# Adaptive NN Control of Uncertain Nonaffine Pure-Feedback Systems with Unknown Time-Delay

Xiaoqiang Li, Dan Wang, Tieshan Li, Zhouhua Peng, Gang Sun and Ning Wang

*Abstract*—Adaptive dynamic surface control is presented for a class of nonaffine pure-feedback systems with unknown time-delay using neural networks. The problem of "explosion of complexity" in the traditional backstepping algorithm is avoided using dynamic surface control (DSC). The effects of unknown time-delays are eliminated by using appropriate Lyapunov-Krasovskii functionals in the design procedure. The proposed control scheme guarantees that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded. A simulation example is presented to demonstrate the method.

## I. INTRODUCTION

In the past decades, backstepping [1] based adaptive control has been extensively investigated for uncertain nonlinear systems by using universal function approximators such as neural networks or fuzzy logic systems to approximate the unknown nonlinearities. Many remarkable results have been obtained in those domains [2-4]. In [1], adaptive backstepping, as a breakthrough in nonlinear control area, was introduced. In the paper, Kanellakopoulos et al. proposed stable controllers for strict-feedback systems and pure-feedback systems. After that, adaptive backstepping approach, a recursive design procedure, has been found to be particularly useful for nonlinear systems with trianguiar structures. Representative work can be found in, to just name a few, [5-7]. In particular, in [5], Zhang, Ge, and Hang gave a controller for strict-feedback nonlinear systems using backstepping design and the proposed controller guarantees the uniform ultimate boundedness of the closed-loop adaptive systems. In [6], Gong and Yao gave a neural network based adaptive robust control design scheme for semi-strict feedback nonlinear systems. Ge and Wang presented a semiglobal uniform ultimate bounded controller for the uncertain nonlinear pure-feedback systems using backstepping design technical in [7].

The backstepping technique has become one of the most popular design methods for a class of nonlinear systems. However, a drawback with the backstepping technique is the problem of "explosion of complexity" [8-12]. In the design procedure of the controller using the backstepping technique, certain nonlinear functions such as virtual controls are differentiated repeatedly. The increasing of the order nof the systems will result in the increasing of the complexity of the controller drastically. In [8], a dynamic surface control (DSC) technique was proposed to overcome this problem by introducing a first-order filtering of the synthetic input at each step of the traditional backstepping approach. DSC for the tracking problem of non-Lipschitz systems was considered in [9]. In [10], a point controller (i. e., the reference input is a constant) was designed using DSC and it drives the system to the semi-global exponential stability. The design procedure of the asymptotical tracking controller for a class of strict-feedback nonlinear systems was given in [11] and the controller guarantees that the solution of the closedloop is uniformly ultimately bounded. After this work, DSC technique has been widely used in solving the difficulty of "explosion of complexity" in the controller design of all kinds of uncertainty systems with cascade structures such as strict-feedback system [12-14], pure-feedback system [15-17] and so on [18-23].

One of the challenging problems in control of nonlinear systems is the nonlinear systems with time-delay. Time-delay is often encountered in various systems, such as in recycled reactors, recycled storage tanks, nuclear reactors, rolling mills and chemical processes, etc [24]. The existence of time-delays can destroy the stability or debase the performance of control systems [25]. Therefore, the stability analysis and controller design of time-delay systems are very important both in theory and in practice. However, the control problem and the stable analysis of the systems with time-delay are difficult. The Lyapunov-Krasovskii method [12,13,26,29] and the Lyapuov-Razumikhin method [27] are widely employed in the controller design of time-delay systems.

Pure-feedback system represents a class of lowertriangular nonlinear systems which has a more representative form than the strict-feedback systems [28]. In the past decade, the control problem of various pure-feedback systems were investigated such as parameter-pure-feedback[1] in which the parameters are uncertain, pure-feedback with uncertain functions [3,16], uncertain nonaffine pure-feedback systems [7,17,28], uncertain nonaffine pure-feedback systems with unknown dead zone [15], with unknown timedelay [29] and with hysteresis input [30].

In this paper, neural network based adaptive control is investigated for a class of nonaffine pure-feedback nonlinear systems with time-delay by combining dynamic surface

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control with backstepping technique. The dynamic surface control(DSC) technique is used to overcome the problem of "explosion of complexity" in the traditional backstepping algorithm. The given appropriate Lyapunov-Krasovskii function eliminates the effects of the unknown time-delay in the system. The analysis based on the Lyapunov stability theorem shows that under the appropriate assumptions, the solution of the closed-loop system is globally uniformly ultimately bounded. In addition, the output of the system is proven to converge to a small neighborhood of the origin.

The rest of the paper is organized as follows: The problem formulation and some preliminary results are presented in section 2. Section 3 gives the adaptive dynamic surface control by using backstepping technique and Nussbaum control gain technique for discussional systems of our work. The stability of the closed-loop system consisting of discussional systems, control law and update laws is analyzed in section 4. An example is given in Section 5 to illustrate the effectiveness of our method. Finally, section 6 concludes this paper.

### **II. PROBLEM FORMULATIONS AND PRELIMINARIES**

We consider a single-input single-output pure-feedback nonlinear system with unknown time -delay described by

$$\begin{cases} \dot{x}_{i} = f_{i}(\bar{x}_{i}, x_{i+1}) + h_{i}(\bar{x}_{i}(t - \tau_{i})), 1 \leq i \leq n - 1\\ \dot{x}_{n} = f_{n}(\bar{x}_{n}, u) + h_{n}(\bar{x}_{n}(t - \tau_{n}))\\ y = x_{1}. \end{cases}$$
(2.1)

where  $\bar{x}_i = [x_1, x_2, \dots, x_i] \in R^i, i = 1, \dots, n, u \in R, y \in R$  are state variables, system input and output, respectively;  $f_i(\cdot), h_i(\cdot)$   $(i = 1, \dots, n)$  are unknown smooth functions;  $\tau_i(i = 1, \dots, n)$  is the unknown time-delay of state.

**Remark 1:** The structure of the discussional system is different from the structure of the discussional system in [29]. The control problem for nonaffine pure-feedback systems free of time-delay has been solved by [1,3,7,16,28].

Our objective is to design a robust adaptive controller for (2.1) such that the closed-loop system is stable and the output y(t) of the system tracks the reference signal  $y_r(t)$ .

According to the mean value theorem[31], function  $f_i(\cdot)$  $(i = 1, \dots, n)$  in (2.1) can be rewritten as:

$$f_i(\bar{x}_i, x_{i+1}) = f_i(\bar{x}_i, x_{i+1}^*) + g_{\lambda_i}(x_{i+1} - x_{i+1}^*)$$
 (2.2)

where  $g_{\lambda_i} = g_i(\bar{x}_i, \lambda_i x_{i+1} + (1 - \lambda_i) x_{i+1}^*) = \frac{\partial f_i(\bar{x}_i, x_{i+1})}{\partial x_{i+1}}, 0 < \lambda_i < 1 \ (1 \le i \le n) \text{ and } x_{n+1} = u$ and  $x_{i+1}^* \in R$ . By choosing  $x_{i+1}^* = 0$  [21-23], (2.2) can be expressed as  $f_i(\bar{x}_i, x_{i+1}) = f_i(\bar{x}_i, 0) + g_{\lambda_i} x_{i+1}$  and  $g_{\lambda_i} = g_i(\bar{x}_i, \lambda_i x_{i+1})$ .

Then the system (2.1) can be transform into the form as follows:

$$\begin{cases} \dot{x}_{i} = f_{i}(\bar{x}_{i}, 0) + g_{\lambda_{i}}x_{i+1} + h_{i}(\bar{x}_{i}(t-\tau_{i})), 1 \le i \le n-1 \\ \dot{x}_{n} = f_{n}(\bar{x}_{n}, 0) + g_{\lambda_{i}}u + h_{n}(\bar{x}_{n}(t-\tau_{n})) \\ y = x_{1}. \end{cases}$$
(2.3)

For the design of the controller, we give some assumptions as follows:

**Assumption 1:** The state  $\bar{x}_n$  of the system (2.1) is available for feedback.

Assumption 2: There exist known constants  $0 < g_{min} \le g_{max}$  such that  $g_{min} \le |g_{\lambda_i}| \le g_{max}$ .

Assumption 3: There exist known function  $h_{im}(\cdot)$  and known constant  $\tau_m$  such that  $h_i(\cdot) \leq h_{im}(\cdot), \tau_i \leq \tau_m$  $(1 \leq i \leq n)$ , that is, function  $h_i(\cdot)$   $(1 \leq i \leq n)$  and  $\tau_i$ are bounded.

**Assumption 4:** The reference signal  $y_r(t)$  is a sufficiently smooth function of t.  $y_r, \dot{y}_r$  and  $y_r^{(2)}$  are bounded.

Before introducing our control design method, we first recall the technique of Nussbaum control gain [32] and the approximation property of the RBF NN [11].

An even differentiable function is called Nussbaum-type function if it has the following properties:

$$\lim_{s \to +\infty} \sup \frac{1}{s} \int_0^s N(\zeta) d\zeta = +\infty$$
 (2.4)

$$\lim_{s \to +\infty} \inf \frac{1}{s} \int_0^s N(\zeta) d\zeta = -\infty$$
 (2.5)

The continuous functions  $\zeta^2 \cos \zeta$ ,  $e^{\zeta^2} \cos(\frac{\pi}{2})$  have those two properties and they are Nussbaum functions. In this paper, Nussbaum function  $\zeta^2 \cos \zeta$  is exploited.

The following lemma regarding the property of Nussbaum function is used in the controller design and stability analysis in next section.

**Lemma 1 [32] :** Let V and  $\kappa$  be smooth functions defined on  $[0, f_f)$  with  $V(t) \ge 0, \forall t \in [0, t_f)$  and  $N(\kappa)$  be an even smooth Nussbaum-type function. The following inequality holds:

$$0 \le V(t) \le c_0 + e^{-c_1 t} \int_0^t (G(x(\tau))N(\kappa) + 1)\dot{\kappa}e^{c_1 \tau} d\tau,$$
(2.6)

where  $c_1 > 0$  and  $t \in [0, t_f)$ , G(x(t)) is a time-varying parameter which takes values in the unknown closed intervals  $I := [l^-, l^+]$  with  $0 \notin I$ , and  $c_0$  represents some suitable constant, the  $V(t), \kappa(t)$  and  $\int_0^t g(x(\tau))N(\kappa)\dot{\kappa}d\tau$  must be bounded on  $[0, t_f)$ .

The RBF neural networks take the form  $\theta^T \xi$  where  $\theta = [\theta_1, \theta_2, \dots, \theta_N]^T$  is called weight vector,  $\xi$  is a vector valued function defined in  $\mathbb{R}^N$ . Denote the components of  $\xi$  by  $\rho_i$   $(i = 1, \dots, N)$ , then  $\rho_i(x)$   $(i = 1, \dots, N)$  is called a basis function. A commonly used basis function is the so-called Gaussian function of the following form:

$$\rho_i(x) = \frac{1}{\sqrt{2\pi\sigma}} \exp(-\frac{\|x-\zeta_j\|^2}{2\sigma^2}), \sigma \ge 0, j = 1, \cdots, N.$$
(2.7)

where  $\zeta_j$   $(j = 1, \dots, N) \in \mathbb{R}^n$  are the constant vectors called the center of the basis function, and  $\sigma$  is a real number called the width of the basis function. According to the approxiamtion property of the RBF network, given a continuous real valued function  $f: \Omega \mapsto \mathbb{R}$  with  $\Omega \in \mathbb{R}^n$  a compact set, and any  $\delta_m > 0$ , by appropriately choosing  $\sigma$ and  $\zeta_j$   $(j = 1, \dots, N) \in \mathbb{R}^n$  for some sufficiently large integer N, there exists an ideal  $\theta^* = [\theta_1^*, \theta_2^*, \dots, \theta_N^*]^T$ such that the RBF network  $\theta^{*T}\xi$  can approximate the given function  $f(x) : \mathbb{R}^m \mapsto \mathbb{R}$  with the approximation error bounded by  $\delta_m$ , i.e.,  $f(x) = \theta^{*T}\xi(x) + \delta^*$  with  $|\delta^*| \leq \delta_m$ , where  $\delta^*$  represents the reconstruction error and  $x \in \Omega_x$ . The ideal weigh vector  $\theta^*$  is an artificial quantity required for analytical purposes.  $\theta^*$  is defined as the value of  $\theta$  that minimizes  $|\delta|$  for all  $x \in \Omega_x \subset R^m$ , i.e.,

$$\theta^* \cong \arg\min \sup_{x \in \mathbb{R}^N} |f(x) - \theta^T \xi(x)|.$$
(2.8)

Assumption 5:  $\theta^*$  is bounded parameter, that is, there exists a constant vector  $\theta_M$  such that  $|\theta^*| \leq \theta_M$ .

## III. ROBUST ADAPTIVE CONTROL DESIGN

In this section, we will incorporate the DSC technique into a neural network based adaptive control design scheme for the *nth*-order system described by (2.3). Similar to traditional backstepping, the design of adaptive control laws is based on the adjusting of the errors  $s_1 = x_1 - y_r, \dots, s_i =$  $x_i - z_i$   $(i = 2, \dots, n)$  where  $z_i$  is the output of a first-order filter with virtual controller  $\alpha_{i-1}$  as the input. Finally, an overall control law u is constructed at step n.

In the controller design procedure, we define  $\theta_i = \hat{\theta}_i - \theta_i^*$  $(i = 1, \dots, n)$ , where  $\hat{\theta}_i$  is the estimation of  $\theta_i^*$ ;  $\Gamma_i$   $(i = 1, \dots, n)$  is a constant matrix satisfying  $\Gamma_i = \Gamma_i^T > 0$ ;  $k_{i1}, k_{i2}(i = 1, \dots, n)$ ,  $\gamma$  are positive constants;  $\lambda_{max}(A)$  denote the largest eigenvalue of a square matrix A; Giving any a compact set  $\Omega_i$ , the set  $\Omega_{s_i}(i = 1, \dots, n)$  is defined as:  $\Omega_{s_i} = \{s_i \mid \mid s_i \mid \geq c_{s_i}, s_i + y_r \in \Omega_i\}$  and  $\Omega_i - \Omega_{s_i} = \{s_i \mid s_i + y_r \in \Omega_{s_i}\}$ ; Function  $q(s_i, c_{s_i})(1 \leq i \leq n)$  is defined as follows and this function will be used in the control design later:

$$q(s_i, c_{s_i}) = \begin{cases} 1, s_i \in \Omega_{s_i} \\ 0, s_i \in \Omega_i - \Omega_{s_i}. \end{cases}$$
(3.1)

where  $c_{s_i}$  is a positive design constant that can be chosen arbitrarily small. Next is the controller design procedure.

Step *i*: At this step, we consider the first equation in (2.3). We define  $s_i = x_i - z_i$  which is called the error surface with  $z_i$  as the desired trajectory and  $z_1 = y_r$ . Then,

$$\dot{s}_i = f_i(\bar{x}_i, 0) + g_{\lambda_i} x_{i+1} + h_i(\bar{x}_i(t - \tau_i)) - \dot{z}_i \qquad (3.2)$$

In order to design the control law, we define a smooth scalar function as follows:

$$V_i = \frac{1}{2}s_i^2 + \frac{1}{2}\tilde{\theta}_i^T \Gamma_i^{-1}\tilde{\theta}_i + \int_{t-\tau_i}^t h_{im}^2(\bar{x}_i(\sigma))d\sigma \qquad (3.3)$$

Differentiating  $V_i$  with respect to time t, we obtain:

$$\dot{V}_{i} = s_{i}\dot{s}_{i} + \tilde{\theta}_{i}^{T}\Gamma_{i}^{-1}\hat{\theta}_{i} + h_{im}^{2}(\bar{x}_{i}(t)) - h_{im}^{2}(\bar{x}_{i}(t-\tau_{i})) \\
= s_{i}[f_{i}(\bar{x}_{i},0) + g_{\lambda_{i}}x_{i+1} + h_{i}(\bar{x}_{i}(t-\tau_{i}))] \\
+ \tilde{\theta}_{i}^{T}\Gamma_{i}^{-1}\dot{\hat{\theta}}_{i} + h_{im}^{2}(\bar{x}_{i}(t)) - h_{im}^{2}(\bar{x}_{i}(t-\tau_{i})) \\
= s_{i}f_{i}(\bar{x}_{i},0) + s_{i}h_{i}(\bar{x}_{i}(t-\tau_{i})) - s_{i}\dot{z}_{i} + s_{i}g_{\lambda_{i}}x_{i+1} \\
+ \tilde{\theta}_{i}^{T}\Gamma_{i}^{-1}\dot{\hat{\theta}}_{i} + h_{im}^{2}(\bar{x}_{i}(t)) - h_{im}^{2}(\bar{x}_{i}(t-\tau_{i})) \\
\leq s_{i}F_{i} + \frac{s_{i}^{2}}{4} + s_{i}g_{\lambda_{i}}x_{i+1} - s_{i}\dot{z}_{i} + \tilde{\theta}_{i}^{T}\Gamma_{i}^{-1}\dot{\hat{\theta}}_{i} \quad (3.4)$$

where

$$F_i = f_i(\bar{x}_i, 0) + \frac{h_{im}^2(\bar{x}_i(t))}{s_i}.$$
(3.5)

Define a compact set  $\Omega_i \subset R$ , let  $\theta_i^{*T}$  and  $\varepsilon_i^*$  be such that for any  $x_i \in \Omega_i$ 

$$F_i = q(s_i, c_{s_i})(\theta_i^{*T}\xi_i + \varepsilon_i^*)$$
(3.6)

where  $|\varepsilon_i^*| \leq \varepsilon$ .

Note that if  $F_i$  is utilized to construct the controller, controller singularity may occur since  $\frac{h_{im}^2(\bar{x}_i(t))}{s_i}$  is not well-defined at  $s_i = 0$ . Therefore, care must be taken to guarantee the boundedness of the control as discussed in [17]. So, we choose the virtual control  $\alpha_{i+1}$  as follows:

$$\alpha_{i+1} = q(s_i, c_{s_i}) N(\kappa_i) [K_i s_i + \hat{\theta}_i^T \xi_i - \dot{z}_i]$$
(3.7)

with

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$$\dot{x}_i = q(s_i, c_{s_i})(K_i s_i^2 + \hat{\theta}_i^T \xi_i s_i - \dot{z}_i s_i)$$
 (3.8)

$$\hat{\theta}_i = q(s_i, c_{s_i}) \Gamma_i(\xi_i s_i - \gamma \hat{\theta}_i)$$
(3.9)

Define a new state variable  $z_{i+1}$  and let  $\alpha_{i+1}$  pass through a first-order filter with time constant  $\beta_{i+1}$  to obtain  $z_{i+1}$ :

$$\beta_{i+1}\dot{z}_{i+1} + z_{i+1} = \alpha_{i+1}, z_{i+1}(0) = \alpha_{i+1}(0)$$
(3.10)

Step n: At this step, we consider the *nth* equation in (2.3), i.e.,

$$\dot{x}_n = f_n(\bar{x}_n, 0) + g_{\lambda_n} u + h_n(\bar{x}_n(t - \tau_n))$$
 (3.11)

We define  $s_n = x_n - z_n$ , then

$$\dot{x}_n = f_n(\bar{x}_n, 0) + g_{\lambda_n} u + h_n(\bar{x}_n(t - \tau_n)) - \dot{z}_n$$
 (3.12)

In order to design the control law, define a smooth scalar function as follows:

$$V_n = \frac{1}{2}s_n^2 + \frac{1}{2}\tilde{\theta}_n^T \Gamma_n^{-1}\tilde{\theta}_n + \int_{t-\tau_n}^t h_{nm}^2(\bar{x}_n(\sigma))d\sigma \quad (3.13)$$

Differentiating  $V_n$  with respect to time t, we obtain

$$\begin{split} \dot{V}_{n} &= s_{n}\dot{s}_{n} + \tilde{\theta}_{n}^{T}\Gamma_{n}^{-1}\hat{\theta}_{n} \\ &+ h_{nm}^{2}(\bar{x}_{n}(t)) - h_{nm}^{2}(\bar{x}_{n}(t-\tau_{n})) \\ &= s_{n}[f_{n}(\bar{x}_{n},0) + g_{\lambda_{n}}u + h_{n}(\bar{x}_{n}(t-\tau_{n}))] \\ &+ \tilde{\theta}_{n}^{T}\Gamma_{n}^{-1}\dot{\hat{\theta}}_{n} + h_{nm}^{2}(\bar{x}_{n}(t)) - h_{nm}^{2}(\bar{x}_{n}(t-\tau_{n})) \\ &= s_{n}f_{n}(\bar{x}_{n},0) + s_{n}h_{n}(\bar{x}_{n}(t-\tau_{n})) - s_{n}\dot{z}_{n} \\ &+ s_{n}g_{\lambda_{n}}u + \tilde{\theta}_{n}^{T}\Gamma_{n}^{-1}\dot{\hat{\theta}}_{n} \\ &+ h_{nm}^{2}(\bar{x}_{n}(t)) - h_{nm}^{2}(\bar{x}_{n}(t-\tau_{n})) \\ &\leq s_{n}F_{n} + \frac{s_{n}^{2}}{4} + s_{n}g_{\lambda_{n}}u - s_{n}\dot{z}_{n} + \tilde{\theta}_{n}^{T}\Gamma_{n}^{-1}\dot{\hat{\theta}}_{n} (3.14) \end{split}$$

where

$$F_n = f_n(\bar{x}_n, 0) + \frac{h_{nm}^2(\bar{x}_n(t))}{s_n}.$$
 (3.15)

Define a compact set  $\Omega_n \subset R^n$ , let  $\theta_n^*$  and  $\varepsilon_n^*$  be such that for any  $\bar{x}_n \in \Omega_n$ 

$$F_n = q(s_i, c_{s_i})(\theta_n^{*T}\xi_n + \varepsilon_n^*)$$
(3.16)

where  $|\varepsilon_n^*| \leq \varepsilon$ .

Similarly, we choose the control input u as follows:

$$u = q(s_n, c_{s_n}) N(\kappa_n) [K_n s_n + \hat{\theta}_n^T \xi_n - \dot{z}_n]$$
(3.17)

with

$$\dot{\kappa}_n = q(s_n, c_{s_n})(K_n s_n^2 + \hat{\theta}_n^T \xi_n s_n - \dot{z}_n s_n)$$
(3.18)

$$\dot{\hat{\theta}}_n = q(s_n, c_{s_n})\Gamma_n(\xi_n s_n - \gamma \hat{\theta}_n)$$
(3.19)

#### IV. STABILITY ANALYSIS

In this section we show that the control law and update law introduced in above design procedure guarantee the uniform ultimate boundedness of the solution of the closedloop system.

**Theorem 1:** Under assumptions 1-5, consider the closedloop system consisting of the plant, the proposed robust adaptive state feedback control law (3.17) and the adaptive laws (3.9) and (3.19), it is guaranteed that all of the signal of the closed-loop system are semi-globally uniformly ultimately bounded and the output y(t) of the given system (2.1) converges to the reference signal  $y_r(t)$ .

The proof is omitted due to the space limit.

## V. SIMULATION EXAMPLE

To illustrate and clarify the proposed design procedure, we apply the adaptive neural network controller developed in section 3 to control a nonlinear system. Consider a nonlinear system

$$\dot{x}_1 = f_1(x_1, x_2) + h_1(x_1(t - \tau_1)) \dot{x}_2 = f_2(x_1, x_2, u) + h_2(x_1(t - \tau_2), x_2(t - \tau_2)) y = x_1$$

$$(4.1)$$

For the purpose of simulation, following plant dynamics are used:  $f_1(\cdot) = (1+x_1^2)x_2+x_1e^{-x_2}$ ,  $h_1(\cdot) = 2x_1^2$ ,  $f_2(\cdot) = cos(x_1x_2) + x_1x_2^2 + u + sin(u)$ ,  $h_2(\cdot) = sin(x_1x_2)$ ,  $\tau_1 = \tau_2 = 1$ . And we choose  $h_1m(\cdot) = 2x_1^2$ ,  $h_2m(\cdot) = 1$  and  $\tau_m = 2$  for satisfying assumption 3. The initial values of states are all 0.4. The initial value of Parameter  $\theta_1$  is set to 0.1 and  $\theta_2$  is set to 0.4. The initial value of Parameter  $\kappa_1$  is set to 1.5 and  $\kappa_2$  is set to 0.4. The initial value of Parameter  $\kappa_1$  is set to 1.5 and  $\kappa_2$  is set to 0.4. The initial value of Parameter  $r_1$  is set to 1.5 and  $\kappa_2$  is set to 0.4. The initial value of Parameter  $\kappa_1$  is set to 0.4. The initial value of Parameter  $\kappa_1$  is set to 1.5 and  $\kappa_2$  is set to 0.4. The initial value of Parameter  $\kappa_1$  is set to 0.4. The output parameter chosen for simulation are :  $K_1(t) = 20$ ,  $K_2(t) = 300$ ,  $\Gamma_1 = \Gamma_2 = 0.0002I$ ,  $\gamma = 0.001$ ,  $c_{s_i} = 0.001$ .

Simulation results are shown in Figures 1-6. Figure 1 shows that the given control input is bounded. Figure 2 gives the output of the closed-loop system and the reference signal. The output of the closed-loop system tracks the reference input fairly well. After a short transient process the output tracks the reference input at a high precision. Figure 3 is the tracking error and the good performance is shown again. The amplitude of the tracking error is about 0.04. Figure 4 shows that the error  $s_2$  is bounded. Figure 5-6 shows that the parameter  $\zeta_i (i = 1, 2)$  is bounded. The simulation results show that the designed controller is workable.

# VI. CONCLUSIONS

Adaptive neural control has been proposed for a class of unknown SISO nonaffine pure-feedback systems with

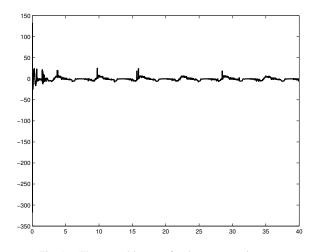


Fig. 1. The control input u for the augmented system

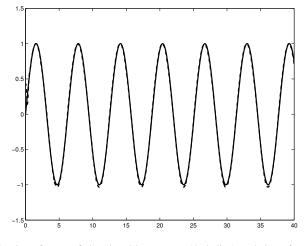


Fig. 2. Output of the closed-loop system(dash line) and the reference signal(solid line)

unknown time-delay. The DSC technique was used in the design procedure for avoiding the problem of explosion of complexity. By using the appropriate Lyapunov-Krasovskii function, the effects of the unknown time-delays are eliminated. Since that the transformed system contain the unknown virtual control coefficients, the technique of Nussbaum gain function control scheme is adopted. The proposed control scheme guarantees that all the signals in the closed-loop system are semi-globally uniformly ultimately bounded. Simulation is given to show the effectiveness of the presented method.

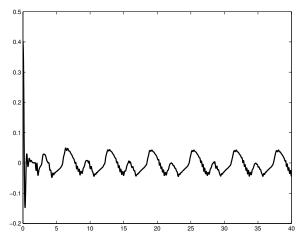


Fig. 3. Tracking error

0.8

0.6

0.4

0.2

-0.2

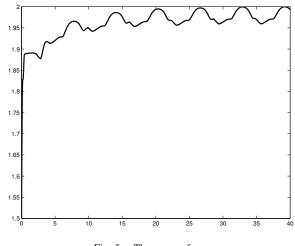


Fig. 5. The curve of  $\kappa_1$ 

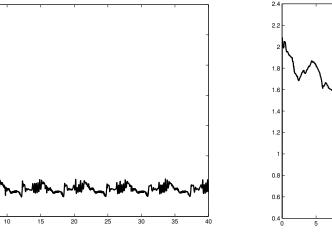


Fig. 4. The curve of  $s_2$ 

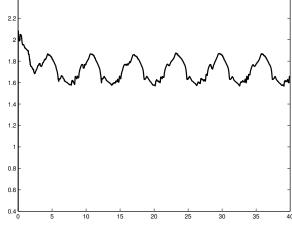


Fig. 6. The curve of the derivative of  $\kappa_2$ 

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