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Passivity-Based Robust Control for Power Systems Subject to Wind Power Variability

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***Abstract*— As wind generation becomes a significant portion of total energy production, wind power variability will introduce more variability in system frequency. This paper presents a method to improve primary control for frequency regulation in large-scale power systems with high wind power penetration. To assure system stability, a passivity-based framework is developed for power systems by introducing a storage function derived from the entropy of individual generators. Tellegen's theorem is invoked to derive the storage function for the entire power network. Given the network parameters and the point of interconnection of the wind farm, a single generator is selected to balance wind power fluctuations. A passive H_∞ controller is synthesized for the selected generator by using a passive reduced-order model of the large-scale power system. Simulation results of a 9-bus test system show the effectiveness of the passive H_∞ controller. This work also suggests several directions for further research.**

I. INTRODUCTION

The frequency of a power system must be maintained nearly constant at nominal frequency to ensure safe and reliable operation. System frequency deviation indicates the imbalance between generation and load. To maintain the frequency in an acceptable range, the generation must be adjusted in real time to meet deviations of the load from predicted values. Under normal conditions, frequency is tightly controlled within a narrow band around the nominal frequency.

Increasing the proportion of wind power to traditional generators can degrade frequency performance. In traditional power systems, load variation is the main disturbance in the frequency control loop under normal operating conditions. When a significant amount of wind energy penetrates the power system, fluctuations in wind generation due to wind variations need to be compensated in addition to load variations, thus increasing the amount of control effort required to maintain system frequency. Moreover, modern variable speed wind turbine generators are often isolated from the grid by power electronic converters, thus contributing almost no inertial response to the overall power system [1]. If more synchronous machines are displaced by wind generation, the system inertia will decrease, making the power system more sensitive to generation-load imbalances.

Improving frequency control for power systems with high wind power penetration is an active area of research. Bevrani illustrates the use of robust control design to automatic generation control [2]. Researchers have also been investigating methods for implementing primary control on the wind generation itself. Control schemes that allow wind turbine generator to participate in inertial response and primary frequency control are proposed in [1], [3], [4], [5]. Strategies have also been proposed to improve frequency control for systems with high wind power penetration using natural gas generation [6], battery systems [7], flywheels [8], etc.

H_∞ methods can be used to synthesize controllers to achieve robust performance and stability in the presence of bounded uncertainties, disturbances and noise [9]. In [10], we apply H_∞ methods to design a robust controller for a single generator to attenuate continuous wind power fluctuations. However, due to the disadvantages of standard H_∞ methods, it produces a high-order dynamic controller which is generally undesirable in practice. Therefore, the approach in [10] is only applicable to small-scale power systems. For large-scale power systems, reduced-order model are often used. One intuitive thought is to use reduced-order model to design robust controller, then attach it to the original system. However, in general there is no guarantee that the controller designed using reduced-order model will stabilize the real system. To resolve this issue, we propose a passivity-based framework, allowing robust control design through a passive reduced-order model of the original system. The resulting passive, low-order controller can achieve both stability and robust performance for the overall system.

The paper is organized as follows. In Section II we present models used in power system primary frequency regulation and a characterization of short-term wind power fluctuations. Section III develops a passive-based framework for power systems by introducing a storage function derived from the entropy of individual generators. In Section IV a passivity-preserving model-order reduction technique is applied to obtain a passive reduced-order model of large-scale power systems. In Section V we synthesize a passive dynamic output feedback controller using dissipative H_∞ methods and the passive reduced-order model obtained from the previous section. Simulation results of a 9-Bus test system are presented in Section VI to demonstrate the effectiveness of the proposed controller. The final section summarizes the contribution of this paper and suggests directions for further research.

II. PROBLEM FORMULATION

The focus of this paper is primary frequency control for continuous wind power fluctuations. The time scale of

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interest is from seconds to a few minutes. Mathematical models of synchronous generators, network power flow and wind power fluctuations are summarized as follows.

A. Synchronous Generator

Synchronous generators are represented with the classical swing equations (1)–(2) [11]

$$\dot{\theta}_i = \omega_i - \omega_0 \quad (1)$$

$$\frac{2H_i}{\omega_0}\dot{\omega}_i = T_{mi} - T_{ei} - K_{Di}\left(\omega_i - \omega_0\right) \quad (2)$$

where θ_i and ω_i are the rotor angle and rotor speed of machine i. T_{mi} and T_{ei} are the per unit mechanical and electrical torque associated with machine i. ω_0 is the nominal speed $2\pi 60\ rad/s$. The relation between per unit power and torque is $P_{mi} = T_{mi}\omega_i/\omega_0$ and $P_{ei} = T_{ei}\omega_i/\omega_0$. As the speed is usually controlled tightly around ω_0, the per unit value of power and torque are approximately equal. The electrical power output P_{ei} of the generator is equal to the sum of all the power flows from bus i to other buses in the network, i.e. $P_{ei} = \sum_{j=1,j\neq i}^{n} f_{ij}$, where f_{ij} is the power flow from bus i to j.

B. DC Power Flow

For the purpose of primary frequency control, we assume that reactive power is compensated locally and bus voltages are tightly controlled, so that the bus voltages can be assumed to be 1 p.u. and we only need to consider real power balance equations for the network. Consider two buses i and j connected through a lossless transmission line of reactance X_{ij}. Let the voltages at two buses be $1\angle\theta_i$ and $1\angle\theta_j$. Then the power transferred from bus i to bus j can be expressed as $f_{ij} = (\theta_i - \theta_j)/X_{ij}$. This is called DC power flow [12].

C. Wind Power Fluctuations

For our study, the wind power is assumed to be an average wind power plus fluctuation components, similar to the model used in [13]. We assume that the wind turbines in this study do not participate in frequency regulation and the power output from wind farms is injected into the grid as "negative load". In [14], the authors show that power systems are more sensitive to the power fluctuations in the medium frequency range (between 0.01 Hz to 1 Hz), and that the majority of wind power fluctuations are located in that region and below. We characterize the wind power fluctuation by its frequency spectrum. For a specific system, such information can be obtained from wind power data [15].

III. Passivity-Based Framework

The power network can be shown to satisfy the generalized Tellegen's theorem [16], [17]. With proper storage function for the power system, we can choose local input and output variables so that when local passive controllers are connected in negative feedback to the overall system, stability is guaranteed.

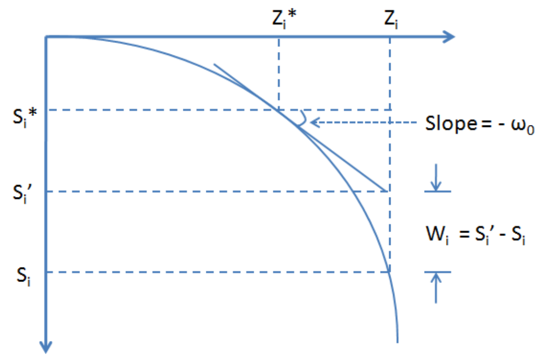

Fig. 1. Entropy versus inventory.

A. Preliminaries

We begin with some important definitions and theorems in passivity literature. A system is said to be *dissipative* with respect to the *supply rate* $\phi\left(u, y\right)$ if there exists a nonnegative real function $W(x)$, called the *storage function*, such that for all $t_1 \geq t_0 \geq 0$, all initial conditions and all controls, the increase in its energy (storage function) $W(x)$ during the interval (t_0, t_1) is no greater than the energy supplied to it via the supply rate. A system is said to be *passive* if it is dissipative with respect to the supply rate $\phi(u(t), y(t)) = u(t)^T y(t)$ [18].

The *negative feedback* connection of two passive systems is passive [19]. A passive system with a positive definite storage function is *stable* in the sense of Lyapunov. A linear system is passive if and only if its transfer function $G(s)$ is positive real [18].

B. Entropy Function and Storage Function

One major challenge in passivity-based control is to find a proper storage function. To obtain a storage function for the power network, we first define an *inventory* or *extensive variable* at each bus. Here, the kinetic energy stored at each bus is chosen as the local inventory $Z_i = \frac{1}{2}J_i\omega_i^2$, where the moment of inertia $J_i = \frac{2H_i}{\omega_0^2}$. The *intensive variable* is chosen to be the speed ω_i ($= 2\pi\times$frequency) at each bus.

We propose the *entropy function* for each generator to be

$$S_i\left(Z_i\right) = -\frac{2}{3}\sqrt{\frac{2}{J_i}}Z_i^{\frac{3}{2}}.$$

This entropy function is concave in Z_i and

$$\frac{\partial S_i\left(Z_i\right)}{\partial Z_i} = -\frac{2}{3}\sqrt{\frac{2}{J_i}}\frac{3}{2}Z_i^{\frac{1}{2}} = -\omega_i.$$

Consider the tangent of the entropy function at the reference speed ω_0 and kinetic energy Z_i^*. Define the storage function as the difference between S_i' and S_i as shown in Fig. 1. We have

$$\begin{aligned} W_i\left(\omega_i^*, Z_i\right) &= S_i^* - S_i - \omega_0\left(Z_i - Z_i^*\right) \quad (3) \\ &= \frac{J_i}{6}\left(\omega_i - \omega_0\right)^2\left(2\omega_i + \omega_0\right) \geq 0, \end{aligned}$$

which proves that the storage function $W_i(Z_i)$ is non-negative and the equality holds when $\omega_i = \omega_0$.

For a constant reference point, we have $d\omega_0/dt = 0$, $dZ_i^*/dt = 0$, and $dS_i^*/dt = 0$. Therefore, differentiating (3) with respect to time gives

$$\frac{dW_i}{dt} = \frac{dS_i^*}{dt} - \frac{\partial S_i}{\partial Z_i}\frac{dZ_i}{dt} - \omega_0 \frac{d\left(Z_i - Z_i^*\right)}{dt} = \overline{\omega}_i \frac{d\overline{Z}_i}{dt},$$

where $\overline{\omega}_i = \omega_i - \omega_0$ and $\overline{Z}_i = Z_i - Z_i^*$ are the deviation variables.

The above derivation is for generator buses. For load buses, there can be two different models. We can either treat the load as motors ("negative" generators), with associated inertia, speed, electrical power and mechanical power, or as just the power extracted from the load bus with the dynamics of the load not modeled. For the first case, the dynamical load bus is essentially the same as a generator bus, therefore it is not discussed further. For the second case, we consider there is no inertia and hence no storage at load buses and wind buses, i.e. the load and wind power are seen as disturbances to the system. An inventory of kinetic energy Z_i is nevertheless assigned to the load and wind buses, with $Z_i = 0$ and $\frac{dZ_i}{dt} = 0$. Then the time derivative of the storage function of all buses is

$$\frac{dW}{dt} = \sum_{i=1}^{n} \frac{dW_i}{dt} = \sum_{i=1}^{n} \overline{\omega}_i \frac{d\overline{Z}_i}{dt}. \tag{4}$$

C. Tellegen's Theorem

As one of the most powerful theorems in network theory, Tellegen's theorem gives a simple relation between magnitudes of network systems that obey the generalized Kirchhoff's laws of electrical circuit theory. The basic assumptions are the conservation of flow of extensive quantities (Kirchhoff's current law, KCL) and the uniqueness of the potentials at the network nodes (Kirchhoff's voltage law, KVL). One formulation for Tellegen's theorem of process systems is presented as follows [16], [17]:

Theorem 3.1 (Tellegen's Theorem): Consider a process network system with extensive variables Z_i, and intensive variables w_i. Let the overbar denote the deviation from the setpoint. If each node satisfies the KCL-like conservation law

$$\frac{d\overline{Z}_i}{dt} = \overline{p}_i + \sum_{j=1, j\neq i}^{n_p} \overline{f}_{ij}, \quad i = 1, ..., n_p$$

where p_i is the production term and f_{ij} is the flow connection, and each loop satisfies the KVL-like law $\sum\limits_{loop} (\overline{w}_i - \overline{w}_j) = 0$, then

$$\sum_{i=1}^{n_p} \overline{w}_i \frac{d\overline{Z}_i}{dt} = \sum_{i=1}^{n_p} \overline{w}_i \overline{p}_i + \sum_{j>i}^{n_p} \sum_{i=1}^{n_p} (\overline{w}_i - \overline{w}_j) \overline{f}_{ij}.$$

For each generator bus *i*, the kinetic energy $Z_i = \frac{1}{2} J_i \omega_i^2$. We have

$$\frac{d\overline{Z}_i}{dt} = \overline{P}_{mi} - \frac{K_{Di}}{\omega_0}\omega_i \overline{\omega}_i - \sum_{j=1, j\neq i}^{n} \overline{f}_{ij}. \tag{5}$$

For each load bus *i* that is treated as power disturbance,

$$\frac{d\overline{Z}_i}{dt} = -\overline{P}_{Li} - \sum_{j=1, j\neq i}^{n} \overline{f}_{ij}. \tag{6}$$

For each wind bus *i* that is treated as power disturbance,

$$\frac{d\overline{Z}_i}{dt} = \overline{P}_{Wi} - \sum_{j=1, j\neq i}^{n} \overline{f}_{ij}. \tag{7}$$

Equations (5)–(7) show that the network satisfies the KCL at each bus. For each loop, we have $\sum\limits_{loop} (\overline{\omega}_i - \overline{\omega}_j) = 0$, i.e. the KVL is also satisfied. By Theorem 3.1 and from (4),

$$\begin{aligned} \frac{dW}{dt} &= \sum_{i=1}^{n} \overline{\omega}_i \frac{d\overline{Z}_i}{dt} = \sum_{i=1}^{n_G} \overline{\omega}_i \overline{P}_{mi} + \sum_{i=1}^{n_W} \overline{\omega}_i \overline{P}_{Wi} - \sum_{i=1}^{n_L} \overline{\omega}_i \overline{P}_{Li} \\ &\quad - \sum_{i=1}^{n_G} \frac{K_{Di}}{\omega_0} \omega_i \overline{\omega}_i^2 - \sum_{j>i}^{n} \sum_{i=1}^{n} (\overline{\omega}_i - \overline{\omega}_j) \overline{f}_{ij}. \end{aligned} \tag{8}$$

To analyze the stability of closed-loop system, the disturbances from the load and wind can be temporarily ignored. It is evident from the first term of the right hand side of (8) that we identify $u_i = \overline{P}_{mi}$ and $y_i = \overline{\omega}_i$ as our local input/output pair. Then by definition, the system is passive with respect to the supply rate $\phi(u, y) = u^T y = \sum\limits_{i=1}^{n_G} \overline{\omega}_i \overline{P}_{mi}$. The proof is omitted due to space limitations.

IV. Passivity-Preserving Model Order Reduction

Passivity-preserving model order reduction is an important topic in circuit simulation, analysis and design area [20]. Standard balanced order reduction methods cannot guarantee that the reduced order model of a passive system remains passive. Phillips *et al.* develop truncated balanced realization (TBR)-like methods that generate guaranteed passive reduced order models in [21]. We are interested in its application in primary frequency control for large-scale power systems. The preservation of passivity of model reduction is extremely important, as it ensures that when the controller designed based on reduced order models are connected to the real system, the stability of the overall system is guaranteed.

A. Conversion of DAEs to ODEs

The state-space model of power systems is described by a set of differential-algebraic equations (DAEs):

$$\dot{x} = A_0 x + B_0 u + E_0 v \tag{9}$$

$$0 = C_0 x + D_0 v + F_0 d \tag{10}$$

where $x = [\omega_1, \theta_2, \omega_2, ..., \theta_{n_G}, \omega_{n_G}]^T$ is the state vector, v is the vector of algebraic variables (i.e. the non-generator bus angles), $u = P_{mi}$ is the control variable for selected machine Gen-i, d is the wind power disturbance. Note that θ_1 has been chosen as the reference angle, hence it is absent from x.

The DAEs (9)–(10) can be transformed into ordinary-differential equations (ODEs) as the matrix D_0 is invertible:

$$\dot{x} = Ax + Bu + Ed, \tag{11}$$

$$y = Cx, \tag{12}$$

where $A = A_0 - E_0 D_0^{-1} C_0$, $B = B_0$, $E = -E_0 D_0^{-1} F_0$. The output/measurement $y = Cx$ is the speed of selected machine *Gen-i*, i.e. the elements in C are all zeros, except the entry corresponding to ω_i is one.

B. Positive-Real Truncated Balanced Realization (PR-TBR)

Given a passive system as in (11)–(12), we apply Algorithm 3 in [21] to obtain a passive reduced-order model with the following state-space representation:

$$\dot{\tilde{x}} = \tilde{A}\tilde{x} + \tilde{B}u + \tilde{E}d, \quad (13)$$

$$y = \tilde{C}\tilde{x}. \quad (14)$$

V. Passivity-Based H_∞ Controller Synthesis

In [22], a method of positive real synthesis technique with H_∞-norm constraint is proposed by Haddad *et al*. The main result is recapitulated below in the form of an algorithm.

Algorithm 1: Given a stabilizable and detectable positive real plant in the form of (11)–(12) (or (13)–(14)), determine a dynamic controller $G_c(s)$ of the form

$$\dot{x}_c = A_c x_c + B_c y \quad (15)$$

$$u = C_c x_c \quad (16)$$

such that

- the closed-loop system is stable;
- the closed-loop transfer function from the disturbance d to the performance variables $z = [\ \eta_1 y,\ \ \eta_2 u\ ]^T = E_1 x + E_2 u$ satisfies $\|G(s)\|_\infty \le \gamma$, where γ is a positive pre-defined value for guaranteed robust performance;
- $-G_c(s)$ is positive real (passive).

The procedure is as follows:

1) Find matrices Q_0 and L with $Q_0 = Q_0^T > 0$ such that

$$A Q_0 + Q_0 A^T = -L L^T$$
$$Q_0 C^T = B.$$

2) Choose proper weights η_1, η_2 and γ such that

$$V_1 = L L^T + B R_2^{-1} B^T - \gamma^{-2} Q_0 R_1 Q_0 > 0$$
$$R_1 \ge C^T R_2^{-2} C,$$

where $R_1 \triangleq E_1^T E_1$, $R_2 \triangleq E_2^T E_2$.

3) Find non-negative definite matrices Q and P satisfying

$$0 = AQ + QA^T + V_1 + \gamma^{-2} Q R_1 Q - Q C^T R_2^{-1} C Q$$

$$0 = (A + \gamma^{-2} Q R_1)^T P + P(A + \gamma^{-2} Q R_1) + R_1$$
$$- P B R_2^{-1} B^T P + \gamma^{-2} P Q C^T R_2^{-1} C Q P.$$

If such Q and P do not exist, go to 2) and re-tune design weights η_1, η_2 and γ.

4) Compute the controller matrices as

$$A_c = A - Q C^T R_2^{-1} C - B R_2^{-1} B^T P + \gamma^{-2} Q R_1$$

$$B_c = Q C^T R_2^{-1}$$

$$C_c = -R_2^{-1} B^T P.$$

One important remark is that Algorithm 1 only works with a passive plant model. Therefore, the passive reduced-order model obtained from the PR-TBR algorithm in Section IV can be used in Algorithm 1 to produce a passive H_∞ controller. This controller can then be used in the original system for robust performance. Stability is guaranteed because both the controller and original system are passive, thus when they are connected together in negative feedback, passivity is preserved in the overall system. The closed-loop system is hence stable in the sense of Lyapunov. This idea is illustrated in Fig. 2.

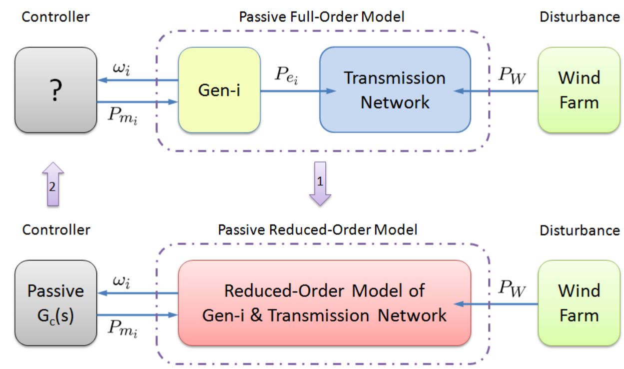

Fig. 2. Passive H_∞ controller design using reduced-order models.

We now propose the following procedure as the main algorithm of this paper.

Algorithm 2: Given a power system model in (9)–(10).

1) Convert the power system model from DAEs to ODEs as in (11)–(12).
2) Apply PR-TBR algorithm to obtain a passive reduced-order model as in (13)–(14).
3) Based on the passive reduced-order model (13)–(14) obtained in 2), apply Algorithm 1 to synthesize a passive H_∞ controller as in (15)–(16).
4) Evaluate the control performance of the resulting controller using the full-order model (11)–(12).

The implementation of Algorithm 2 involves several linear matrix inequalities. We use `CVX` [23], a package for specifying and solving convex programs to solve this problem.

VI. Case Study

A. IEEE 9-Bus Test System

The IEEE 9-bus test system [24] is modified to include a wind farm that has an average output of 190 MW (about 25% of the total system capacity). The power fluctuations are about $\pm 5\%$ of the average power. The point of interconnection of the wind farm to the grid is *bus 9*. Therefore the nearby machine *Gen-3* is selected to balance the fluctuation of wind power so as to minimize the effects of that fluctuation on other parts of the system. The other machines Gen-1 and Gen-2 remain the standard proportional droop control of their mechanical power, i.e. $\overline{P}_{mi} = -\frac{S_i}{S_N R_D \omega_0}\overline{\omega}_i$, for $i = 1, 2$, where $R_D = 0.05$ is the droop.

B. Results of Implementing Algorithm 2

The original system has 5 states and 6 algebraic variables. After the conversion of DAEs to ODEs, the full-order system

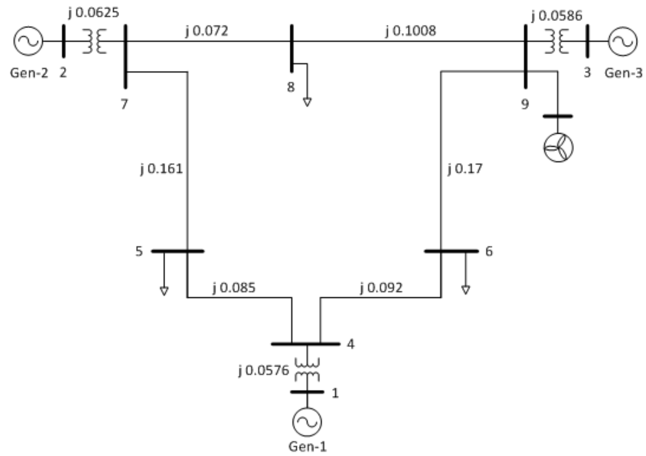

Fig. 3. Schematic of the 9-bus test system.

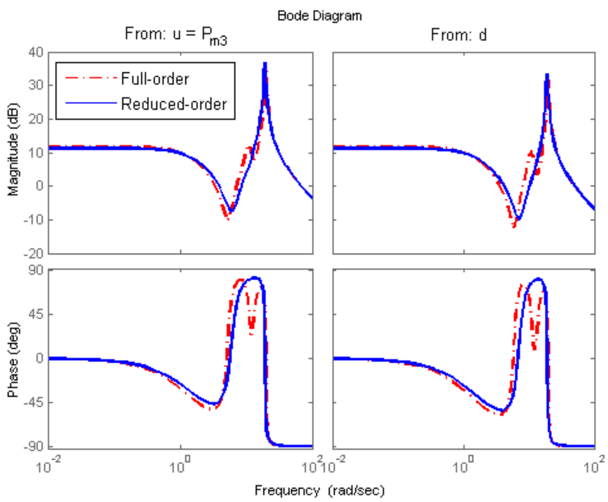

Fig. 4. Comparison of Bode plots of the full-order model and passive reduced-order model.

is of 5^{th}-order. Algorithm PR-TBR gives a 3^{rd}-order passive reduced-order model:

$$\tilde{A} = \begin{bmatrix} -0.3 & 18.01 & -0.647 \\ 18.01 & -0.779 & 5.903 \\ -0.647 & 5.903 & -1.456 \end{bmatrix}, \tilde{B} = \begin{bmatrix} 7.913 \\ 0 \\ 0 \end{bmatrix},$$

$$\tilde{E} = \begin{bmatrix} 5.52 \\ 0.05207 \\ 0.8028 \end{bmatrix}, \tilde{C} = \begin{bmatrix} 7.9135 & 0 & 0 \end{bmatrix}. \quad (17)$$

The bode plot of the full-order model and reduced-order model is shown in Fig. 4.

With the passive reduced-order model in (17), and the choice of weights $\eta_1 = 0.12$, $\eta_2 = 0.1$ and $\gamma = 2$, a passive H_∞ controller is synthesized using Algorithm 1. Its transfer function is

$$G_c(s) = -\frac{8460s^2 + 5579s + 2.411e005}{s^3 + 6349s^2 + 1.442e004s + 2.284e005}. \quad (18)$$

The minus sign stands for negative feedback interconnection. The bode plot of $-G_c$ is shown in Fig. 5.

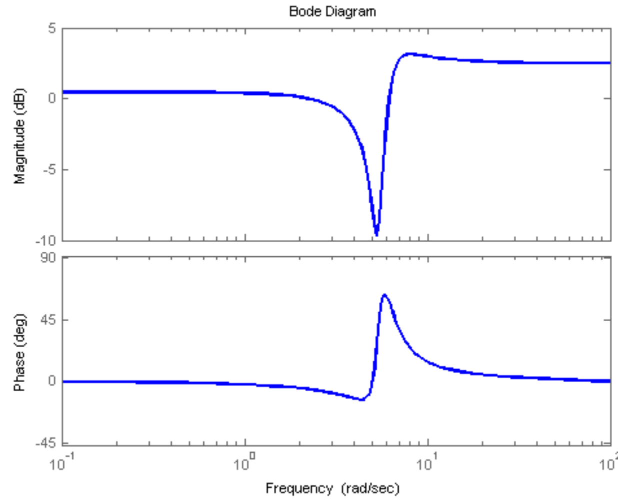

Fig. 5. Bode plot of the passive controller.

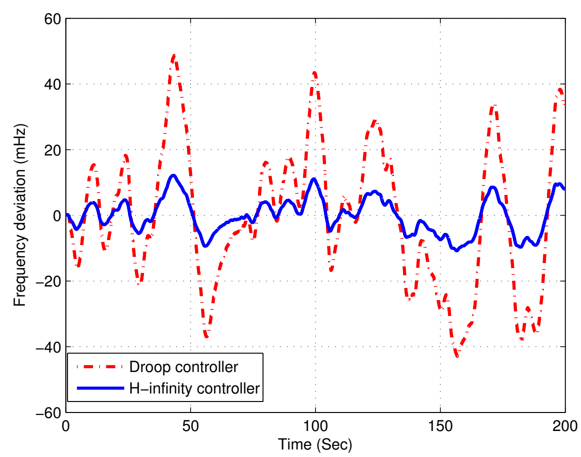

Fig. 6. Comparison of frequency performance for different controllers.

Figure 6 shows the frequency deviations caused by wind power fluctuations with conventional droop controller and passive H_∞ controller in (18). We can see that the new controller can effectively narrow the band of frequency deviations caused by wind power fluctuations.

To evaluate the control action, mechanical power deviations of Gen-3 are plotted in Fig. 7 (lower figure). The required P_{m3} by the H_∞ controller has a larger band than by the conventional governor. The rate of change of mechanical power also increases for Gen-3. However, the required mechanical power changes become less for Gen-1 and Gen-2 as seen in the upper and middle figures of Fig. 7. This shows that by installing H_∞ controller on Gen-3, it becomes more responsible for attenuating the frequency deviation caused by wind power fluctuations and thus requires more control action. By installing passive robust controller on the generator close to the point of interconnection of the wind farm to compensate the wind power fluctuations locally, the rest of the network is less affected.

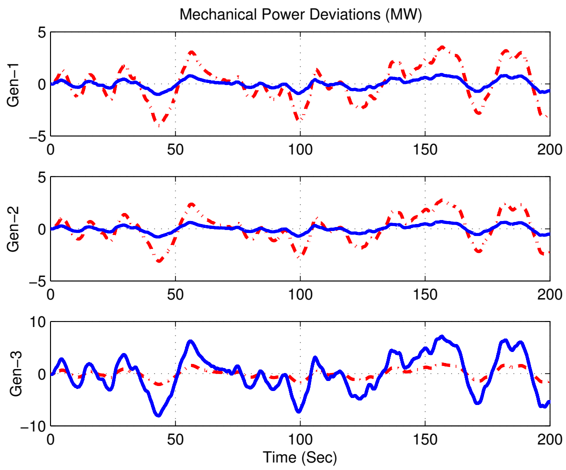

Fig. 7. Mechanical power deviations of Gen-1, Gen-2, Gen-3, with conventional governor (red dashdot) and H-infinity controller (solid blue).

VII. CONCLUSIONS AND FUTURE WORK

As wind power introduces more disturbances into power systems, the primary frequency controller of conventional generator needs to be redesigned to attenuate frequency deviations caused by wind power fluctuations. While standard H_∞ methods tend to produce complex controllers and hence are not suitable for large-scale power systems, this paper presents a passivity-based framework and an algorithm that allows the synthesis of a passive H_∞ controller based on a passivity-preserving reduced-order model. When the passive controller is connected to the full-order system in negative feedback, the closed-loop system remains stable, thanks to the special properties of passive systems. One contribution of this paper is the derivation of a proper storage function for power networks using the entropy of individual generators and Tellegen's theorem. Also, this paper summarizes and combines the positive-real truncated balanced realization (PR-TBR) method and the positive real synthesis technique with H_∞ constraint to produce a low-order dynamic controller with robust performance for large-scale power systems.

Although the main focus of this paper is on single controller design to compensate wind power variability, the proposed passivity-based framework can also support decentralized control of multiple generators. We are currently investigating the application of passivity-based methods to design decentralized robust frequency controllers for power systems with high penetration of wind power generation.

APPENDIX

The parameters of the 9-bus test system in Section VI are:

S_N=100 MVA	ω_0=120π rad/s	f_0=60 Hz
S_1=247.5 MVA	H_1=23.64 MW·s/MVA	K_{D1}=0.0125
S_2=192 MVA	H_2=6.4 MW·s/MVA	K_{D2}=0.0068
S_3=128 MVA	H_3=3.01 MW·s/MVA	K_{D3}=0.0048.

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