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A Simple Tunable Method for Profile Control – Least-squares Configuration

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Abstract—**For multiple-input and multiple-output (MIMO) process control, when inputs and outputs are equal in number, a widely adopted scheme in practice is to first decouple the process using a low-frequency square pre-compensator, and then 'tune' the decoupled loops as single control loops. Within the MIMO class, there are also problems having more process variables to be controlled than the available control inputs. This is referred to as the Profile Control (PC) problem. For PC problems, pre-compensation decoupling cannot apply. Applicable methods for PC include Receding-Horizon based real-time optimization and Singular Value Decomposition (SVD) based mode-decoupling configuration. We describe in this paper a simple alternative, the least-squares (LS) configuration, which employs an intuitive post-compensator, and retains the easy-to-tune decoupling control feature. Under mild conditions, we show convergence of this scheme to the steady-state LS solution. We also discuss its robustness against output disturbances, and its flexibility to accommodate actuator range limitations.**

I. Introduction

For multiple-input and multiple-output (MIMO) process control problems with equal number of control inputs and controlled outputs, a widely adopted strategy is to first reduce the (cross-coupling) interactions between the inputs and the outputs. With reduced interactions, the modified MIMO process can often achieve the so-called Diagonal Dominance (DD) condition, under which a controller can be designed and operated as a collection of multiple parallel single control loops [1], [2]. A first step in reducing MIMO interaction is to ensure proper pairing between the control inputs and the process outputs. Physical justifications, Bristol's Relative Gain Array (RGA) [2], and Singular Value Analysis [2] are among the tools available for input-output pairing. Given a proper pairing, interactions may be further reduced by placing a square compensator in front of the process to significantly weaken the cross-couplings, thereby achieving DD. The presence of DD in the low-frequency region is often sufficient to assure process stability after closing the individual 'decoupled' loops. An often adopted further simplification is to use the inverse of the steady-state process gain matrix as the decoupling pre-compensator, making the compensator a constant matrix[1].

With reduced interactions, each 'decoupled' individual loop can usually be closed with a controller that involves integral action, such as the Proportional and Integral (PI) controller. As such, the controlled process variables can satisfy the set points at steady state, provided that the control actions are within the actuators' range. Dynamic tuning of these PI controllers is fairly straightforward, due to reduced loop interactions.

However, when it is desired to control more process variables using fewer control inputs, this class of MIMO problems is referred to as Profile Control (PC). For PC, the above described pre-compensation scheme would not work. For one, the process gain matrix would be non-square – rendering square-inverse pre-compensator inapplicable. Secondly, by manipulating a smaller number of controls, one cannot expect that the process output variables comply with arbitrary set-points. For example, by manipulating two control inputs one can only influence a two-dimensional subspace in a higher-dimensional process output space. This two-dimensional subspace is the 'Range Space' of the n x 2 steady-state process gain matrix, K, denoted by $R[K]$, where $n > 2$. Among generically applicable PC control methods, there are Receding-Horizon based real-time optimization [2], [4] and SVD-based mode-decoupling scheme [2]. Both come with varied degrees of sophistication and complexity.

In this paper, we discuss a simple alternative, based on the conventional least-squares (LS) concept. In the sense of LS, the best attainable performance with respect to the specified set-points is the projection of these set-points onto $R[K]$. Geometrically, Fig. 1 depicts this projection operation and the orthogonality property that determines the best solution.

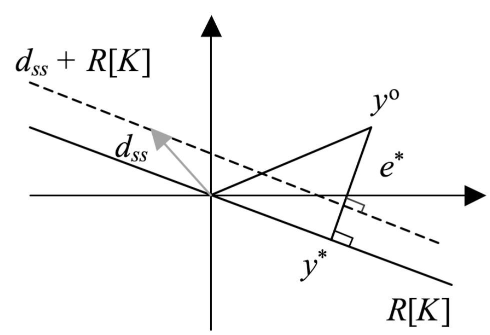

Fig. 1. Least-squares, projection and orthogonality.

Let y^o denote the specified set-points, y^* the attainable performance with LS error e^*, and let u^* be the controls that would produce y^*. Then, the orthogonality property dictates

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[1] When the process is relatively poor-conditioned, or the gain matrix is not accurately known, there are 'regularization' techniques that can reduce the risk of uncertainty-induced side effects [3].

$$(K u)^T e^* = u^T K^T e^* = 0, \qquad (1)$$

where u stands for arbitrary controls, and superscript T designates matrix/vector transposition. Since $e^* = y^o - y^*$, the above becomes $u^T K^T (y^o - K u^*) = 0$. With u being arbitrary, this requires $K^T (y^o - K u^*) = 0$, leading to the well-known static LS solution

$$\begin{aligned} u^* &= [(K^T K)^{-1} K^T]\, y^o \equiv K^+ y^o \\ y^* &= K u^* = K K^+ y^o, \end{aligned} \qquad (2)$$

where we have introduced the pseudo-inverse notation, $K^+ \equiv (K^T K)^{-1} K^T$, which maps the set-point y^o to the (static) LS control solution, u^*, and $K K^+$ is the Projection Operator mapping y^o to the attainable y^*. Fig. 2(a) describes this static LS solution with a block diagram.

A major question remains: *How do we translate this static LS solution to a feedback control solution in a dynamic process setting?* Fig. 2(b) illustrates the proposed dynamic control scheme, which can achieve steady-state LS error, while maintaining the ease of controller tuning and loop decoupling, as described in the following sections.

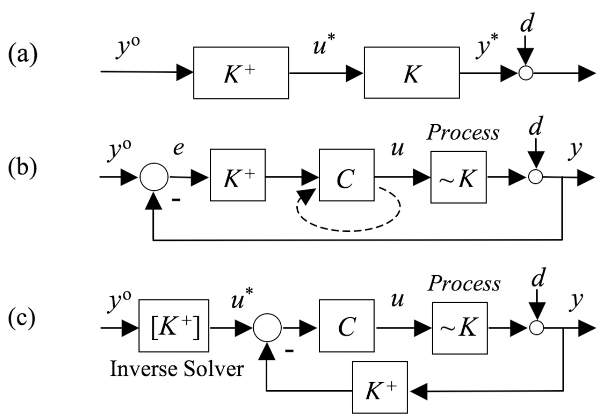

Fig. 2. LS solution and feedback control configurations.

II. STEADY-STATE LS PERFORMANCE AND ROBUSTNESS

If we assume 1) open-and-closed loop stability, 2) existence of integral-control-action (I-action) in controller, C, and 3) LS solution u^* being within the actuators' range, we can then show the convergence of the proposed dynamic feedback control to the LS steady state performance.

Since the loops are (asymptotically) stable, steady state can be reached under set-point control. At steady state, the inputs to the I-action controller, C, must be 0. That is,

$$\begin{aligned} & K^+ e_{ss} = 0 = (K^T K)^{-1} K^T e_{ss}, \\ \Rightarrow\ & K^T e_{ss} = 0 \end{aligned} \qquad (3)$$

which implies that $e_{ss} \equiv y^o - y_{ss}$ is the LS error e^*, in the absence of output disturbance. This is because $K^T e_{ss} = 0$ means that e_{ss} is orthogonal to $R[K]$, the range space of K, thereby satisfying the definition of LS solution.

When subjected to a bounded output disturbance, d, with steady-state bias, d_{ss}, the proposed control scheme has the 'smallest' steady-state error among all schemes facing the same d_{ss}. In fact, the controller will accommodate the component (of d_{ss}), $d_{ss}^{\|}$, that lies in $R[K]$, even though 'blinded' to the component d_{ss}^{+} that is orthogonal to $R[K]$. That is, the proposed scheme preserves the LS-property and is 'LS-robust,' with respect to output disturbance d_{ss}, as Fig. 1 illustrates.

III. LOOP DECOUPLING, TUNING, AND ACTUATOR LIMITS

With a reasonably accurate K, steady-state decoupling is essentially accomplished for the 2 x 2 control loops from the output of the controller, C, going around and back to the controller's input, because $K^+ K = (K^T K)^{-1} K^T K = I$. Steady-state decoupling often leads to low-frequency DD, which in practice facilitates ease of controller tuning and loop stability. The stably tuned control loops will accommodate the disturbance component $d^{\|}$, while leaving its orthogonal counterpart, d^{+}, unattenuated.

A very useful interpretation of the proposed LS configuration is obtained by moving K^+ to the feedback path and to the forward set-point path (Figure 2(c)). This gives a post-compensation interpretation of the feedback scheme. The 'transformed' set-points now clearly show their connection with the actuators. Moreover, we can replace the pseudo-inverse K^+ in the set-point path with a more general inverse problem solver that could take the actuators' finite range into consideration. Quadratic Programming (QP) is one such inverse solver. Alternatively, a simple Constrained Estimation algorithm as described in [5] can be applied here.

IV. SUMMARY

As a practical alternative to Receding-Horizon based on-line optimization and SVD-based mode-decoupling, a least-squares (LS) based feedback configuration is discussed as a simple, tunable method for Profile Control. Extension to weighted LS is straightforward.

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