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Control Signal Constraints and Filter Order Selection for PI and PID Controllers

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***Abstract*— Large control signal derivatives or inter-sample differences may harm actuators. An optimization constraint limiting such variations, related to measurement noise, is derived. Using the constraint, optimal PI, PID and measurement filters with different orders are designed for several processes and compared to the optimal linear controller of high order found via Youla parametrization. Simulations of load disturbance rejections and measurement noise sensitivities are shown and conclusions on filter order selection for PI and PID controllers are drawn.**

I. Introduction

Proportional-Integral (PI) and Proportional-Integral-Derivative (PID) controllers have been used for decades and are still today the most commonly used controllers in for instance process industry, see [1]. Several tuning methods exist and during the recent years a considerable research effort has been made for improvements, both in robustness and disturbance rejection. However, in practice, as noted in [2], PI controllers are often chosen over PID controllers even though a considerable improvement can be made by adding a derivative part. One reason mentioned is the noise sensitivity introduced by the derivative part. This may give undesirable control signal variations, leading to expensive wear of actuators. Controller de-tuning is one remedy, used in e.g., λ-tuning and internal model control, see [3] and [4], while another is to set controller parameter bounds, see [5].

For PID, derivative filters of order one is commercial standard, where the time constant has a preset relation to derivative time [6]. However, as shown in [2], the filter cut-off frequency may have a significant impact on both performance and noise sensitivity and should thus be part of the design procedure of the controller. Recently, four-parameter tunings have emerged. In [7] and [8] an upper bound on the $\mathcal{H}_\infty$-norm of the transfer function from measurement noise to control signal is used while tuning PID controllers with first order filters. The $\mathcal{H}_2$-norm of the transfer function is used in [9] together with PI and PID controllers with order one roll-off by using appropriate filters. In [10], PID is used with a second order filter together with $\mathcal{H}_\infty$-norm constraints on the transfer functions from noise to control signal and its derivative.

Light filtering of measured signal implies ability to react fast to load disturbances but gives often undesirable variations in control signal due to noise, while substantial filtering yields the opposite if the same controller settings are used. There is hence a trade-off and a need for quantifying performance gains and noise rejection abilities for different filter orders when constraints are set on control signal behaviour.

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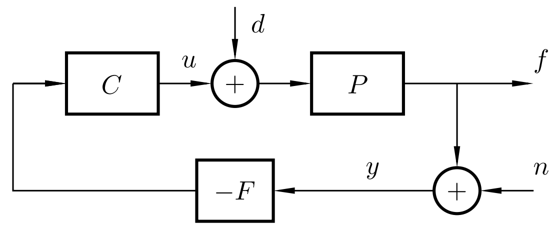

Fig. 1. Process P, controller C, and measurement filter F.

In this paper, such a behaviour constraint is presented, involving the closed loop transfer function from noise to control signal, that is related to measurement noise and practical considerations. Optimal PI(D) controllers and noise filters with different orders together with optimal linear high order controllers will be designed. Simulations of load disturbances and noise rejection abilities will be shown for the different control structures.

II. Specifications on Control Signal

A. *Control Signal Inter-Sample Amplitude Difference*

The control structure considered can be found in Figure 1 with process P and controller C with the measurement filter F such that CF is at least proper. Controlled process output is denoted f and measured output y, while the control signal, measurement noise and load disturbance are denoted u, n and d, respectively. As mentioned in Section I, a highly varying control signal is undesirable due to e.g., wear of actuators. It is important to emphasize that in most cases it is not the size of the control signal amplitude, assuming it is in actuator range, that may be harmful. It is rapid fluctuations in the control signal that may cause most damage.

Fluctuations in the control signal can be seen by e.g. large derivatives or large inter-sample differences. Since the control signal is in discrete time with constant value between sampling instants, the approximation of the derivative, i.e., $\Delta u/h$, where Δu is the inter-sample amplitude difference and h is the sampling period, will be used. Assuming that the measurement noise is white with zero mean and standard deviation σ_n, then the discrete time derivative of the control signal will be zero mean with standard deviation

$$\frac{\sigma_{\Delta u}}{h} = \left\| \frac{z-1}{zh} \frac{CF}{1+PCF} \right\|_2 \sigma_n, \tag{1}$$

where $\sigma_{\Delta u}$ is the standard deviation of Δu.

The above measure may be used to constrain control signal movement, and two different view points may be taken,

1) Considering control signal derivative, as used in velocity limiters, see [11].

2) Considering inter-sample differences associated with e.g., full control signal range.

The two view points are application and user dependent and differ only in a scaling factor of h^{-1} in Eq. (1), and are thus identical. In the sequel, the inter-sample difference view point will be used, removing h^{-1} from both sides.

Since Δu is a stochastic process, a measure of the control signal activity is how large part of the distribution of Δu is outside a certain limit, $\pm\Delta u_{\text{limit}}$. Assuming that α percent of the control activity is allowed outside the limit and using that Δu is normal, leads to the relation

$$\Delta u_{\text{limit}} = \sigma_{\Delta u}\lambda_{\alpha/2}, \tag{2}$$

where $\lambda_{\alpha/2}$ is a quantile for a normal distribution, giving

$$\left\| \frac{z-1}{z} \frac{CF}{1+PCF} \right\|_2 = \frac{\Delta u_{\text{limit}}}{\sigma_n} \cdot \frac{1}{\lambda_{\alpha/2}}.$$

Since equality does not have to be fulfilled at controller design, the constraint becomes

$$\left\| \frac{z-1}{z} \frac{CF}{1+PCF} \right\|_2 \le \frac{\Delta u_{\text{limit}}}{\sigma_n} \cdot \frac{1}{\lambda_{\alpha/2}}.$$

This constraint relates directly to measurement signal quality, i.e., measurement noise, and allowed control signal movement. When used at feedback system design, it can specify how active the control signal may be.

1) Constraint Properties: Some properties of the constraint may be remarked,

- smaller α, i.e., less accepted activity outside the limits, yields larger $\lambda_{\alpha/2}$ and a tighter constraint.
- more noise, i.e., larger σ_n, yields tighter constraint.
- larger inter-sample difference acceptance, i.e., larger Δu_{limit}, yields softer constraint.

2) Specifying Constraint Limit: The noise variance σ_n may be estimated using standard techniques on measurement data, see e.g., [11], while the inter-sample amplitude limit may be related to control signal range and actuator properties. Selecting α will then determine the contraint. However, these specifications may be scaled in relation to each other yielding the same upper limit. For simplicity, Δu_{limit} and σ_n are set equal, and all specifications are collected in α, leading to the simplified constraint

$$\left\| \Delta_z \frac{CF}{1+PCF} \right\|_2 \le \frac{1}{\lambda_{\alpha/2}}, \tag{3}$$

where $\Delta_z = (z-1)/z$ has been introduced.

B. Control Signal Energy

The constraint in section II-A limits rapid variations in the control signal. However, also control signal energy and low frequency variations due to measurement noise should be considered. As in [9], the variance of the control signal amplitude due to noise is used, i.e.,

$$\frac{\sigma_u^2}{\sigma_n^2} = \left\| \frac{CF}{1+PCF} \right\|_2^2 \le \eta^2.$$

The limit η is application dependent, and for simplicity chosen to 1 when investigating the constraint in section II-A, yielding no energy amplification of the noise.

III. Feedback Structures

A. PID and Measurement Filter

The considered PI(D) controllers are on parallel form, i.e.,

$$C(s) = K_C\left(1 + 1/T_i s + T_d s\right),$$

where the integral and derivative parts are discretized using forward and backward differences, respectively, with sampling period h. For comparison, three different measurement filters will be used such that the controllers have roll-offs of orders 0–2 in continuous time, i.e., a PI without filter will also be compared. Using roll-off less than 2 would not be possible if continuous time was considered, using that Δ_z/h corresponds to s, since the transfer function in Eq. (3), with a proper process, must be strict proper in this case. The filters are restricted to have at most two tuning parameters, yielding few optimization variables. A natural choice is the damping ζ and time constant T_f of the filters, and thus the three different filters are chosen as

$$\begin{aligned} &F_1(s) = \frac{1}{sT_f+1}, \qquad F_2(s) = \frac{1}{s^2T_f^2 + 2\zeta T_f s + 1}, \\ &F_3(s) = F_1(s)F_2(s), \end{aligned} \tag{4}$$

and sampled using zero-order hold technique. Note that the third order filter does not have full degree of freedom when choosing poles since only two parameters may be specified.

B. Youla Parametrization

For evaluation of designed PI(D) controllers and measurement filters, the optimal linear controller of high order, i.e., the Youla parametrization, also known as Q-parametrization, will be used. Additionally, this controller will give a performance bound. Consider the generalized process in Figure 2,

$$G = \begin{bmatrix} G_{\mathbf{zw}} & G_{\mathbf{z}u} \\ G_{y\mathbf{w}} & G_{yu} \end{bmatrix},$$

where $\mathbf{z}$ is controlled outputs, y measured output, u controlled input and $\mathbf{w}$ exogenous inputs, with negative SISO feedback K. Closing the loop yields

$$\begin{aligned} H_{\mathbf{zw}} &= G_{\mathbf{zw}} - G_{\mathbf{z}u}K\left(I + G_{yu}K\right)^{-1} G_{y\mathbf{w}} \\ &= G_{\mathbf{zw}} - G_{\mathbf{z}u}QG_{y\mathbf{w}}, \end{aligned}$$

where $Q = K\left(I + G_{yu}K\right)^{-1}$. If G is stable, then $H_{\mathbf{zw}}$ is stable for all stable transfer functions Q.

Choosing G properly, many convex control costs and specifications, including those used in this paper, may be set on the individual elements of $H_{\mathbf{zw}}$. They are convex in Q due to the affine relationship between $H_{\mathbf{zw}}$ and Q. However, since Q may be any stable transfer function, the search space when optimizing over Q is infinite dimensional. For numerical computations, Q is parametrized as a FIR filter,

$$Q(z) = \sum_{i=0}^{N-1} q_i z^{-i},$$

where N is the length and the convexity properties are found in the q_i coefficients. This methodology is found in e.g., [12] and [13], and implemented in the toolbox QTOOL, see [14], which is used when solving the optimization problem

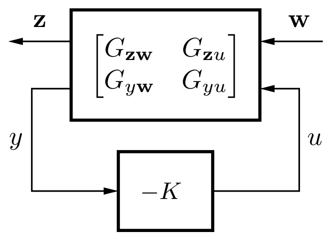

Fig. 2. Setup with generalized process G, SISO negative feedback controller K, exogenous inputs $\mathbf{w}$, control input u, controlled outputs $\mathbf{z}$ and measured output y.

to be stated. Finding K from Q is a direct calculation as the mapping is unique. Compared to PI(D) control, no additional measurement filter will be designed, it is incorporated in K directly. Choosing N large, then K, and thus $H_{\mathbf{zw}}$, may be shaped almost arbitrarily as long as the constraints are respected.

IV. Optimization Problem

A. Optimization Problem Formulation

The integrated absolute error (IAE) at a load disturbance step is used as objective function to minimize, see e.g., [3]. Robustness towards multiplicative and inverse multiplicative uncertainties, see [15], may be achieved by constraining the $\mathcal{H}_\infty$-norm of the sensitivity functions,

$$S = \frac{1}{1+KP}, \qquad T = \frac{KP}{1+KP},$$

where K is the feedback transfer function, e.g., controller and measurement filter. Frequency independent upper limits M_S and M_T as in e.g., [3], hold the number of optimization parameters to select reasonably small.

With a load disturbance step applied at process input at initial time when the system is in steady state, the optimization problem may be stated as

$$\begin{aligned}
&\underset{\mathcal{K}}{\text{minimize}} && h\sum_{k=0}^{\infty}|f(k)| && \text{(5a)}\\
&\text{subject to} && \|S\|_\infty \le M_S && \text{(5b)}\\
& && \|T\|_\infty \le M_T && \text{(5c)}\\
& && \|KS\|_2 \le \eta && \text{(5d)}\\
& && \|\Delta_z KS\|_2 \le 1/\lambda_{\alpha/2}, && \text{(5e)}
\end{aligned}$$

where $\mathcal{K}$ contains PI(D) and measurement filter parameters or FIR coefficients at Youla parametrization. In the case of PI(D) control, K is factorized as $K = CF$ as in Section II.

B. Additional Constraints on PI(D) Measurement Filters

Two constraints on the filter parameters are set. They will restrain the PI(D) and measurement filter from being a richer structure compared to how they are normally used.

1) Damping ζ: In [7], a ζ around 0.4 is used as a rule of thumb for a second order filter. However, a small ζ may give oscillations in the control signal due to the amplitude peak and in general, a measurement filter may only perform attenuation of the measured signal. The filters defined in Eq. (4) have static gain 1 and if the damping coefficient is restricted to be greater than $1/\sqrt{2}$, this is fulfilled. Additionally, to have only one break point of the filter, defined by T_f, an upper limit of 1 is set on ζ, giving the following constraint in the optimization problem,

$$\zeta \in \left[1/\sqrt{2},\ 1\right].$$

2) Time constant T_f: The time constant of the measurement filter must be smaller than the inverse of the largest modulus of the controller zeros. That is, filtering is only present at higher frequencies than e.g., the derivative action start frequency of a PID controller. Since the system is sampled, the filter cut-off frequency must be lower than the Nyquist frequency. Thus, the filter time constant is restrained to be in the interval

$$T_f \in \left[\frac{h}{\pi},\ T_i\right], \text{ or} \tag{6a}$$

$$T_f \in \left[\frac{h}{\pi},\ \left|\frac{1}{2T_d} + \sqrt{\frac{1}{4T_d^2} - \frac{1}{T_iT_d}}\right|^{-1}\right], \tag{6b}$$

for PI and PID, respectively. The upper limits are derived from continuous time controllers and hold approximately for discrete time versions if high enough sampling rate.

C. General Process for Youla Parametrization

The considered process P in Figure 1, with control signal u and load disturbance d, process output f, and measured output y, may be written in state-space form as

$$\begin{aligned}
\mathbf{x}_{k+1} &= \mathbf{A}\mathbf{x}_k + \mathbf{B}u_k + \mathbf{B}d_k\\
f_k &= \mathbf{C}\mathbf{x}_k + \mathbf{D}u_k + \mathbf{D}d_k\\
y_k &= f_k + n_k.
\end{aligned}$$

Letting $u_k^d = u_k + d_k$, we can define the general signals

– Control input: u_k
– Exogenous inputs: $\mathbf{w}_k = \begin{bmatrix} d_k & n_k \end{bmatrix}^T$
– Controlled outputs: $\mathbf{z}_k = \begin{bmatrix} f_k & u_k^d & \Delta u_k \end{bmatrix}^T$
– Measured output: y_k

which gives the following generalized process G, using the state vector $\mathbf{x}_{k+1}^e = \begin{bmatrix} \mathbf{x}_{k+1}^T & u_k \end{bmatrix}^T$,

$$\mathbf{x}_{k+1}^e = \begin{bmatrix} \mathbf{A} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix}\mathbf{x}_k^e + \begin{bmatrix} \mathbf{B} & \mathbf{0} & \mathbf{B} \\ 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} d_k \\ n_k \\ u_k \end{bmatrix}$$

$$\begin{bmatrix} \mathbf{z_k} \\ y_k \end{bmatrix} = \begin{bmatrix} \mathbf{C} & 0 \\ \mathbf{0} & 0 \\ \mathbf{0} & -1 \\ \mathbf{C} & 0 \end{bmatrix}\mathbf{x}_k^e + \begin{bmatrix} \mathbf{D} & 0 & \mathbf{D} \\ 1 & 0 & 1 \\ 0 & 0 & 1 \\ \mathbf{D} & 1 & \mathbf{D} \end{bmatrix}\begin{bmatrix} d_k \\ n_k \\ u_k \end{bmatrix}.$$

The closed loop transfer function from $\mathbf{w}_k$ to $\mathbf{z}_k$ is then

$$H_{\mathbf{zw}} = \begin{bmatrix} PS & -T \\ S & -KS \\ -\Delta_z T & -\Delta_z KS \end{bmatrix},$$

containing all relevant transfer functions for stability analysis and to solve the optimization problem in Eq. (5). $H_{\mathbf{zw}}$ may be used directly in QTool, see [14], by associating the objective cost in time domain and constraints in frequency domain with the corresponding matrix elements. The resulting optimization problem is solved using Yalmip [16] and SeDuMi [17] with built-in functions in QTool [14].

V. Process Batch

As pointed out in [3], PI(D) control is not suitable for all processes. An appropriate process batch was given with process dynamics ranging from first to eighth order, with or without time delays. The batch includes integrating, non-oscillative, and oscillative processes as well as processes with non-minimum phase zeros. A subset of the batch has been used in e.g., [7] and [8] to evaluate four parameter designs for PID. The optimization problem posed in section IV-A has been solved for the batch for PI and PID control with different filter orders and for Youla parametrized controllers.

VI. Performance Comparison

The optimization problem for PI(D) and measurement filter is solved using the Optimization Toolbox™, Control System Toolbox™ and Simulink® in MATLAB®, see [18], [19] and [20]. Two processes from the batch serve as examples, using sampling time $h = 0.02$ and the constraints

$$M_S = M_T = 1.4,\ \eta = 1,$$
$$\alpha = 15\% \ \text{ yielding } \ 1/\lambda_{\alpha/2} = 0.97.$$

With load steps of amplitude 1 and measurement noise with standard deviation of 0.025, the performances of the different feedback designs will be shown.

A. *Example I:* $P_1(s) = 1/((s+1)(0.5s+1))$

Optimization results for P_1, i.e., IAE, constraint function values and controller parameters, can be found in Table I and Figure 3 shows load disturbance and noise responses. For low order processes such as P_1, a PI controller is often considered sufficient. With no filter action, K_C is the high frequency gain, yielding it sensitive to the constraint in Eq. (5e). However, adding filter action of order 1, the inverse of the integral gain, i.e., T_i/K_C, a good estimate of IAE for closed loop systems with essentially monotone load step response, may be decreased. This effect is also seen in Figure 4, showing the feedback transfer functions, where the small filter time constant makes the amplitude curve drop at high frequencies. The phase tends to $-180°$ due to the filter sampling. Increasing noise filter order to 2 has small effect on the responses, see figures 3 and 4. The optimal second order filter has damping $\zeta = 1/\sqrt{2}$, i.e., as low as possible,

TABLE I
Optimal controllers for $P_1(s) = 1/\left((s+1)(0.5s+1)\right)$.

Type$_{\text{order}}$	IAE	$\lVert S\rVert_\infty$	$\lVert T\rVert_\infty$	$\lVert KS\rVert_2$	$\lVert\Delta_z KS\rVert_2$
PI$_0$	1.15	1.40	1.10	0.70	0.97
PI$_1$	0.77	1.40	1.04	0.91	0.97
PI$_2$	0.76	1.40	1.05	1.00	0.97
PID$_1$	0.77	1.40	1.04	0.91	0.97
PID$_2$	0.55	1.40	1.13	1.00	0.38
PID$_3$	0.59	1.40	1.12	1.00	0.30
Youla	0.47	1.40	1.17	1.00	0.43

	K_C	T_i	T_d	T_f	ζ
PI$_0$	0.69	0.65	–	–	–
PI$_1$	1.38	1.07	–	0.022	–
PI$_2$	1.39	1.05	–	0.012	0.71
PID$_1$	1.38	1.07	0	0.022	–
PID$_2$	2.11	1.06	0.27	0.150	0.71
PID$_3$	2.06	1.11	0.29	0.087	0.98

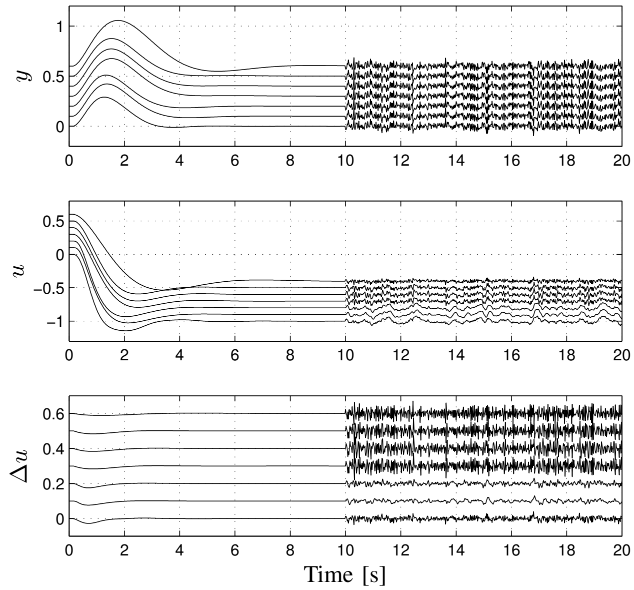

Fig. 3. Top to bottom, responses for (indices denote filter order): PI$_0$, PI$_1$, PI$_2$, PID$_1$ (PI$_1$), PID$_2$, PID$_3$, and Youla param. controller at step load disturbance and measurement noise for $P_1(s)$. Upper: Output y. Middle: Control signal u. Lower: Inter-sample control signal differences Δu. Biases with steps of 0.1 have been added for separation.

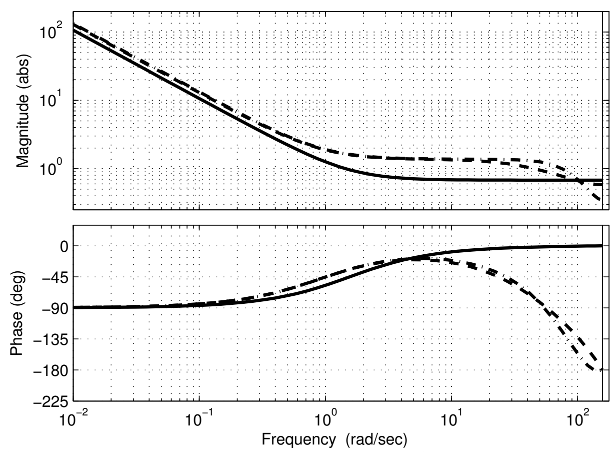

Fig. 4. Optimal PI controllers and measurement filters (indices denote filter order), PI$_0$ (–), PI$_1$ (- -) and PI$_2$ (-·-) for $P_1(s)$.

which has a more distinct cut-off than a first order filter. Due to the increased roll-off, the filter time constant may be halved compared to first order filter, still fulfilling the high frequency emphasizing constraint in Eq. (5e). However, this increases the constraint function in Eq. (5d), see Table I.

For PID control, with a first order filter, i.e., no roll-off, and a $T_d > 0$, T_i/K_C has to be increased a considerable amount compared to a PI with first order filter to fulfill the control signal constraints. This is due to the additional constraint in Eq. (6b), yielding that with T_d close to 0 and first order filter, the feedback will essentially be a PI without any filter action. For a first order filter, it is thus optimal to choose $T_d = 0$, recovering a PI controller while using the constraint in Eq. (6a) instead. However, increasing filter order, i.e., roll-off in the feedback, derivative action may be allowed, increasing performance significantly, see Table I and Figure 3. The optimal second order filter has $\zeta = 1/\sqrt{2}$, thus trying to save as much of the phase and gain at mid-frequencies as possible. However,

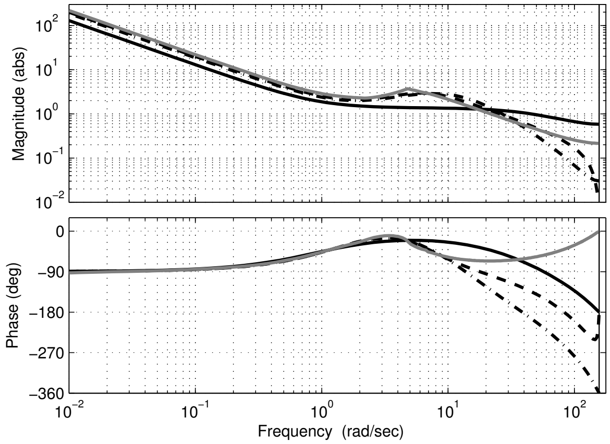

Fig. 5. Optimal PID controllers and measurement filters (indices denote filter order), PID_1 (–) (PI_1), PID_2 (- -), and PID_3 (-·-) and Youla parametrized controller (grey) for $P_1(s)$.

the increased mid-frequency gain requires larger filter time constant, $T_f = 0.150$, such that enough attenuation is given at high frequencies to hold the control signal constraints. This implies that control effort is shifted towards lower frequencies, i.e., the control signal energy constraint is active instead of the control signal inter-sample difference, see Table I and responses in Figure 3. This decreases e.g., wear on actuators. Third order filter decreases the inter-sample differences further, but to the cost of increased IAE. The damping of the optimal third order filter is large and a smaller filter time constant is possible due to the higher roll-off.

To evaluate PI(D) control performance, a Youla parametrized controller with $N = 1000$, corresponding to a 20 s. long FIR filter, was designed. Performance results can be found in Table I and Figure 3 and controller transfer function is shown in Figure 5. There are strong similarities between PID control with filter order higher than 1 and the Youla controller. The same constraints are active and the magnitude of the feedback transfer function has the same characteristics. Due to the high order, the Youla controller is able to give a higher peak and phase advance at mid-frequencies and also phase advance at high frequencies. This contributes to the only 15% better IAE value than the optimal PID with second order filter, showing that PID is close to optimal for this process when control signal and robustness constraints are set.

B. Example II: $P_2(s) = 1/(s+1)^4$

Increased complexity of the process, comparing P_1 and P_2, often requires increased complexity of the controller for good performance. PI and PID controllers and a Youla parametrized controller were designed for P_2 and the results are found in Table II while step and noise responses are seen in Figure 6. Pure PI control is not able to be sufficiently aggressive for the control signal constraints to be active and hence, adding a measurement filter will not increase performance. However, adding derivative gain and a first order filter and thus increasing controller complexity, decreases IAE but at the same time increases noise sensitivity seen by the active constraint of inter-sample control signal amplitude. Compared to P_1, it is however possible to have PID control with a first order measurement filter, although

TABLE II

OPTIMAL CONTROLLERS FOR $P_2(s) = 1/(s+1)^4$.

$\mathrm{Type}_{\mathrm{order}}$	IAE	$\|S\|_\infty$	$\|T\|_\infty$	$\|KS\|_2$	$\|\Delta_z KS\|_2$
$\mathrm{PI}_{0,1,2}$	5.24	1.40	1.00	0.43	0.61
PID_1	4.45	1.40	1.00	0.68	0.97
PID_2	3.05	1.40	1.04	1.00	0.37
PID_3	3.13	1.40	1.04	1.00	0.21
Youla	2.34	1.40	1.04	1.00	0.31
	K_C	T_i	T_d	T_f	ζ
$\mathrm{PI}_{0,1,2}$	0.43	2.26	–	–	–
PID_1	0.80	3.41	1.16	1.37	–
PID_2	0.95	2.44	1.19	0.21	0.71
PID_3	0.93	2.46	1.20	0.14	0.71

the derivative gain is small since the filter cancels much of the gain, see Figure 7 for the feedback transfer functions. The phase of the feedback at high frequencies tend to $-180°$ due to sampling effects. The upper limit on T_f from Eq. (6b) is approximately 2, which the optimal value is close to. With orders 2 and 3 of the filter, a larger derivative action may be used since the filters are able to decrease feedback gain at high frequencies. The optimal filters have low damping, $\zeta = 1/\sqrt{2}$ and significantly smaller T_f, 0.21 and 0.14, respectively. This yields as much as possible of the derivative phase advance and gain can be used to increase performance, as seen in Table II, which also shows that control action is shifted to lower frequencies compared to a first order filter. This effect is also seen in figures 6 and 7.

The Youla parametrized controller with $N = 1200$ has again the same magnitude characteristics as a PID with higher order filter, see Figure 7. It is able to give larger amplification and phase advance at mid-frequencies than a derivative part due to its high order, which yields a 23% better performance than the optimal PID controller. Optimizations without the lower bound on ζ have been performed, yielding the resulting measurement filter to give a peak to the feedback similar to the Youla parametrized controller and results in [10]. However, in this case, the filter is more than a noise attenuating filter.

VII. GENERAL RESULTS AND CONCLUSIONS

For low order simple processes, e.g., first and well damped second order dynamics, PI control is sufficiently aggressive for at least one of the control signal constraints to be active. It has also been noted that, in general, if no filter is used, the control signal inter-sample constraint is active while control signal amplitude constraint is far from active. Adding a filter increases performance significantly and yields more control signal energy in mid-frequencies, increasing the control signal amplitude constraint function, as was seen for P_1. However, for higher order and oscillative processes, PI control will in general not give active control signal constraints if not set very hard. It has also been noted that the performance difference between no filter and first order filter for PI control is significant when control signal constraints are active, while the difference between first and second order filter is small, which was seen for P_1.

PID control, with its derivative action, is for the process batch in general able to have at least one of the control

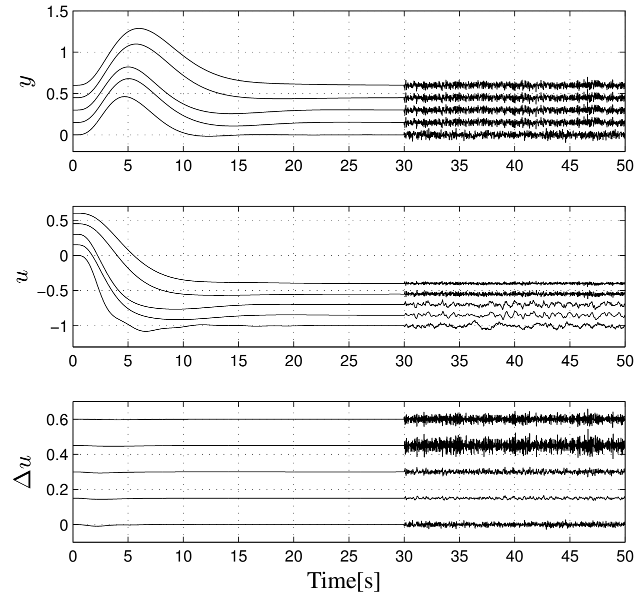

Fig. 6. Top to bottom, responses for (indices denote filter order): $\mathrm{PI}_{0,1,2}$, PID_1, PID_2, PID_3, and Youla parametrized controller at step load disturbance and measurement noise for $P_2(s)$. Upper: Output y. Middle: Control signal u. Lower: Inter-sample control signal differences Δu. Biases with steps of 0.15 have been added for separation.

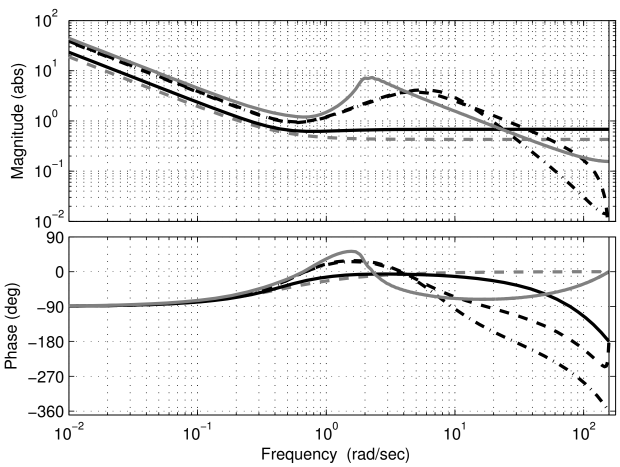

Fig. 7. Optimal PI(D) controllers and measurement filters (indices denote filter order), $\mathrm{PI}_{0,1,2}$ (gray - -), PID_1 (–), PID_2 (- -) and PID_3 (-·-) and Youla parametrized controller (grey –) for $P_2(s)$.

signal constraints active due to the derivative action. A PID controller with a first order filter often have very small or no derivative action since the filter cancels it to hold the control signal constraints. When using second or third order filters, which in general has as low damping as possible to save phase advance and gain, performance is increased. Due to the roll-off, smaller filter time constants may be used, noise sensitivity is decreased, and control signal energy is shifted to mid-frequencies where it is less harmful for actuators. The difference between second and third order filters is however slight. This filter effect was seen in both examples.

Youla parametrized controllers, that due to high orders have the ability to choose the most important frequencies in the feedback, emphasizes the importance of increased gain and phase advance at mid frequencies with a strong peak and roll-off in the feedback, as shown in the examples. For the constraints set in section IV-A and considered processes without large time delays, the magnitude of the Youla parametrized controller is very similar to a PID controller with roll-off apart from having a slightly more defined peak at mid-frequencies, compare to [10]. However, this is not realizable by the PI(D) controllers due to the constraint on ζ. Processes with large time delays yield the Youla parametrized controllers to give feedback similar to dead-time compensating control.

Numerical values of α and η are application dependent and the values set in this paper may be used as starting point. As seen from the simulations, the control signal inter-sample amplitude constraint will limit e.g., wear on actuators, and together with the characteristics of Youla parametrized controllers, it is concluded that measurement filters should be chosen such that roll-off is present in the feedback.

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