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PI Control of First Order Time-Delay Systems via Eigenvalue Assignment

Sun Yi, Patrick W. Nelson, and A. Galip Ulsoy

Abstract— **A new approach to design PI controllers for time-delay systems is presented. A time-delay can limit and degrade the achievable performance of the controlled system, and even induce instability. This paper presents a new method, based on the Lambert W function [20], for design of PI feedback controllers as an alternative to the well-known Smith predictor. PI controllers for first-order plants with time-delays are designed by obtaining the rightmost (i.e., dominant) eigenvalues in the infinite eigenspectrum of time-delay systems, and assigning them to desired positions in the complex plane. The process is possible due to a novel property of the Lambert W function. Using the controllers designed by using the presented approach, system performance can be improved as well as successfully stabilized. Also, sensitivity analysis of the rightmost eigenvalues is conducted to show that robustness compares favorably to the Smith predictor.**

I. INTRODUCTION

Time-delay systems (TDS), where time-delays exist between the applications of input to the system and their resulting effect, can be represented by delay differential equations (DDEs) [10]. The principal difficulty in studying DDEs results from the fact that the time-delay terms in DDEs always lead to an infinite spectrum of eigenvalues. Recently, based on the analytical solution to systems of DDEs in terms of the Lambert W function, an approach to control systems of DDEs has been developed [20]. Using the Lambert W function-based approach, for a given time-delay system, the free and forced solution is derived in terms of system parameters. From the solution, the stability of the system is determined [16], and controllability/observability is analyzed [17]. Linear feedback controllers are then designed via eigenvalue assignment to stabilize unstable systems [18], to achieve robust stability, and/or to meet time-domain specifications [19]. For a system of ordinary differential equations (ODEs), which is delay-free, this serial process would be standard. However, time-delay systems have an infinite eigenspectrum, and such a process has not been previously feasible. By obtaining and assigning the rightmost (i.e., dominant) eigenvalues in the infinite eigenspectrum, time-delay systems can be analyzed and controlled systematically using the Lambert W function.

In this paper, we make use of the Lambert W function-based approach to design PI controllers for time-delay systems. Many controllers in industrial processes only have PI action [12] and such controllers are widely used, for example, in automotive controllers [21]. Using the Lambert W function, PI controllers are designed by assigning the rightmost eigenvalues to desired positions in the complex plane. The designed controller can improve performance as well as successfully stabilize unstable systems. Also, sensitivity of the rightmost eigenvalue with respect to system parameters (including delays) is studied analytically, and via this analysis the robustness of the controller is shown to compare favorably to Smith predictor-based controllers.

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II. PI CONTROL FOR SYSTEMS WITH TIME-DELAYS

A first-order plant with a pure time-delay is commonly described by

$$G(s) = G_P(s)e^{-sh} = \frac{K_M}{\tau_M s + 1}e^{-sh} \tag{1}$$

where τ_M is the time constant, h is the time-delay, and K_M represents the steady state gain.

Consider a proportional-plus integral controller:

$$G_S = \bar{K}_P + \frac{\bar{K}_I}{s} \tag{2}$$

A primary goal of this paper is to choose the gains, $\bar{K}_P$ and $\bar{K}_I$, such that a stable closed-loop system with desirable performance is obtained. Barred variables ($\bar{\cdot}$) denote gains selected by using the Smith predictor to distinguish them from gains selected by using the Lambert W function. It has been shown that PI controllers are sufficient for all systems that have first-order transfer functions [1], and used for numerous industrial processes [12]. However, it has been well-known that the longer the time-delay, the more difficult it is to stabilize the system. Moreover, the delay term in the closed-loop characteristic equation complicates the stability analysis and the design of the controller to guarantee stability [22].

The stability analysis for the system (1) with the controller in Eq. (2) has been conducted using bifurcation methods (see, e.g., [11] and the references therein). The gains in Eq. (2) are chosen based on the stability regions in the $\bar{K}_P$-$\bar{K}_I$ space [12], often combined with the Nyquist method [5]. Alternatively, such problems have been addressed by using an observer based controller with a discrete model [9], H_∞ feedback control with a discrete event based model to improve robustness to disturbances and modeling uncertainties [6], the Smith predictor [8] and its adaptive version [21], nonlinear adaptive controllers [14], and Padé approximation approaches [4]. Among those, PI controllers with

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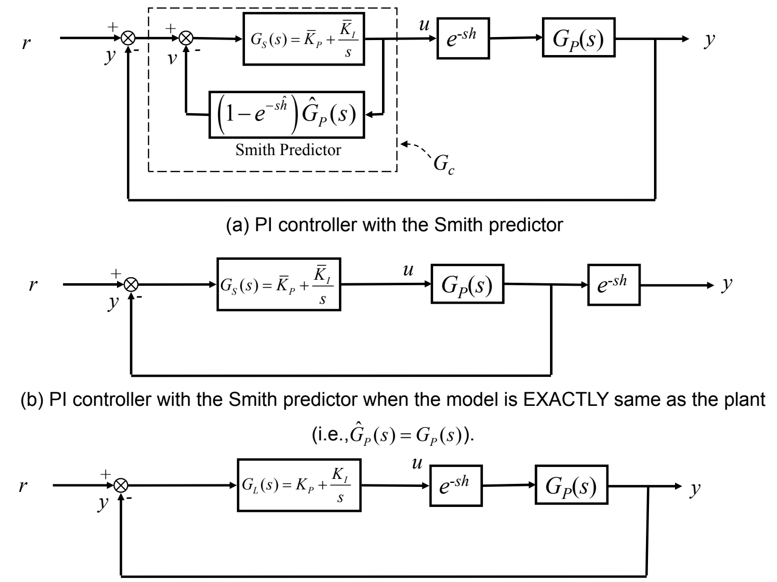

(a) PI controller with the Smith predictor

(b) PI controller with the Smith predictor when the model is EXACTLY same as the plant (i.e., $\hat{G}_P(s) = G_P(s)$).

(c) PI controller using the Lambert W function-based approach without the Smith predictor

Fig. 1. Block diagrams for a closed-loop system with the Smith Predictor-based control (a), its equivalent system when the response of the plant is predicted perfectly (b), and the closed-loop system with the Lambert W function-based control (c).

the Smith predictor successfully improved the performance in simulation and experimental results, and was compared to an experimental tuning method, the classical Ziegler-Nichols method step response method, in [8].

A. Use of the Smith Predictor

The Smith predictor in Fig. 1-(a) results in a delayed response of a delay-free system by moving the time-delay outside the feedback loop only when the model in the Smith predictor, $\hat{G}_P(s)$, is ideally the same as the plant, $G_P(s)$. Then, the controller $G_S(s)$ in (2) can be designed considering only the delay-free plant, $G_P(s)$ (see Fig. 1-(b)). This is the main advantage of the Smith predictor control.

For example, in order to meet given time domain specifications, the desired eigenvalues can be chosen from the desired natural frequency, ω_n, and the desired damping ratio, ζ as

$$\lambda_d = -\omega_n \zeta \pm \omega_n \sqrt{1-\zeta^2} i = -\sigma \pm \omega_d i \tag{3}$$

Assuming no time-delay, $\bar{K}_P$ and $\bar{K}_I$ are chosen as

$$\bar{K}_I = \frac{\omega_n^2 \tau_M}{K_M}, \ \bar{K}_P = \frac{2\zeta\omega_n\tau_M - 1}{K_M} \tag{4}$$

by substituting the desired values into the closed-loop characteristic equation of the system in Fig. 1-(b):

$$s^2 + \left(\frac{1 + K_M \bar{K}_P}{\tau_M}\right) s + \frac{K\bar{K}_I}{\tau_M} = 0 \tag{5}$$

This means that the controller, $G_S(s)$, can be designed considering only the non-delayed part, $\hat{G}_P(s)$, of the plant ignoring the time-delay, e^{-sh}. This method is, however, based on pole-zero cancellation and, thus, the stability is vulnerable to uncertainty in system parameters [2]. Careful modeling and parameter identification are crucial for successful application [3]. Furthermore, the Smith predictor cannot handle disturbances and nonzero initial conditions. Problems caused by parameter mismatches were studied in [15] and shortcomings have been discussed in [22]. It is well known that the stability of the controllers using the Smith predictor is sensitive with respect to delay uncertainties [7]. This is discussed more in detail in Section III.

B. Use of the Lambert W Function-Based Approach

In this subsection, as an alternative to the Smith predictor, a design approach for the PI controller is developed via rightmost eigenvalue assignment using the Lambert W function. The open-loop transfer function with a PI controller is

$$G_{open} = \frac{K_M}{\tau_M s + 1} e^{-sh} \left(K_P + \frac{K_I}{s}\right) = \frac{y}{e} \tag{6}$$

where $e = -y + r$. Then, the closed-loop system as in Fig. 1-(c) in the time-domain becomes

$$\begin{aligned} \ddot{y} &= -\frac{1}{\tau_M}\dot{y} - \frac{K_M K_P}{\tau_M}\dot{y}(t-h) - \frac{K_M K_I}{\tau_M} y(t-h) \\ &\quad + \{K_M K_P s + K_M K_I\} r(t-h) \end{aligned} \tag{7}$$

By defining $x_1 \equiv y, x_2 \equiv \dot{y}$, the equation can be re-written as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{1}{\tau_M}x_2 - \frac{K_M K_P}{\tau_M}x_2(t-h) - \frac{K_M K_I}{\tau_M}x_1(t-h) \\ &\quad + \{K_M K_P s + K_M K_I\} r(t-h) \end{aligned} \tag{8}$$

Then, we obtain the closed-loop system in state-space form ignoring the reference input to focus on stability, which is given by:

$$\dot{\mathbf{x}}(t) =$$
$$\underbrace{\begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{1}{\tau_M} \end{bmatrix}}_{\equiv \mathbf{A}} \mathbf{x}(t) - \underbrace{\begin{bmatrix} 0 & 0 \\ \dfrac{K_M K_I}{\tau_M} & \dfrac{K_M K_P}{\tau_M} \end{bmatrix}}_{\equiv \mathbf{A_d}} \mathbf{x}(t-h) \tag{9}$$

From the roots of the characteristic equation of the system (9), the eigenvalues of the system are obtained. However, due to the time-delay, e^{-sh}, the system is infinite-dimensional and, thus, there exists an infinite number of eigenvalues. The principal difficulty in analyzing and controlling systems with time-delays arises from this transcendental character, and the determination of this eigenspectrum typically requires numerical, approximate, or other approaches [10]. Obtaining and controlling the entire infinite eigenspectrum is not as straightforward as for systems of ODEs. Instead, for DDEs, like Eq. (9), it is desired to locate the dominant eigenvalues, which are rightmost in the complex plane, and to assign them to desired positions. The Lambert W function-based approach is an efficient tool for doing this and in the subsequent section the approach is applied to the system (9) to design the PI controller and compared with the Smith predictor. With the coefficient matrices, $\mathbf{A}$ and $\mathbf{A_d}$ defined in Eq. (9), the solution matrix, $\mathbf{S}_0$ is computed as

$$\mathbf{S}_0 = \frac{1}{h}\mathbf{W}_0(\mathbf{A_d} h \mathbf{Q}_0) + \mathbf{A} \tag{10}$$

where the unknown matrix $\mathbf{Q}_0$ is obtained by solving

$$\mathbf{W}_0(\mathbf{A_d} h \mathbf{Q}_0) e^{\mathbf{W}_0(\mathbf{A_d} h \mathbf{Q}_0) + \mathbf{A}h} = \mathbf{A_d} h \tag{11}$$

Then after setting an equation so that the desired positions for eigenvalues are equal to those of $\mathbf{S}_0$ as $\lambda_i(\mathbf{S}_0) = \lambda_{i,desired}$ for $i = 1, \cdots, n$, where, $\lambda_i(\mathbf{S}_0)$ is i^{th} eigenvalue of the matrix $\mathbf{S}_0$, the gains, K_P and K_I are obtained by solving the equation numerically. For detailed explanation on the Lambert W function-based approach, including a method for eigenvalue assignment, refer to [20]. Desired positions for rightmost eigenvalues, $\lambda_{i,desired}$ in assigning them can be chosen from the desired natural frequency and damping ratio using the relation in Eq. (3).

III. ILLUSTRATIVE EXAMPLES AND SENSITIVITY ANALYSIS

In this section, the Lambert W function-based approach is applied to two different cases: an open-loop unstable plant and an open-loop stable plant. The results are compared to the Smith predictor-based approach with respect to stability and robustness against mismatches of parameters.

A. Open-Loop Unstable Plants

If the system (1) is unstable, the characteristic equation of the closed-loop system with the Smith predictor in Fig. 1-(a) retains the spectrum of the unstable pole of the open-loop system. Therefore, the Smith predictor cannot stabilize the system [2]. For example, for an unstable system

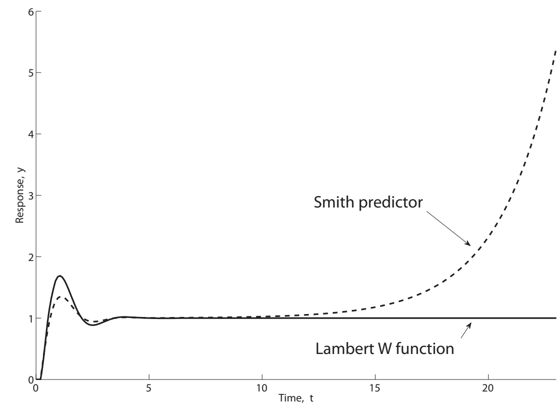

Fig. 2. Simulated responses of the unstable system (12) controlled with the Smith predictor (dashed line) and the Lambert W function-based approach (solid line) using Simulink. Because the Smith predictor is based on unstable pole-zero cancelation, it fails to stabilize the system.

$$G(s) = G_P(s)e^{-sh} = \frac{1}{0.5s - 0.2}e^{-0.2s} \tag{12}$$

the gains of the PI controller, using the approach in Subsection II-A, $\bar{K}_I = 3.1250$ and $\bar{K}_P = 1.4500$ when the desired natural frequency is $\omega_n = 2.5$ and the desired damping ratio is $\zeta = 0.5$. With those gains one can assign the eigenvalues of the system to be $-1.2500 \pm 2.1651i$, which are stable, theoretically. However, due to initial conditions, disturbance, and errors in simulation, this control leads to instability as seen in Fig. 2. Figure 2 shows responses simulated using Simulink. Even though there is no disturbance or no initial condition mismatch, due to errors in numerical integration, the Smith predictor cannot successfully stabilize the unstable plant (12).

On the other hand, if the Lambert W function-based approach in the subsection II-B is applied, the gains are $K_I = 1.4309$ and $K_P = 1.2232$ for the same desired rightmost eigenvalues and because it stabilizes the system without pole-zero cancellation, the designed controller safely stabilizes the system (see Fig. 2). As mentioned earlier, the recently developed prediction-based approach by modifying the Smith predictor, FSA has also unsolved problems regarding approximation and, thus, can fail to stabilize unstable systems. It was shown that the Padé approximation does not guarantee stability of controlled system due to its inaccuracy [18]. Therefore, the Lambert W function has an advantage over such methods in stabilizing unstable systems.

B. Open-Loop Stable Plants

When the time constant, τ_M, is 0.5, and the time delay, h, is 0.2 with $K_M = 1$ for the plant in Eq. (1), the gains for PI control, for several desired natural frequencies and damping ratios, are obtained by using the Smith predictor-based approach and the Lambert W function-based approach,

TABLE I

GAINS, K_I AND K_P, OF PI CONTROLLER: OBTAINED BY USING THE LAMBERT W FUNCTION APPROACH VIA RIGHTMOST EIGENVALUE ASSIGNMENT

		Smith predictor (from Eq. (4))		Lambert W function	
ω_n	ζ	$\bar{K}_I$	$\bar{K}_P$	K_I	K_P
1.0	0.3	0.5000	-0.7000	0.6214	-0.5143
1.1	0.5	0.6050	-0.4500	0.6884	-0.2439
1.5	0.5	1.1250	-0.2500	1.1751	0.0155
2.5	0.5	3.1250	0.2500	2.5629	0.6013

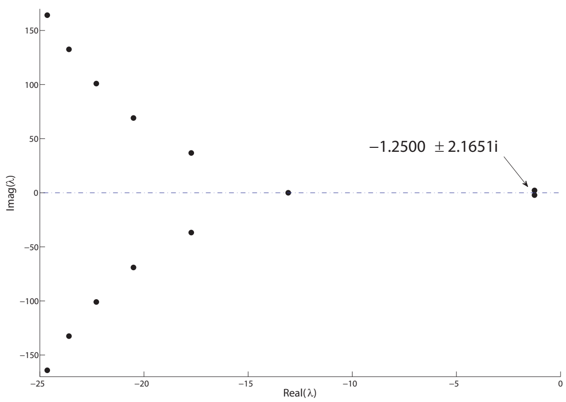

Fig. 3. Eigenspectrum of the closed-loop system in Fig. 1-(c). By using the Lambert W function-based approach in the subsection II-B, the rightmost eigenvalues of the infinite eigenspectrum are assigned to the desired positions, $-1.2500 \pm 2.1651i$.

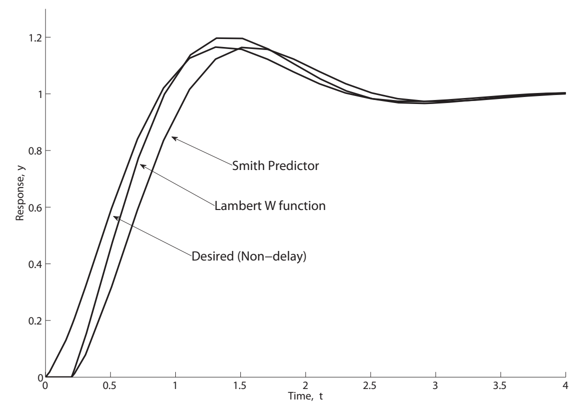

Fig. 4. Responses of systems controlled by using the Smith predictor and the Lambert W function.

and given in Table I. The Smith predictor moves the time-delay outside the feedback loop as seen in Fig. 1-(b) and makes the number of poles finite (in this case 2) by canceling the other (infinite number of) eigenvalues. On the other hand, the Lambert W function-based approach, without the cancellation, obtains the rightmost eigenvalues and assigns them to the desired positions. For example, when the desired natural frequency $\omega_n = 2.5$ and the desired damping ratio is $\zeta = 0.5$ (thus, the desired positions for the rightmost eigenvalues are $-1.2500 \pm 2.1651i$), the eigenspectrum of the closed-loop system is depicted in Fig. 3. The rightmost eigenvalues are placed exactly on the desired positions, which are obtained from Eq. (3), and others are to the left of them. The response of the closed systems by using the Smith predictor (Fig. 1-(a)) and the Lambert W function-based approach are shown in Fig. 4. Although the systems in Fig. 1-(c) has an infinite number of eigenvalues, by assigning the rightmost (thus, dominant) eigenvalues to the desired positions, one can meet time-domain specifications [19]. As for robustness, if the system (1) is open-loop stable, the Smith predictor works as desired and does not have apparent mismatch problems unlike the example in Subsection III-A. However, as seen in Fig. 1, the Lambert W function-based controller has a form simpler than the Smith predictor-based one. Also, for the Smith predictor-based control in Fig. 1-(a), the feedback signal for the controller, $G_S(s)$, uses $y \cdot e^{sh}$, which is the output, y, predicted one time-delay ahead. Therefore, it is reasonable to consider sensitivity with respect to mismatch of parameters, and compare the robustness of the two different control methods. Sensitivity is discussed in the next two subsequent subsections. In the literature, it has been shown that estimation of time-delay in continuous linear time-invariant systems is more difficult than other system parameters [13]. Also, it is well known that control using the Smith predictor is sensitive with respect to delay mismatches [7]. Thus, the sensitivity analysis is focused on the time-delay. However, the analysis for other parameters can be conducted in a similar way and results are discussed.

C. Sensitivity of the Smith Predictor

In this subsection the sensitivity of the closed-loop eigenvalues to small variance in the time-delay, h, is considered for the Smith predictor-based controller.

The transfer function of the closed-loop system with the Smith predictor in Fig. 1-(a) can be represented as

$$\frac{y}{r} = \frac{G_S G_P e^{-sh}}{1 + \left(1 - e^{-s\hat{h}}\right) G_S \hat{G}_P + G_S G_P e^{-sh}} \qquad (13)$$

Assuming all parameters except the time delay are wellmatched, i.e., $\hat{G}_P = G_P$, the denominator will be

$$\begin{aligned} DE(s) &\equiv 1 + \left(1 - e^{-s\hat{h}}\right) G_S \hat{G}_P + G_S G_P e^{-sh} \\ &= 1 + G_S G_P - G_S G_P e^{-s\hat{h}} + G_S G_P e^{-sh} \\ &= 1 + G_S G_P - G_S G_P e^{-sh}(e^{-s\delta} - 1) \end{aligned} \tag{14}$$

where $\delta \equiv \hat{h} - h$. Then substituting the terms

$$G_P \equiv \frac{B_P}{A_P} = \frac{K_M}{\tau_M s + 1}, \quad G_S \equiv \frac{B_C}{A_C} = \frac{\bar{K}_p s + \bar{K}_I}{s} \tag{15}$$

yields

$$\begin{aligned} DE(s) &= 1 + G_S G_P - G_S G_P e^{-s\hat{h}} + G_S G_P e^{-sh} \\ &= 1 + \frac{B_P B_C}{A_P A_C} - \frac{B_P B_C}{A_P A_C} e^{-s\hat{h}} + \frac{B_P B_C}{A_P A_C} e^{-sh} \end{aligned} \tag{16}$$

Setting $DE(s) = 0$ yields the closed-loop characteristic equation:

$$\begin{aligned} Ch(s) &= A_P A_C + B_P B_C - B_P B_C e^{-s\hat{h}} + B_P B_C e^{-sh} \\ &= (\tau_M s + 1)(s) + K_M\left(\bar{K}_P s + \bar{K}_I\right) \\ &\quad - K_M\left(\bar{K}_P s + \bar{K}_I\right) e^{-s\hat{h}} \\ &\quad + K_M\left(\bar{K}_P s + \bar{K}_I\right) e^{-sh} = 0 \end{aligned} \tag{17}$$

Differentiating both sides of the characteristic equation with respect to the time-delay, h, yields

$$\begin{aligned} \frac{\partial}{\partial h} Ch(s) &= 2\tau_M s \frac{\partial s}{\partial h} + \frac{\partial s}{\partial h} + K_M \bar{K}_P \frac{\partial s}{\partial h} + K_M \bar{K}_I \\ &\quad - K_M\left(\bar{K}_P \tfrac{\partial s}{\partial h} + \bar{K}_I\right) e^{-s\hat{h}} \\ &\quad - K_M\left(\bar{K}_P s + \bar{K}_I\right) e^{-s\hat{h}} \left(-\hat{h}\right) \frac{\partial s}{\partial h} \\ &\quad + K_M\left(\bar{K}_P \frac{\partial s}{\partial h} + \bar{K}_I\right) e^{-sh} \\ &\quad + K_M\left(\bar{K}_P s + \bar{K}_I\right) e^{-sh} \left\{(-h)\frac{\partial s}{\partial h} - s\right\} \end{aligned} \tag{18}$$

Because $\frac{\partial}{\partial h} Ch(s) = 0$, we get the sensitivity with respect to the time-delay, h, given by

$$\frac{\partial s}{\partial h} = \frac{K_M\left(\bar{K}_P s + \bar{K}_I\right) e^{-sh}(s) - K_M \bar{K}_I}{2\tau_M s + 1 + K_M \bar{K}_P} \tag{19}$$

By substituting the eigenvalues, λ, for s in Eq. (19), one can get the sensitivity of the eigenvalues with respect to small variance in the time-delay. The sensitivity in Eq. (19) represents an incremental change in the positions of the eigenvalues corresponding to an incremental change in the time-delay. Numerical values for several cases, with the same parameter set as in Subsection III-B, are given in Table II.

D. Sensitivity of the Lambert W Function

From the state equation (9) of the closed-loop system in 1-(c), the sensitivity of the eigenvalue with respect to the time-delay, h, is obtained in a way similar to the previous derivation. The characteristic equation of the system (9) is

$$s^2 + \frac{1}{\tau_M} s + \frac{K_M K_P}{\tau_M} s e^{-sh} + \frac{K_M K_I}{\tau_M} e^{-sh} = 0 \tag{20}$$

In a similar way to the previous section, by differentiating both sides the sensitivity of the eigenvalues with respect to change in the time-delay, h, is given by

$$\frac{\partial s}{\partial h} = \frac{s^2 K_M K_P e^{-sh} + s K_M K_I e^{-sh}}{2s\tau_M + 1 + K_M K_P e^{-sh} - (K_M K_P s + K_M K_I) e^{-sh} h} \tag{21}$$

The numerical values of the sensitivity with respect to h for the Lambert W function-based approach and the Smith predictor are compared in Table II. As seen in Table II, for several rightmost eigenvalues, which are arbitrarily chosen, the Lambert W function-based approach shows smaller values of the sensitivity and, thus, improvement in robustness. For comparison, 'improvement' is calculated as the ratio of decrease in real parts of the sensitivities to the real part of the sensitivity of the Smith predictor (i.e., $(\Re(S1) - \Re(S2))/\Re(S1) \times 100$). Because in the Smith predictor control the feedback signal for the controller ($G_S(s)$ in Fig. 1-(a)) uses the "predicted" output, y, (e.g., ye^{sh}) by canceling signals [22], it is sensitive with respect to infinitesimal delay mismatches [7]. On the other hand, for the Lambert W function-based approach, the feedback signal for the controller ($G_L(s)$ in Fig. 1-(c)) is not predicted and does not require any cancelation. This may result in improvement in robustness as seen in Table II.

For other parameters in Eq. (1), sensitivity analysis can be conducted in a way similar to the time-delay, h. For example, for K_M and τ_M, the obtained sensitivities are summarized in Tables III. The sensitivity with respect to K_M for the Lambert W function also has smaller values than the Smith predictor. Thus, the Lambert W function-based approach enhances robustness. However, it does not mean the presented approach always renders less sensitive controllers. In Table III, for the sensitivity with respect to τ_M, the results are mixed. Thus, use of the Lambert W function does not always reduce sensitivity of the rightmost eigenvalues. Therefore, for stable first-order plants, after comparing sensitivity with respect to parameters (especially, ones having larger variance, for example, due to difficulty in estimating) one can choose more suitable method to design PI controllers that is more robust against variance in system parameters.

IV. CONCLUDING REMARKS AND FUTURE WORK

In this paper, we have studied a new approach to design of PI controllers for time-delay systems as an alternative to the Smith predictor. PI controllers are the most common controller and for first order time-delay systems. Choosing the gains in the PI controllers corresponding to desired system performance is essential to achieve the control goals. The presented approach based on the Lambert W function enables one to choose the gains by assigning the rightmost eigenvalue to the desired positions, which are obtained from the desired natural frequency and damping ratio. Unlike prediction-based methods, the approach is not dependent on

TABLE II

SENSITIVITY COMPARISON WITH RESPECT TO h: THE LAMBERT W FUNCTION-BASED APPROACH SHOWS SMALLER VALUES OF THE SENSITIVITY AND IMPROVEMENT IN ROBUSTNESS

ω_n	ζ	Rightmost eigenvalue, λ	Sensitivity, $\frac{\partial\lambda}{\partial h}$		Improvement (%)
			Smith predictor ($S1$)	Lambert W function ($S2$)	$\frac{\Re(S1)-\Re(S2)}{\Re(S1)} \times 100$
1.0	0.3	$-0.3000 \pm 0.9539i$	$0.8697 \mp 0.1245i$	$0.7431 \mp 0.3423i$	14.59
1.1	0.5	$-0.5500 \pm 0.9526i$	$1.2191 \pm 0.4722i$	$0.9606 \pm 0.0662i$	21.20
1.5	0.5	$-0.7500 \pm 1.2990i$	$1.8135 \pm 0.9045i$	$1.4319 \pm 0.4189i$	21.04
2.5	0.5	$-1.2500 \pm 2.1651i$	$4.0803 \pm 2.6198i$	$3.2842 \pm 2.4937i$	19.51

TABLE III

COMPARISON OF THE SENSITIVITY: WITH RESPECT TO K_M, THE LAMBERT W FUNCTION-BASED APPROACH SHOWS SMALLER VALUES OF THE SENSITIVITY AND AN IMPROVEMENT IN ROBUSTNESS. ON THE OTHER HAND, WITH RESPECT TO τ_M: UNLIKE h AND K_M, THE SENSITIVITY SHOWS MIXED RESULTS.

ω_n	ζ	Sensitivity, $\frac{\partial\lambda}{\partial K_M}$		Improvement	Sensitivity, $\frac{\partial\lambda}{\partial \tau_M}$		Improvement
		Smith predictor	Lambert W function	(%)	Smith predictor	Lambert W function	(%)
1.0	0.3	$0.8797 \pm 0.6350i$	$0.5495 \pm 0.6061i$	37.53	$0.4525 \mp 1.0170i$	$0.4930 \mp 0.6794i$	-8.96
1.1	0.5	$0.6824 \pm 0.8857i$	$0.3845 \pm 0.7864i$	43.66	$1.0714 \mp 0.9286i$	$0.9675 \mp 0.5443i$	9.70
1.5	0.5	$0.5823 \pm 1.0599i$	$0.2354 \pm 0.9663i$	59.56	$1.4258 \mp 1.4201i$	$1.4364 \mp 0.8225i$	-0.74
2.5	0.5	$0.4085 \pm 1.6487i$	$-0.2070 \pm 1.6364i$	49.33	$2.1361 \mp 3.0292i$	$2.9950 \mp 1.9965i$	-40.21

pole-zero cancellation and, thus, unstable systems with time delays can be successfully stabilized as seen in Section III. Also, because prediction of the plant is not required, the obtained controller has a simpler form as seen in Fig. 1 and is more robust in the presence of delay mismatches.

This research can be extended to more complex cases. For example, the approach is applicable to higher order systems with time delays and to design of PI with derivative (PID) control. Those theoretical studies and implementation on physical systems are being conducted by the authors.

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