2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Globally Asymptotically Stable Formation Control of Three Agents

Qin Wang, Yu-Ping Tian, and Yao-Jin Xu

Abstract— **This paper considers the formation-shape control of three agents moving in the plane. By adding an adaptive vector perturbation to any agent's movement direction, a novel control strategy is proposed. It is shown that the proposed novel bidirectional control law can not only guarantee the global asymptotical stability of the desired formation shape, but also ensure the collision avoidance of agents between each other. Simulation results are provided to illustrate the effectiveness of the control algorithm.**

Keywords: formation control; multi-agents system; global asymptotic stability

I. INTRODUCTION

Formation control of multi-robot networks is an area of ongoing research in control systems. Formation problems are particularly interesting due to their broad range of applications in teams of UAVs performing military reconnaissance and surveillance missions in hostile environments, satellite formations for high-resolution Earth and deep-space imaging, and submarine swarms for oceanic exploration and mapping. A fundamental task for multi- agent formation is formation shape control.

In this paper, the desired formation shape is controlled by maintaining inter-agent relative distances. Works in this framework have been studied in the context of graph rigidity where a series of results have appeared in recent literature [1]-[5]. Olfati-Saber and Murray [1] showed that proposed control algorithm based on the inter-agents potential only have local validity for small perturbations around the desired formation due to multiple equilibria in the designed nonlinear system [6], and the stability analysis of [1] is not rigorous. Motivated by [1], in recent paper [4], [7], [8], Krick *et al.* [4] provided a complete analysis showing that the crucial property to achieve local stability is that the graph corresponding to the target formation be infinitesimally rigid. Under the assumption of infinitesimal rigidity, the set of equilibria of the gradient dynamics corresponding to the target formation becomes a three-dimensional equilibrium manifold. Dimarogonas and Johansson [7], [8] examined the stabilization issue for distance-based formations, and they stated that the multi-agent system is globally stable with respect to the desired formation with negative gradient control laws if and only if the formation graph is a tree. Cao *et al.* [9] designed a gradient-like control law which could cause any initially non-collinear triangular formation to converge exponentially fast to a desired triangular formation. Therefore, the global asymptotical stability of the desired formation shape remains to be a challenging open problem. For example, Summersy *et al.* [10] investigated the undesired formation-shape of four agents with complete graph. They showed that under certain acuteness conditions on the desired formation shape, any possible undesired equilibrium shape is unstable, thereby the desired shape is almost globally asymptotically stable.

This work is supported by National Nature Science Foundation under grants 60974041, 60934006 and Research Fund for the Doctoral Program of Higher Education of China under grant 20090092110021.

All authors are with School of Automation, Southeast University, 2 Sipailou Road, Nanjing, 210096, P.R. China (E-mail:wangqinyzu@126.com, yptian@seu.edu.cn, xuyaojin2003@163.com)

In this paper, we study the undirected triangle formation control problem. By adding a constant vector perturbation to any agent's movement direction, a novel control strategy is proposed. Based on the proposed controller, the collinear set discussed in [4], [9] is not an invariant manifold. Furthermore, we demonstrate that the desired formation is globally asymptotically stable, and the collision between each agent can be avoided, too.

The rest of the paper is organized as follows: the problem statement is given in Section 2. The control law and stability analysis are presented in Section 3. Simulation is included in Section 4 and the results are summarized in Section 5.

II. PROBLEM STATEMENT

We consider a formation comprising three agents, each agent is described by a single integrator model:

$$\dot{r}_i = v_i, \tag{1}$$

where $r_i = [r_{xi}, r_{yi}]^T$ denotes the position of agent i, $v_i = [v_{xi}, v_{yi}]^T$ denotes the velocity input of agent i, $i = 1, 2, 3$; let $r = [r_1^T, r_2^T, r_3^T]^T$, $v = [v_1^T, v_2^T, v_3^T]^T$, $r_{ij} = r_i - r_j$.

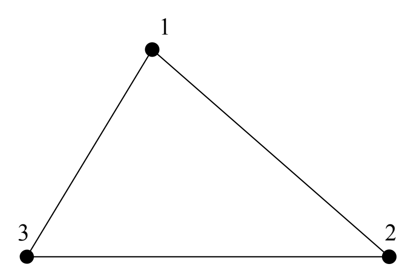

Fig. 1. Formation consisting of three agents

Figure 1 illustrates an undirected formation in the plane consisting of three mobile autonomous agents labeled 1, 2, 3. Each agent can only communicate with a specific subset $N_i \subset N$. By convention, $i \notin N_i$. The desired formation can be encoded in terms of an undirected graph, from now on called the formation graph $G = (N, E)$, whose set of vertices $N = \{1, 2, 3\}$ is indexed by the team members,

978-1-4577-0079-8/11/$26.00 ©2011 AACC

and whose set of edges $E = \{(i,j) \in N \times N | \; j \in N_i\}$ contains pairs of vertices that represent inter-agent formation specifications. Each edge $(i,j) \in E$ is assigned to a scalar parameter $d_{ij} = d_{ji}$, representing the desired distance which agents i, j should converge to. Here, d_{ij}, $(i,j) \in E$ is a positive constant. Denote by $\alpha_{ij} = ||r_{ij}||^2$ the distance of any pair of agents in the group. The formation potential function between agents i and j with $j \in N_i$ is defined as in references [7] and [8]:

$$V_{ij} = \frac{(\alpha_{ij} - d_{ij}^2)^2}{\alpha_{ij}}, \quad V_i = \sum_{j \in N_i} V_{ij}(\alpha_{ij}). \tag{2}$$

Obviously, V_i is zero if all the neighbors are located apart from agent i by the distance required by the desired formation, and goes to infinity if any of the neighbors approaches agent i with zero-distance. We also define

$$\rho_{ij} \triangleq \frac{\partial V_{ij}(\alpha_{ij})}{\partial \alpha_{ij}} = \frac{\alpha_{ij}^2 - d_{ij}^4}{\alpha_{ij}^2}. \tag{3}$$

Note that $\rho_{ij} = \rho_{ji}$, $j \in N_i$.

The formation control problem is to design a distance-based undirected control law

$$v_i = h_i(r_{ij}), \quad j \in N_i,$$

such that for any initial condition $r_i(0) \in R^2, i = 1, 2, 3,$ three agents can achieve the globally asymptotically stable formation, i.e.

$$\lim_{t \to \infty} (||r_{ij}|| - d_{ij}) = 0, \quad j \in N_i,$$

and no collision happens between each two agents, or there does not exist a time $t = t_1 > 0$ so that

$$||r_{ij}(t)|| = 0,$$

where $(i,j) \in E$.

Remark 1: Compared to the exiting works [3], [9] where the desired distances between agents have to satisfy the triangle inequality, our approach doesn't require the condition. In other words, the desired formation-shape in this paper can be a line or a triangle. Formation control of agents moving in a line has important application in practice such as small satellites formation SAR in Line to observe earth, so the achievement of line formation also has practical significance.

III. CONTROLLER DESIGN AND STABILITY ANALYSIS

A. Gradient Method

Early work with formation shape control includes [4], [7]-[9]. They all proposed a negative gradient control algorithm for the formation control problem. The control law was as follows:

$$v_i = -\nabla_{r_i} V_i = -\sum_{j \in N_i} \nabla_{r_i} V_{ij}(||r_{ij}||), \; i = 1, 2, \cdots, N, \tag{4}$$

where $V_{ij}(r_{ij})$ is a suitable formation potential function between agents i and j.

In [9], Cao *et al.* proposed a directed triangle formation control scheme. Under the proposed control law (4), the complete set of equilibrium points of the overall system is:

$$E = E_1 \cup M, \tag{5}$$

where $E_1 = \{r \,|\, ||r_{ij}|| = d_{ij}\}$ is the desired equilibrium points set, $M = \{r \,|\, r \in L, r_{12}\rho_{12} = r_{23}\rho_{23} = r_{31}\rho_{31}\}$ denotes the set that three agents positions are collinear and move at the same velocity, $L = \{r | \, \text{rank}[r_{12} \, r_{23} \, r_{31}] < 2\}$ is the set corresponding to three agent positions in the plane which are collinear.

It is shown in [9] that when the three agents are initially collinear, they always remain collinear forever and the formation can't converge to the desired triangle formation.

In [4], Krick *et al.* designed an n-agent undirected formation control law based on the negative gradient control algorithm, too. When the number of agents is 3, the complete set of equilibrium points of the overall system is the same as equation (5), but M denotes the set that three agents positions are collinear and stationary. However, when the the number of agents is more than 3, in addition to the unexpected set M, there exists other unexpected equilibrium points set.

In [7], [8], Dimarogonas and Johansson showed that if the formation graph is a tree, the set $E_1 = E$ is the unique desired equilibria set of the overall system. When the formation graph contains cycle, it is not a tree and the desired formation is thus not globally stable due to multiple equilibria in the designed nonlinear system.

Therefore, when the formation graph contains cycle, how to design a global stabilizer is a challenging and meaningful work.

B. Design of Global Stabilizer

In order to achieve the globally asymptotically stable formation, we design a novel undirected formation control strategy based on relative positions by adding a constant vector perturbation to any agent's movement direction. The control law v_i is as follows:

$$\begin{aligned} v_1 &= -\nabla_{r_1} V_1 - 2k_{12}\rho_{12}a - 2|k'_{12}\rho_{12}|s \\ &= -2(r_{12} + k_{12}a)\rho_{12} - 2r_{13}\rho_{13} - 2s|k'_{12}\rho_{12}|, \end{aligned} \tag{6}$$

$$v_2 = -\nabla_{r_2} V_2 = -2r_{23}\rho_{23} - 2r_{21}\rho_{21}, \tag{7}$$

$$v_3 = -\nabla_{r_3} V_3 = -2r_{31}\rho_{31} - 2r_{32}\rho_{32}, \tag{8}$$

where a is a unit constant vector perturbation added to the movement direction of agent 1, $0 < k_{12} < k'_{12}$, $\nabla_{r_1} V_1 = \left[\frac{\partial V_1}{\partial r_{x1}} \; \frac{\partial V_1}{\partial r_{y1}} \right]^T$, $s = [\text{sgn}(\nabla_{r_1} V_1)_x \; \text{sgn}(\nabla_{r_1} V_1)_y]^T$.

Under the controllers (6)-(8), because of the non-zero constant vector perturbation, the collinear set L discussed in [9] is not an invariant manifold. In other words, even if the agents are initially collinear, they will not remain collinear forever.

To understand why L is not invariant, first note that for any two vectors $p, q \in R^2$, $\det[p \; q] = p^T G q$, where

$$G = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}.$$

From this fact and $r_{12}+r_{23}+r_{31}=0$ it follows that

$$\det[r_{12}\ r_{23}] = -\det[r_{12}\ r_{31}] = -\det[r_{23}\ r_{31}].$$

Hence, the definition of L imply that

$$L = \{r_{ij}|\ \det[r_{12}\ r_{23}] = 0\}.$$

But along with (6)-(8), we have

$$\begin{array}{l}\frac{d}{dt}\det[r_{12}\ r_{23}] = \frac{d}{dt}(r_{12}^T G r_{23})\\ = -4(\rho_{12}+\rho_{23}+\rho_{31})\det[r_{12}\ r_{23}]\\ -2k_{12}\rho_{12}(a+s\mathrm{sgn}(k_{12}\rho_{12}))^T G r_{23}.\end{array}$$

Thus, if $\det[r_{12}\ \ r_{23}] = 0$ at $t=0$, $\det[r_{12}\ \ r_{23}]$ isn't identically equal to zero for any $t>0$ because of $a + s\mathrm{sgn}(k_{12}\rho_{12}) \neq 0$. Therefore L is not invariant as claimed.

Furthermore, observe that the equilibrium points of the overall system are those values of the r for which

$$v = -2R^T\rho = 0, \tag{9}$$

where

$$\rho = [\rho_{12}\ \rho_{13}\ \rho_{23}]^T,$$

$$R^T = \begin{bmatrix} r_{12}+k_{12}a+k'_{12}\mathrm{sgn}(\rho_{12})s & r_{13} & 0\\ r_{21} & 0 & r_{23}\\ 0 & r_{31} & r_{32}\end{bmatrix}.$$

Remark 2: Under the existing controllers based on negative gradient algorithm [4], the overall system can also be written as the equation (9), and the matrix R is the rigidity matrix discussed in [5]. When the three agents are collinear, the rank of R is 2, then $v=0$, we can't have $\rho=0$, every point in the manifold E is an equilibrium point of the overall systems, and the undesired equilibrium points of the overall system exist.

Then we analyze the rank of matrix R under the proposed controllers (6)-(8), it is crucial in the stability analysis.

Let $R_1 = [r_{12}^T + k_{12}a + k'_{12}\mathrm{sgn}(\rho_{12})s\ \ \ r_{21}^T\ \ \ 0]^T$, $R_2 = [0\ \ \ r_{23}^T\ \ \ r_{32}^T]^T$, $R_3 = [r_{13}^T\ \ \ 0\ \ \ r_{31}^T]^T$. Because $k_{12}a + k'_{12}\mathrm{sgn}(\rho_{12})s$ isn't equal to zero and its value is related to state, it is obvious that vectors R_1, R_2 and R_3 are not associated with each other. Therefore, whether the three agents are collinear or not, the rank of matrix R is 3, and it is a full row-rank matrix. From the equation (9), we have $\rho=0$, which implies every point in the manifold E_1 is an equilibrium point of the overall systems.

Later in the paper it will be shown that the converse is also true. In other words, the complete equilibria set of the overall system is E_1 which is the unique desired equilibria set.

C. Stability Analysis

In this section we show that under the proposed gradient control with perturbation, the equilibrium corresponding to the desired formation is unique and globally asymptotically stable.

Theorem 1: Assume that the system (1) evolves under the control law (6)-(8), with the potential function V_{ij} as equation (2). Then the desired formation is globally asymptotically stable, and collision between each agent is avoided.

Proof: To present the proof of Theorem 1, we need to do some preparing work. Let the Lyapunov function candidate be chosen as the following nonnegative function:

$$V(r_{ij}(t)) = \sum_{i=1}^{3} V_i = \sum_{i=1}^{3}\sum_{j\in N_i} V_{ij}(\alpha_{ij}). \tag{10}$$

From the definition of V, we know that the function V is continuous, but not continuously differentiable everywhere due to the proposed discontinuous controller. Therefore, we introduce the LaSalles invariant principle for autonomous non-smooth systems by Shevitz and Paden [11] to analyze the stability of the overall system. And the uniqueness of solutions is guaranteed by the definition of Filippov solutions, along with the measurability assumption of $f(x)$ in [12].

Since V is smooth and hence regular, while its generalized gradient [13] is a singleton which is equal to its usual gradient everywhere in the state space: $\partial V = \nabla V = \nabla \sum_{i=1}^{3} V_i$. Due to V_i being symmetric with respect to r_{ij} and the fact that $r_{ij} = -r_{ji}$, it has

$$\nabla_{r_{ij}} V_{ij} = \nabla_{r_i} V_{ij} = -\nabla_{r_j} V_{ij}.$$

Because the proposed controller is discontinuous, we use the Theorem 2.2 in [11] to calculate the time derivative of $V(r_{ij}(t))$. Then, we have

$$\begin{aligned}\dot{V}(r_{ij}) &= 2\sum_{i=1}^{3}(\nabla_{r_i}V_i)^T\dot{r}_i\\ &\subset \sum_{i=1}^{3}(\nabla_{r_i}V_i)^T K[v_i] \qquad (11)\\ &= -2\sum_{i=1}^{3}||\nabla_{r_i}V_i||^2 - 4k_{12}\rho_{12}(\nabla_{r_1}V_1)^T a\\ &\quad -4k'_{12}|\rho_{12}|(\nabla_{r_1}V_1)^T K[s],\end{aligned}$$

where where $K[v_i]$ is called Filipov set-valued mapping, it is defined in detail in [12]. In the above analysis (11) we have used Theorem 1(7) in [14] to calculate the inclusions of the Filippov set. Since $K[\mathrm{sgn}(x)]x = |x|$ [14], the choice of equations (6)-(8) results in

$$\begin{aligned}\dot{V}(r_{ij}) &= -2\sum_{i=1}^{3}||\nabla_{r_i}V_i||^2 + 4|k_{12}\rho_{12}|(\nabla_{r_1}V_1)^T a\\ &\quad -4|k'_{12}\rho_{12}|[|(\nabla_{r_1}V_1)_x| + |(\nabla_{r_1}V_1)_y|]\\ &\le -2\sum_{i=1}^{3}||\nabla_{r_i}V_i||^2 + 4|k_{12}\rho_{12}|\cdot||\nabla_{r_1}V_1||\cdot||a||\\ &\quad -4|k'_{12}\rho_{12}|[|(\nabla_{r_1}V_1)_x| + |(\nabla_{r_1}V_1)_y|]\\ &\le -\sum_{i=1}^{3}||\nabla_{r_i}V_i||^2 \le 0, \qquad (12)\end{aligned}$$

so that the generalized derivative of V reduces to a singleton. The equation (12) implies that V is nonincreasing across the trajectories of the closed-loop system, i.e., $V(r_{ij}(t)) \leq V(r_{ij}(0))$ for all $t \geq 0$. Therefore, we have the following lemma.

Lemma 2: Consider system (1) driven by the controllers (6)-(8). Then the set $S = \{r_{ij} \mid V(r_{ij}) \leq V_0 < \infty\}$ is positively invariant for the trajectories of the closed-loop system.

Proof: The set $\{r_{ij}\}$, which makes $V(r_{ij}) \leq V_0 < \infty$, for any constant $V_0 > 0$, is closed by continuity. From equation (2) and (10), we know $||r_{ij}||$ is bounded, then the set $S = \{r_{ij} \mid V(r_{ij}(t)) \leq V_0 < \infty\}$ is compact. Moreover, since V is nonincreasing, we have that $V(r_{ij}(t)) \leq V(r_{ij}(0))$, here, $V(r_{ij}(0)) \leq V_0$. According to the definition of positively invariant set in [15], we can conclude that $S = \{r_{ij} \mid V(r_{ij}(t)) \leq V_0 < \infty\}$ is positively invariant set for the trajectories of the closed-loop system.□

The next result involves the fact that with this choice of formation potential, communicating agents do not collide and there is a minimum separation distance between them when the system starts within S:

Lemma 3: Consider system (1) driven by the controllers (6)-(8), with potential function as in (2), and starting from a set of initial conditions $S = \{r_{ij} \mid V(r_{ij}) \leq V_0 < \infty\}$. Then it holds that

$$\frac{-\sqrt{V_0}+\sqrt{V_0+4d_{ij}^2}}{2} \leq ||r_{ij}(t)|| \leq \frac{\sqrt{V_0}+\sqrt{V_0+4d_{ij}^2}}{2}, \tag{13}$$

for all $(i, j) \in E$ and all $t \geq 0$.

Proof: For any $r_{ij}(0) \in S$, the time derivative of $V(r_{ij}(t))$ remains non-positive for all $t \geq 0$, by virtue of (12). Hence $V(r_{ij}(t)) \leq V(r_{ij}(0)) \leq V_0 < \infty$ for all $t \geq 0$. Moreover, since $V(t) = \sum_{i=1}^{3} \sum_{j \in N_i} V_{ij}(\alpha_{ij})$, we have that $V_{ij}(\alpha_{ij}) \leq V_0$, so that

$$\frac{-\sqrt{V_0}+\sqrt{V_0+4d_{ij}^2}}{2} \leq ||r_{ij}(t)|| \leq \frac{\sqrt{V_0}+\sqrt{V_0+4d_{ij}^2}}{2}.$$

It is easily seen that $\frac{-\sqrt{V_0}+\sqrt{V_0+4d_{ij}^2}}{2}$ is strictly positive. Therefore, no collision happens between any two agents.□

Lemmas 2 and 3, along with the non-smooth LaSalles invariant principle [11] imply that the system converges to the largest invariant subset of the set $\Omega = \left\{ r_{ij} \,\middle|\, 0 \in \dot{V}(r_{ij}(t)) \right\}$, and all agents eventually stop at steady state.

Next, we will show that in the invariant set Ω, all the agents' velocities are equal to zero, and the complete equilibria set of the overall systems is E_1.

Since at steady state, we have

$$\dot{V}(r_{ij}(t)) = W_1 + W_2,$$

where

$$W_1 = -2\sum_{i=1}^{3} ||\nabla_{r_i} V_i||^2,$$

$$W_2 = -4|\rho_{12}|[k'_{12}(|(\nabla_{r_1} V_1)_x| + |(\nabla_{r_1} V_1)_y|) - k_{12}(\nabla_{r_1} V_1)^T a].$$

Since $W_1 \leq 0,\ W_2 \leq 0$, we have that

$$0 \in \dot{V}(r_{ij}(t)),\ \text{i.e., } \mathrm{W}_1 = 0,\ \mathrm{W}_2 = 0,$$

so that

$$\bar{v}_i = 0 \ or \ \bar{v}_i = 0,\ \rho_{12} = 0,\ i = 1, 2, 3,$$

where $\bar{v}_i = -\nabla_{r_i} V_i = -\sum_{j \in N_i} \nabla_{r_i} V_{ij}(\alpha_{ij}) = -\sum_{j \in N_i} 2\rho_{ij} r_{ij}$. Therefore, we have

$$\Omega = \{r_{ij} \mid \bar{v}_i = 0 \text{ or } \bar{v}_i = 0, \rho_{12} = 0, i = 1, 2, 3\}.$$

In Ω, the agent dynamics become the following two situations:

Case 1: $\bar{v}_i = 0, \rho_{12} = 0$.

From the equations (6)-(8), it is obvious that we have $\rho_{23} = 0,\ \rho_{31} = 0$.

Case 2: $\bar{v}_i = 0$.

Since $v_2 = v_3 = 0, v_{23} = v_2 - v_3 = 0$, we have that r_{23} is a constant vector, due to the fact that

$$v_2 = -2r_{23}\rho_{23} - 2r_{21}\rho_{21} = 0,$$

then r_{21} is a constant vector, so that $v_{21} = 0$ and $v_1 = 0$. Because $\bar{v}_i = 0$, $v = 0$, and the matrix R is a full row-rank matrix, along with the equation (9), we have $\rho = 0$, it is equal to $\rho_{12} = 0,\ \rho_{23} = 0,\ \rho_{31} = 0$.

From equation (3), we have $\alpha_{ij} = d_{ij}^2$, i.e., $||r_{ij}|| = d_{ij}$, then $\Omega = \{r_{ij} \mid ||r_{ij}|| = d_{ij}\}$, for all $(i, j) \in E$. Moreover, we deduce that no solution other than $||r_{ij}|| = d_{ij}$ can stay forever in Ω. Hence, the desired formation is globally asymptotically stable, and collision between each agent is avoided.□

IV. SIMULATIONS

In this section we provide some simulation examples to support the derived results. The equations of motion are given by (1).

Case 1: When the agents' initial state are:

$$r_1(0) = [0, 0]^T,\ r_2(0) = [1, 1]^T,\ r_3(0) = [2, 2]^T.$$

Obviously, they are initially collinear. And the desired distance between a pair of agents are $d_{12} = d_{23} = d_{31} = 3$. Under the negative gradient control law proposed in [4, 9], they remain collinear forever, and the formation can't converge to the desired one. However, when the control law is given by (6)-(8), and the constant vector perturbation added to the movement direction of agent 1 is $a = [\sin\pi/6,\ \cos\pi/6]^T$, the movement trajectories of agents and the distance $||r_{ij}||$ shown in Figures 2 and 3 demonstrate that three agents achieve the desired triangular formation. Figures 4 and 5 demonstrate that the velocity v_{xi} along the x-axis and v_{yi} along the y-axis tend to zero at the stable state.

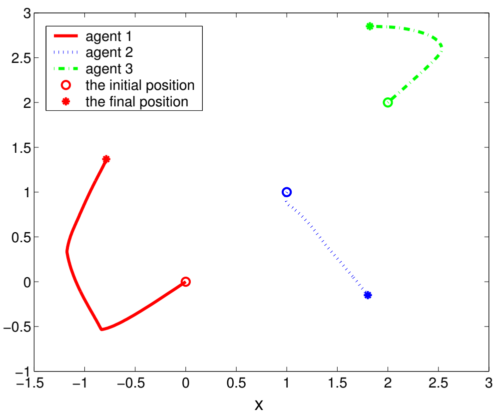

Fig. 2. Movement trajectories of agents

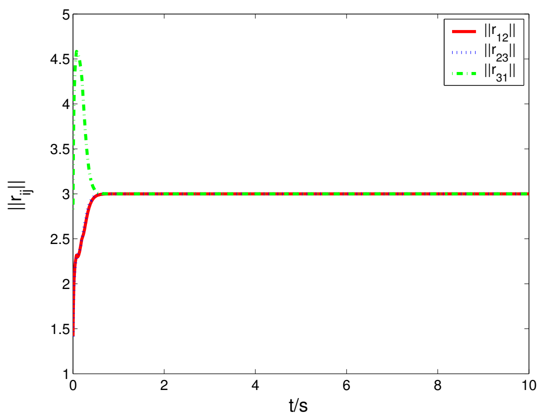

Fig. 3. Distance between any two agents $||r_{ij}||$

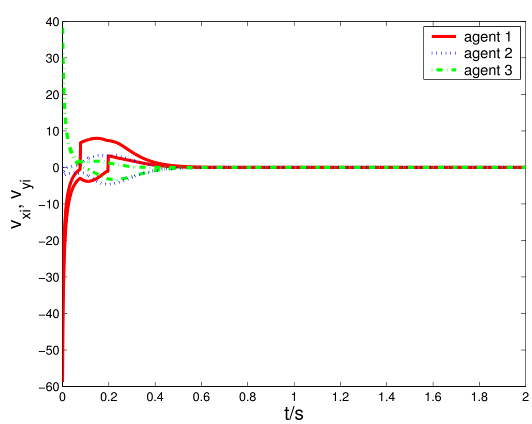

Fig. 4. Agents' velocity along the x-axis v_{xi} and the y-axis v_{yi}

Case 2: In this case, the agents' initial positions are:

$$r_1 = [0.5,\, 0.6]^T,\ r_2 = [0.7,\, 0.6]^T,\ r_3 = [0.6,\, 0.8]^T.$$

They are close enough to each other. The desired distance is $d_{12} = 1, d_{23} = 2, d_{31} = 3$. It is obvious that the desired final position of the agents are collinear. Such a collinear formation with a desired distance constraint can not be defined under the control laws proposed in the existing references, e.g., [4], [7]-[9]. Now, we apply the controller proposed in this paper, the constant vector perturbation is chosen as same as case 1. From the movement trajectories of agents in Figures 6 and the distance $||r_{ij}||$ in Figures 7, we can conclude that three agents achieve the desired collinear formation and never collide between each other. Based on Figures 8 and 9, the velocity v_{xi} along the x-axis and v_{yi} along the y-axis converge to zero at the stable state.

In summary, whether the initial states are collinear or not, the proposed control scheme can guarantee that the three agents achieve the desired formation shape, and the collision between each pair of agents is avoided.

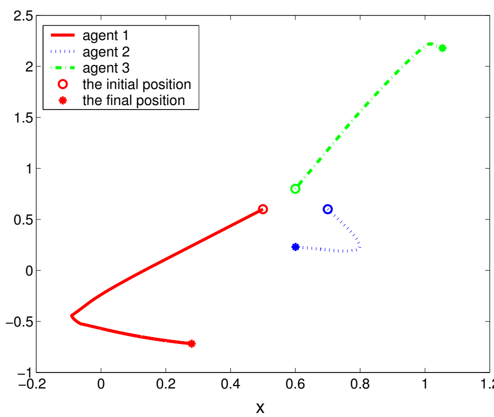

Fig. 5. Movement trajectories of agents

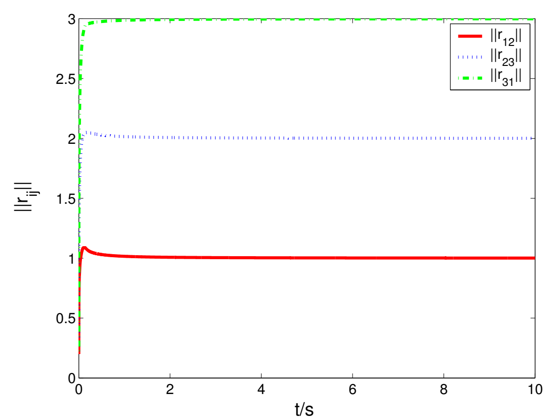

Fig. 6. Distance between any two agents $||r_{ij}||$

V. CONCLUSIONS

In this paper, we have proposed a novel control law that maintains the formation shape of three autonomous agents

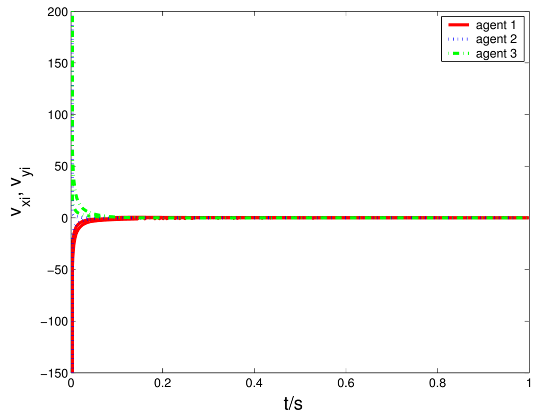

Fig. 7. Agents' velocity along the x-axis v_{xi} and the y-axis v_{yi}

in the plane. The proposed controllers can ensure the global asymptotical stability of the desired formation and collision avoidance of agents between each other. Extension of the proposed control scheme to systems with more than three agents is a very interesting and challenging problem for the future study.

REFERENCES

[1] R. Olfati-Saber and R. Murray, "Distributed cooperative control of multiple vehicle formation using structural potential functions", *in Proceedings of the 15th IFAC World Congress*, Barcelona, Spain, 2002, pp. 1-7.

[2] J. Baillieul and A. Suri, "Information patterns and hedging Brocketts theorem in controlling vehicle formations", *43rd IEEE Conference Decision and Control*, Paradise Island, Bahamas, 2004, pp. 556-563.

[3] B.D.O. Anderson, C. Yu, and S. Dasgupta, "Control of a three coleader persistent formation in the plane", *Systems and Control Letters*, vol. 56, no. 10, pp. 573-578, 2007.

[4] L. Krick, M. Broucke, and B. Francis, "Stabilization of Infinitesimally Rigid Formations of Multi-Robot Networks", *International Journal of Control*, vol. 82, no. 3, pp. 423-439, 2009.

[5] C. Yu, J. M. Hendrickx, B. Fidan, and V. Blondel, "Three and higher dimensional autonomous formation: rigidity, persistence and structural persistence", *Automatica*, vol. 43, no. 3, pp. 387-402, 2007.

[6] M. C. Gennaro, L. Iannelli, and F. Vasca, "Formation control and collision avoidance in mobile agent systems", *in Proceedings of the IEEE International Symposium on Intelligent Control and 13th Mediterranean Conference on Control and Automation*, Limassol, Cyprus, 2005, pp. 27-29.

[7] D. V. Dimarogonas, and K. H. Johansson, "On the stability of distance-based formation control", *in proceedings of the 47th IEEE Conference on Decision and Control*, Cancun, Mexico, 2008, pp. 1200-1205.

[8] D. V. Dimarogonas, and K. H. Johansson, "Further results on the stability of distance-based Multi-Robot Formations", *2009 American Control Conference*, St. Louis, MO, 2009, pp. 2972-2977.

[9] M. Cao, A. S. Morse, C. Yu, B.D.O. Anderson, and S. Dasgupta, "Controlling a triangular formation of mobile autonomous agents", *in Proceedings of the IEEE Conference on Decision and Control*, New Orleans, LA, USA, 2007, pp. 3603-3608.

[10] T. H. Summersy, C. Yu, B. D. O. Anderson,and S. Dasgupta, "Formation shape control: Global asymptotic stability of a four-agent formation", *48th IEEE Conference on Decision and Control and 28th Chinese Control Conference*, Shanghai, P.R. China, 2009, pp. 3002-3007.

[11] D. Shevitz, B. Paden. Lyapunov stability theory of nonsmooth systems. *IEEE Transactions Automatic Control*, vol. 49, no. 9, pp. 1910-1914, 1994.

[12] A. Filippov. *Differential Equations With Discontinuous Right-Hand Sides*, Norwell, MA: Kluwer, 1988.

[13] F. Clarke, *Optimization and Nonsmooth Analysis*, Reading, MA: Addison-Wesley, 1983.

[14] B. Paden, and S. S. Sastry, "A calculus for computing Filippov's differential inclusion with application to the variable structure control of robot manipulators", *IEEE Transactions on Circuits System*, vol. 34, no. 1, pp. 73-82, 1987.

[15] M. Krstic, I. Kanellakopoulos, and P. V. Kokotocic, *Nonlinear and Adaptive Control Design*, New York: Wiley, 1995, pp. 22-25.