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# One Step Prediction-Based Packet Dropout Compensation for Networked Control Systems

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***Abstract*— This paper studies the problem of network-induced delay and packet dropout compensation for continuous-time networked control systems (NCSs). By proposing the one step prediction-based packet dropout compensation method, new model for NCSs with packet dropout and network-induced long delay is presented. Then, a packet dropout compensation threshold time based Lyapunov functional is proposed, and $H_\infty$ controller design method is presented. Even for NCSs without packet dropout compensation, the obtained result is still less conservative than the existing ones. This paper proves also that some existing results can be improved by using the convex analysis method. Numerical examples are given to illustrate the merits and effectiveness of the proposed methods.**

## I. INTRODUCTION

Networked control systems are spatially distributed systems in which the system components are connected by shared communication networks. Introducing communication networks into control systems will lead to many advantages. However, it will inevitably lead to network-induced delay, packet dropout, sampling, etc., which should be taken into full consideration in NCSs analysis and design.

The problems of stability analysis and stabilization for NCSs have been paid much attention [1]–[6]. Network-induced delay and packet dropout have received increasing attentions recently. By combining network-induced delay and packet dropout into one item $\tau(t)$, [7] presented stability analysis and state feedback controller design for continuous-time NCSs. In [8], network-based $H_\infty$ output tracking performance analysis and controller design were studied. In [9], a Lyapunov-Krasovskii functional was proposed to drive some delay-dependent stability criteria. By uniformly dividing the discrete constant delay interval into multiple segments, [10] presented new discrete delay-dependent stability criteria for both retarded systems and neutral systems. Sampled-data control systems have attracted much attention from research communities [11], [12], and there have also been considerable research efforts on $H_\infty$ control for NCSs [13], [14].

For continuous-time NCSs or sampled-data control systems, define $\tau(t) = t - t_k$, then for $t \in [t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$ or $t \in [t_k,\ t_{k+1})$, the control input $u(t)$ can be written as $u(t) = Kx(t - \tau(t))$, and $u(t) = Kx(t - \tau(t))$ was used in [7], [8], [11]–[14]. If there are large number of packet dropout during $t \in [t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$ or $t \in [t_k,\ t_{k+1})$, using such control input may lead to deterioration of system performance since the problem of packet dropout compensation was not considered. The problem of network-induced delay and packet dropout compensation was studied in [15]–[17], and such compensation methods are based on discretized or discrete-time NCSs. *For continuous-time NCSs, how to compensate the negative influences of network-induced delay and packet dropout during the time interval* $[t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$ *is not studied in the literature.*

The convex analysis method was adopted in [18]–[20] to deal continuous-time systems with time-varying delay. For discrete-time NCSs with network-induced delay and packet dropout, [21] proved that the convex analysis method will help to improve some existing results, however, the problem of packet dropout compensation was not studied. *For continuous-time NCSs with packet dropout, this paper proves theoretically that some existing results can be improved by using the convex analysis method, and packet dropout compensation is taken into full consideration.*

For NCSs, the network-induced delay may be shorter than the mean delay at most time, and longer than the mean delay at least time, and packet dropout may also demonstrate such non-uniform distribution characteristic. [22]–[24] studied systems with non-uniformly distributed delay, *however, the non-uniform distribution characteristic of packet dropout was not considered. On the other hand, for NCSs with non-uniformly distributed packet dropout, the problem of network-induced delay and packet dropout compensation is not studied in the literature.*

This paper is devoted to proposing new network-induced delay and packet dropout compensation method for continuous-time NCSs. By proposing the one step prediction-based packet dropout compensation method and taking the non-uniform distribution characteristic of packet dropout into full consideration, this paper presents new model for NCSs, and new $H_\infty$ controller design method is proposed. When transferring non-linear matrix inequalities into linear matrix inequalities (LMIs), a new searching algorithm, which is proved to be less conservative, is proposed. For continuous-

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time systems with delay and packet dropout, this paper proves theoretically that some existing results can be improved by using the convex analysis method.

This paper is organized as follows. By proposing the one step prediction-based packet dropout compensation method, Section 2 presents new model for continuous-time NCSs. Section 3 is dedicated to $H_\infty$ controller design for NCSs with packet dropout compensation. Section 4 improves some existing results by using the convex analysis method. The results of numerical simulation are presented in Section 5. Conclusions are drawn in Section 6.

*Notation:* Throughout this paper, $M^T$ represents the transpose of matrix $M$. $I$ and 0 represent identity matrices and zero matrices with appropriate dimensions, respectively. $E$ stands for the expectation operation. $*$ denotes the entries of matrices implied by symmetry. Matrices, if not explicitly stated, are assumed to have appropriate dimensions.

## II. MODELLING NCSS WITH PACKET DROPOUT COMPENSATION

Consider a linear time-invariant system described by

$$\begin{cases} \dot{x}(t) = Ax(t) + B_1u(t) + B_2\omega(t) \\ z(t) = Cx(t) + Du(t) \\ x(t_0 + \tau_0) = x_0 \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^m$, $z(t) \in \mathbb{R}^r$, and $\omega(t) \in \mathbb{R}^q$ are the state vector, control input vector, controlled output, and disturbance input, respectively; $\omega(t)$ is assumed to belong to $L_2[t_0,\ \infty)$, $t_0 \geq 0$; $x_0 \in \mathbb{R}^n$ denotes the initial condition; $A$, $B_1$, $B_2$, $C$, and $D$ are known constant matrices of appropriate dimensions.

In this paper, we assume that the system (1) is controlled through a network, network-induced delay and packet dropout will occur in both sensor-to-controller and controller-to-actuator channels, $x(t)$ is online measurable, the sensor is clock-driven, while the controller and actuator are event-driven.

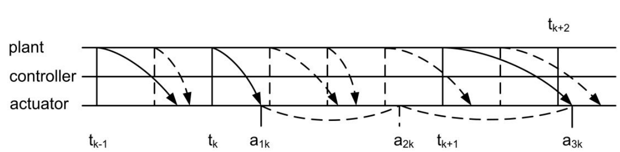


Fig. 1. NCSs with delay and packet dropout

Suppose that $h$ is the length of a sampling period, $\delta$ is a given constant, $\delta = 1,\ 2,\ \cdots$, and $\delta - 1$ is the upper bound of consecutive packet dropout. Fig. 1 depicts the transmission of control inputs for NCSs with network-induced delay and packet dropout, where the dashed lines denote that the control inputs are dropped. As shown in Fig. 1, the control inputs which are based on the plant states at the instants $t_k,\ t_{k+1},\ \cdots\ (k = 0,\ 1,\ 2,\ \cdots)$ are transmitted to the actuator successfully, while the ones which are based on the plant states between the instants $t_k$ and $t_{k+1}$ are dropped.

Let $\tau_k$ be the time from the instant $t_k$ when a sensor samples data from the plant to the instant when an actuator transmits data to the plant. Then

$$u(t) = Kx(t_k) \tag{2}$$

where $t \in [t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$, $k = 0,\ 1,\ 2,\ \cdots$, and $K$ is the state feedback controller gain which will be designed.

*Remark 1:* For continuous-time NCSs or sampled-data control systems, the control input similar to (2) was adopted in [7], [8], [11]–[14]. However, the problem of network-induced delay and packet dropout compensation was not considered. Network-induced delay and packet dropout compensation was studied in [15]–[17], and such compensation methods are based on discretized or discrete-time NCSs. *For continuous-time NCSs, how to compensate the negative influences of network-induced delay and packet dropout during the time interval* $[t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$ *is not studied in the literature.*

In the following, we will propose the one step prediction-based packet dropout compensation method, and new NCSs model will be presented.

As shown in Fig. 1, $a_{1k}$ and $a_{3k}$ are equal to $t_k + \tau_k$ and $t_{k+1} + \tau_{k+1}$, respectively. Divide the time interval $[a_{1k},\ a_{3k})$ as $[a_{1k},\ a_{2k})$ and $[a_{2k},\ a_{3k})$. Suppose $\tau_m \leq \tau_k \leq \tau_M$, then $t_{k+1} + \tau_{k+1} - t_k - \tau_k \leq \delta h + \tau_M - \tau_m$. Define $\beta = (\delta h + \tau_M - \tau_m)/2$, $\bar{\tau} = (\tau_M + \tau_m)/2$, then $\bar{\tau} + \beta = \tau_M + \frac{\delta h}{2} > \tau_k$, that is $t_k + \bar{\tau} + \beta > t_k + \tau_k$, so we choose $a_{2k} = t_k + \bar{\tau} + \beta$.

The main idea of the one step prediction-based packet dropout compensation method is: if $t \in [t_k + \tau_k,\ t_k + \bar{\tau} + \beta)$, the most recently received control input will be used; if $t \in [t_k + \bar{\tau} + \beta,\ t_{k+1} + \tau_{k+1})$, the predicted control inputs will be used. In fact, if $t_{k+1} + \tau_{k+1} < t_k + \bar{\tau} + \beta$, only the most recently received control input is adopted during the time interval $[t_k + \tau_k,\ t_{k+1} + \tau_{k+1})$.

For $t \in [t_k + \bar{\tau} + \beta,\ t_{k+1} + \tau_{k+1})$, the one step prediction-based plant state is described as

$$\hat{x}(t_k) = (\Phi + \Gamma L_0)x(t_k) \tag{3}$$

and the corresponding control input is

$$L_0\hat{x}(t_k) = L_0(\Phi + \Gamma L_0)x(t_k) \tag{4}$$

where $\Phi = e^{Ah}$, $\Gamma = \int_0^h e^{As} ds B_1$, $L_0$ is the known controller gain, and the selection criterion of $L_0$ will be discussed in Remark 3.

Then the control input $u(t)$ can be chosen as

$$u(t) = \begin{cases} Lx(t_k), & t \in [t_k + \tau_k,\ t_k + \bar{\tau} + \beta) \\ L_0\hat{x}(t_k), & t \in [t_k + \bar{\tau} + \beta,\ t_{k+1} + \tau_{k+1}) \end{cases} \tag{5}$$

where $L$ is the controller gain which will be designed in this paper.

*Remark 2:* As we can see, the one step prediction-based method is adopted in (3). In fact, $\hat{x}(t_k)$ can be achieved by using $l$ ($l = 1,\ 2,\ \cdots,\ \sigma$) steps prediction, that is $\hat{x}(t_k) = (\Phi + \Gamma L_0)^l x(t_k)$, where $\sigma = \lfloor (\bar{\tau} + \beta - \tau_m)/h \rfloor$, and $\sigma$ is the largest integer smaller than or equal to $(\bar{\tau} + \beta - \tau_m)/h$. For convenience of computation, the one step prediction-based compensation method is adopted in this paper.

Define $\tau(t) = t - t_k$, then $t_k = t - \tau(t)$ and the control input $u(t)$ in (5) is written as

$$u(t) = \begin{cases} Lx(t-\tau(t)), & \tau(t) \in [\tau_k,\ \bar{\tau}+\beta) \\ \hat{u}(t), & \tau(t) \in [\bar{\tau}+\beta,\ t_{k+1}-t_k+\tau_{k+1}) \end{cases} \tag{6}$$

where $\hat{u}(t) = L_0(\Phi+\Gamma L_0)x(t-\tau(t))$, and (6) can be written as the following uniform representation

$$u(t) = \begin{cases} Lx(t-\tau(t)), & \tau(t) \in [\tau_m,\ \bar{\tau}+\beta) \\ \hat{u}(t), & \tau(t) \in [\bar{\tau}+\beta,\ \tau_M+\delta h) \end{cases} \tag{7}$$

As shown in (7), the system will choose to use the most recently received control input or the predicted one based on $\bar{\tau}+\beta$, so $\bar{\tau}+\beta$ is named as packet dropout compensation threshold time in this paper. On the other hand, network-induced delay and packet dropout are included in $\tau(t)$, by taking the non-uniform distribution characteristic of $\tau(t)$ into full consideration, we will present a new model for NCSs with network-induced delay and packet dropout compensation.

Define $\widetilde{\Gamma}_1 = [\tau_m,\ \bar{\tau}+\beta)$, $\widetilde{\Gamma}_2 = [\bar{\tau}+\beta,\ \tau_M+\delta h)$. Suppose the probability of $\tau(t) \in \widetilde{\Gamma}_1$ is $\bar{\lambda}$, where $\bar{\lambda} \in (0,\ 1]$, then the probability of $\tau(t) \in \widetilde{\Gamma}_2$ is $1-\bar{\lambda}$, and such statistic characteristic can be described by the following formula:

$$\begin{cases} Prob\{\tau(t) \in \widetilde{\Gamma}_1\} = \bar{\lambda} \\ Prob\{\tau(t) \in \widetilde{\Gamma}_2\} = 1-\bar{\lambda} \end{cases} \tag{8}$$

Define a stochastic variable $\lambda(t)$

$$\lambda(t) = \begin{cases} 1, & \tau(t) \in \widetilde{\Gamma}_1 \\ 0, & \tau(t) \in \widetilde{\Gamma}_2 \end{cases} \tag{9}$$

By using the Bernoulli distributed white sequence to describe stochastic variable $\lambda(t)$, one gets

$$\begin{cases} Prob\{\lambda(t)=1\} = E\{\lambda(t)\} = \bar{\lambda} \\ Prob\{\lambda(t)=0\} = 1-E\{\lambda(t)\} = 1-\bar{\lambda} \end{cases} \tag{10}$$

Take the non-uniform distribution characteristic of $\tau(t)$ into consideration, then the control input $u(t)$ in (7) can be rewritten as

$$\begin{aligned} u(t) = \ & \lambda(t)Lx(t-\tau_1(t)) \\ & +(1-\lambda(t))L_0(\Phi+\Gamma L_0)x(t-\tau_2(t)) \end{aligned} \tag{11}$$

where

$$\tau_1(t) = \begin{cases} \tau(t), & \tau(t) \in \widetilde{\Gamma}_1 \\ \bar{\tau}_1, & \tau(t) \in \widetilde{\Gamma}_2 \end{cases} \tag{12}$$

$$\tau_2(t) = \begin{cases} \tau(t), & \tau(t) \in \widetilde{\Gamma}_2 \\ \bar{\tau}_2, & \tau(t) \in \widetilde{\Gamma}_1 \end{cases} \tag{13}$$

$\bar{\tau}_1$, $\bar{\tau}_2$ are constants and $\bar{\tau}_1 \in \widetilde{\Gamma}_1$, $\bar{\tau}_2 \in \widetilde{\Gamma}_2$.

Combining the original system (1) and the control input presented in (11) together, one gets the following closed-loop NCS

$$\begin{cases} \dot{x}(t) = & \phi_1(t) + (\lambda(t)-\bar{\lambda})\phi_2(t) + B_2\omega(t) \\ z(t) = & \phi_3(t) + (\lambda(t)-\bar{\lambda})\phi_4(t), \\ & t \in [t_k+\tau_k,\ t_{k+1}+\tau_{k+1}) \end{cases} \tag{14}$$

where

$$\begin{aligned} \phi_1(t) = \ & Ax(t) + \bar{\lambda}B_1Lx(t-\tau_1(t)) \\ & +(1-\bar{\lambda})B_1L_0(\Phi+\Gamma L_0)x(t-\tau_2(t)) \\ \phi_2(t) = \ & B_1[Lx(t-\tau_1(t)) - L_0(\Phi+\Gamma L_0)x(t-\tau_2(t))] \\ \phi_3(t) = \ & Cx(t) + \bar{\lambda}DLx(t-\tau_1(t)) \\ & +(1-\bar{\lambda})DL_0(\Phi+\Gamma L_0)x(t-\tau_2(t)) \\ \phi_4(t) = \ & D[Lx(t-\tau_1(t)) - L_0(\Phi+\Gamma L_0)x(t-\tau_2(t))] \end{aligned}$$

The following inequality will be used in the sequel.

*Lemma 1 [25]:* For any symmetric positive definite matrix $M \in \mathbb{R}^{n*n}$, scalars $r_1 < r_2$, a vector function $x : [r_1,\ r_2] \to \mathbb{R}^n$ such that the integrals in the following are well defined, then

$$\begin{aligned} & -(r_2-r_1)\int_{r_1}^{r_2} x^T(s)Mx(s)ds \\ & \le -\Big(\int_{r_1}^{r_2} x(s)ds\Big)^T M \Big(\int_{r_1}^{r_2} x(s)ds\Big) \end{aligned} \tag{15}$$

## III. CONTROLLER SYNTHESIS FOR NCSs WITH PACKET DROPOUT COMPENSATION

This section is devoted to proposing new controller synthesis method for NCSs with packet dropout compensation.

*Theorem 1:* For given positive scalars $\bar{\lambda}$, $h$, $\delta$, $\tau_M$, $\tau_m$ and $\mu_1$, $\mu_2$, $\mu_3$, if there exist symmetric positive definite matrices $W$, $\widetilde{Q}$, $\widetilde{Z}_1$, $\widetilde{Z}_2$, $\widetilde{Z}_3$, $\widetilde{N}_1$, $\widetilde{N}_2$, matrix $V_1$, scalar $\gamma > 0$, such that

$$\begin{bmatrix} \widetilde{\Pi}^i_{11} & \widetilde{\Pi}_{12} \\ * & \widetilde{\Pi}_{22} \end{bmatrix} < 0 \tag{16}$$

where $i = 1,\ 2,\ 3,\ 4$, $\widetilde{\Pi}^i_{11} = \widetilde{\Omega} - 2\tilde{\theta}_i$,

$$\widetilde{\Omega} = \begin{bmatrix} \widetilde{\Omega}_{11} & \widetilde{\Omega}_{12} & \widetilde{\Omega}_{13} & \widetilde{Z}_1 & 0 & 0 & B_2 \\ * & -2\widetilde{Z}_2 & 0 & \widetilde{Z}_2 & \widetilde{Z}_2 & 0 & 0 \\ * & * & -2\widetilde{Z}_3 & 0 & \widetilde{Z}_3 & \widetilde{Z}_3 & 0 \\ * & * & * & \widetilde{\Omega}_{44} & 0 & 0 & 0 \\ * & * & * & * & \widetilde{\Omega}_{55} & 0 & 0 \\ * & * & * & * & * & \widetilde{\Omega}_{66} & 0 \\ * & * & * & * & * & * & -\gamma I \end{bmatrix}$$

$$\begin{aligned} \widetilde{\Omega}_{11} =& AW + WA^T + \widetilde{Q} - \widetilde{Z}_1 \\ \widetilde{\Omega}_{12} =& \bar{\lambda}B_1V_1^T \\ \widetilde{\Omega}_{13} =& (1-\bar{\lambda})B_1L_0(\Phi+\Gamma L_0)W \\ \widetilde{\Omega}_{44} =& -\widetilde{Q} + \widetilde{N}_1 - \widetilde{Z}_1 - \widetilde{Z}_2 \\ \widetilde{\Omega}_{55} =& \widetilde{N}_2 - \widetilde{N}_1 - \widetilde{Z}_2 - \widetilde{Z}_3 \\ \widetilde{\Omega}_{66} =& -\widetilde{N}_2 - \widetilde{Z}_3 \\ \widetilde{\theta}_1 =& [0\ \ I\ \ 0\ \ 0\ \ -I\ \ 0\ \ 0]^T\widetilde{Z}_2[0\ \ I\ \ 0\ \ 0\ \ -I\ \ 0\ \ 0] \\ \widetilde{\theta}_2 =& [0\ \ -I\ \ 0\ \ I\ \ 0\ \ 0\ \ 0]^T\widetilde{Z}_2[0\ \ -I\ \ 0\ \ I\ \ 0\ \ 0\ \ 0] \\ \widetilde{\theta}_3 =& [0\ \ 0\ \ I\ \ 0\ \ 0\ \ -I\ \ 0]^T\widetilde{Z}_3[0\ \ 0\ \ I\ \ 0\ \ 0\ \ -I\ \ 0] \\ \widetilde{\theta}_4 =& [0\ \ 0\ \ -I\ \ 0\ \ I\ \ 0\ \ 0]^T\widetilde{Z}_3[0\ \ 0\ \ -I\ \ 0\ \ I\ \ 0\ \ 0] \end{aligned}$$

$$\widetilde{\Pi}_{12} = \begin{bmatrix} WA^T & WA^T & WA^T & 0 & 0 & 0 & WC^T & 0 \\ \bar{\lambda}\widetilde{\Upsilon}_1 & \bar{\lambda}\widetilde{\Upsilon}_1 & \bar{\lambda}\widetilde{\Upsilon}_1 & \widetilde{\Upsilon}_1 & \widetilde{\Upsilon}_1 & \widetilde{\Upsilon}_1 & \bar{\lambda}\widetilde{\Upsilon}_2 & \widetilde{\Upsilon}_2 \\ \widetilde{\Upsilon}_3 & \widetilde{\Upsilon}_3 & \widetilde{\Upsilon}_3 & \widetilde{\Upsilon}_4 & \widetilde{\Upsilon}_4 & \widetilde{\Upsilon}_4 & \widetilde{\Upsilon}_5 & \widetilde{\Upsilon}_6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ B_2^T & B_2^T & B_2^T & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

$$
\begin{aligned}
\widetilde{\Pi}_{22} =& diag(\mathcal{X}_1,\ \mathcal{X}_2,\ \mathcal{X}_3,\ \hat{\lambda}^{-1}\mathcal{X}_1,\ \hat{\lambda}^{-1}\mathcal{X}_2,\\
&\qquad \hat{\lambda}^{-1}\mathcal{X}_3,\ -\gamma I,\ -\hat{\lambda}^{-1}\gamma I)\\
\hat{\lambda} =& \bar{\lambda}(1-\bar{\lambda})\\
\widetilde{\Upsilon}_1 =& V_1 B_1^T\\
\widetilde{\Upsilon}_2 =& V_1 D^T\\
\widetilde{\Upsilon}_3 =& (1-\bar{\lambda})W(\Phi+\Gamma L_0)^T L_0^T B_1^T\\
\widetilde{\Upsilon}_4 =& -W(\Phi+\Gamma L_0)^T L_0^T B_1^T\\
\widetilde{\Upsilon}_5 =& (1-\bar{\lambda})W(\Phi+\Gamma L_0)^T L_0^T D^T\\
\widetilde{\Upsilon}_6 =& -W(\Phi+\Gamma L_0)^T L_0^T D^T\\
\mathcal{X}_1 =& \tau_m^{-2}(\mu_1^2\widetilde{Z}_1 - 2\mu_1 W)\\
\mathcal{X}_2 =& (\bar{\tau}+\beta-\tau_m)^{-2}(\mu_2^2\widetilde{Z}_2 - 2\mu_2 W)\\
\mathcal{X}_3 =& (\tau_M+\delta h-\bar{\tau}-\beta)^{-2}(\mu_3^2\widetilde{Z}_3 - 2\mu_3 W)
\end{aligned}
$$

then with the control law

$$
\begin{aligned}
u(t) =& \lambda(t)Lx(t-\tau_1(t)) + (1-\lambda(t))\\
&\times L_0(\Phi+\Gamma L_0)x(t-\tau_2(t)), \quad L = V_1^T W^{-1} \quad (17)
\end{aligned}
$$

the system described by (14) is asymptotically mean-square stable with an $H_\infty$ norm bound $\gamma$.

*Proof:* Let us consider the following packet dropout compensation threshold time (that is $\bar{\tau}+\beta$) based Lyapunov functional

$$
V(t) = V_1(t) + V_2(t) + V_3(t) \quad (18)
$$

where

$$
\begin{aligned}
V_1(t) =& x^T(t)Px(t)\\
V_2(t) =& \int_{t-\tau_m}^{t} x^T(s)Qx(s)ds + \int_{t-\bar{\tau}-\beta}^{t-\tau_m} x^T(s)N_1x(s)ds\\
&+ \int_{t-\tau_M-\delta h}^{t-\bar{\tau}-\beta} x^T(s)N_2x(s)ds\\
V_3(t) =& \tau_m \int_{-\tau_m}^{0}\int_{t+s}^{t} \dot{x}^T(r)Z_1\dot{x}(r)drds\\
&+ \beta_1 \int_{-\bar{\tau}-\beta}^{-\tau_m}\int_{t+s}^{t} \dot{x}^T(r)Z_2\dot{x}(r)drds\\
&+ \beta_2 \int_{-\tau_M-\delta h}^{-\bar{\tau}-\beta}\int_{t+s}^{t} \dot{x}^T(r)Z_3\dot{x}(r)drds
\end{aligned}
$$

$\beta_1 = \bar{\tau}+\beta-\tau_m$, $\beta_2 = \tau_M+\delta h-\bar{\tau}-\beta$, matrices $P$, $Q$, $N_1$, $N_2$, $Z_1$, $Z_2$, $Z_3$ are symmetric positive definite with appropriate dimensions. The rest of the proof is omitted due to page limitation. □

*Remark 3:* If the problem of packet dropout compensation is not considered, the control input presented in (11) is reduced to $u(t) = \lambda(t)Kx(t-\tau_1(t)) + (1-\lambda(t))Kx(t-\tau_2(t))$, where $K$ is the controller gain which will be designed. On the other hand, if $L_0$ in (11) is a variable, the conditions in Theorem 1 are nonlinear matrix inequalities, which are difficult to be solved, so we choose $L_0$ in (11) as $L_0 = K$, and the design of controller gain $K$ is similar to Theorem 1. Compared with the methods without packet dropout compensation, the one step prediction-based control input presented in (11) will introduce less conservatism.

*Remark 4:* The bounding inequalities similar to $-Z_i^{-1} \le P^{-1}Z_iP^{-1} - 2P^{-1}$ were used in [8] to transfer nonlinear matrix inequalities into LMIs, and the bounding inequality similar to $-Z_i^{-1} \le \mu^2P^{-1}Z_iP^{-1} - 2\mu P^{-1}$ was introduced in [4]. Obviously, if an appropriate $\mu$ is chosen, $-Z_i^{-1} \le \mu^2P^{-1}Z_iP^{-1} - 2\mu P^{-1}$ will introduce less conservatism than $-Z_i^{-1} \le P^{-1}Z_iP^{-1} - 2P^{-1}$. However, how to choose an appropriate $\mu$ was not discussed in [4]. The inequalities $-Z_i^{-1} \le \mu^2P^{-1}Z_iP^{-1} - 2\mu P^{-1}$ were adopted also in [21], and a searching algorithm was presented to choose a local optimal $\mu$, which will introduce less conservatism than the corresponding method in [4]. The inequalities $-Z_i^{-1} \le \mu_i^2P^{-1}Z_iP^{-1} - 2\mu_i P^{-1}$ ($i = 1, 2, 3$) are introduced in Theorem 1 of this paper. If $\mu_1 = \mu_2 = \mu_3 = \mu$, $-Z_i^{-1} \le \mu_i^2P^{-1}Z_iP^{-1} - 2\mu_i P^{-1}$ reduces to $-Z_i^{-1} \le \mu^2P^{-1}Z_iP^{-1} - 2\mu P^{-1}$, so if appropriate $\mu_i$ are chosen, $-Z_i^{-1} \le \mu_i^2P^{-1}Z_iP^{-1} - 2\mu_i P^{-1}$ will lead to less conservatism than the corresponding inequalities in [4], [8], [21].

The following searching algorithm describes the method of choosing appropriate $\mu_i$.

*Algorithm 1:*

*Step 1:* For given $\bar{\lambda}$, $h$, $\delta$, $\tau_M$, $\tau_m$, choose the initial values $\mu_{i,0} > 0$ ($i = 1, 2, 3$) and the final values $\mu_{i,ult} > 0$ ($\mu_{i,ult} < \mu_{i,0}$) for $\mu_i$, set appropriate step lengths $\mu_{i,dec} > 0$; choose a large enough $H_\infty$ norm bound $\gamma_{opt}$ and set $\mu_{i,opt} = \mu_{i,0}$; set $\mu_1$=$\mu_{1,0}$.

*Step 2:* Set $\mu_2$=$\mu_{2,0}$.

*Step 3:* Set $\mu_3$=$\mu_{3,0}$.

*Step 4:* Solve the LMIs presented in (16), if $\gamma < \gamma_{opt}$, set $\gamma_{opt} = \gamma$, $\mu_{i,opt} = \mu_i$ and go to step 5; otherwise, go to step 5 directly.

*Step 5:*

```
    Set μ3 = μ3 − μ3,dec;
if μ3 ≥ μ3,ult
    go to step 4
else
    μ2 = μ2 − μ2,dec
    if μ2 ≥ μ2,ult
        go to step 3
    else
        μ1 = μ1 − μ1,dec
        if μ1 ≥ μ1,ult
            go to step 2
        else
            go to step 6
        endif
    endif
endif
```

*Step 6:* Output the locally optimal $\mu_{i,opt}$ and $\gamma_{opt}$.

By using Algorithm 1, one can get the locally optimal $\mu_{i,opt}$, which will lead to less conservatism than the existing methods.

## IV. COMPARISON WITH THE EXISTING RESULTS

In this section, we will prove that some existing results can be further improved.

The nominal system of the system (1) in [9] can be described as

$$\begin{cases} \dot{x}(t) = Ax(t) + Bx(t - \tau(t)) \\ x(t) = \sigma(t), \quad t \in [-\tau_M, \ 0], \end{cases} \tag{19}$$

The following theorem improves the result of Proposition 2 in [9].

*Theorem 2:* For given scalars $\tau_m$ and $\tau_M$, the system (19) with $0 \le \tau_m \le \tau(t) \le \tau_M$ is asymptotically stable, if there exist some matrices $P > 0$, $\begin{bmatrix} Q_1 & Q_2 \\ Q_2^T & Q_3 \end{bmatrix} > 0$, $\begin{bmatrix} Q_4 & Q_5 \\ Q_5^T & Q_6 \end{bmatrix} > 0$, $R_1 > 0$, $R_2 > 0$, and $S > 0$ of appropriate dimensions such that

$$\Xi - [0 \ \ I \ \ 0 \ \ 0 \ \ 0 \ \ -I]^T S [0 \ \ I \ \ 0 \ \ 0 \ \ 0 \ \ -I] < 0 \tag{20}$$

$$\Xi - [0 \ \ -I \ \ 0 \ \ I \ \ 0 \ \ 0]^T S [0 \ \ -I \ \ 0 \ \ I \ \ 0 \ \ 0] < 0 \tag{21}$$

where $\Xi$ is the same as the one in Proposition 2 of [9].

*Proof:* By using the convex analysis method, the inequality (11) of [9] can be rewritten as

$$\begin{aligned} &-(\tau_M - \tau_m) \int_{t-\tau_M}^{t-\tau_m} \dot{x}^T(s) S \dot{x}(s) ds \\ &= -(\tau_M - \tau(t)) \int_{t-\tau(t)}^{t-\tau_m} \dot{x}^T(s) S \dot{x}(s) ds \\ &\quad - (\tau(t) - \tau_m) \int_{t-\tau(t)}^{t-\tau_m} \dot{x}^T(s) S \dot{x}(s) ds \\ &\quad - (\tau_M - \tau(t)) \int_{t-\tau_M}^{t-\tau(t)} \dot{x}^T(s) S \dot{x}(s) ds \\ &\quad - (\tau(t) - \tau_m) \int_{t-\tau_M}^{t-\tau(t)} \dot{x}^T(s) S \dot{x}(s) ds \end{aligned}$$

Define $\rho = (\tau(t) - \tau_m)/(\tau_M - \tau_m)$, then

$$-(\tau(t) - \tau_m) \int_{t-\tau_M}^{t-\tau(t)} \dot{x}^T(s) S \dot{x}(s) ds \le -\rho \varphi_5^T S \varphi_5 \tag{22}$$

where $\varphi_5 = [x(t - \tau(t)) - x(t - \tau_M)]$. Similarly, one gets

$$-(\tau_M - \tau(t)) \int_{t-\tau(t)}^{t-\tau_m} \dot{x}^T(s) S \dot{x}(s) ds \le -(1 - \rho) \varphi_6^T S \varphi_6 \tag{23}$$

where $\varphi_6 = [x(t - \tau_m) - x(t - \tau(t))]$. The rest of the proof is similar to Theorem 1 in this paper and Proposition 2 in [9], it is omitted due to page limitation. □

*Theorem 3:* If the condition of Proposition 2 in [9] is satisfied, then the conditions of Theorem 2 are also feasible.

*Proof:* From $S > 0$, one gets

$$-[0 \ \ I \ \ 0 \ \ 0 \ \ 0 \ \ -I]^T S [0 \ \ I \ \ 0 \ \ 0 \ \ 0 \ \ -I] < 0 \tag{24}$$

$$-[0 \ \ -I \ \ 0 \ \ I \ \ 0 \ \ 0]^T S [0 \ \ -I \ \ 0 \ \ I \ \ 0 \ \ 0] < 0 \tag{25}$$

if $\Xi < 0$ in Proposition 2 of [9] is satisfied, we have

$$\Xi - [0 \ \ I \ \ 0 \ \ 0 \ \ 0 \ \ -I]^T S [0 \ \ I \ \ 0 \ \ 0 \ \ 0 \ \ -I] < 0 \tag{26}$$

$$\Xi - [0 \ \ -I \ \ 0 \ \ I \ \ 0 \ \ 0]^T S [0 \ \ -I \ \ 0 \ \ I \ \ 0 \ \ 0] < 0 \tag{27}$$

that is, the conditions of Theorem 2 are satisfied, this completes the proof. □

*Remark 5:* By using the convex analysis method, it is easy to prove theoretically that the results in [14], [22] can be further improved, the detailed proof is similar to Theorem 1 and Theorem 3 in this paper, it is omitted here.

TABLE I

MATIS BASED ON DIFFERENT METHODS

| Methods | MATIs |
|---|---|
| [5] | 0.00045 |
| [1] | 0.7805 |
| [7] | 0.8871 |
| [9] | 1.0081 |
| Theorem 2 | 1.0239 |

## V. NUMERICAL EXAMPLES

*Example 1:* To illustrate the merits of the proposed packet dropout compensation method, we consider the following NCS

$$\begin{aligned} \dot{x}(t) &= \begin{bmatrix} 0 & 1 \\ 0 & -0.1 \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ 0.1 \end{bmatrix} u(t) + \begin{bmatrix} -0.2 \\ 0.1 \end{bmatrix} \omega(t) \\ z(t) &= [0.2 \ \ 0.3] x(t) + 0.5 u(t) \end{aligned} \tag{28}$$

If we choose $B_2 = C = D = 0$, the system (28) will be reduced to the one presented in [5] with the controller gain $K = [-3.75 \quad -11.5]$. For Theorem 1 of this paper, suppose the sampling period $h = 0.5s$, $\bar{\lambda} = 0.55$, $\tau_M = 0.5s$, $\tau_m = 0.1s$, $\alpha_1 = \alpha_2 = \alpha_3 = 1$. As discussed in Remark 3, if the packet dropout compensation method is not adopted, one can get the controller gain $K = [-0.0012 \ \ 0.7777]$. Choose $L_0 = K = [-0.0012 \ \ 0.7777]$ for Theorem 1, one can see that the maximum admissible number of consecutive packet dropout is 22 ($\delta = 23$), and the upper bound of $\tau(t)$ is $\tau_M + \delta h = 12s$. Using some existing stability criteria, one can get the maximum allowable transfer intervals (MATIs) that guarantee the asymptotic stability of the considered system (see Table 1).

As shown in Table 1, even for given controller gain $K = [-3.75 \quad -11.5]$, Theorem 2 is less conservative than the existing results. If the controller design and packet dropout compensation method proposed in Theorem 1 are adopted, less conservative result is obtained.

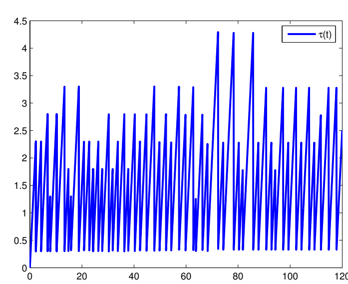


Fig. 2. Curve of $\tau(t)$

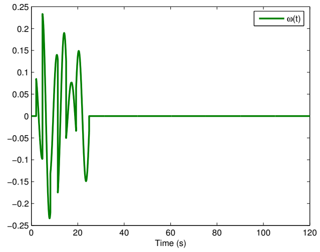


Fig. 3. Curve of disturbance input $\omega(t)$

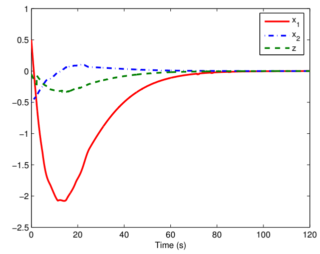


Fig. 4. Curves of plant state and controlled output

*Example 2:* This example is given to illustrate the effectiveness of the proposed compensation method. Consider the NCS presented in (28). Suppose $\bar{\lambda} = 0.8$, $h = 0.5s$, $\delta = 12$, $\tau_M = 0.5s$, $\tau_m = 0.1s$, $\alpha_1 = \alpha_2 = \alpha_3 = 1$. If the packet dropout compensation method is not adopted, by using the method similar to Theorem 1, one can get the controller gain $K = [-0.0728\ -0.6861]$. Choose $L_0 = K = [-0.0728\ -0.6861]$ for (11), then the predicted control input is $L_0(\Phi + \Gamma L_0)x(t) = [-0.0703\ \ -0.6646]x(t)$. Solve the LMIs in Theorem 1, one gets $L = [-0.0736\ \ -0.6932]$.

Suppose the initial state of the system is $x_0 = [0.5\ \ -0.5]^T$, $\tau(t)$ is given in Fig. 2, disturbance input $\omega(t)$ is presented in Fig. 3, then curves of plant state and controlled output corresponding to Theorem 1 are pictured in Fig. 4, which illustrate the effectiveness of the proposed controller design.

## VI. CONCLUSIONS

The problem of network-induced delay and packet dropout compensation for continuous-time NCSs has been studied in this paper. By proposing the one step prediction-based packet dropout compensation method, new NCSs model has been established. Combined with the non-uniform distribution characteristic of packet dropout, this paper proposes new $H_\infty$ controller design method. Some existing results are improved by using the convex analysis method. When transferring non-linear matrix inequalities into LMIs, a new searching algorithm, which is proved to be less conservative, is proposed. Numerical examples have illustrated the merits and effectiveness of the proposed methods.

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