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An Improved Approach to Fuzzy Model Construction and Servo Control with Constraints based on Error Dynamics

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Abstract— **This paper presents an improved approach to servo control for nonlinear systems with constraints on inputs and states using the fuzzy model-based control approach. In our previous paper, servo control based on error dynamics using error vector and input difference was proposed. Although the approach can achieve servo control with constraints on both of inputs and states, error dynamics with time-varying extra terms cannot be applied. In this paper, time-varying cancellation input is newly introduced. By using the cancellation input, servo control for error dynamics with time varying extra terms is achieved. We derive servo controller design conditions and constraint conditions on inputs and states in the form of LMI. Design examples illustrate the utility of this approach.**

I. INTRODUCTION

Recently, a lot of researches on fuzzy model-based control has been reported [1]–[4]. Most of them deal with Takagi-Sugeno (T-S) fuzzy model [5] and LMI-based designs[6]. By employing the T-S fuzzy model, which utilizes local linear system description for each rule, we can devise a control methodology to fully take advantages of linear control theory. However, most of literature on fuzzy model based control have mainly dealt with the regulation problem to discuss stability analysis or fuzzy controller design satisfying a variety of control performances and constraint conditions [7].

In our previous paper [8], servo and model following control for a class of nonlinear systems using the fuzzy model-based control approach was discussed. In order to achieve servo control, firstly, augmented T-S fuzzy system was constructed by differentiating the original nonlinear dynamics with respect to time and then the dynamic fuzzy servo controller which can make outputs of the nonlinear systems converge to target points was designed. But, this approach cannot deal with nonlinear systems with constraint on control inputs since time derivatives of control inputs were used instead of control inputs in the augmented T-S fuzzy system. In [9], input difference between current input and constant steady input was introduced and servo control with input constraint was achieved. But, this approach cannot be applied to nonlinear systems with constraint on states. To overcome these problems, servo control for nonlinear systems with constraints on both of inputs and states was proposed in [10]. In this approach, original nonlinear dynamics is converted into error dynamics by using error vector between current state and target point and input difference between current input and steady input. Constant terms in the error dynamics, which are caused by constant target point, are cancelled by steady input. Then, T-S fuzzy model is constructed from error dynamics. Although this approach employs constant steady input to cancel extra constant terms, unfortunately, extra terms may contain not only constant ones but also time-varying ones. In such a case, this approach cannot be applied.

This paper presents an improved approach to servo control for nonlinear systems with constraints using the fuzzy model-based control approach. Time-varying cancellation input is newly introduced to construct a different input difference from [9], [10]. By using the cancellation input, servo control for error dynamics with time-varying extra terms is achieved. We derive servo controller design conditions and constraint conditions on inputs and states in the form of LMI. Design examples illustrate the utility of this approach.

II. LINEAR SERVO CONTROL

In [11], servo control utilizing time-derivative of linear system is discussed. In this section, we explain the servo control based on the error dynamics for continuous-time linear systems. Consider the following linear system.

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{A}\boldsymbol{x}(t) + \boldsymbol{B}\boldsymbol{u}(t) \tag{1}$$

where $\boldsymbol{x}(t) = [x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T$ is the state vector and $\boldsymbol{u}(t) = [u_1(t)\ u_2(t)\ \cdots\ u_m(t)]^T$ is the input vector. For the above linear system, we consider the servo control problem, that is, the control problem to make the state $\boldsymbol{x}(t)$ converge to the certain target point $\boldsymbol{r}$, where $\boldsymbol{r}$ is the constant vector, $\boldsymbol{r} = [r_1, r_2, \cdots, r_n]^T$. We assume that all states are measurable.

Firstly, we define the error vector $\boldsymbol{e}(t)$ and its time derivative as follows:

$$\boldsymbol{e}(t) = \boldsymbol{x}(t) - \boldsymbol{r} \tag{2}$$

$$\dot{\boldsymbol{e}}(t) = \dot{\boldsymbol{x}}(t) \tag{3}$$

Next, we introduce the following input difference between current input $\boldsymbol{u}(t)$ and steady input $\boldsymbol{u}_f$ which is a bias input to keep the state $\boldsymbol{x}(t)$ at a target point $\boldsymbol{r}$ after making the state converge to the target point.

$$\boldsymbol{e}_u(t) = \boldsymbol{u}(t) - \boldsymbol{u}_f \tag{4}$$

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where $\boldsymbol{u}_f = [u_1(\infty),\ u_2(\infty),\ \cdots u_m(\infty)]^T$. Then, by substituting (2), (3) and (4) into (1), we can obtain the following error dynamics.

$$\begin{aligned}\dot{\boldsymbol{e}}(t) &= \boldsymbol{A}(\boldsymbol{e}(t)+\boldsymbol{r})+\boldsymbol{B}(\boldsymbol{e}_u(t)+\boldsymbol{u}_f) \\ &= \boldsymbol{A}\boldsymbol{e}(t)+\boldsymbol{B}\boldsymbol{e}_u(t)+\boldsymbol{A}\boldsymbol{r}+\boldsymbol{B}\boldsymbol{u}_f \qquad (5)\end{aligned}$$

Note that $\boldsymbol{A}\boldsymbol{r}+\boldsymbol{B}\boldsymbol{u}_f=\boldsymbol{0}$ since $\boldsymbol{e}(t)$ and $\boldsymbol{e}_u(t)$ go to $\boldsymbol{0}$ if servo control for the linear system (1) can be achieved. In other words, we have to select $\boldsymbol{u}_f$ such that $\boldsymbol{A}\boldsymbol{r}+\boldsymbol{B}\boldsymbol{u}_f=\boldsymbol{0}$ with the preassigned $\boldsymbol{r}$ is satisfied. Therefore, (5) can be rewritten as

$$\dot{\boldsymbol{e}}(t) = \boldsymbol{A}\boldsymbol{e}(t)+\boldsymbol{B}\boldsymbol{e}_u(t) \qquad (6)$$

Finally, we design the following controller to stabilize the error dynamics (6).

$$\boldsymbol{e}_u(t) = -\boldsymbol{K}\boldsymbol{e}(t) \qquad (7)$$

where $\boldsymbol{K}$ is a feedback gain. By substituting (7) into (6), we can obtain the following linear control system.

$$\dot{\boldsymbol{e}}(t) = (\boldsymbol{A}-\boldsymbol{B}\boldsymbol{K})\,\boldsymbol{e}(t) \qquad (8)$$

The feedback gain $\boldsymbol{K}$ is determined by solving Theorem 1.

Theorem 1: If there exist positive definite matrix $\boldsymbol{X}$ and matrix $\boldsymbol{M}$ satisfying (9) and (10), then the error system (6) can be stabilized by the controller (7).

$$\boldsymbol{X} > \boldsymbol{0}, \qquad (9)$$

$$\boldsymbol{A}\boldsymbol{X}+\boldsymbol{X}\boldsymbol{A}^T-\boldsymbol{B}\boldsymbol{M}-\boldsymbol{M}^T\boldsymbol{B}^T < \boldsymbol{0}, \qquad (10)$$

where $\boldsymbol{K}=\boldsymbol{M}\boldsymbol{X}^{-1}$.

The controller (7) can be converted into the following equation.

$$\boldsymbol{u}(t) = -\boldsymbol{K}(\boldsymbol{x}(t)-\boldsymbol{r})+\boldsymbol{u}_f \qquad (11)$$

By using the designed controller, we can make the state $\boldsymbol{x}(t)$ of the linear system (1) converge to the target point $\boldsymbol{r}$.

III. Nonlinear Servo Control based on Error Dynamics

In this section, we propose the servo control based on error dynamics for continuous-time nonlinear systems with constraints on inputs and states.

Consider the following continuous-time nonlinear system.

$$\dot{\boldsymbol{x}}(t) = \boldsymbol{f}(\boldsymbol{x}(t))+\boldsymbol{g}(\boldsymbol{x}(t))\boldsymbol{u}(t) \qquad (12)$$

where $\boldsymbol{x}(t)=[x_1(t)\ x_2(t)\ \cdots\ x_n(t)]^T$ is the state vector and $\boldsymbol{u}(t)=[u_1(t)\ u_2(t)\cdots\ u_m(t)]^T$ is the input vector. We assume that $\boldsymbol{f}$ and $\boldsymbol{g}$ are known.

A. Fuzzy Model Construction based on Error Dynamics

We recall the error vector $\boldsymbol{e}(t)$ and its time derivative.

$$\boldsymbol{e}(t) = \boldsymbol{x}(t)-\boldsymbol{r} \qquad (13)$$

$$\dot{\boldsymbol{e}}(t) = \dot{\boldsymbol{x}}(t) \qquad (14)$$

Next, we newly define the following input difference between current input $\boldsymbol{u}(t)$ and cancellation input $\boldsymbol{u}_c(t)$.

$$\boldsymbol{e}_u(t) = \boldsymbol{u}(t)-\boldsymbol{u}_c(t) \qquad (15)$$

where $\boldsymbol{u}_c(t)=[u_{c1}(t),\ u_{c2}(t),\ \cdots u_{cm}(t)]^T$. The roll of the cancellation input will be explained later. We assume that the cancellation input remains finite and satisfy the following condition.

$$\boldsymbol{u}_c(t)\rightarrow\boldsymbol{u}_f \quad (t\rightarrow\infty) \qquad (16)$$

where $\boldsymbol{u}_f$ is the steady input, that is, a bias input to keep the state $\boldsymbol{x}(t)$ at a target point $\boldsymbol{r}$ after making the state converges to the target point.

By substituting (13), (14) and (15) into (12), we can obtain the following error dynamics.

$$\begin{aligned}\dot{\boldsymbol{e}}(t) &= \boldsymbol{f}(\boldsymbol{e}(t)+\boldsymbol{r})+\boldsymbol{g}(\boldsymbol{e}(t)+\boldsymbol{r})(\boldsymbol{e}_u(t)+\boldsymbol{u}_c(t)) \\ &= \boldsymbol{F}_1(\boldsymbol{e}(t),\boldsymbol{r})\boldsymbol{e}(t)+\boldsymbol{F}_2(\boldsymbol{e}(t),\boldsymbol{r}) \\ &\quad +\boldsymbol{g}(\boldsymbol{e}(t)+\boldsymbol{r})\boldsymbol{e}_u(t)+\boldsymbol{g}(\boldsymbol{e}(t)+\boldsymbol{r})\boldsymbol{u}_c(t) \qquad (17) \\ &= \boldsymbol{F}_1(\boldsymbol{e}(t),\boldsymbol{r})\boldsymbol{e}(t)+\boldsymbol{F}_2(\boldsymbol{e}(t),\boldsymbol{r}) \\ &\quad +\boldsymbol{g}(\boldsymbol{e}(t)+\boldsymbol{r})\boldsymbol{e}_u(t) \\ &\quad +\boldsymbol{G}_1(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t))\boldsymbol{e}(t)+\boldsymbol{G}_2(\boldsymbol{e}(t),\boldsymbol{r})\boldsymbol{u}_c(t) \qquad (18) \\ &= \hat{\boldsymbol{F}}(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t))\boldsymbol{e}(t)+\boldsymbol{g}(\boldsymbol{e}(t)+\boldsymbol{r})\boldsymbol{e}_u(t) \\ &\quad +\hat{\boldsymbol{H}}(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t)) \qquad (19)\end{aligned}$$

where, for (17),

$$\begin{aligned}\hat{\boldsymbol{F}}(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t))\boldsymbol{e}(t) &= \boldsymbol{F}_1(\boldsymbol{e}(t),\boldsymbol{r})\boldsymbol{e}(t) \\ \hat{\boldsymbol{H}}(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t)) &= \boldsymbol{F}_2(\boldsymbol{e}(t),\boldsymbol{r})+\boldsymbol{g}(\boldsymbol{e}(t)+\boldsymbol{r})\boldsymbol{u}_c(t).\end{aligned}$$

For (18),

$$\begin{aligned}\hat{\boldsymbol{F}}(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t))\boldsymbol{e}(t) &= \boldsymbol{F}_1(\boldsymbol{e}(t),\boldsymbol{r})\boldsymbol{e}(t) \\ &\quad +\boldsymbol{G}_1(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t))\boldsymbol{e}(t) \\ \hat{\boldsymbol{H}}(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t)) &= \boldsymbol{F}_2(\boldsymbol{e}(t),\boldsymbol{r})+\boldsymbol{G}_2(\boldsymbol{e}(t),\boldsymbol{r})\boldsymbol{u}_c(t)\end{aligned}$$

$\hat{\boldsymbol{F}}(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t))\boldsymbol{e}(t)$ and $\boldsymbol{g}(\boldsymbol{e}(t)+\boldsymbol{r})\boldsymbol{e}_u(t)$ are functions which linearly depend on $\boldsymbol{e}(t)$ and $\boldsymbol{e}_u(t)$, respectively. This means that these functions go to $\boldsymbol{0}$ when $\boldsymbol{e}(t)$ and $\boldsymbol{e}_u(t)$ go to $\boldsymbol{0}$. In order to achieve servo control, the following condition has to be satisfied at all times $t\geq 0$.

$$\hat{\boldsymbol{H}}(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t)) = \boldsymbol{0} \qquad (20)$$

The cancellation input $\boldsymbol{u}_c(t)$ is selected such that (16) and (20) with the preassigned $\boldsymbol{r}$ are satisfied.

By using the cancellation input $\boldsymbol{u}_c(t)$ satisfying (20), (19) can be rewritten as

$$\dot{\boldsymbol{e}}(t) = \hat{\boldsymbol{F}}(\boldsymbol{e}(t),\boldsymbol{r},\boldsymbol{u}_c(t))\boldsymbol{e}(t)+\boldsymbol{g}(\boldsymbol{e}(t)+\boldsymbol{r})\boldsymbol{e}_u(t) \qquad (21)$$

By applying sector nonlinearity concept [7] to (21), we can obtain the following T-S fuzzy model.

$$\dot{\boldsymbol{e}}(t) = \sum_{i=1}^{r} h_i(\boldsymbol{e}(t), \boldsymbol{r}, \boldsymbol{u}_c(t)) \left(\hat{\boldsymbol{A}}_i \boldsymbol{e}(t) + \hat{\boldsymbol{B}}_i \boldsymbol{e}_u(t)\right) \quad (22)$$

Example

Consider the following nonlinear system.

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ x_1(t)^2 + x_1(t)u(t) \end{bmatrix} \quad (23)$$

We assume that the target point $\boldsymbol{r} = [r\ 0]^T$. Then, the following error dynamics can be obtained from (13), (14) and (15).

$$\begin{aligned} \begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{bmatrix} &= \begin{bmatrix} e_2(t) \\ (e_1(t)+r)^2 + (e_1(t)+r)(e_u(t)+u_c(t)) \end{bmatrix} \\ &= \begin{bmatrix} e_2(t) \\ e_1^2(t) + 2re_1(t) + r^2 + (e_1(t)+r)e_u(t) \\ \quad +(e_1(t)+r)u_c(t) \end{bmatrix} \end{aligned} \quad (24)$$

For (24), the following cancellation input $u_c(t)$ is selected.

$$u_c(t) = -\frac{r^2}{e_1(t)+r}$$

Note that the assumption $e_1(t) \neq -r$ has to be considered. Then, (24) can be rewritten by

$$\begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{bmatrix} = \begin{bmatrix} e_2(t) \\ e_1^2(t) + 2re_1(t) + (e_1(t)+r)e_u(t) \end{bmatrix}$$

By applying sector nonlinearity concept to the nonlinear terms with the constraint $|e_1(t)| \leq d$, the T-S fuzzy model is constructed as follows:

$$\dot{\boldsymbol{e}}(t) = \sum_{i=1}^{2} h_i(e_1(t)) \left(\hat{\boldsymbol{A}}_i \boldsymbol{e}(t) + \hat{\boldsymbol{B}} e_u(t)\right) \quad (25)$$

where

$$\hat{\boldsymbol{A}}_1 = \begin{bmatrix} 0 & 1 \\ d+2r & 0 \end{bmatrix}, \quad \hat{\boldsymbol{A}}_2 = \begin{bmatrix} 0 & 1 \\ -d+2r & 0 \end{bmatrix},$$
$$\hat{\boldsymbol{B}}_1 = \begin{bmatrix} 0 \\ d+r \end{bmatrix}, \quad \hat{\boldsymbol{B}}_2 = \begin{bmatrix} 0 \\ -d+r \end{bmatrix},$$
$$h_1(e_1(t)) = \frac{e_1(t)+d}{2d}, \quad h_2(e_1(t)) = \frac{d-e_1(t)}{2d}$$

On the other hand, (24) can be also regarded as the following nonlinear dynamics.

$$\begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{bmatrix} = \begin{bmatrix} e_2(t) \\ e_1^2(t) + 2re_1(t) + r^2 + (e_1(t)+r)e_u(t) \\ \quad +u_c(t)e_1(t) + ru_c(t) \end{bmatrix} \quad (26)$$

For (26), the following cancellation input $u_c(t)$ can be selected.

$$u_c(t) = -r$$

Then, (26) can be rewritten by

$$\begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{bmatrix} = \begin{bmatrix} e_2(t) \\ e_1^2(t) + re_1(t) + (e_1(t)+r)e_u(t) \end{bmatrix}$$

By applying sector nonlinearity concept to the nonlinear terms with the constraint $|e_1(t)| \leq d$, the T-S fuzzy model is constructed as follows:

$$\dot{\boldsymbol{e}}(t) = \sum_{i=1}^{2} h_i(e_1(t)) \left(\hat{\boldsymbol{A}}_i \boldsymbol{e}(t) + \hat{\boldsymbol{B}} e_u(t)\right) \quad (27)$$

where

$$\hat{\boldsymbol{A}}_1 = \begin{bmatrix} 0 & 1 \\ d+r & 0 \end{bmatrix}, \quad \hat{\boldsymbol{A}}_2 = \begin{bmatrix} 0 & 1 \\ -d+r & 0 \end{bmatrix},$$
$$\hat{\boldsymbol{B}}_1 = \begin{bmatrix} 0 \\ d+r \end{bmatrix}, \quad \hat{\boldsymbol{B}}_2 = \begin{bmatrix} 0 \\ -d+r \end{bmatrix},$$
$$h_1(e_1(t)) = \frac{e_1(t)+d}{2d}, \quad h_2(e_1(t)) = \frac{d-e_1(t)}{2d}$$

Note that the constructed fuzzy models are different depending on the cancellation input.

B. Servo Controller Design

To stabilize the T-S fuzzy model (22), we employ the following PDC controller.

$$\boldsymbol{e}_u(t) = -\sum_{i=1}^{r} h_i(\boldsymbol{e}(t), \boldsymbol{r}, \boldsymbol{u}_c(t)) \hat{\boldsymbol{K}}_i \boldsymbol{e}(t) \quad (28)$$

where $\hat{\boldsymbol{K}}_i$ is a feedback gain. By substituting (28) into (22), we can obtain the following fuzzy control system.

$$\begin{aligned} \boldsymbol{e}(t) \ &= \ \sum_{i=1}^{r}\sum_{j=1}^{r} h_i(\boldsymbol{e}(t), \boldsymbol{r}, \boldsymbol{u}_c(t)) h_j(\boldsymbol{e}(t), \boldsymbol{r}, \boldsymbol{u}_c(t)) \\ &\quad \times \left(\hat{\boldsymbol{A}}_i - \hat{\boldsymbol{B}}_i \hat{\boldsymbol{K}}_j\right) \boldsymbol{e}(t) \end{aligned} \quad (29)$$

The feedback gain $\hat{\boldsymbol{K}}_i$ is determined by solving Theorem 2.

Theorem 2: If there exist positive definite matrix $\boldsymbol{X}$ and matrix $\hat{\boldsymbol{M}}_i$ satisfying (30), (31) and (32), then the fuzzy system (22) can be stabilized by the fuzzy controller (28).

$$\boldsymbol{X} > \boldsymbol{0}, \quad (30)$$
$$\hat{\boldsymbol{A}}_i \boldsymbol{X} + \boldsymbol{X} \hat{\boldsymbol{A}}_i^T - \hat{\boldsymbol{B}}_i \hat{\boldsymbol{M}}_i - \hat{\boldsymbol{M}}_i^T \hat{\boldsymbol{B}}_i^T < \boldsymbol{0}, \ \forall i, \quad (31)$$
$$\begin{aligned} &\hat{\boldsymbol{A}}_i \boldsymbol{X} + \boldsymbol{X} \hat{\boldsymbol{A}}_i^T + \hat{\boldsymbol{A}}_j \boldsymbol{X} + \boldsymbol{X} \hat{\boldsymbol{A}}_j^T \\ &\quad -\hat{\boldsymbol{B}}_i \hat{\boldsymbol{M}}_j - \hat{\boldsymbol{M}}_j^T \hat{\boldsymbol{B}}_i^T - \hat{\boldsymbol{B}}_j \hat{\boldsymbol{M}}_i - \hat{\boldsymbol{M}}_i^T \hat{\boldsymbol{B}}_j^T < \boldsymbol{0}, \quad (32) \\ &\qquad \forall i, \ i < j, \end{aligned}$$

where $\hat{\boldsymbol{K}}_i = \hat{\boldsymbol{M}}_i \boldsymbol{X}^{-1}$.

Based on (15) and the controller (28), the following servo controller can be obtained.

$$\begin{aligned} \boldsymbol{u}(t) \ &= \ \boldsymbol{e}_u(t) + \boldsymbol{u}_c(t) \\ &= \ -\sum_{i=1}^{r} h_i(\boldsymbol{e}(t), \boldsymbol{r}, \boldsymbol{u}_c(t)) \hat{\boldsymbol{K}}_i (\boldsymbol{x}(t) - \boldsymbol{r}) + \boldsymbol{u}_c(t) \end{aligned} \quad (33)$$

By using (33), $\boldsymbol{e}(t)$ goes to $\boldsymbol{0}$. Therefore, we can make the state $\boldsymbol{x}(t)$ of the nonlinear system (12) converge to $\boldsymbol{r}$.

C. Constraint Conditions on States and Inputs

State $x_{\ell_1}(t)$ and control input $u_{\ell_2}(t)$ are obtained as follows, respectively.

$$
\begin{aligned}
x_{\ell_1}(t) &= \boldsymbol{C}^1_{\ell_1}\boldsymbol{e}(t) + r_{\ell_1}, \quad \ell_1 = 1, 2, \cdots n \qquad (34)\\
u_{\ell_2}(t) &= \boldsymbol{C}^2_{\ell_2}\boldsymbol{e}_u(t) + u_{c\ell_2}(t), \quad \ell_2 = 1, 2, \cdots m \quad (35)
\end{aligned}
$$

where $\boldsymbol{C}^1_{\ell_1}$ and $\boldsymbol{C}^2_{\ell_2}$ are vectors whose ℓ_1th or ℓ_2th element is 1 and all the other elements are 0 in order to determine which state or control input is constrained. By using the following theorems, $|x_{\ell_1}(t) - r_{\ell_1}| \le \mu_{1\ell_1}$ and $|u_{\ell_2}(t) - u_{c\ell_2}(t)| \le \mu_{2\ell_2}$ are enforced. This means that state x_{ℓ_1} and control input $u_{\ell_2}(t)$ are limited in the following ranges.

$$
\begin{aligned}
-\mu_{1\ell_1} + r_{\ell_1} \le \quad & x_{\ell_1}(t) \quad \le \mu_{1\ell_1} + r_{\ell_1}\\
-\mu_{2\ell_2} + u_{c\ell_2}(t) \le \quad & u_{\ell_2}(t) \quad \le \mu_{2\ell_2} + u_{c\ell_2}(t)
\end{aligned}
$$

For satisfying the above input constraint, also the cancellation input $u_{c\ell_2}(t)$ should be constraint. Fortunately, the constraint can be achieved by (20) and the above-mentioned constraint on the state.

Theorem 3: Assume that initial condition $\boldsymbol{e}(0)$ is known. The constraint $|x_{\ell_1}(t) - r_{\ell_1}| \le \mu_{1\ell_1}$ is enforced at all times $t \ge 0$ if the following LMIs hold.

$$
\begin{bmatrix} 1 & \boldsymbol{e}^T(0) \\ \boldsymbol{e}^T(0) & \boldsymbol{X} \end{bmatrix} \ge \boldsymbol{0}, \quad \begin{bmatrix} \boldsymbol{X} & (\boldsymbol{C}^1_{\ell_1}\boldsymbol{X})^T \\ \boldsymbol{C}^1_{\ell_1}\boldsymbol{X} & \mu^2_{1\ell_1}\boldsymbol{I} \end{bmatrix} \ge \boldsymbol{0},
$$

Theorem 4: Assume that initial condition $\boldsymbol{e}(0)$ is known. The constraint $|u_{\ell_2}(t) - u_{c\ell_2}(t)| \le \mu_{2\ell_2}$ is enforced at all times $t \ge 0$ if the following LMIs hold.

$$
\begin{bmatrix} 1 & \boldsymbol{e}^T(0) \\ \boldsymbol{e}^T(0) & \boldsymbol{X} \end{bmatrix} \ge \boldsymbol{0}, \quad \begin{bmatrix} \boldsymbol{X} & (\boldsymbol{C}^2_{\ell_2}\hat{\boldsymbol{M}}_i)^T \\ \boldsymbol{C}^2_{\ell_2}\hat{\boldsymbol{M}}_i & \mu^2_{2\ell_2}\boldsymbol{I} \end{bmatrix} \ge \boldsymbol{0},
$$

The proofs of Theorems 3 and 4 are similar to those of Theorems 11 and 12 in [7].

IV. Design Example

To illustrate the utility of this servo control approach, we show two simulation examples.

A. Example 1

Consider (23) again. Two kinds of T-S fuzzy models are already constructed in Section III-A. Assume that $\boldsymbol{r} = [1\ 0]^T$, $d = 0.9$ and initial state is $\boldsymbol{x}(0) = [0.2\ \ 0]^T$. By solving servo controller design conditions in Theorem 2, each servo controller can be designed. For (25), feedback gains are obtained as follows.

$$
\boldsymbol{K}_1 = [8.08 \quad 6.70], \ \boldsymbol{K}_2 = [19.86 \quad 17.53]
$$

For (27), feedback gains are obtained as follows.

$$
\boldsymbol{K}_1 = [1.57 \quad 2.03], \ \boldsymbol{K}_2 = [1.92 \quad 2.89]
$$

Figures 1 and 2 show the simulation results using designed servo controllers based on (25) and (27), respectively. The selection of cancellation inputs strongly affects control performances.

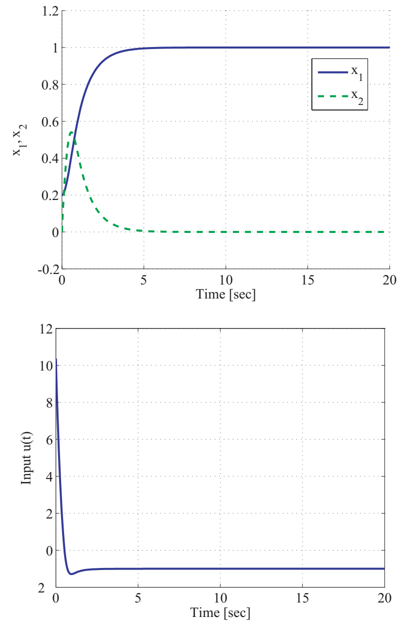

Fig. 1. Simulation result based on (25).

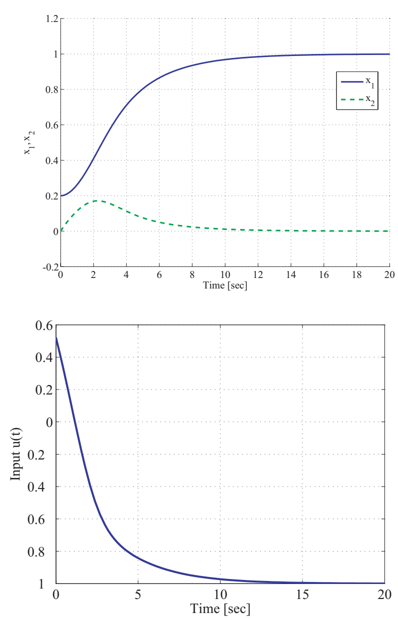

Fig. 2. Simulation result based on (27).

B. Example 2

Consider the following nonlinear system.

$$
\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} x_2(t) \\ x_1^2(t) - \sin x_1(t) u(t) \end{bmatrix} \qquad (36)
$$
$$
|u(t)| \le 15
$$

We assume that the target point $\boldsymbol{r} = [r\ 0]^T$. Then, the error dynamics can be obtained as follows.

$$
\begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{bmatrix} = \begin{bmatrix} e_2(t) \\ e_1^2(t) + 2re_1(t) + r^2 - \sin(e_1(t) + r)e_u(t) \\ \quad - \sin(e_1(t) + r)u_c(t) \end{bmatrix}
$$

The following cancellation input $u_c(t)$ is selected.

$$u_c(t) = \frac{r^2}{\sin(e_1(t) + r)}$$

Then, we can obtain the following error dynamics.

$$\begin{bmatrix} \dot{e}_1(t) \\ \dot{e}_2(t) \end{bmatrix} = \begin{bmatrix} e_2(t) \\ e_1^2(t) + 2re_1(t) - \sin(e_1(t)+r)e_u(t) \end{bmatrix}$$

By applying sector nonlinearity concept to the nonlinear terms with the constraint $|e_1(t)| \le d < r$, the T-S fuzzy model is constructed as follows:

$$\dot{\boldsymbol{e}}(t) = \sum_{i=1}^{4} h_i(e_1(t)) \left(\hat{\boldsymbol{A}}_i \boldsymbol{e}(t) + \hat{\boldsymbol{B}} e_u(t) \right) \tag{37}$$

where

$$\hat{\boldsymbol{A}}_1 = \hat{\boldsymbol{A}}_2 = \begin{bmatrix} 0 & 1 \\ d+2r & 0 \end{bmatrix}, \quad \hat{\boldsymbol{A}}_3 = \hat{\boldsymbol{A}}_4 = \begin{bmatrix} 0 & 1 \\ -d+2r & 0 \end{bmatrix},$$

$$\hat{\boldsymbol{B}}_1 = \hat{\boldsymbol{B}}_3 = \begin{bmatrix} 0 \\ -\sin(-d+r) \end{bmatrix}, \quad \hat{\boldsymbol{B}}_2 = \hat{\boldsymbol{B}}_4 = \begin{bmatrix} 0 \\ -1 \end{bmatrix},$$

$$h_1(e_1(t)) = \frac{e_1(t)+d}{2d} \times \frac{-\sin(e_1(t)+r)+1}{-\sin(-d+r)+1}$$

$$h_2(e_1(t)) = \frac{e_1(t)+d}{2d} \times \frac{-\sin(-d+r)+\sin(e_1(t)+r)}{-\sin(-d+r)+1}$$

$$h_3(e_1(t)) = \frac{d-e_1(t)}{2d} \times \frac{-\sin(e_1(t)+r)+1}{-\sin(-d+r)+1}$$

$$h_4(e_1(t)) = \frac{d-e_1(t)}{2d} \times \frac{-\sin(-d+r)+\sin(e_1(t)+r)}{-\sin(-d+r)+1}$$

We select $\boldsymbol{r} = [1 \ \ 0]^T$ as the target point. Assume that $|e_1(t)| \le 0.8$. Then, $1.0 \le u_c(t) \le 5.0$. In order to achieve the input constraint $|u(t)| \le 15$, the input difference has to satisfy $|e_u(t)| \le 10$. Unfortunately, the control result using the controller designed by solving only Theorem 2 without Theorems 3 and 4 did not satisfy the input constraint.

By solving Theorems 2, 3 and 4 with initial state $\boldsymbol{x}(0) = [0.3 \ \ 0]^T$, $\mu_{11} = 0.8$ and $\mu_{21} = 10$, we can obtain the following feedback gains.

$$\boldsymbol{K}_1 = [-14.15 \ \ -8.54], \ \boldsymbol{K}_2 = [-7.46 \ \ -11.09]$$
$$\boldsymbol{K}_3 = [-10.21 \ \ -11.66], \ \boldsymbol{K}_4 = [-4.79 \ \ -12.89]$$

Figures 3 and 4 show the control result and the control input. By using the designed controller, the state $\boldsymbol{x}(t)$ converges to the target point, and $e_1(t)$ $(= x_1(t) - r)$ and $u(t)$ are in the required range.

V. Conclusions

This paper has presented improved approach to servo control for nonlinear systems using the fuzzy model-based control approach. Time-varying cancellation input has been newly introduced. By using the cancellation input, servo control for error dynamics with time varying extra terms has been achieved. Moreover we have derived servo controller design conditions and constraint conditions on inputs and states in the form of LMI. Design examples have illustrated the utility of this approach. Our future work is to apply this approach to real complicated systems.

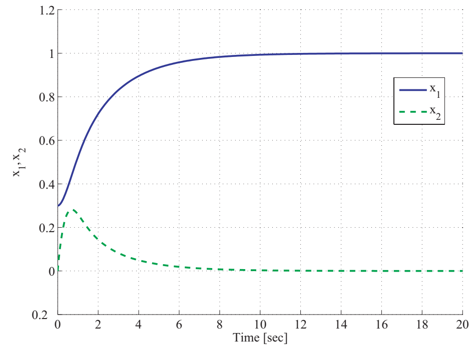

Fig. 3. Control result.

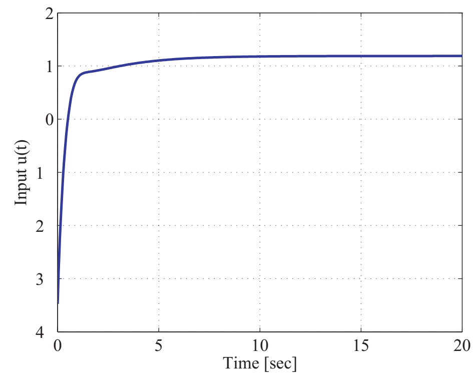

Fig. 4. Control input.

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