for LGJMS

Assume that the dynamics of each target can be characterized by one of M hypothesized models with model set $\mathcal{M} = \{1, 2, \cdots, M\}$. Although all the targets share a common model set, any two targets may be in different motion status from time to time. The target motion model can be described by

$$x_{k+1} = F_k(r_{k+1})x_k + w_k(r_{k+1}) \tag{6}$$

where $x_k \in \mathbb{R}^n$ is the target state at time k, $F_k(r_{k+1})$ denotes the transition matrix of model r_{k+1}. $w_k(r_{k+1})$ is the additive zero-mean white Gaussian noise with covariance $Q_k(r_{k+1})$. r_{k+1} specifies the target motion model which is in effect during the time interval $[k, k+1)$. The event that model r is in effect during the sampling period $[k, k+1)$ is denoted by M_{k+1}^r. The evolution of motion models follows a discrete-time Markov chain with known transition probability $\pi_{ij} = \Pr\{r_{k+1} = j | r_k = i\}$ and initial motion model probabilities $p_{1,r} = \Pr\{M_1^r\}$ for $i, j, r \in \mathcal{M}$. For notational simplicity, we denote $F_k(r)$ and $Q_k(r)$ by F_k^r and Q_k^r for $r_{k+1} = r$, respectively.

The objective of the BFG approximation is to express the dynamics of the LGJMS (6) with the following linear Gaussian model

$$x_{k+1} = \Phi_k x_k + w_k \tag{7}$$

where w_k is a zero-mean white Gaussian random vector with covariance matrix Σ_k. To be precise, we want to replace the LGJMS (6) with a single BFG distribution (7) such that the distribution of x_k has the same mean and covariance under both models, i.e.,

$$\mathrm{E}\{x_k | \mathcal{A}\} = \mathrm{E}\{x_k | \mathcal{B}\} \tag{8}$$

$$\mathrm{Cov}\{x_k | \mathcal{A}\} = \mathrm{Cov}\{x_k | \mathcal{B}\} \tag{9}$$

where "$\mathcal{A}$" and "$\mathcal{B}$" refer to models (6) and (7), respectively. "E" and "Cov" denote the expectation and the covariance operators, respectively.

Thus, the central problem is to determine the matrices Φ_k and Σ_k. As shown in [17], they can be derived as follows

$$p_{k+1,r} = \sum_{i=1}^{M} \pi_{ir} p_{k,i} \tag{10}$$

$$\Phi_k = \sum_{r=1}^{M} p_{k+1,r} F_k^r \tag{11}$$

$$\Theta_{k+1} = \sum_{r=1}^{M} p_{k+1,r} \left[F_k^r (\Theta_k + \varepsilon_k \varepsilon_k^T) [F_k^r]^T + Q_k^r \right] - \Phi_k \varepsilon_k \varepsilon_k^T \Phi_k^T \tag{12}$$

$$\Sigma_k = \Theta_{k+1} - \Phi_k \Theta_k \Phi_k^T \tag{13}$$

$$\varepsilon_{k+1} = \Phi_k \varepsilon_k \tag{14}$$

where $p_{k+1,r}$ is the probability of the event that model r is effect during the sampling interval $[k, k+1)$, and

$$\varepsilon_k \triangleq \mathrm{E}\{x_k | \mathcal{A}\} \tag{15}$$

$$\Theta_k \triangleq \mathrm{Cov}\{x_k | \mathcal{A}\} \tag{16}$$

It is observed that the recursive formulas have been obtained for calculating the system matrix Φ_k and the covariance matrix Σ_k. In the following, we aim to obtain the GM-PHD smoother based on the linear Gaussian model (7) instead of the LGJMS (6). It should be pointed out that this scheme does not provide an optimal estimation for LGJMS but provides an accurate predictor alternative of an IMM estimator [17].

B. GM-PHD smoother based on BFG approximation

Assume that the state dynamics and measurements of each target can be modeled as

$$f(x_k | x_{k-1}) = \mathcal{N}(x_k; \Phi_{k-1} x_{k-1}, \Sigma_{k-1}) \tag{17}$$

$$h(z_k | x_k) = \mathcal{N}(z_k; H_k x_k, R_k) \tag{18}$$

where Φ_{k-1} and Σ_{k-1} are calculated by the above BFG approximation at each time step. H_k and R_k denote the measurement matrix and the covariance matrix of the measurement noise, respectively. Note that H_k and R_k do not evolve with time according to the switching parameter r_k. This is reasonable since the measurements from the sensors remain the same with respect to the system states.

To derive Gaussian mixture implementations of the PHD recursion, the intensities of the birth and spawning RFSs are assumed to be

$$\gamma_k(x) = \sum_{j=1}^{J_{\gamma,k}} w_{\gamma,k}^j \mathcal{N}(x; m_{\gamma,k}^j, P_{\gamma,k}^j) \tag{19}$$

$$\beta_{k|k-1}(x|\xi) = \sum_{l=1}^{J_{\beta,k}} w_{\beta,k}^l \mathcal{N}(x; F_{\beta,k}^l \xi + d_{\beta,k}^l, Q_{\beta,k}^l) \tag{20}$$

where $J_{\gamma,k}$, $w_{\gamma,k}^j$, $m_{\gamma,k}^j$ and $P_{\gamma,k}^j$ are given parameters that determine the shape of the birth intensity. $J_{\beta,k}$, $w_{\beta,k}^l$, $F_{\beta,k}^l$, $d_{\beta,k}^l$ and $Q_{\beta,k}^l$ are given parameters that determine the shape of the spawning intensity. It should be mentioned that these intensities are not assumed to be mode-dependent as in [10] since the LGJMS has been replaced by the linear Gaussian model.

Under the above assumptions, the PHD smoothing recursion (3)-(5) can be carried out as follows.

BFG Approximation Step: Given the mode probability $p_{k,i}$, the mean ε_k and the covariance Θ_k at time step k, determine the matrices Φ_k and Σ_k

$$p_{k+1,r} = \sum_{i=1}^{M} \pi_{ir} p_{k,i} \tag{21}$$

$$\Phi_k = \sum_{r=1}^{M} p_{k+1,r} F_k^r \tag{22}$$

$$\Theta_{k+1} = \sum_{r=1}^{M} p_{k+1,r} \left[F_k^r (\Theta_k + \varepsilon_k \varepsilon_k^T) [F_k^r]^T + Q_k^r \right] - \Phi_k \varepsilon_k \varepsilon_k^T \Phi_k^T \tag{23}$$

$$\Sigma_k = \Theta_{k+1} - \Phi_k \Theta_k \Phi_k^T \tag{24}$$

$$\varepsilon_{k+1} = \Phi_k \varepsilon_k \tag{25}$$

Prediction Step: Given that the posterior intensity is a Gaussian mixture

$$\nu_{k|k} = \sum_{j=1}^{J_k} w_k^j \mathcal{N}(x_k; m_{k|k}^j, P_{k|k}^j) \tag{26}$$

then the predicted intensity is also a Gaussian mixture with the form

$$\nu_{k+1|k} = \nu_{s,k+1|k} + \nu_{\beta,k+1|k} + \gamma_{k+1}(x_{k+1}) \tag{27}$$

where $\gamma_{k+1}(x)$ is given by (19), and

$$\nu_{s,k+1|k} = p_s \sum_{j=1}^{J_k} w_k^j \mathcal{N}(x_{k+1}; m_{s,k+1|k}^j, P_{s,k+1|k}^j) \tag{28}$$

$$\nu_{\beta,k+1|k} = \sum_{j=1}^{J_k} \sum_{l=1}^{J_{\beta,k+1}} w_k^j w_{\beta,k+1}^l \mathcal{N}(x_{k+1}; m_{\beta,k+1|k}^{j,l}, P_{\beta,k+1|k}^{j,l}) \tag{29}$$

$$m_{s,k+1|k}^j = \Phi_k m_{k|k}^j \tag{30}$$

$$P_{s,k+1|k}^j = \Phi_k P_{k|k}^j \Phi_k^T + \Sigma_k \tag{31}$$

$$m_{\beta,k+1|k}^{j,l} = F_{\beta,k+1}^l m_{k|k}^j + d_{\beta,k+1}^l \tag{32}$$

$$P_{\beta,k+1|k}^{j,l} = F_{\beta,k+1}^l P_{k|k}^j [F_{\beta,k+1}^l]^T + Q_{\beta,k+1}^l \tag{33}$$

Update Step: Given that the predicted intensity can be represented as the form of

$$\nu_{k+1|k} = \sum_{i=1}^{J_{k+1|k}} w_{k+1|k}^i \mathcal{N}(x_{k+1}; m_{k+1|k}^i, P_{k+1|k}^i) \tag{34}$$

then the posterior intensity is updated as

$$\nu_{k+1|k+1} = (1-p_d)\nu_{k+1|k} + \sum_{z \in Z_{k+1}} \nu_{d,k+1}(x_{k+1}; z) \tag{35}$$

where

$$\nu_{d,k+1}(x_{k+1}; z) = \sum_{i=1}^{J_{k+1|k}} w_{k+1}^i(z) \times \mathcal{N}(x_{k+1}; m_{k+1|k+1}^i(z), P_{k+1|k+1}^i) \tag{36}$$

$$w_{k+1}^i(z) = \frac{p_d w_{k+1|k}^i q_{k+1}^i(z)}{\kappa_{k+1}(z) + p_d \sum_{l=1}^{J_{k+1|k}} w_{k+1|k}^l q_{k+1}^l(z)} \tag{37}$$

$$q_{k+1}^i(z) = \mathcal{N}(z; \hat{z}_{k+1|k}^i, H_{k+1} P_{k+1|k}^i H_{k+1}^T + R_{k+1}) \tag{38}$$

$$m_{k+1|k+1}^i(z) = m_{k+1|k}^i + K_{k+1}^i (z - \hat{z}_{k+1|k}^i) \tag{39}$$

$$\hat{z}_{k+1|k}^i = H_{k+1} m_{k+1|k}^i \tag{40}$$

$$P_{k+1|k+1}^i = (I - K_{k+1}^i H_{k+1}) P_{k+1|k}^i \tag{41}$$

$$K_{k+1}^i = P_{k+1|k}^i H_{k+1}^T \times (H_{k+1} P_{k+1|k}^i H_{k+1}^T + R_{k+1})^{-1} \tag{42}$$

Smoothing Step: Assume that the smoothed PHD at time step $t+1$ are Gaussian mixtures, i.e.,

$$\nu_{t|t} = \sum_{i=1}^{J_{t|t}} w_{t|t}^i \mathcal{N}(x_t; m_{t|t}^i, P_{t|t}^i) \tag{43}$$

$$\nu_{t+1|k+1} = \sum_{j=1}^{J_{t+1|k+1}} w_{t+1|k+1}^j \mathcal{N}(x_{t+1}; m_{t+1|k+1}^j, P_{t+1|k+1}^j) \tag{44}$$

Substituting (43) and (44) into the smoothed PHD recursion (5) leads to

$$\begin{aligned} \nu_{t|k+1} &= (1-p_s) \sum_{i=1}^{J_{t|t}} w_{t|t}^i \mathcal{N}(x_t; m_{t|t}^i, P_{t|t}^i) \\ &+ p_s \sum_{i=1}^{J_{t|t}} \sum_{j=1}^{J_{t+1|k+1}} w_{t|t}^i w_{t+1|k+1}^j \mathcal{N}(x_t; m_{t|t}^i, P_{t|t}^i) \\ &\times \int \frac{\mathcal{N}(x_{t+1}; m_{t+1|k+1}^j, P_{t+1|k+1}^j)}{\nu_{t+1|t}} \\ &\times \mathcal{N}(x_{t+1}; \Phi_t x_t, \Phi_t P_{t|t}^i \Phi_t^T + \Sigma_t) dx_{t+1} \end{aligned} \tag{45}$$

As discussed in [15], the second term on the right hand side of (45) can be implemented using the Rauch-Tung-Striebel type smoothing algorithm [19]. More precisely,

$$\begin{aligned} \nu_{t|k+1} &= (1-p_s) \sum_{i=1}^{J_{t|t}} w_{t|t}^i \mathcal{N}(x_t; m_{t|t}^i, P_{t|t}^i) \\ &+ p_s \sum_{i=1}^{J_{t|t}} \sum_{j=1}^{J_{t+1|k+1}} w_{t|k+1}^{i,j} \mathcal{N}(x_{t+1}; m_{t|k+1}^{i,j}, P_{t|k+1}^{i,j}) \end{aligned} \tag{46}$$

where

$$w_{t|k+1}^{i,j} = \frac{w_{t|t}^i w_{t+1|k+1}^j \mathcal{N}(m_{t+1|k+1}^j; m_{t+1|t}^i, P_{t+1|t}^i)}{\gamma_{t+1}(m_{t+1|k+1}^j) + \sum_{l=1}^{J_{t|t}} \mathcal{N}(m_{t+1|k+1}^j; m_{t+1|t}^l, P_{t+1|t}^i)} \tag{47}$$

$$m_{t+1|t}^i = \Phi_t m_{t|t}^i \tag{48}$$

$$P_{t+1|t}^i = \Phi_t P_{t|t}^i \Phi_t^T + \Sigma_t \tag{49}$$

$$m_{t|k+1}^{i,j} = m_{t|t}^i + D_t^i (m_{t+1|k+1}^j - \Phi_t m_{t|t}^i) \tag{50}$$

$$P_{t|k+1}^{i,j} = P_{t|t}^i + D_t^i (P_{t+1|k+1}^j - \Phi_t P_{t|t}^i \Phi - \Sigma_t)[D_t^i]^T \tag{51}$$

$$D_t^i = P_{t|t}^i \Phi_t [\Phi_t P_{t|t}^i \Phi_t^T + \Sigma_t]^{-1} \tag{52}$$

Remark 1: It is worth noting that the pruning scheme is required before and after the smoothing step since the number of Gaussian components increase without bound as time progresses. A simple pruning procedure has been provided by truncating components that have weak weights to mitigate this problem. For the detail, see [5].

Remark 2: An advantage of the BFG approximation is that it is restricted to linear dynamic models but places no such restriction on the measurement equation. Thus it is possible to handle nonlinear measurements by using nonlinear filters. This coincides with the requirements in the target tracking community: the target dynamics are often described by a linear kinematics model while measurements are nonlinear with respect to the target states.

IV. Simulation results

In this section, we present a numerical example to compare the proposed smoother with the corresponding filter in terms of tracking accuracy and computational cost.

Tracking model: Consider a two-dimensional scenario with an unknown and time-varying number of targets, which is similar to the example provided in [10]. The target state is denoted by $x_k = [p_{x,k}\ \dot{p}_{x,k}\ p_{y,k}\ \dot{p}_{y,k}]^T$, where $(p_{x,k}\ p_{y,k})$ represents the Cartesian coordinates in the horizontal plane and $(\dot{p}_{x,k}\ \dot{p}_{y,k})$ represents its velocities. The target dynamics is described by the coordinated turn model

$$x_k = \begin{bmatrix} 1 & \frac{\sin(\omega T)}{\omega} & 0 & -\frac{1-\cos(\omega T)}{\omega} \\ 0 & \cos(\omega T) & 0 & -\sin(\omega T) \\ 0 & \frac{1-\cos(\omega T)}{\omega} & 1 & \frac{\sin(\omega T)}{\omega} \\ 0 & \sin(\omega T) & 0 & \cos(\omega T) \end{bmatrix} x_{k-1} + w_{k-1}(\omega) \tag{53}$$

where ω denotes the turn rate and $T = 1$ is the sampling time period. $w_{k-1}(\omega)$ is zero-mean white Gaussian noise with covariance matrix

$$Q(\omega) = \sigma^2(\omega) \begin{bmatrix} \frac{T^3}{3} & \frac{T^2}{2} & 0 & 0 \\ \frac{T^2}{2} & T & 0 & 0 \\ 0 & 0 & \frac{T^3}{3} & \frac{T^2}{2} \\ 0 & 0 & \frac{T^2}{2} & T \end{bmatrix} \tag{54}$$

Three models corresponding to different turn rates are used. Model 1 is a coordinated turn model with a turn rate of $0°/\mathrm{s}$ and $\sigma(0) = 5$. Model 2 is a coordinated turn model with a clockwise turn rate of $4°/\mathrm{s}$ and $\sigma(4) = 20$. Model 3 is a coordinated turn model with a counterclockwise turn rate of $4°/\mathrm{s}$ and $\sigma(4) = 20$. The switching between three models is governed by a first order Markov chain with known transition probability matrix

$$\Pi = \begin{bmatrix} 0.8 & 0.1 & 0.1 \\ 0.1 & 0.8 & 0.1 \\ 0.1 & 0.1 & 0.8 \end{bmatrix} \tag{55}$$

The measurement consisting of range and bearing is given by

$$z_k = \begin{bmatrix} \sqrt{(p_{x,k} - s_x)^2 + (p_{y,k} - s_y)^2} \\ \arctan[(p_{x,k} - s_x)/(p_{y,k} - s_y)] \end{bmatrix} + v_k \tag{56}$$

where $[s_x, s_y]$ is the location of the sensor, and the measurement noise v_k is assumed to be zero-mean white Gaussian with $R = \mathrm{diag}\{100^2\ \ (\pi/180)^2\}$. The sensor is located at $[6, 10]$ km, and the average number of clutter returns per unit volume is taken as $\lambda_c = 1.04 \times 10^{-4}$ $(\mathrm{rad\ km})^{-1}$. In this work, the cubature Kalman filter (CKF) [20] and the Rauch-Tung-Striebel type cubature Kalman smoother (CKS) are used to handle nonlinear measurements.

The number of targets is time-varying due to target appearance and disappearance in the scene at any time. The spontaneous birth RFS is Poisson with intensity

$$\gamma_k(\xi) = 0.1\big[\mathcal{N}(\xi; m_\gamma^1, P_\gamma) + \mathcal{N}(\xi; m_\gamma^2, P_\gamma)\big] \tag{57}$$

where $m_\gamma^1 = [10,\ 0,\ 20,\ 0]^T$, $m_\gamma^2 = [0,\ 0,\ 30,\ 0]^T$ and $P_\gamma = \mathrm{diag}\{10^6,\ 10^4,\ 10^6,\ 10^4\}$.

The intensity of the Poisson RFS of spawn births is given by

$$\beta_{k|k-1}(x|\xi) = 0.05\mathcal{N}(x; \xi, Q_\beta) \tag{58}$$

where $Q_\beta = \mathrm{diag}\{10^4, 400, 10^4, 400\}$.

Simulation results: In the simulations, the survival and the detection probabilities are set to $p_s = 0.99$ and $p_d = 0.98$, respectively. The pruning threshold has been taken as $T_{\mathrm{Th}} = 0.01$, the merging threshold $U_{\mathrm{Th}} = 5$, the weight threshold $w_{\mathrm{Th}} = 0.5$ and the maximum number of Gaussian terms $J_{\max} = 10$ (see [5] for the meanings of these parameters). The criterion known as optimal subpattern assignment (OSPA) metric is used for performance evaluation. It has been shown in [21] that the OSPA metric joint captures the differences in cardinality and individual elements between two finite sets.

Target 1 starts at time $k = 1$ with initial position at $[10, 20]$ km and ends at time $k = 100$; Target 2 is spawned from target 1 at time $k = 30$ and ends at time 70; Target 3 starts at time $k = 5$ with initial position at $[0, 30]$ km and ends at time $k = 85$; Target 4 is spawned from target 3 at time $k = 40$ and ends at time 80. To verify the performance of the proposed smoother with a lag of 1 time step, 100 Monte Carlo runs are performed.

The position estimates of the proposed smoother and the corresponding filter for one trial shown in Fig. 1 indicate that the smoother provides more accurate tracking performance. We compare the performance of the proposed GM-PHD smoother with that of the GM-PHD filters, namely, the BFG-based GM-PHD filter [18] and the multiple model GM-PHD filter without interacting [10]. The OSPA distance for $p = 2$ and $c = 200$ is shown in Fig. 2 (see [21] for the meanings of these parameters). It can be seen from Fig. 2 that the proposed smoother gives the best estimates. Specially, the BFG-based GM-PHD filter performs better than the multiple model GM-PHD filter without interacting. This is expected since the former provides an accurate predictor of the IMM estimator. Note here that we do not compare our results with that of the particle implementation due to its high computational expense.

To assess the computational cost of the proposed method, we compute the averaged CPU time in MATLAB 7.1 on a 2.80 GHz 4 CPU Pentium-based computer operating under Windows XP (Professional). The proposed smoother consumes approximately 3.17 s per sample run over 100 time steps, while the BFG-based GM-PHD filter and the multiple model GM-PHD filter require 1.94 s and 11.72 s, respectively. From the above comparisons, a modest conclusion can be drawn that the proposed method achieves better performance with less computational cost than the multiple model scheme without interacting. In addition, the proposed GM-PHD smoother outperforms the corresponding filter with a moderate increase in the computational load.

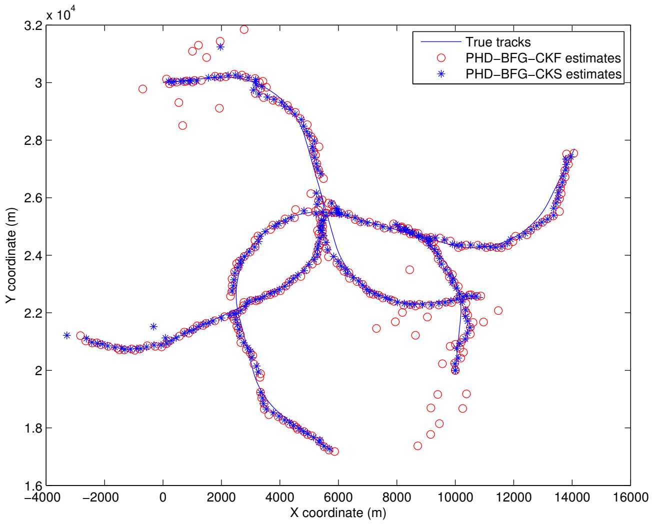

Fig. 1: Position estimates of the PHD-BFG-CKF and the PHD-BFG-CKS.

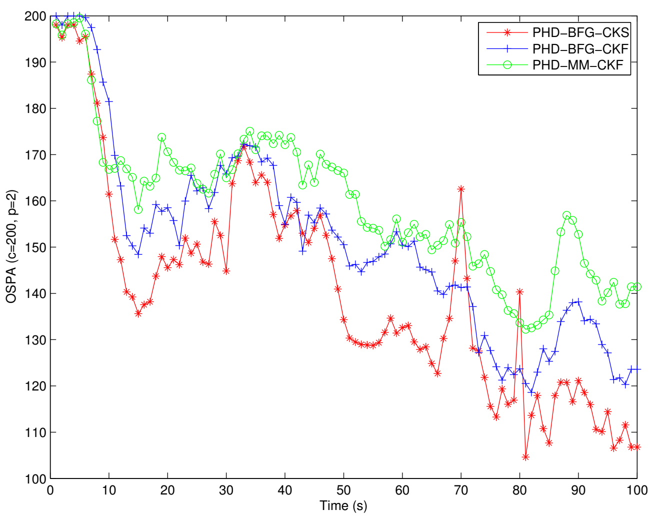

Fig. 2: Performance comparison with respect to OSPA metric.

V. Conclusion

In this paper, a novel GM-PHD smoother for tracking an unknown and time-varying number of targets that follow jump Markov models has been proposed. The mechanism of the smoother differs from the previously proposed multiple model GM-PHD filters without interacting. Instead, the BFG distribution is used to approximate the dynamics of the LGJMS, which has been shown to be in close agreement with the performance of an IMM estimator. Therefore, the proposed approach achieves better estimates than that of the multiple model scheme without interacting. Moreover, the requirement of multiple model estimation is avoided and a low computational cost is consumed.

References

[1] R. Mahler, Statistical Multisource-Multitarget Information Fusion, Norwood, MA: Artech House, 2007.

[2] R. Mahler, Multitarget Bayes filtering via first-order multitarget moments, IEEE Transactions on Aerospace and Electronic Systems, vol. 39, no. 4, pp. 1152-1178, 2003.

[3] H. Sidenbladh, Multi-target particle filtering for the probability hypothesis density, Proceedings of International Conference on Information Fusion, Cairns, Australia, 2003, 800-806.

[4] B. N. Vo, S. Singh, and A. Doucet, Sequential Monte Carlo methods for multi-target filtering with random finite sets, IEEE Transactions on Aerospace and Electronic Systems, vol. 41, no. 4, pp. 1224-1245, 2005.

[5] B. N. Vo, and W. K. Ma, The Gaussian mixture probability hypothesis density filter, IEEE Transactions on Signal Processing, vol. 54, no. 11, pp. 4091-4104, 2006.

[6] D. E. Clark, J. Bell, Convergence results for the particle PHD filter, IEEE Transactions on Signal Processing, vol. 54, no. 7, pp.2652-2661, 2006.

[7] D. E. Clark, B. N. Vo, Convergence analysis of the Gaussian mixture PHD filter, IEEE Transactions on Signal Processing, vol. 55, no. 4, pp.1204-1211, 2007.

[8] B. N. Vo, A. Pasha, and H. D. Tuan, A Gaussian mixture PHD filter for nonlinear jump Markov models, Proceedings of the 45th IEEE Conference on Decision and Control, San Diego, CA, USA, Dec. 13-15, 2006, 3162-3167.

[9] K. Punithakumar, T. Kirubarajan, and A. Sinha, Multiple model probability hypothesis density filter for tracking maneuvering targets, IEEE Transactions on Aerospace and Electronic Systems, vol. 44, no. 1, pp. 87-98, 2008.

[10] S. A. Pasha, B. N. Vo, H. D. Tuan, and W. K. Ma, A Gaussian mixture PHD filter for jump Markov system models, IEEE Transactions on Aerospace and Electronic Systems, vol. 45, no. 3, pp. 919-936, 2009.

[11] S. A. Pasha, H. D. Tuan, and P. Apkarian, The LFT based PHD filter for nonlinear jump Markov models in multi-target tracking, Proceedings of the 48th IEEE Conference on Decision and Control and 28th Chinese Control Conference, Shanghai, P.R. China, Dec. 2009, 5478-5483.

[12] X. R. Li and V. P. Jilkov, Survey of maneuvering target tracking. Part V: multiple-model methods, IEEE Transactions on Aerospace and Electronic Systems, vol. 41, no. 4, pp. 1255-1321, 2005.

[13] O. Erdinc, P. Willett, and Y. Bar-Shalom, The bin-occupancy filter and its connection to the PHD filters, IEEE Transactions on Signal Processing, vol. 57, no. 11, pp. 4232-4246, 2009.

[14] N. Nandakumaran, K. Punithakumar, and T. Kirubarajan, Improved multi-target tracking using probability hypothesis density smoothing, Proceedings of the SPIE Conference on Signal and Processing of Small Targets, vol. 6699, Aug. 2007.

[15] N. Nandakumaran, R. Tharmarasa, T. Lang, and T. Kirubarajan, Gaussian mixture probability hypothesis density smoothing with multistatic sonar, Proceedings of the SPIE Conference on Signal Processing, Sensor Fusion and Target Recognition, vol. 6968, Mar. 2008.

[16] N. Nandakumaran, T. Kirubarajan, Maneuvering target tracking using probability hypothesis density smoothing, Proceedings of the SPIE Conference on Signal Processing, Sensor Fusion and Target Recognition, vol. 7336, Mar. 2009.

[17] M. L. Hernandez, B. Ristic, A. Farina, T. Sathyan, and T. Kirubarajan, Performance measure for Markovian switching systems using best-fitting Gaussian distributions, IEEE Transactions on Aerospace and Electronic Systems, vol. 44, no. 2, pp. 724-747, 2008.

[18] W. Li, Y. Jia, Gaussian mixture PHD filter for jump Markov models based on best-fitting Gaussian approximation, Signal Processing, vol. 91, no. 4, pp. 1036-1042, 2011.

[19] H. E. Rauch, F. Tung, and C. T. Striebel, Maximum likelihood estimates of linear dynamic systems, AIAA Journal, vol. 3, no. 8, pp. 1445-1450, 1965.

[20] I. Arasaratnam, and S. Haykin, Cubature Kalman filters, IEEE Transactions on Automatic Control, vol. 54, no. 6, 1254-1269, 2009.

[21] D. Schuhmacher, B. T. Vo, and B. N. Vo, A consistent metric for performance evaluation of multi-object filters, IEEE Transactions on Signal Processing, vol. 56, no. 8, pp. 3447-3457, 2008.