2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

A Multivariable MRAC Design for Aircraft Systems under Failure and Damage Conditions

Jiaxing Guo and *Gang Tao*
Department of Electrical and Computer Engineering
University of Virginia
Charlottesville, VA 22904

Yu Liu
Cummins Technical Center
Columbus, IN 47201

Abstract— **This paper studies the multivariable model reference adaptive control (MRAC) design for aircraft systems under simultaneous actuator failures and airframe damage. A modeling study of the aircraft under failure and damage conditions is conducted, which captures the key characteristics of the aircraft dynamics under such hazardous conditions. The key design conditions for the multivariable MRAC design are studied for nominal and post-hazard aircraft systems, and the invariance of these essential conditions is concluded under realistic failure and damage conditions. A multivariable MRAC scheme is developed to ensure stability and asymptotic output tracking for the aircraft in the presence of uncertain actuator failures and airframe damage. Simulation results are presented to demonstrate the application of the proposed adaptive control scheme to the NASA Generic Transport Model (GTM).**

Keywords: Actuator failure, airframe damage, adaptive control, output feedback, output tracking.

I. INTRODUCTION

The research of adaptive aircraft control under adverse conditions has remained an important topic in the past decades. Adaptive control methodologies are capable of autonomously adjusting the controller parameters to maintain satisfactory performance when unforeseen changes in the system dynamics occur. These unique features provide the potential to improve flight safety when hazards such as actuator failures and airframe damage occur.

Adaptive control of aircraft systems under actuator failures has remained a major focus for the past decades and various adaptive control designs for actuator failure compensation have been proposed (for instance, [1], [2], [3], [8], [10]). A challenge of successful actuator failure compensation is uncertainties of the failures, including their onset time instants, failure patterns, and magnitudes. An adaptive control design is able to employ the built-in redundant actuators to achieve control objectives while rejecting the disturbances caused by out-of-control actuators. The challenge of aircraft control under damage conditions is that the system dynamics will be subject to uncertain changes, which may lead to large parametric variations and dynamic coupling. A non-adaptive controller may not be able to ensure satisfactory tracking performance or stability under such conditions. The adaptive control is capable of compensating for large parametric uncertainties while ensuring stability and tracking. Several adaptive control designs for the modeling and control of damaged aircraft have been proposed, such as those in [4], [5], [6], and others.

A topic that is of both theoretical and practical importance is the adaptive control of simultaneous airframe damage and actuator failures. Actuator failures may appear with the airframe damage frequently. With such damage and failure scenarios, both parametric and failure uncertainties as well as dynamic coupling will be present in the system. In this paper we will study the multivariable MRAC design using output feedback for output tracking (OFOT) for the aircraft under both failure and damage conditions. The modeling of aircraft under both failures and damage will be studied. Case studies are conducted to investigate two key conditions for the multivariable MRAC design, i.e., infinity zero structure and the signs of the leading principal minors of the high frequency gain matrix, for aircraft under both failures and damage. The proposed adaptive control scheme can ensure closed-loop stability and asymptotic output tracking in the presence of uncertain failures and damage. This linearization-based control design is further applied to the nonlinear NASA Generic Transport Model (GTM) to demonstrate its efficacy for improving flight safety in the presence of failure and damage conditions.

The paper is organized as follows. In Section II, MRAC of aircraft under failures and damage is formulated. Section III provides a generic linearized aircraft model to investigate the invariance properties of an aircraft model with failure and damage conditions. In Section IV, the multivariable MRAC scheme is developed. Simulation results of the linearized and nonlinear NASA GTM are presented in Section V.

II. PROBLEM STATEMENT

Consider a linear system with structural damage and lock-in-place actuator failures described as

$$\begin{aligned} &\dot{x}(t) = Ax(t) + Bu(t) + f_0, \quad y(t) = Cx(t) \\ &u(t) = (I_m - \sigma)v(t) + \sigma\bar{u}, \end{aligned} \tag{1}$$

where $x(t) \in R^n, y(t) \in R^M, u(t) \in R^m$ are state, output, and input vector signals with $m > M$ (actuation redundancy), $v(t) \in R^m$ is a commanded control input vector signal, $\bar{u} \in R^m$ is an unknown constant failure vector. We assume that, within a time interval $[t_{k-1}, t_k)$,

Corresponding author: Gang Tao, gt9s@virginia.edu.
This research was supported by NASA under the grant NNX08AB99A

with $t_0 = 0$, $t_N = \infty$, and $t_k, k = 2, \ldots, N-1$, being unknown due to the uncertainties of damage and failures, the damage and failure pattern is fixed, such that $A = A_k$, $B = B_k$, $C = C_k$, $f_0 = f_{0k}$ with A_k, B_k, C_k, and f_{0k} being unknown constant parameters, and $\sigma = \sigma_k = \mathrm{diag}\{\sigma_{kf1}, \sigma_{kf2}, \ldots, \sigma_{kfm}\}$, with $\sigma_{kfj} = 1$ if the jth actuator fails or $\sigma_{kfj} = 0$ otherwise, $j = 1, \ldots, m$. Therefore, within each time interval $[t_{k-1}, t_k)$, the model (1) can represent a specific damage and failure scenario.

The linear system (1) with the dynamics offset f_0 is motivated from linearization of a nonlinear aircraft system.

Aircraft application. The nonlinear aircraft system with damage and failures can be denoted as

$$\begin{aligned} &\dot{x}(t) = f(x(t), u(t)), \quad y(t) = Cx(t), \\ &u(t) = (I_m - \sigma)v(t) + \sigma\bar{u}, \end{aligned} \tag{2}$$

where the structures of $f(x, u)$ are different before and after damage and the parameter σ changes due to different actuator failure patterns. To compensate the damage and failures, we apply a linearization-based control design to the nonlinear aircraft system. Since there exist uncertainties for the system with damage, the equilibrium point is not available. We linearize the system at a chosen operating point (x_0, u_0), which may not be an equilibrium:

$$\begin{aligned} &\Delta\dot{x} = A\Delta x + B\Delta u + f_0, \quad \Delta y = C\Delta x, \\ &\Delta u = (I_m - \sigma)\Delta v + \sigma\Delta\bar{u}, \end{aligned} \tag{3}$$

where $f_0 = f(x_0, u_0)$ is a dynamics offset which may not be 0, $\Delta x = x - x_0$, $\Delta u = u - u_0$, $\Delta v = v - u_0$, $\Delta\bar{u} = \bar{u} - u_0$, and A, B, f_0, σ are piecewise constant parameters due to different damage and failure patterns. After we develop the control law Δv for the linearized system (3), we can apply $v = \Delta v + u_0$ to the nonlinear system (2) to assess the linearization-based adaptive control design.

Actuation redundancy and grouping. Redundant actuators are widely employed in the aircraft flight control systems. For example, an aircraft has a group of two rudder segments and a group of four elevator segments. We divide the m actuators into M groups: $u(t) = [u_1^T, u_2^T, \ldots, u_M^T]^T$, where $u_i = [u_{i1}, u_{i2}, \ldots, u_{in_i}]^T$ with $i = 1, 2, \ldots, M$ and $m = n_1 + n_2 + \cdots + n_M$. Assuming that the actuators in each group have the same physical characteristics, we can apply a proportional actuation scheme.

Proportional actuation scheme. For the ith group of actuators with the lock-in-place failures:

$$u_i = v_i + \mathrm{diag}\{\sigma_{f_{i1}}, \ldots, \sigma_{f_{in_i}}\}(\bar{u}_i - v_i), i = 1, \ldots, M, \tag{4}$$

with the commanded control input $v_i = [v_{i1}, \ldots, v_{in_i}]^T$ and the failure constant $\bar{u}_i = [\bar{u}_{i1}, \ldots, \bar{u}_{in_i}]^T$, a proportional actuation scheme is used: $v_{ij}(t) = \alpha_{ij} v_{i0}(t)$ for $i = 1, \ldots, M$ and $j = 1, \ldots, n_i$. Then, there exists $H \in R^{m\times M}$ to achieve

$$v(t) = Hv_0(t), \tag{5}$$

where $v_0(t) = [v_{10}, v_{20}, \ldots, v_{M0}]^T \in R^M$ and $H = \mathrm{diag}\{H_1, H_2, \ldots, H_M\}$ with $H_i = [\alpha_{i1}, \alpha_{i2}, \ldots, \alpha_{in_i}]^T$ for $i = 1, \ldots, M$. Applying (5) to the system (1), we have

$$y(t) = G(s)(I_m - \sigma)H[v_0](t) + G(s)\sigma[\bar{u}](t) + G_{f_0}(s)[h_f](t) \tag{6}$$

where $G(s) = C(sI_n - A)^{-1}B$, $G_{f_0}(s) = C(sI_n - A)^{-1}f_0$, and $h_f(t)$ is a unit step function. We denote $G_{ij}(s), i = 1, \ldots, M, j = 1, \ldots, n_i$, as the column vectors of $G(s)$ corresponding to the control inputs u_{ij}. Suppose that, within each time interval $[t_{k-1}, t_k), k = 1, 2, \ldots, N$, there are p_{ki} failed actuators in the ith group of actuators, that is

$$u_{ij} = \bar{u}_{ij}, i \in \{1, \ldots, M\}, j \in \{j_{i1}, \ldots, j_{ip_{ki}}\} \subset \{1, \ldots, n_i\}.$$

To compensate the damage and actuator failures, we need to assume that there is at least one working actuator in each group for the damage and failure condition within $[t_{k-1}, t_k), k = 1, 2, \ldots, N$, such that $0 \le p_{ki} < n_i$ and the columns of B matrix corresponding to the healthy actuators in each group are not 0 (since a column of B with all entries being 0 means that the corresponding actuator is lost caused by airframe damage). The system (6) can be described as

$$y(t) = G_a(s)[v_0](t) + \bar{y}(t), \quad \text{where} \tag{7}$$

$$G_a(s) = \left[\sum_{j \ne j_{11}, \ldots j_{1p_{k1}}} G_{1j}(s), \ldots, \sum_{j \ne j_{M1}, \ldots, j_{Mp_{kM}}} G_{Mj}(s)\right],$$

$$\bar{y}(t) = \sum_{i=1,\ldots,M} \sum_{j = j_{i1}, \ldots, j_{ip_{ki}}} G_{ij}(s)[\bar{u}_{ij}](t) + G_{f_0}(s)[h_f](t).$$

Control objective. The control objective is to design an output feedback control law $v_0(t) = [v_{10}, v_{20}, \ldots, v_{M0}]^T$ for the system (7) to make all the closed-loop signals bounded and the output $y(t)$ track a given reference signal $y_m(t)$ generated from a reference model system

$$y_m(t) = W_m(s)[r](t), \tag{8}$$

where $W_m(s) \in R^{M\times M}$ is a stable transfer matrix and $r(t) \in R^M$ is a bounded reference input signal.

Assumptions. To proceed the control design, for all damage and failure patterns $(A, B, C, f_0, \sigma) = (A_k, B_k, C_k, f_{0k}, \sigma_k), k = 1, 2, \ldots, N$, we assume: (A1) $G_a(s)$ has full rank and all zeros of $G_a(s)$ have negative real parts; (A2) an upper bound $\bar{\nu}_0$ of the observability indices of all $G_a(s)$ is known; (A3) for $G_a(s) = P_l^{-1}(s)Z_{la}(s)$, $G(s)\sigma = P_l^{-1}(s)Z_{lb}(s)$, and $G_{f_0}(s) = P_l^{-1}(s)Z_f(s)$, the transfer function matrices $Z_{la}^{-1}(s)Z_{lb}(s)$ and $Z_{la}^{-1}(s)Z_f(s)$ are proper; (A4) there is a known modified left interactor matrix $\xi_m(s)$ for all $G_a(s)$, which is invariant for all the damage and failure patterns, and $W_m(s) = \xi_m^{-1}(s)$; (A5) all leading principal minors $\Delta_i, i = 1, 2 \ldots, M$, of the high frequency gain matrix $K_{pa} = \lim_{t\to\infty} \xi_m(s)G_a(s)$ are nonzero and the signs are known and invariant for all the damage and failure patterns.

Next, we will present a generic linearized aircraft model to investigate the invariance of the interactor matrix (A4) and signs of the high frequency gain matrix (A5).

III. System Invariance of An Aircraft Model

The nonlinear aircraft system with damage and actuator failures is denoted as (2), where the state signal is $x = [u_b, w_b, q_b, \theta, v_b, r_b, p_b, \phi, \psi]^T$ and the output signal is chosen as $y = [\theta, \psi]^T$ with u_b, v_b, w_b being the body-axis velocity components, p_b, q_b, r_b being the body-axis components of the angular velocity, θ, ϕ, ψ being the Euler pitch, roll, and yaw angle of the aircraft body axes with respect to the reference axes. For control of the aircraft system (2), we only manipulate elevators and rudders, while fixing the other control inputs (ailerons, throttles, etc.) as the operating point values. Hence, the control signal with redundancy is given as $u = [\delta_{e_{l_1}}, \delta_{e_{l_2}}, \delta_{e_{r_1}}, \delta_{e_{r_2}}, \delta_{r_u}, \delta_{r_l}]^T$, where $\delta_{e_{l_1}}$ and $\delta_{e_{l_2}}$ are the left two elevator segment deflections, $\delta_{e_{r_1}}$ and $\delta_{e_{r_2}}$ are the right two elevator segment deflections, and δ_{r_u} and δ_{r_l} are the upper and lower rudder segment deflections.

A. *Linearization of the Nonlinear Aircraft Model*

We choose a wings-level flight condition (x_0, u_0), where $x_0 = [u_{d0}, w_{d0}, 0, \theta_0, 0, 0, 0, 0, \psi_0]^T$, as the operating condition for the nonlinear aircraft system (2) with damage and failures, which may not be an equilibrium point due to the structural uncertainty. Then, the linearized system of (2) is given as (3), where $A = \frac{\partial f}{\partial x}\Big|_{(x_0,u_0)}$, $B = \frac{\partial f}{\partial u}\Big|_{(x_0,u_0)}$, and

$$C = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}. \tag{9}$$

Applying the actuation scheme $\Delta v = H \Delta v_0$ to the elevator group and the rudder group, we obtain the system with damage and failures as

$$\Delta y(t) = G_a(s)[\Delta v_0](t) + \Delta \bar{y}(t), \tag{10}$$

where $G_a(s) = C(sI - A)^{-1} B_a$ with $B_a = [B_{a1}, B_{a2}] = B(I - \sigma)H \in R^{9\times 2}$. Based on the generic nonlinear aircraft models before and after damage given in [5] and [7], we will derive the structures of A and B_a for the following four damage and failure cases:

(A, B_a) **for the nominal case**. A and B are decoupled:

$$A = \begin{bmatrix} A_1^{(4\times4)} & 0^{(4\times5)} \\ 0^{(5\times4)} & A_4^{(5\times5)} \end{bmatrix}, \quad B = \begin{bmatrix} B_1^{(3\times4)} & 0^{(3\times2)} \\ 0^{(1\times4)} & 0^{(1\times2)} \\ 0^{(3\times4)} & B_4^{(3\times2)} \\ 0^{(2\times4)} & 0^{(2\times2)} \end{bmatrix}, \tag{11}$$

$$A_1 = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \\ 0 & 0 & 1 & 0 \end{bmatrix}, A_4 = \begin{bmatrix} a_{55} & a_{56} & a_{57} & a_{58} & 0 \\ a_{65} & a_{66} & a_{67} & a_{68} & 0 \\ a_{75} & a_{76} & a_{77} & a_{78} & 0 \\ 0 & \tan\theta_0 & 1 & 0 & 0 \\ 0 & 1/\cos\theta_0 & 0 & 0 & 0 \end{bmatrix}.$$

Since no failures happen ($\sigma = 0$), we have $B_a = BH$.

(A, B_a) **for the failure but no damage case**. When there is no damage, the matrices A and B are given as (11). We assume that there are q_e healthy elevators with $0 < q_e \le 4$ and q_r healthy rudders with $0 < q_r \le 2$, that is

$$B_a = B(I - \sigma)H, \ \sigma = \mathrm{diag}\{\sigma_{e_1}, \sigma_{e_2}, \sigma_{e_3}, \sigma_{e_4}, \sigma_{r_1}, \sigma_{r_2}\} \tag{12}$$

where $\sigma_{e_i} = 0$ with $i \in \{j_1, \ldots, j_{q_e}\} \subseteq \{1, 2, 3, 4\}$ and $\sigma_{e_i} = 1$ otherwise, and $\sigma_{r_i} = 0$ with $i \in \{l_1, l_{q_r}\} \subseteq \{1, 2\}$ and $\sigma_{r_i} = 1$ otherwise.

(A, B_a) **for the damage but no failure case**. When damage occurs, the longitudinal and lateral-directional dynamics are coupled. The parameters A and B become to be

$$A = \begin{bmatrix} A_{d1}^{(4\times4)} & A_{d2}^{(4\times5)} \\ A_{d3}^{(5\times4)} & A_{d4}^{(5\times5)} \end{bmatrix}, \quad B = \begin{bmatrix} B_{d1}^{(3\times4)} & B_{d2}^{(3\times2)} \\ 0^{(1\times4)} & 0^{(1\times2)} \\ B_{d3}^{(3\times4)} & B_{d4}^{(3\times2)} \\ 0^{(2\times4)} & 0^{(2\times2)} \end{bmatrix}, \tag{13}$$

$$A_{d_1} = \begin{bmatrix} a_{d_{11}} & a_{d_{12}} & a_{d_{13}} & a_{d_{14}} \\ a_{d_{21}} & a_{d_{22}} & a_{d_{23}} & a_{d_{24}} \\ a_{d_{31}} & a_{d_{32}} & a_{d_{33}} & a_{d_{34}} \\ 0 & 0 & 1 & 0 \end{bmatrix} A_{d_2} = \begin{bmatrix} a_{d_{15}} & a_{d_{16}} & a_{d_{17}} & a_{d_{18}} & 0 \\ a_{d_{25}} & a_{d_{26}} & a_{d_{27}} & a_{d_{28}} & 0 \\ a_{d_{35}} & a_{d_{36}} & a_{d_{37}} & a_{d_{38}} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A_{d_3} = \begin{bmatrix} a_{d_{51}} & a_{d_{52}} & a_{d_{53}} & a_{d_{54}} \\ a_{d_{61}} & a_{d_{62}} & a_{d_{63}} & a_{d_{64}} \\ a_{d_{71}} & a_{d_{72}} & a_{d_{73}} & a_{d_{74}} \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} A_{d_4} = \begin{bmatrix} a_{d_{55}} & a_{d_{56}} & a_{d_{57}} & a_{d_{58}} & 0 \\ a_{d_{65}} & a_{d_{66}} & a_{d_{67}} & a_{d_{68}} & 0 \\ a_{d_{75}} & a_{d_{76}} & a_{d_{77}} & a_{d_{78}} & 0 \\ 0 & a_{d_{86}} & 1 & 0 & 0 \\ 0 & \frac{1}{\cos\theta_0} & 0 & 0 & 0 \end{bmatrix}.$$

Since no failure occurs, it follows that $B_a = BH$.

(A, B_a) **for the both damage and failure case**. In this case, the remaining actuators after damage still suffer from the lock-in-place failures. Since damage occurs, A and B become to be the damaged values (13). To compensate the damage and failures, we assume that there are q_e healthy elevators with $0 < q_e \le 4$ and q_r healthy rudders with $0 < q_r \le 2$. The healthy actuator in this case means that it is not totally lost due to damage (so that the corresponding column of B matrix is not 0) and does not suffer from the lock-in-place failure. With such an actuator failure and damage pattern, parameter B_a is given as

$$B_a = B(I - \sigma)H, \ \sigma = \mathrm{diag}\{\sigma_{e_1}, \sigma_{e_2}, \sigma_{e_3}, \sigma_{e_4}, \sigma_{r_1}, \sigma_{r_2}\} \tag{14}$$

where $\sigma_{e_i} = 0$ with $i \in \{j_1, \ldots, j_{q_e}\} \subseteq \{1, 2, 3, 4\}$ and $\sigma_{e_i} = 1$ otherwise, and $\sigma_{r_i} = 0$ with $i \in \{l_1, l_{q_r}\} \subseteq \{1, 2\}$ and $\sigma_{r_i} = 1$ otherwise.

Then, we can investigate the two invariance properties for $G_a(s)$ based on the structures of A and B_a.

B. *Invariance of Infinity Zero Structures*

To determine the infinity zero structure for the system, we need to study the relative degrees for the entries of the transfer matrix $G_a(s)$, which can be calculated as

$$G_a(s) = \frac{1}{\alpha(s)}(E_{n-1}s^{n-1} + E_{n-2}s^{n-2} + \cdots + E_1 s + E_0), \tag{15}$$
$$\alpha(s) = \det(sI - A) \triangleq s^n + \alpha_{n-1}s^{n-1} + \cdots + \alpha_1 s + \alpha_0,$$
$$E_{n-1} = CB_a, \ E_{n-2} = CAB_a + \alpha_{n-1}CB_a, \ \ldots,$$
$$E_0 = CA^{n-1}B_a + \alpha_{n-1}CA^{n-2}B_a + \cdots + \alpha_1 CB_a.$$

For the aircraft systems with simultaneous damage and actuator failures, we will consider the following two scenarios:

- actuator failures occur before damage happens;
- damage occurs before failures happen.

We first obtain the infinity zero structure for the no damage and no failure case, i.e. the nominal case.

Infinity zero structure of the nominal case. The parameters A and B are given as (11) and $B_a = BH$. With the matrix C given as (9), we have coefficients for $G_a(s)$ as

$$E_{n-1}=0,\ E_{n-2}=\mathrm{diag}\{\sum_{i=1}^{4} b_{3ei},\ \frac{1}{\cos\theta_0}(b_{6r1}+b_{6r2})\}. \quad (16)$$

Therefore, we can choose an interactor matrix for $G_a(s)$ as $\xi_m(s) = \mathrm{diag}\{(s+1)^2, (s+1)^2\}$, so that the high frequency gain matrix can be obtained as $K_{pa} = \lim_{s\to\infty} \xi_m(s)G_a(s) = CAB_a$. Since the parameters $b_{3e1}, \ldots, b_{3e4}$, b_{6r1}, and b_{6r2} are the control gains from elevators to pitch acceleration and rudders to yaw acceleration, the signs of these parameters can be obtained: $b_{3e1}, \ldots, b_{3e4} < 0$, $b_{6r1} < 0$, and $b_{br2} < 0$. The operating point is chosen as $\theta_0 \in (-\pi/2, \pi/2)$. That is the signs of the leading principal minors are $\mathrm{sign}(\Delta_1) = -1, \mathrm{sign}(\Delta_2) = 1$.

Case I: failures occur before damage happens. When failures occur, the matrix A is still the nominal value given in (11), but the matrix B_a changes to (12) with certain failure patterns. E_{n-1} and E_{n-2} for $G_a(s)$ are calculated as

$$E_{n-1}=0,\ E_{n-2}=\mathrm{diag}\{\sum_{i=j_1,\ldots,j_{q_e}} b_{3ei},\ \frac{1}{\cos\theta_0}\sum_{i=l_1,\, l_{q_r}} b_{6ri}\}.$$

Therefore, we can choose $\xi_m(s)$ as $\xi_m(s) = \mathrm{diag}\{(s+1)^2, (s+1)^2\}$, so that $K_{pa} = \lim_{s\to\infty} \xi_m(s)G_a(s) = CAB_a$. Since there is no damage, the parameters $b_{3ei} < 0, i \in \{j_1, \ldots, j_{q_e}\}$ and $b_{6ri} < 0, i \in \{l_1, l_{q_r}\}$, it follows that $\mathrm{sign}(\Delta_1) = -1, \mathrm{sign}(\Delta_2) = 1$.

After damage happens, the aircraft suffers from the simultaneous failures and damage condition. The matrices A and B_a become to be the damaged values (13) and (14), while the failure patterns $\{j_1, \ldots, j_{q_e}\}$ and $\{l_1, l_{q_r}\}$ for B_a do not change. The coefficients are calculated as $E_{n-1} = 0$ and

$$E_{n-2} = \begin{bmatrix} \sum_{i=j_1,\ldots,j_{q_e}} b_{d3ei} & \sum_{i=l_1,\, l_{q_r}} b_{d3ri} \\ \frac{1}{\cos\theta_0}\sum_{i=j_1,\ldots,j_{q_e}} b_{d6ei} & \frac{1}{\cos\theta_0}\sum_{i=l_1,\, l_{q_r}} b_{d6ri} \end{bmatrix}.$$

Therefore, the interactor matrix can still be $\xi_m(s) = \mathrm{diag}\{(s+1)^2, (s+1)^2\}$, and $K_{pa} = E_{n-2}$. When the shift of center of gravity caused by damage is small, the signs of parameters in B_a may not change and the coupled terms in K_{pa} may not be large enough to have impact on the sign of the second leading principal minor, which will be assessed by simulation studies. That is $\mathrm{sign}(\Delta_1) = -1, \mathrm{sign}(\Delta_2) = 1$.

Case II: damage occurs before failures happen. When damage occurs first, the matrices A and B change to the damaged values given as (13) and $B_a = BH$. So, we have the coefficients $E_{n-1} = 0$ and $E_{n-2} = CAB_a$. Similar to the case I, we can choose $\xi_m(s)$ as $\xi_m(s) = \mathrm{diag}\{(s+1)^2, (s+1)^2\}$, and $K_{pa} = CAB_a$. If the shift of center of gravity is small, from the structure of CAB_a, we have $\mathrm{sign}(\Delta_1) = -1, \mathrm{sign}(\Delta_2) = 1$.

Then, after failures happen, some of the remaining actuators are locked. So the matrix B_a with certain failure patterns $\{j_1, \ldots, j_{q_e}\}$ and $\{l_1, l_{q_r}\}$ becomes to be (14). The coefficients are calculated as $E_{n-1} = 0$ and $E_{n-2} = CAB_a$. Similar to the analysis in case I, we can conclude that the interactor matrix is $\xi_m(s) = \mathrm{diag}\{(s+1)^2, (s+1)^2\}$, and the signs are invariant: $\mathrm{sign}(\Delta_1) = -1, \mathrm{sign}(\Delta_2) = 1$ when the shift of center of gravity caused by damage is small.

Summary. For these two patterns, the interactor matrix $\xi_m(s)$ is invariant and the signs of K_{pa} are invariant if the shift of center of gravity (due to damage) is small.

IV. Adaptive Control Scheme

In this section, we will develop an adaptive control scheme for (7) to achieve the desired output tracking performance.

Output feedback controller structure. We choose an output feedback controller for (7) as

$$\begin{aligned} v_0(t) &= \Theta_1^T(t)\omega_1(t) + \Theta_2^T(t)\omega_2(t) \\ &\quad + \Theta_{20}(t)y(t) + \Theta_3(t)r(t) + \Theta_4(t), \end{aligned} \quad (17)$$

where $\omega_1(t) = F(s)[v_0](t)$, $\omega_2 = F(s)[y](t)$, $F(s) = \frac{A_0(s)}{\Lambda(s)}$, $A_0(s) = [I, sI, \ldots, s^{\bar{\nu}_0-2}I]^T$, $\Lambda(s)$ is a monic stable polynomial of degree $\bar{\nu}_0 - 1$, with the upper bound $\bar{\nu}_0$ of the observability indices of $G_a(s)$, and $\Theta_1(t)$, $\Theta_2(t)$, $\Theta_{20}(t)$, $\Theta_3(t)$, and $\Theta_4(t)$ are the estimates of some nominal plant-model matching parameters Θ_1^*, Θ_2^*, Θ_{20}^*, and Θ_4^*. In particular, the term $\Theta_4(t)$ is for compensation of the actuator failures and the unknown offset f_0 due to the damage.

Plant-model matching. To derive adaptive laws for (17), we need to ensure there exist Θ_1^*, Θ_2^*, Θ_{20}^*, Θ_3^* and Θ_4^* to achieve plant-model matching condition.

Lemma 1: *There exist parameters* Θ_1^*, Θ_2^*, Θ_{20}^*, Θ_3^* *and* Θ_4^*, *such that, when* $\Theta_1(t) = \Theta_1^*$, $\Theta_2(t) = \Theta_2^*$, $\Theta_{20}(t) = \Theta_{20}^*$, $\Theta_3(t) = \Theta_3^*$, *and* $\Theta_4(t) = \Theta_4^*$, *the controller (17) ensures closed-loop signal boundedness and* $\lim_{t\to\infty}(y - y_m) = 0$.

Proof: Applying the nominal controller

$$v_0^*(t)=\Theta_1^{*T}\omega_1(t)+\Theta_2^{*T}\omega_2(t)+\Theta_{20}^*y(t)+\Theta_3^*r(t)+\Theta_4^* \quad (18)$$

to the system (7), we obtain

$$y(t) = G_a(s)[v_0^*](t) + \bar{y}(t). \quad (19)$$

Substituting (19) into (18), it follows that

$$\begin{aligned} v_0^*(t) &= (I - \Theta_1^{*T}F(s) - \Theta_2^{*T}F(s)G_a(s) - \Theta_{20}^*G_a(s))^{-1} \\ &\quad \times(\Theta_2^{*T}F(s)[\bar{y}](t) + \Theta_{20}^*\bar{y}(t) + \Theta_3^*r(t) + \Theta_4^*). \end{aligned} \quad (20)$$

There exist $\Theta_1^*, \Theta_2^*, \Theta_{20}^*$, and $\Theta_3^* = K_{pa}^{-1}$ [9], such that

$$\begin{aligned} &I - \Theta_1^{*T}F(s) - \Theta_2^{*T}F(s)G_a(s) - \Theta_{20}^*G_a(s) \\ &= \Theta_3^*W_m^{-1}(s)G_a(s). \end{aligned} \quad (21)$$

Substituting (20) in (19) and from (21), we have

$$\begin{aligned} y(t) &= W_m(s)[r](t) + W_m(s)K_{pa}(\Theta_2^{*T}F(s)[\bar{y}](t) + \Theta_{20}^*\bar{y}(t) \\ &\quad + \Theta_3^*\xi_m(s)[\bar{y}](t) + \Theta_4^*) \\ &\triangleq W_m(s)[r](t) + f_p(t). \end{aligned} \quad (22)$$

From the reference (8), we have $e(t)=y(t)-y_m(t)$ as

$$e(t) = W_m(s)K_{pa}[\frac{\Lambda(s)I - \Theta_1^{*T}A_0(s)}{\Lambda(s)} Z_{la}^{-1}(s)Z_{lb}(s)[\bar{u}] + \frac{\Lambda(s)I - \Theta_1^{*T}A_0(s)}{\Lambda(s)} Z_{la}^{-1}(s)Z_f(s)[h_f] + \Theta_4^*](t).$$

From (A5) where $Z_{la}^{-1}Z_{lb}(s)$ and $Z_{la}^{-1}Z_f(s)$ are proper, it can be concluded that $\frac{\Lambda(s)I-\Theta_1^{*T}A_0(s)}{\Lambda(s)}Z_{la}^{-1}(s)Z_{lb}(s)$ and $\frac{\Lambda(s)I-\Theta_1^{*T}A_0(s)}{\Lambda(s)}Z_{la}^{-1}(s)Z_f(s)$ are proper. Since $\bar{u}(t)$ and $h_f(t)$ are step functions, $W_m(s)$ and $\Lambda(s)$ are stable, and $G_a(s)$ is minimum phase, there exists a Θ_4^* to ensure that $\lim_{t\to\infty} e(t) = \lim_{t\to\infty} f_p(t) = 0$. ∇

Since the parameters are unknown due to uncertain damage and failures, the adaptive control law (17) is employed.

Tracking error equation. Operating both sides of (21) on $v_0(t)$ and from the system transfer function (7), we obtain

$$v_0(t) = \Theta_1^{*T}\omega_1(t) + \Theta_2^{*T}\omega_2(t) + \Theta_{20}^* y(t) - \Theta_2^{*T}F(s)[\bar{y}](t) - \Theta_{20}^*\bar{y}(t) + \Theta_3^*\xi_m(s)[y](t) - \Theta_3^*\xi_m(s)[\bar{y}](t). \quad (23)$$

Substituting (17) in (23), we obtain the tracking error as

$$e(t) = y(t) - y_m(t) = W_m(s)K_{pa}[\widetilde{\Theta}^T\omega](t) + f_p(t), \quad (24)$$

where $\widetilde{\Theta} = \Theta - \Theta^*$, $\Theta = [\Theta_1^T, \Theta_2^T, \Theta_{20}, \Theta_3, \Theta_4]^T$, $\Theta^* = [\Theta_1^{*T}, \Theta_2^{*T}, \Theta_{20}^*, \Theta_3^*, \Theta_4^*]^T$, and $\omega = [\omega_1^T, \omega_2^T, y^T, r^T, 1]^T$. To deal with the uncertainty of K_{pa}, we use its LDS decomposition $K_{pa} = L_sD_sS$, where $S = S^T > 0$, L_s is a unit lower triangular matrix, and $D_s = \mathrm{diag}\{\mathrm{sign}[\Delta_1]\gamma_1, \ldots, \mathrm{sign}[\frac{\Delta_M}{\Delta_{M-1}}]\gamma_M\}$ with arbitrarily chosen $\gamma_i > 0$, $i = 1, \ldots, M$ [9]. Since the signs of the leading principal minors $\Delta_i, i = 1, 2, \ldots, M$, are invariant, we can choose a uniform D_s for the possible damage and failure patterns as a gain matrix which will be used in the adaptive laws. Ignoring the decaying term f_p, and substituting the LDS decompensation in (24) and operating both sides of (24) by $h(s)I_M$, where $h(s) = 1/f_h(s)$ with $f_h(s)$ being a stable and monic polynomial of degree equals to the degree of $\xi_m(s)$, we have

$$L_s^{-1}\xi_m(s)h(s)[e](t) = D_s\, S\, h(s)[\tilde{\Theta}^T\omega](t). \quad (25)$$

To parameterize the unknown matrix L_s, we introduce $\Theta_0^* = L_s^{-1} - I = \{\theta_{ij}^*\}$, where $\theta_{ij}^* = 0$ for $i = 1, 2, \ldots, M$ and $j \geq i$. Then we have

$$\bar{e}(t) + [0, \theta_2^{*T}\eta_2(t), \ldots, \theta_M^{*T}\eta_M(t)]^T = D_sSh(s)[\widetilde{\Theta}^T\omega](t), \quad (26)$$

where $\bar{e}(t) = \xi_m(s)h(s)[e](t) = [\bar{e}_1(t), \ldots, \bar{e}_M(t)]^T$, $\eta_i(t) = [\bar{e}_1(t), \ldots, \bar{e}_{i-1}(t)]^T$, $\theta_i^* = [\theta_{i1}^*, \ldots, \theta_{ii-1}^*]^T, i = 2, \ldots, M$.

Estimation error. We introduce an estimation error signal

$$\epsilon(t) = [0, \theta_2^T(t)\eta_2(t), \ldots, \theta_M^T(t)\eta_M(t)]^T + \Psi(t)\xi(t) + \bar{e}(t), \quad (27)$$

where $\theta_i(t), i = 2, 3, \ldots, M$ are the estimates of θ_i^*, and $\Psi(t)$ is the estimate of $\Psi^* = D_sS$, and

$$\xi(t) = \Theta^T(t)\zeta(t) - h(s)[\Theta^T\omega](t), \ \zeta(t) = h(s)[\omega](t). \quad (28)$$

From (26)–(28), we can derive that

$$\epsilon(t) = [0, \tilde{\theta}_2^T(t)\eta_2(t), \tilde{\theta}_3^T(t)\eta_3(t), \ldots, \tilde{\theta}_M^T(t)\eta_M(t)]^T + D_s\, S\tilde{\Theta}^T(t)\zeta(t) + \tilde{\Psi}(t)\xi(t), \quad (29)$$

where $\tilde{\theta}_i(t) = \theta_i(t) - \theta_i^*$, $i = 2, 3, \ldots, M$ and $\tilde{\Psi}(t) = \Psi(t) - \Psi^*$.

Adaptive laws. With the estimation error model (29), we choose the adaptive laws

$$\dot{\theta}_i(t) = -\frac{\Gamma_{\theta i}\epsilon_i(t)\eta_i(t)}{m^2(t)}, i = 2, 3, \ldots, M \quad (30)$$

$$\dot{\Theta}^T(t) = -\frac{D_s\epsilon(t)\zeta^T(t)}{m^2(t)}, \quad \dot{\Psi}(t) = -\frac{\Gamma\epsilon(t)\xi^T(t)}{m^2(t)}, \quad (31)$$

where the signal $\epsilon(t) = [\epsilon_1(t), \epsilon_2(t), \ldots, \epsilon_M(t)]^T$ is computed from (27), $\Gamma_{\theta i} = \Gamma_{\theta i}^T > 0$, $i = 2, 3, \ldots, M$, and $\Gamma = \Gamma^T > 0$ are adaptation gain matrices, and

$$m(t) = (1 + \zeta^T(t)\zeta(t) + \xi^T(t)\xi(t) + \sum_{i=2}^{M}\eta_i^T(t)\eta_i(t))^{1/2}.$$

Stability analysis. The adaptive laws (30)–(31) ensure that $\theta_i(t) \in L^\infty$, $i = 2, 3, \ldots, M$, $\Theta(t) \in L^\infty$, $\Psi(t) \in L^\infty$, $\frac{\epsilon(t)}{m(t)} \in L^2 \cap L^\infty$, $\dot{\theta}_i(t) \in L^2 \cap L^\infty$, $i = 2, 3, \ldots, M$, $\dot{\Theta}(t) \in L^2 \cap L^\infty$, and $\dot{\Psi}(t) \in L^2 \cap L^\infty$. The proof of these results is standard by using the positive definite function

$$V = \frac{1}{2}(\sum_{i=2}^{m}\tilde{\theta}_i^T\Gamma_{\theta i}^{-1}\tilde{\theta}_i + \mathrm{tr}[\tilde{\Psi}^T\Gamma^{-1}\tilde{\Psi}] + \mathrm{tr}[\tilde{\Theta}S\tilde{\Theta}^T]). \quad (32)$$

Based on these properties, we can prove the following desired closed-loop system properties.

Theorem 1: *The multivariable MRAC scheme with the output feedback control law (17) updated by the adaptive laws (30)–(31), when applied to the plant (7), guarantees the closed-loop signal boundedness and asymptotic output tracking:* $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$, *for any initial conditions.*

The first step of the proof of this theorem is to express a filtered version of the plant output $y(t)$ in a feedback framework which has a small gain due to the L^2 properties of $\dot{\Theta}(t), \dot{\theta}_i(t)$, and $\frac{\epsilon(t)}{m(t)}$. This step leads to the closed-loop signal boundedness. The asymptotic tracking property follows from the complete parametrization of the error equation (27), the L^2 properties, and the signal boundedness.

V. AIRCRAFT FLIGHT CONTROL APPLICATION

In this section, we will apply the above MRAC design to a linearized NASA generic transport model (GTM) with damage and actuator failures. Then, it will be applied to the nonlinear GTM to assess the effectiveness of the proposed linearization-based control design.

Failure and damage patterns. In this study, we will consider the following two damage and failure patterns:

- lock-in-place failures of $\delta_{e_{l1}}$, $\delta_{e_{r2}}$, δ_{r_u} occur first, then damage with a loss of the two left elevators happens;
- damage with a loss of the two left elevators happens first, then failures of the actuators $\delta_{e_{r2}}$ and δ_{r_u} happen.

Simulation results for linearized GTM. Applying the adaptive control law $\Delta v_0(t)$ given as (17) to the linearized system (10), we have the following results.

Fig. 1 shows the first damage and failure pattern: after 300 seconds, $\delta_{el1} = 3\text{deg}$, $\delta_{er2} = -6\text{deg}$, and $\delta_{ru} = 4\text{deg}$; then after 600 seconds, the entire two left elevators are lost from the stabilizer. From Fig. 1 , we can see that $\Delta y = [\Delta\theta, \Delta\psi]^T$ (solid) tracks the reference $\Delta y_m = [\Delta\theta_m, \Delta\psi_m]^T$ (dotted) after the damage and the failures occur. The second damage and failure pattern is shown in Fig. 2: after 300 seconds, the two left elevator segments are lost from the stabilizer; then after 600 seconds, the right inside elevator is locked at $\delta_{er2} = -6\text{deg}$, and after 650 seconds, the upper rudder is locked at $\delta_{ru} = 4\text{deg}$. From Fig. 2, we can see that Δy (solid) tracks the reference Δy_m (dotted).

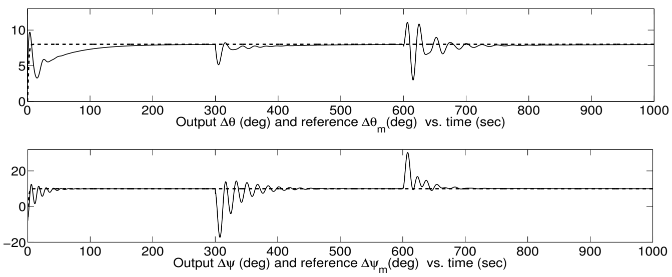

Fig. 1. Aircraft outputs (solid) vs. reference outputs (dotted) (Case I).

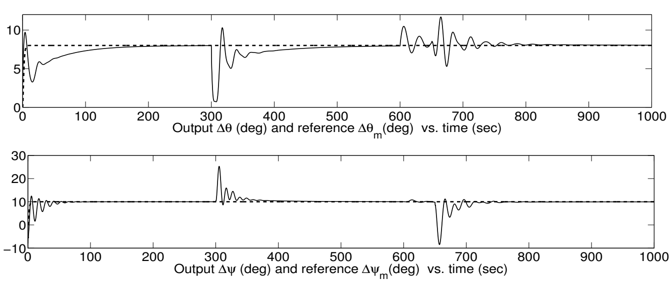

Fig. 2. Aircraft outputs (solid) vs. reference outputs (dotted) (Case II).

Simulation results for GTM. We have obtained the control law $\Delta v = H\Delta v_0(t)$ for the linearized system (3), then we apply $v(t) = \Delta v + u_0$ to the nonlinear GTM (2) to assess the effectiveness of this linearization-based design.

Fig. 3 shows the simulation result for a damage and failure pattern: after 200 seconds, $\delta_{el1} = 2\text{deg}$ and $\delta_{ru} = -2\text{deg}$; then after 500 seconds, the entire left elevators are lost from the stabilizer. From Fig. 3, we can see that the GTM output perturbation $\Delta y(t)$ (solid) tracks the reference output signal $\Delta y_m(t)$ (dotted). Thus, we can conclude that this linearization-based design is applicable for the nonlinear GTM with damage and actuator failures around a small neighborhood of the operating point (x_0, u_0).

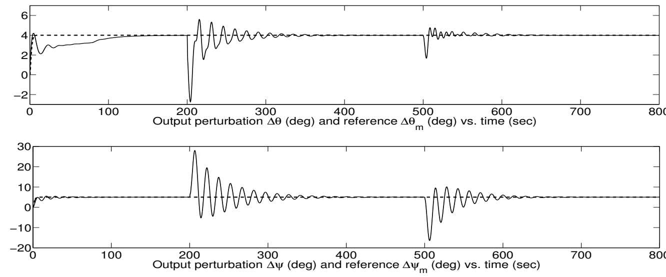

Fig. 3. Aircraft outputs (solid) vs. reference outputs (dotted).

VI. CONCLUSIONS

In this paper, the modeling and control of aircraft under simultaneous failure and damage conditions have been studied. An extensive generic analysis of the linearized aircraft models under failures and damage has been conducted. It has been shown that two essential conditions for multivariable MRAC designs, namely, the interactor matrix and the signs of leading principal minors of the high frequency gain matrix, can remain invariant under realistic failure and damage conditions. A multivariable MRAC scheme has been developed for the aircraft under the hazardous conditions, without the need of explicit detection of actuator failures and structure damage. The stability analysis has shown that the proposed MRAC scheme is capable of ensuring closed-loop stability and asymptotic output tracking in the presence of uncertain failures and damage. Simulation studies of the linearized GTM and the nonlinear GTM have been conducted to show the effectiveness of the proposed MRAC scheme. Further research of this topic includes the multivariable MRAC design when the infinity zero structure or the control direction is altered by the damage.

REFERENCES

[1] F. Ahmed-Zaid, P. Ioannou, K. Gousman, and R. Rooney, "Accommodation of failures in the F-16 aircraft using adaptive control," *IEEE Control Systems Magazine*, vol. 11, no. 1, pp. 73–78, 1991.

[2] M. Bodson and J. E. Groszkiewicz, "Multivariable adaptive algorithms for reconfigurable flight control," *IEEE Transactions on Control Systems Technology*, vol. 5, no. 2, pp. 217–229, 1997.

[3] J. D. Boskovic and R. K. Mehra. Stable multiple model adaptive flight control for accommodation of a large class of control effector failures. In *Proceedings of the 1999 American Control Conference*, pages 1920–1924, 1999.

[4] Nguyen, N., Krishnakumar, K., Kaneshige, J., and Nespeca, P., "Flight Dynamics and Hybrid Adaptive Control of Damaged Aircraft," *Journal of Guidance, Control, and Dynamics*, Vol. 31, No. 3, 2008, pp. 751–764.

[5] Bacon, B. J. and Gregory, I. M., "General equations of motion for a damaged asymmetric aircraft," *Proceedings of 2007 AIAA GNC Conference*, No. AIAA-2007-6306, Hilton Head, SC, August 2007.

[6] Liu, Y., G. Tao, and S. M. Joshi, "Modeling and model reference adaptive control of aircraft with asymmetric damages," *Journal of Guidance, Control, and Dynamics*, vol. 33, no. 5, 2010.

[7] Stengel, R.F., *Flight Dynamics*, Princeton University Press, 2004.

[8] Tang, X., G. Tao, and S. M. Joshi, "Adaptive actuator failure compensation for nonlinear MIMO systems with an aircraft control application," *Automatica*, vol. 43, no. 11, 2007.

[9] Tao, G., *Adaptive Control Design and Analysis*, John Wiley and Sons, New York, 2003.

[10] G. Tao, S. Chen, X. Tang, and S. M. Joshi. *Adaptive Control of Systems with Actuator Failures*. Springer-Verlag, London, 2004.