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Multiple Vehicle Bayesian-Based Domain Search with Intermittent Information Sharing

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Abstract— **This paper focuses on the development of Bayesian-based domain search strategies for distributed multiple autonomous vehicles with intermittent information sharing. Multi-sensor fusion based on observations from neighboring vehicles is implemented via binary Bayesian filtering. We will prove that, under appropriate sensor models, the belief of whether objects exist or not will converge to the true state. An uncertainty map based on these probabilities is constructed to guide the vehicles' motion. It will be shown that all objects in the search domain will be detected. Different motion control schemes are numerically tested to illustrate the effectiveness of the proposed strategy.**

I. Introduction

In recent years, MAVs have been increasingly used to perform operations that were traditionally carried out by humans. Furthermore, the use of distributed MAV systems facilitates improved information sharing and increases system robustness against sensor failure. In this paper, we consider domain search problems using distributed MAVs with intermittent communications in a Bayesian framework. The objective is to find each object within the domain and fix its position in space. We assume that the number and positions of objects of interest are unknown beforehand. Each vehicle is only capable of taking observations within its sensory range. This is consistent with the sensor models used in [1]–[3] and applicable to large-scale domain search. It is also assumed that each vehicle can only communicate with neighboring vehicles within its limited communication range.

Because a vehicle sensor's sensing capability is limited, false alarms and missed detections are inevitable and the system performance is indeterministic [4]. In order to reduce this uncertainty due to sensor perception, or equivalently, to maximize the probability of detecting an object, all the available observations a vehicle has access to (i.e., taken by the vehicle itself and its neighboring vehicles) should be fused together. We will prove that given sensors with a detection probability greater than 0.5, the search uncertainty will converge to a small neighborhood of zero, i.e., all unknown objects of interest are found with 100% confidence. Although this may seem intuitive, there lacks a rigorous mathematical proof in the literature due to the probabilistic nature of the problem.

We first review some related literature. In [5], the Dempster-Shafer evidential method is utilized for domain search with multiple uninhabited aerial vehicles (UAVs) under global communications. The objective is to minimize the environment uncertainty in a finite amount of search time. With the same goal, in [6], the authors present an agent-based negotiation scheme for a multi-UAV search operation with limited sensory and communication ranges. In [7], the authors use the Modified Bayes Factor to model the level of confidence of target existence for an UAV search task in an uncertain environment. In [8], the problem of searching an area containing both regions of opportunity and hazard with multiple cooperative UAVs is considered. In [9], the authors use a density function to represent the frequency of random events taking place over the mission domain. The goal is to maximize coverage using mobile sensors with limited ranges and minimum communication cost. An alternative approach related to search in an uncertain environment is Simultaneous Localization and Mapping (SLAM) [10]. In [4], the occupancy grid mapping algorithm is addressed, which is often used after solving a SLAM problem to generate robot navigation path from the raw sensor endpoints.

The work presented here is analogous to the binary Bayesian filtering and the occupancy grid mapping algorithm [4], which are very popular mapping techniques to deal with the uncertainty in sensor perception in intelligent robotics. However, we seek to *1) find the conditions that guarantees satisfactory detection results using multiple vehicles with limited-range sensors and intermittent communications, and 2) provide a rigorous mathematical proof for these conditions that are also consistent with intuition.* This is a nontrivial problem given limited theoretical results existing in the literature and its significance for effective sensor management, especially when the sensing and communication resources are limited.

The paper is organized as follows. A Bernoulli type sensor model is introduced in Section II. In Section III, the binary Bayesian filtering is used to update the probability of object existence. We will prove in Section IV that the expected probability of object presence will eventually converge to one if there is actually an object or zero if there is none, under appropriate sensor assumptions. In Section V, an information uncertainty map based on these probabilities is constructed to guide the vehicles' motion. A coverage metric is defined to evaluate the search task. In Section VI, a simulation-based study is provided to test two different vehicle motion control strategies under the proposed framework. We conclude the paper with a summary of current and future work in Section VII.

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II. Problem Formulation

A. Problem Setup

Let $\mathcal{D} \in \mathbb{R}^2$ be a domain in which objects are located. Assume there are N_v autonomous vehicles $\mathcal{V}_i$, $i = 1, 2, \cdots, N_\text{v}$, searching for an unknown number of objects of interest within $\mathcal{D}$. Denote the position of vehicle $\mathcal{V}_i$ as $\mathbf{q}_i(t)$. Each vehicle $\mathcal{V}_i$ satisfies the following first order discrete-time equation of motion

$$\mathbf{q}_i(t+1) \quad = \quad \mathbf{q}_i(t) + \mathbf{u}_i(t),$$

where $\mathbf{u}_i \in \mathbb{R}^2$ is the control input. Assume that a vehicle pair can only share information whenever they are within the communication range ρ. The set $\mathcal{N}_i(t) = \{\mathcal{V}_j | \|\mathbf{q}_i(t) - \mathbf{q}_j(t)\| \le \rho\}$ defines vehicle $\mathcal{V}_i$'s neighbors at time t including $\mathcal{V}_i$ itself. This model is consistent with the intermittent communication structure in [11], [12].

We discretize $\mathcal{D}$ into N_tot unit cells, let $\tilde{\mathbf{c}}$ be an arbitrary cell in $\mathcal{D}$ and $\tilde{\mathbf{q}}$ be the centroid of $\tilde{\mathbf{c}}$. Let $1 \le N_\text{o} \le N_\text{tot}$ be the total number of objects, which are i.i.d. distributed over $\mathcal{D}$. By choosing a fine enough grid in the discretization, a cell is guaranteed to contain at most one object. Denote the position of the static object $\mathcal{O}_j$, $j \in \{1, 2, \ldots, N_\text{o}\}$ as $\mathbf{p}_j$. Both N_o and $\mathbf{p}_j$ are unknown beforehand. Let $X(\tilde{\mathbf{c}})$ be a binary state random variable, where 0 corresponds to object absent, and 1 corresponds to object present.

B. Sensor Model

We assume a sensor model with limited sensory capability for each vehicle $\mathcal{V}_i$, which follows a Bernoulli distribution within its sensory domain $\mathcal{W}_i(t)$, and gives binary outputs: object 'absent' or 'present' $\forall \tilde{\mathbf{c}} \in \mathcal{W}_i(t)$.

To be consistent with the binary state, we define a binary observation variable $Y_i(\tilde{\mathbf{c}})$ for each vehicle $\mathcal{V}_i$, where 0 corresponds to a negative observation indicating object absent, and 1 corresponds to a positive observation indicating object present.

Given a state $X(\tilde{\mathbf{c}}) = j$, the conditional probability mass function f of the Bernoulli observation distribution of vehicle $\mathcal{V}_i$ is given by

$$\begin{aligned} & f_{Y_i}(Y_i = k | X = j; \tilde{\mathbf{c}}) \\ &= \begin{cases} \beta_j^i & \text{if } k = j \\ 1 - \beta_j^i & \text{if } k \neq j \end{cases}, \; j, k = 0, 1. \end{aligned} \tag{1}$$

The following matrix gives the general conditional probability matrix associated with each vehicle $\mathcal{V}_i$:

$$B_i = \begin{bmatrix} \text{Prob}(Y_i = 0 | X = 0; \tilde{\mathbf{c}}) & \text{Prob}(Y_i = 0 | X = 1; \tilde{\mathbf{c}}) \\ \text{Prob}(Y_i = 1 | X = 0; \tilde{\mathbf{c}}) & \text{Prob}(Y_i = 1 | X = 1; \tilde{\mathbf{c}}) \end{bmatrix} \tag{2}$$

where $\text{Prob}(Y_i = k | X = j; \tilde{\mathbf{c}})$, $j, k = 0, 1$, is the probability of vehicle $\mathcal{V}_i$ having observation k given that the actual state at cell $\tilde{\mathbf{c}}$ is $X = j$ and is given by the Bernoulli observation distribution (1). For the sake of simplicity, we assume that $\beta_j^i = \beta^i$ for both states $X = j$, $j = 0, 1$. Clearly, $\beta^i \in [0, 1]$.

III. Bayes Probability Updates

In this section, we employ the binary Bayesian filtering to update the probability of object presence at $\tilde{\mathbf{c}}$ of vehicle $\mathcal{V}_i$ based on all the observations available at the current time step and the prior probability. Define $\bar{Y}_t^i(\tilde{\mathbf{c}}) = \{Y_{j,t}(\tilde{\mathbf{c}}), \; \mathcal{V}_j \in \mathcal{N}_i(t)\}$ as the observation sequence taken by all the vehicles in $\mathcal{N}_i(t)$. Given $\bar{Y}_t^i(\tilde{\mathbf{c}})$, Bayes' rule gives, for each vehicle $\mathcal{V}_i$,

$$\begin{aligned} & P_i(X = 1 | \bar{Y}_t^i; \tilde{\mathbf{c}}, t+1) \\ &= \alpha_i P_i(\bar{Y}_t^i | X = 1; \tilde{\mathbf{c}}) P_i(X = 1; \tilde{\mathbf{c}}, t), \end{aligned}$$

where $P_i(X = 1 | \bar{Y}_t^i; \tilde{\mathbf{c}}, t+1)$ is the posterior probability of object presence at cell $\tilde{\mathbf{c}}$ updated by $\mathcal{V}_i$ after the observation sequence $\bar{Y}_t^i(\tilde{\mathbf{c}})$ has been taken. Because the observations taken by different vehicles are i.i.d., we have $P_i(\bar{Y}_t^i | X = 1; \tilde{\mathbf{c}}) = \Pi_{j \in \mathcal{N}_i(t)} \text{Prob}(Y_{j,t} | X = 1; \tilde{\mathbf{c}})$. The quantity $P_i(X = 1; \tilde{\mathbf{c}}, t)$ is the prior probability of object presence, and α_i serves as a normalizing function which ensures $\sum_{j=0}^{1} P_i(X = j | \bar{Y}_t^i; \tilde{\mathbf{c}}, t+1) = 1$.

According to the law of total probability, we have

$$\begin{aligned} & P_i(X = 1 | \bar{Y}_t^i; \tilde{\mathbf{c}}, t+1) = \\ & \frac{P_i(X = 1; \tilde{\mathbf{c}}, t)}{P_i(X = 1; \tilde{\mathbf{c}}, t) + \Pi_{j \in \mathcal{N}_i(t)} (\frac{1}{\beta^j} - 1)^{2 y_{j,t}(\tilde{\mathbf{c}}) - 1} (1 - P_i(X = 1; \tilde{\mathbf{c}}, t))}, \end{aligned} \tag{3}$$

where $y_{j,t}(\tilde{\mathbf{c}})$ is the dummy variable for the random variable $Y_{j,t}(\tilde{\mathbf{c}})$.

IV. Convergence Analysis

In this section, we discuss the conditions for convergence for the sequence $\{P_i(X = 1 | \bar{Y}_t^i; \tilde{\mathbf{c}}, t+1)\}$ when β^i is a deterministic parameter within $[0, 1]$.

For the sake of simplicity, denote $P_i(X = 1 | \bar{Y}_t^i; \tilde{\mathbf{c}}, t+1)$ as P_{t+1}, $P_i(X = 1; \tilde{\mathbf{c}}, t)$ as P_t, and $\Pi_{j \in \mathcal{N}_i(t)} (\frac{1}{\beta^j} - 1)^{2 y_{j,t}(\tilde{\mathbf{c}}) - 1}$ as S_t, Equation (3) then simplifies to the following non-autonomous nonlinear discrete-time system

$$P_{t+1} = \frac{P_t}{P_t + S_t (1 - P_t)}. \tag{4}$$

Note that S_t is a random variable dependent on $\bar{Y}_t^i(\tilde{\mathbf{c}})$. Let $|\mathcal{N}_i(t)|$ be the cardinality of $\mathcal{N}_i(t)$, then $\bar{Y}_t^i(\tilde{\mathbf{c}})$ has $2^{|\mathcal{N}_i(t)|}$ possible combinations at each time step t for cell $\tilde{\mathbf{c}}$. Let $s_t^1, s_t^2, \cdots, s_t^{2^{|\mathcal{N}_i(t)|}}$ be the realizations of S_t corresponding to each of the $2^{|\mathcal{N}_i(t)|}$ different observation sequences. The probability of having each particular observation sequence $\bar{Y}_t^i(\tilde{\mathbf{c}}) = \{Y_{j,t}(\tilde{\mathbf{c}}) = y_{j,t}(\tilde{\mathbf{c}}), \; \mathcal{V}_j \in \mathcal{N}_i(t)\}$ given $X(\tilde{\mathbf{c}}) = 1$ is: $\Pi_{j \in \mathcal{N}_i(t)} (\beta^j)^{y_{j,t}(\tilde{\mathbf{c}})} (1 - \beta^j)^{(1 - y_{j,t}(\tilde{\mathbf{c}}))}$.

Consider the following conditional expectation

$$\begin{aligned} & E[1 - P_{t+1} | P_t] = E[\frac{S_t (1 - p_t)}{p_t + S_t (1 - p_t)} | P_t = p_t] \\ &= \sum_{m=1}^{2^{|\mathcal{N}_i(t)|}} \frac{s_t^m (1 - p_t)}{p_t + s_t^m (1 - p_t)} \text{Prob}(S_t = s_t^m), \end{aligned} \tag{5}$$

where p_t is the dummy variable for P_t. Let us investigate the value of s_t^m and the corresponding $\text{Prob}(S_t = s_t^m)$ from $m = 1$ to $2^{|\mathcal{N}_i(t)|}$.

- $m = 1$ corresponds to the observation sequence $\{1, 1, \cdots, 1\}$, we have $s_t^1 = \Pi_{j \in \mathcal{N}_i(t)} (\frac{1}{\beta^j} - 1)$ and $\text{Prob}(S_t = s_t^1) = \Pi_{j \in \mathcal{N}_i(t)} \beta^j$
- $m = k + 1$, $k = 1, \cdots, |\mathcal{N}_i(t)|$ correspond to the observation sequence where only the k_th vehicle in vehicle $\mathcal{V}_i$'s neighborhood observes a 0. Define C_k^n as the binomial coefficient. Because there are $C_1^{|\mathcal{N}_i(t)|}$ such observation sequences with different

$$E[1-P_{t+1}|P_t] = \Bigg(\frac{\Pi_{j\in\mathcal{N}_i(t)}(1-\beta^j)}{1-\epsilon+\Pi_{j\in\mathcal{N}_i(t)}(\frac{1}{\beta^j}-1)\epsilon} + \sum_{k=1}^{|\mathcal{N}_i(t)|} \frac{\Pi_{j\in\mathcal{N}_i(t)}(1-\beta^j)}{(\frac{1}{\beta^k}-1)(1-\epsilon)+\Pi_{j\in\mathcal{N}_i(t),\ \neq k}(\frac{1}{\beta^j}-1)\epsilon} + \ldots + \sum_{k=1}^{C_2^{|\mathcal{N}_i(t)|}} \frac{\Pi_{j\in\mathcal{N}_i(t)}(1-\beta^j)}{(\frac{1}{\beta^q}-1)(\frac{1}{\beta^r}-1)(1-\epsilon)+\Pi_{j\in\mathcal{N}_i(t),\ \neq q,r}(\frac{1}{\beta^j}-1)\epsilon} + \ldots + \frac{\Pi_{j\in\mathcal{N}_i(t)}(1-\beta^j)}{\Pi_{j\in\mathcal{N}_i(t)}(\frac{1}{\beta^j}-1)(1-\epsilon)+\epsilon} \Bigg)\epsilon. \tag{6}$$

orders, the value of k is in the set $[1, |\mathcal{N}_i(t)|]$. Hence, we have $s_t^{k+1} = \left(\Pi_{j\in\mathcal{N}_i(t),\ \neq k}(\frac{1}{\beta^j}-1)\right)\left(\frac{\beta^k}{1-\beta^k}\right)$ and $\text{Prob}(S_t = s_t^{k+1}) = \left(\Pi_{j\in\mathcal{N}_i(t),\ j\neq k}\beta^j\right)(1-\beta^k)$

- $m = k+1+|\mathcal{N}_i(t)|,\ k = 1, \cdots, C_2^{|\mathcal{N}_i(t)|}$ correspond to the the observation sequences where two of the vehicles, e.g., the q_{th} and r_{th} vehicle, observe a 0. Because there are $C_2^{|\mathcal{N}_i(t)|}$ such observation sequences, k is within $[1, C_2^{|\mathcal{N}_i(t)|}]$. Therefore, we have
$$s_t^{k+1+|\mathcal{N}_i(t)|} = \left(\Pi_{j\in\mathcal{N}_i(t),\ \neq q,r}(\frac{1}{\beta^j}-1)\right)\left(\frac{\beta^q}{1-\beta^q}\right)\left(\frac{\beta^r}{1-\beta^r}\right)$$
and $\text{Prob}(S_t = s_t^{k+1+|\mathcal{N}_i(t)|}) = \left(\Pi_{j\in\mathcal{N}_i(t),\ j\neq q,r}\beta^j\right)(1-\beta^q)(1-\beta^r)$
- And so on for other values of m
- $m = 2^{|\mathcal{N}_i(t)|}$ correspond to the observation sequence $\{0, 0, \cdots, 0\}$, we have $s_t^m = \Pi_{j\in\mathcal{N}_i(t)}\left(\frac{\beta^j}{1-\beta^j}\right)$ and $\text{Prob}(S_t = s_t^m) = \Pi_{j\in\mathcal{N}_i(t)}(1-\beta^j)$

Suppose $p_t = 1-\epsilon$, where $\epsilon \in [0, \frac{1}{2})$ is some constant, Equation (5) can be rewritten as Equation (6) if not all sensing parameters $\beta^j = 1$, and $E[1-P_{t+1}|P_t] = 0$ when all $\beta^j = 1,\ j \in \mathcal{N}_i(t)$. Consider the following condition:
Sensing Condition 1: $\beta^i \in (\frac{1}{2}, 1],\ i = 1, 2, \cdots, N_{\text{v}}$.

This condition requires that all vehicle sensors are more likely to take correct measurements.

Now assume that ϵ is a small number in the neighborhood of zero, under Sensing Condition 1, $\Pi_{j\in\mathcal{N}_i(t)}(\frac{1}{\beta^j}-1)\epsilon$ is also a small number close to zero. Hence, Equation (6) can be approximated as

$$E[1-P_{t+1}|P_t] \approx \Bigg(\Pi_{j\in\mathcal{N}_i(t)}(1-\beta^j) + \sum_{k=1}^{|\mathcal{N}_i(t)|}\Pi_{j\in\mathcal{N}_i(t),\ \neq k}(1-\beta^j)\beta^k + \ldots + \sum_{k=1}^{C_2^{|\mathcal{N}_i(t)|}}\Pi_{j\in\mathcal{N}_i(t),\ \neq q,r}(1-\beta^j)\beta^q\beta^r + \ldots + \Pi_{j\in\mathcal{N}_i(t)}\beta^j\Bigg)\epsilon. \tag{7}$$

Observe the expression within the bracket in Equation (7), it gives the total probability of all possible observation sequences taken by the vehicles in $\mathcal{N}_i(t)$ given $X(\tilde{\mathbf{c}}) = 1$, and is therefore equal to 1. If $\beta^j = \beta,\ \forall \mathcal{V}_j \in \mathcal{N}_i(t)$, the expression gives the total probability of a binomial distribution with parameters β and $|\mathcal{N}_i(t)|$. Hence, $E[1-P_{t+1}|P_t = 1-\epsilon] \approx \epsilon$ and we have the following lemma.

Lemma IV.1. Under Sensing Condition 1, if an object is present, given that the prior probability of object present $P_i(X = 1; \tilde{\mathbf{c}}, t)$ of vehicle $\mathcal{V}_i$ is within a small neighborhood of radius ϵ from 1 at time step t, the conditional expectation of the posterior probability $P_i(X = 1|\bar{Y}_t^i; \tilde{\mathbf{c}}, t+1)$ will remain in this neighborhood at the next time step $t+1$. If all the sensors are "perfect" with zero error probability, i.e., $\beta^j = \beta = 1$, then the conditional expectation of $P_i(X = 1|\bar{Y}_t^i; \tilde{\mathbf{c}}, t+1)$ is 1.

Following a similar derivation, we get a lemma for the posterior probability of object absence $P_i(X = 0|\bar{Y}_t^i; \tilde{\mathbf{c}}, t+1)$ given $X(\tilde{\mathbf{c}}) = 0$. To summarize the above results, we have the following theorem.

Theorem IV.1. For $\beta^i \in (\frac{1}{2}, 1],\ i = 1, 2, \cdots, N_{\text{v}}$, if there is an object absent (respectively, present), given that $P_i(X = 0; \tilde{\mathbf{c}}, t)$ (respectively, $P_i(X = 1; \tilde{\mathbf{c}}, t)$) is within a small neighborhood of 1 at time step t, the conditional expectation of $P_i(X = 0|\bar{Y}_t^i; \tilde{\mathbf{c}}, t+1)$ (respectively, $P_i(X = 1|\bar{Y}_t^i; \tilde{\mathbf{c}}, t+1)$) will remain in this neighborhood at the next time step. If $\beta^i = \beta = 1$, then the conditional expectation is 1.

This theorem gives a weak result because it implies that only if the initial prior probability is close to the true state, given "good" sensors with detection probabilities greater than 0.5, the belief of whether objects exist or not will remain near the true state. We next derive a stronger result under the case of homogeneous sensor properties. Next, consider the following condition.
Sensing Condition 2: $\beta^i = \beta \in (\frac{1}{2}, 1],\ i = 1, 2, \cdots, N_{\text{v}}$.

This condition implies that all the vehicles have identical sensors with the same detection probability $\beta \in (\frac{1}{2}, 1]$.

Under Sensing Condition 2, the term within the bracket in Equation (6) is equivalent to the following expression:

$$g(\beta, \epsilon, |\mathcal{N}_i(t)|) = \sum_{k=0}^{|\mathcal{N}_i(t)|}\frac{C_k^{|\mathcal{N}_i(t)|}(1-\beta)^{|\mathcal{N}_i(t)|}}{(\frac{1}{\beta}-1)^k(1-\epsilon)+(\frac{1}{\beta}-1)^{|\mathcal{N}_i(t)|-k}\epsilon},\ \beta \neq 1. \tag{8}$$

Lemma IV.2. The function $g(\beta, \epsilon, |\mathcal{N}_i(t)|) < 1$ when $\beta \in (\frac{1}{2}, 1)$, $\epsilon \in (0, \frac{1}{2})$, $|\mathcal{N}_i(t)| \geq 1$ and equals to 1 when $\epsilon = 0$, $\beta \in (\frac{1}{2}, 1)$, $|\mathcal{N}_i(t)| \geq 1$.

Proof. For brevity, let $n = |\mathcal{N}_i(t)|$. Proving that $g(\beta, \epsilon, |\mathcal{N}_i(t)|)$ is less than 1 is equivalent to prove that

$$\sum_{k=0}^{n}\frac{C_k^n(1-\beta)^n}{(\frac{1}{\beta}-1)^k(1-\epsilon)+(\frac{1}{\beta}-1)^{n-k}\epsilon} < \sum_{k=0}^{n}\frac{1}{n+1},\ \text{or,}$$

$$\sum_{k=0}^{n}\frac{(n+1)C_k^n(1-\beta)^n - \left[(\frac{1}{\beta}-1)^k(1-\epsilon)+(\frac{1}{\beta}-1)^{n-k}\epsilon\right]}{\left[(\frac{1}{\beta}-1)^k(1-\epsilon)+(\frac{1}{\beta}-1)^{n-k}\epsilon\right](n+1)} < 0.$$

Because $\beta \in (\frac{1}{2}, 1)$, or $(\frac{1}{\beta}-1) \in (0, 1)$, we have

$$\sum_{k=0}^{n}\frac{(n+1)C_k^n(1-\beta)^n - \left[(\frac{1}{\beta}-1)^k(1-\epsilon)+(\frac{1}{\beta}-1)^{n-k}\epsilon\right]}{\left[(\frac{1}{\beta}-1)^k(1-\epsilon)+(\frac{1}{\beta}-1)^{n-k}\epsilon\right](n+1)}$$

$$< \sum_{k=0}^{n} \frac{(n+1)C_k^n(1-\beta)^n - \left[(\frac{1}{\beta}-1)^k(1-\epsilon)+(\frac{1}{\beta}-1)^{n-k}\epsilon\right]}{\left[(\frac{1}{\beta}-1)^n(1-\epsilon)+(\frac{1}{\beta}-1)^n\epsilon\right](n+1)}.$$

Since $\left[(\frac{1}{\beta}-1)^n(1-\epsilon)+(\frac{1}{\beta}-1)^n\epsilon\right](n+1) > 0$, if

$$\sum_{k=0}^{n}(n+1)C_k^n(1-\beta)^n - \left[(\frac{1}{\beta}-1)^k(1-\epsilon)+(\frac{1}{\beta}-1)^{n-k}\epsilon\right] < 0,$$

then $g(\beta,\epsilon,n)$ is less than 1. Note that

$$\sum_{k=0}^{n}(n+1)C_k^n(1-\beta)^n = (n+1)(2-2\beta)^n,$$

and

$$\sum_{k=0}^{n}(\frac{1}{\beta}-1)^k = \sum_{k=0}^{n}(\frac{1}{\beta}-1)^{n-k} = \frac{1-(\frac{1}{\beta}-1)^{n+1}}{2-\frac{1}{\beta}},$$

Therefore, to prove the lemma, we only need to prove that

$$(n+1)(2-2\beta)^n - \frac{1-(\frac{1}{\beta}-1)^{n+1}}{2-\frac{1}{\beta}} < 0. \tag{9}$$

Next, we use the principle of mathematical induction to prove the inequality in Equation (9).

When $n=1$, the left hand side of Equation (9) is given by $\frac{-(2\beta-1)^2}{\beta}$ and is hence less than 0.

Assume that for $n=m$,

$$(m+1)(2-2\beta)^m - \frac{1-(\frac{1}{\beta}-1)^{m+1}}{2-\frac{1}{\beta}} < 0,$$

therefore, when $n=m+1$, we have

$$(m+2)(2-2\beta)^{m+1} - \frac{1-(\frac{1}{\beta}-1)^{m+2}}{2-\frac{1}{\beta}} <$$

$$(m+2)(2-2\beta)\frac{1-(\frac{1}{\beta}-1)^{m+1}}{(m+1)(2-\frac{1}{\beta})} - \frac{1-(\frac{1}{\beta}-1)^{m+2}}{2-\frac{1}{\beta}}. \tag{10}$$

Skipping the detailed derivations, we obtain that the right hand side of Equation (10) is equal to the following expression,

$$\frac{(1-2\beta)m+(3-4\beta)+(\frac{1}{\beta}-1)^{m+2}\left[(1-2\beta)m+(1-4\beta)\right]}{(m+1)(2-\frac{1}{\beta})},$$

and it can be shown that the numerator is always less than 0 and the denominator is always larger than 0 for $\beta \in (\frac{1}{2},1)$ and $m \geq 1$.

To see why this is true, first when $m=1$ and $\beta \in (\frac{1}{2},1)$, the numerator equals to the following expression

$$(4-6\beta)+\left(\frac{1}{\beta}-1\right)^3(2-6\beta) < 0.$$

Next, we take derivative of the numerator with respect to m, which gives

$$(1-2\beta)+(m+2)\left(\frac{1}{\beta}-1\right)^{m+1}\left[(1-2\beta)m+(1-4\beta)\right]$$

$$+\left(\frac{1}{\beta}-1\right)^{m+2}(1-2\beta) < 0.$$

Therefore, the numerator is a monotonically decreasing function for $m \geq 1$ with a negative value at $m=1$.

When $\epsilon = 0$, $g(\beta,\epsilon,n)$ reduces to

$$\sum_{k=0}^{n}\frac{C_k^n(1-\beta)^n}{(\frac{1}{\beta}-1)^k} = \sum_{k=0}^{n}C_k^n\beta^k(1-\beta)^{n-k} = 1.$$

This completes the proof. ∎

Therefore, from Lemma IV.2, the expectation $E[1-P_{t+1}|P_t = 1-\epsilon]$ is always less than $\epsilon > 0$. Hence, we have the following lemma.

Lemma IV.3. Under Sensing Condition 2, if there is an object present, given that the prior probability of object presence $P_i(X=1;\tilde{\mathbf{c}},t)$ is within a neighborhood of one with radius $\epsilon \in [0,\frac{1}{2})$, then the conditional expectation of the posterior probability $E[P_i(X=1|\bar{Y}_t^i;\tilde{\mathbf{c}},t+1)]$ converges to 1.

Same lemma follows for the update sequence $E[P_i(X=0|\bar{Y}_t^i;\tilde{\mathbf{c}},t+1)]$. Therefore, we have the following theorem.

Theorem IV.2. For $\beta^i = \beta \in (\frac{1}{2},1],\ i=1,2,\cdots,N_{\mathrm{v}}$, if an object is present (respectively, absent), then $E[P_i(X=1|\bar{Y}_t^i;\tilde{\mathbf{c}},t+1)]$ converges to 1 (respectively, 0).

V. Uncertainty Map and Coverage Metric

A. Uncertainty Map

In this section, we use an information-based approach to construct an uncertainty map for a multi-cell search domain, which will be used to guide the vehicles within the domain. The information entropy function of a probability distribution is used to evaluate uncertainty [13]. Let $P_i(\tilde{\mathbf{c}},t)$ be the probability distribution for object existence at cell $\tilde{\mathbf{c}}$ of vehicle $\mathcal{V}_i$ at time t and it is given by $P_i(\tilde{\mathbf{c}},t) = \{1-P_i(X=1;\tilde{\mathbf{c}},t), P_i(X=1;\tilde{\mathbf{c}},t)\}$. We define the information entropy for $P_i(\tilde{\mathbf{c}},t)$ as:

$$H_i(P_i(\tilde{\mathbf{c}},t)) = -P_i(X=1;\tilde{\mathbf{c}},t)\ln P_i(X=1;\tilde{\mathbf{c}},t)$$
$$-(1-P_i(X=1;\tilde{\mathbf{c}},t))\ln(1-P_i(X=1;\tilde{\mathbf{c}},t)). \tag{11}$$

If $P_i(X=1;\tilde{\mathbf{c}},t)=0$, we set the term $P_i(X=1;\tilde{\mathbf{c}},t)\ln P_i(X=1;\tilde{\mathbf{c}},t)=0$ by convention. It also follows that $\lim_{P_i(X=1;\tilde{\mathbf{c}},t)\to 0} P_i(X=1;\tilde{\mathbf{c}},t)\ln P_i(X=1;\tilde{\mathbf{c}},t)=0$. The same applies for $(1-P_i(X=1;\tilde{\mathbf{c}},t))\ln(1-P_i(X=1;\tilde{\mathbf{c}},t))$ when $P_i(X=1;\tilde{\mathbf{c}},t)=1$. $H_i(P_i(\tilde{\mathbf{c}},t)) \geq 0$ measures the uncertainty level of object presence or absence of vehicle $\mathcal{V}_i$ at cell $\tilde{\mathbf{c}}$ at time t. The greater the value of H_i, the larger the uncertainty is. $H_i(P_i(\tilde{\mathbf{c}},t))=0$ is the desired uncertainty level. The maximum value attainable by $H_i(P(\tilde{\mathbf{c}},t))$ is $H_{\max} = 0.6931$ when $P(X_i(\tilde{\mathbf{c}})=1;t)=0.5$. The information entropy distribution at time step t over the domain forms an uncertainty map at that time instant. The vehicles are guided towards cells with higher uncertainty. Two different vehicle motion control schemes will be investigated and their performance will be compared in Section VI. The overall goal is to search the entire domain and detect all the objects of interest until zero uncertainty is achieved. From Theorem IV.1, we know that given the true state, the expected posterior probability of object existence $\forall\tilde{\mathbf{c}} \in \mathcal{D}$ will be bounded within a small neighborhood of 1 with radius ϵ if the priors are given by $1-\epsilon$. This corresponds to an upper bound on the uncertainty level $H_i^u = -\epsilon\ln\epsilon-(1-\epsilon)\ln(1-\epsilon)$. Moreover, from Theorem IV.2, it is guaranteed that the expected posterior probability converges to 1, which is equivalent to $H_i \to 0$, $\forall\tilde{\mathbf{c}} \in \mathcal{D}$. This is to say, there is no uncertainty about object presence/absence at every cell in the search domain.

B. Coverage Metric

Now, let us define the coverage metric used in this paper to evaluate the progress of the search task. Associate each vehicle $\mathcal{V}_i$ with the following search cost function:

$$\mathcal{J}_i(t) = \frac{\sum_{\tilde{\mathbf{c}}\in\mathcal{D}} H_i(P_i(\tilde{\mathbf{c}},t))}{H_{\max}A_{\mathcal{D}}}. \quad (12)$$

The cost $\mathcal{J}_i(t)$ is proportional to the sum of uncertainty over $\mathcal{D}$ and $A_{\mathcal{D}}$ is the area of the domain. According to this definition, we have $0 \leq \mathcal{J}_i(t) \leq 1$. If $H_i(P_i(\tilde{\mathbf{c}},t_s)) = 0$ at some $t = t_s$ for all $\tilde{\mathbf{c}} \in \mathcal{D}$, then $\mathcal{J}_i(t_s) = 0$ and the entire domain has been satisfactorily covered and we know with 100% certainty that there are no more objects yet to be found.

VI. Vehicle Motion Control Scheme

A. General Motion Control Scheme

According to the search metric (12), the upper bound on the uncertainty level H_i^u results in $\mathcal{J}_i^u(t_f) = \frac{H_i^u}{H_{\max}} = \delta \geq 0$ at some time $t_f > 0$. This is equivalent to say that the attained accuracy of the search task is $1-\delta$. Furthermore, 100% certainty can be obtained if Sensing Condition 2 is satisfied. Therefore, under any vehicle motion control scheme that covers every cell within $\mathcal{D}$, $\mathcal{J}_i \to \delta$, i.e., all the objects of interest will be guaranteed to be found with desired uncertainty. Here, we seek vehicle motion control strategies that take advantage of the uncertainty map. Two feasible vehicle motion control schemes will be presented, and their performance is compared in simulations.

Let us briefly introduce the mathematical model for β^i used in these control schemes. The key feature of this model is that the sensing capability is limited (see [1]–[3], [11], [12] for more details), other sensor models can be used and the following control schemes still hold. Define $s_i = \|\mathbf{q}_i(t) - \tilde{\mathbf{q}}\|$ as the relative distance between the vehicle's position $\mathbf{q}_i(t)$ and the centroid $\tilde{\mathbf{q}}$ of the observed cell $\tilde{\mathbf{c}}$. Let β^i be dependent on s_i and is given by the following fourth-order polynomial function within the sensor range r_i and $b_n = 0.5$ otherwise,

$$\beta^i(s) = \begin{cases} \frac{M_i}{r_i^4}\left(s^2 - r_i^2\right)^2 + b_n & \text{if } s \leq r_i \\ b_n & \text{if } s > r_i \end{cases}, \quad (13)$$

where $M_i + b_n$ gives the peak value of β^i if the cell $\tilde{\mathbf{c}}$ being observed is located at the sensor vehicle's location. The parameter r_i is the range of the sensor. The sensing capability decreases with range and becomes 0.5 outside of the limited sensory range $\mathcal{W}_i$, implying that the sensor returns an equally likely observation of "absent" or "present" regardless of the true state. This sensor model guarantees the realization of Sensing Condition 1. To satisfy Sensing Condition 2, one may assume an identical value $\beta > 0.5$ within $\mathcal{W}_i$ and 0.5 outside of it for all the vehicles.

B. Memoryless Motion Control Scheme

We first consider a motion control scheme that guides the vehicles based on only the uncertainty map at current time step, that is, the control scheme is memoryless. For the sake of simplicity, we assume that there is no speed limit on the vehicles, i.e., a vehicle is able to move to any cell within $\mathcal{D}$ from its current location.

Consider the set

$$\mathcal{Q}_H^i(t) = \{\tilde{\mathbf{c}} \in \mathcal{D} : \text{argmax}_{\tilde{\mathbf{c}}} H_i(P_i(\tilde{\mathbf{c}},t))\},$$

which is the set of cells with highest search uncertainty level H_i within $\mathcal{D}$. Next, let $\tilde{\mathbf{q}}_c^i(t)$ be the centroid of the cell that vehicle $\mathcal{V}_i$ is currently located at and define the subset $\mathcal{Q}_d^i(t) \subseteq \mathcal{Q}_H^i(t)$ as

$$\mathcal{Q}_d^i(t) = \{\tilde{\mathbf{c}} \in \mathcal{Q}_H^i(t) : \text{argmin}_{\tilde{\mathbf{c}}} \|\tilde{\mathbf{q}}_c^i(t) - \tilde{\mathbf{q}}\|\}.$$

The set $\mathcal{Q}_d^i(t)$ contains the cells which have both the shortest distance from the current cell and the highest uncertainty.

At every time step, a vehicle $\mathcal{V}_i$ takes observations at all the cells within its sensory range. In general, $\beta^i \neq \beta^j$. If $\mathcal{V}_i$ and $\mathcal{V}_j$ have same distance to the centroid of a certain cell $\tilde{\mathbf{c}}$, we may have $\beta^i = \beta^j$. The posterior probabilities at these cells are updated according to Equation (3) based on all the fused observations. The uncertainty map is then updated. At the next time step, the vehicle will choose the next cell to go to from $\mathcal{Q}_d(t)$ based on the updated uncertainty map. Note that $\mathcal{Q}_d(t)$ may have more than one cell. Let N_{Hd} be the number of cells in $\mathcal{Q}_d(t)$, the sensor will randomly pick a cell from $\mathcal{Q}_d(t)$ with probability $\frac{1}{N_{\text{Hd}}}$. This process is repeated until H_i is within a small neighborhood of zero with radius ϵ for every cell $\tilde{\mathbf{c}} \in \mathcal{D}$.

C. Motion Control Scheme with Memory

Now we develop a motion control scheme that takes into account both the current probability information, uncertainty map and the sensing history. Let us first consider the following condition:

Condition C1: $H_i(P_i(\tilde{\mathbf{c}},t)) \leq H_i^u, \ \forall \tilde{\mathbf{c}} \in \mathcal{W}_i(t)$.

For every vehicle $\mathcal{V}_i$, the motion control scheme with memory is given as follows:

$$\mathbf{u}_i^*(t) = \begin{cases} \bar{\mathbf{u}}_i(t) & \text{if } \mathbf{C1} \text{ does not hold} \\ \bar{\bar{\mathbf{u}}}_i(t) & \text{if } \mathbf{C1} \text{ holds} \end{cases} \quad (14)$$

where

$$\bar{\mathbf{u}}_i(t) = \bar{k}_i \sum_{\tilde{\mathbf{c}}\in\mathcal{W}_i(t)} \Bigg(\Big[(2P_i(X=1;\tilde{\mathbf{c}},t)-1)^2 - 1\Big]^2 \cdot \underbrace{\sum_{\tau=0}^{t} \Big(\beta^i(\tau+1) - \beta^i(\tau)\Big)}_{\text{Memory Term}} \Bigg),$$

is the *nominal control law* inspired by its deterministic continuous counterparts in [1], [11], [12], where both the current probability of object presence $P_i(X=1;\tilde{\mathbf{c}},t)$ and the sensing capability β^i up to the current time step are used, and

$$\bar{\bar{\mathbf{u}}}_i(t) \quad = \quad -\bar{\bar{k}}_i(\mathbf{q}_i(t) - \tilde{\mathbf{q}}_i^*)$$

is the *perturbation control law*. The centroid $\tilde{\mathbf{q}}_i^*$ of cell $\tilde{\mathbf{c}}_i^*$ is chosen from the set $\mathcal{Q}_i(t) = \{\tilde{\mathbf{c}} \in \mathcal{D} : H_i(P_i(\tilde{\mathbf{c}},t)) > H_i^u\}$, which is based on the uncertainty information at the current time step and only available to vehicle $\mathcal{V}_i$ itself.

D. Simulation-based Performance Comparison

Next we provide numerical simulations to compare the performances of both motion control schemes. We assume

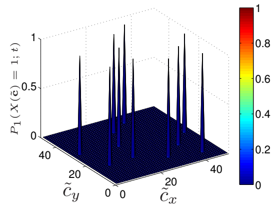
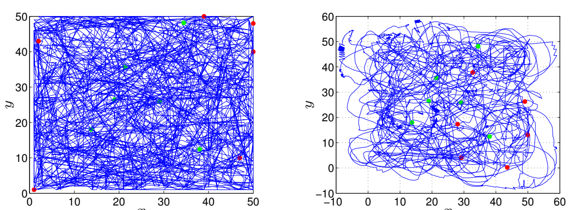


Fig. 1. Probability of object presence of $\mathcal{V}_1$ at time step $t = 1200$.

a square domain $\mathcal{D}$ with size 50×50. The parameter M_i is set as 0.4 and $H_i^u = 0.02$ corresponding to $\epsilon = 0.0002$. There are 10 objects with a randomly selected deployment.

Figure 1 shows the probability of object presence according to vehicle $\mathcal{V}_1$ at time step $t = 1200$ under both control schemes. All the peaks represent the position of the objects with probability 1. The probability of object presence as estimated by other vehicles is similar to that shown in Figure 1. This indicates that all the unknown objects of interest have been found.

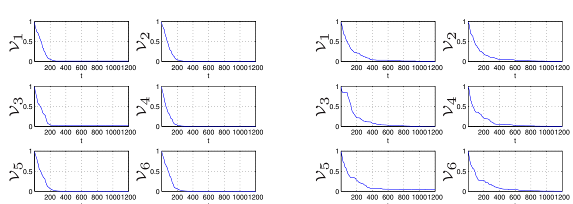
Figure 2(a) shows the trajectories of all the vehicles under the motion control scheme without memory. Figure 2(b) shows the trajectories under the motion control scheme with memory.



(a) (b)

Fig. 2. Fleet motion (green dots represent for vehicles' initial positions and red dots for final positions) under motion control scheme (a) without memory, and (b) with memory.

Figure 3(a) shows $\mathcal{J}_i(t),\ i = 1, \cdots, 6$, respectively under the motion control scheme without memory. Figure 3(b) shows $\mathcal{J}_i(t)$ under the motion control scheme with memory. Here we set $\bar{k}_i = 1, \bar{\bar{k}}_i = 0.025$. In both cases, all the cost functions converge to zero at time step $t = 1200$, which is consistent with the result shown in Figure 1.



(a) (b)

Fig. 3. Cost function for vehicle $\mathcal{V}_1 - \mathcal{V}_6$ under motion control scheme (a) without memory, and (b) with memory.

Comparing the simulation results, there is more redundancy in vehicle trajectories under the memoryless motion control. This is because the controller is only dependent on the current uncertainty map and does not take into account the history of the paths that the vehicles traveled before. However, the reduction of uncertainty is faster under the memoryless control scheme because it is a global controller that always seeks the cell with highest uncertainty within the entire search domain. If fuel efficiency is a priority, one may want to avoid using a memoryless motion controller that spreads all over the domain. On the contrary, if time is a limited resource, one may prefer a memoryless motion controller in order to achieve the desired detection certainty quicker.

VII. Conclusion

In this paper, we developed a Bayes probability update rule for domain search problems using distributed MAVs with limited sensory range and intermittent communications. The expected probability of object existence is guaranteed to be within a desired uncertainty level of the actual state under appropriate sensor model. It is further proved that this belief will converge to the true state given "good" homogenous sensors across the sensor network. Two vehicle motion control strategies are proposed. Currently, we are investigating similar convergence results for heterogeneous sensor networks. Future research will focus on the tracking of mobile objects.

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