2011 American Control Conference
on O'Farrell Street, San Francisco, CA, USA
June 29 - July 01, 2011

Multirate Distributed Model Predictive Control of Nonlinear Systems

Mohsen Heidarinejad, Jinfeng Liu, David Muñoz de la Peña, James F. Davis and Panagiotis D. Christofides

Abstract— **In this work, we consider the design of a distributed model predictive control scheme using multirate sampling for large-scale nonlinear systems composed of several coupled subsystems. Specifically, we assume that the states of each local subsystem can be divided into fast sampled states (which are available every sampling time) and slowly sampled states (which are available every several sampling times). The distributed model predictive controllers are connected through a shared communication network and cooperate in an iterative fashion, at time instants in which full system state measurements (both fast and slow) are available and the network closes, to guarantee closed-loop stability. When the communication network is open, the distributed controllers operate in a decentralized fashion based only on local subsystem fast sampled state information to improve closed-loop performance. In the proposed design, the controllers are designed via Lyapunov-based model predictive control. Sufficient conditions under which the state of the closed-loop system is ultimately bounded in an invariant region containing the origin are derived. The theoretical results are demonstrated through a nonlinear chemical process example.**

I. Introduction

With the rapid growth in the area of network technology, augmentation of local (point-to-point) process control systems with additional networked sensors and actuators has become a subject of increasing importance. Such an augmentation can significantly improve the efficiency, flexibility, robustness and fault tolerance of an industrial control system while reducing the installation, reconfiguration and maintenance expenses at the cost of coordination and design/redesign of the various control systems employed in the new architecture [1], [2], [3]. Model predictive control (MPC) is a principal framework to deal with the design and coordination of control systems because of its ability to account for process/controller interactions in the calculation of the control actions. MPC is an online optimization-based approach, which takes advantage of a system model to predict its future evolution starting from the current system state along a given prediction horizon. Using model predictions, a future manipulated input trajectory is optimized by minimizing a typically quadratic cost function involving penalties on the system state and control action. Once a future input trajectory is calculated, only the first step input value is applied to the system and the input evaluation process is repeated at the next sampling time; this is the so-called receding horizon scheme.

Typically, MPC is studied within a centralized control architecture in which all the manipulated inputs are calculated in a single MPC. Because in the evaluation of the control actions by MPC online optimization problems need to be solved, the evaluation time of the MPC strongly depends on the number of manipulated inputs. As the number of manipulated inputs increases, the evaluation time of a centralized MPC may increase significantly. This may impede the ability of centralized MPC to carry out real-time calculations within the limits imposed by process dynamics and operating conditions.

Distributed MPC (DMPC) is a feasible alternative to overcome the increasing computational complexity of centralized MPC. In a DMPC architecture, the manipulated inputs are computed by solving more than one control (optimization) problems in separate processors in a coordinated fashion. With respect to available results in this direction, several DMPC methods have been proposed in the literature; please see [4], [5], [6] for reviews of results in this area. Specifically, in [7], a distributed control method for weakly-coupled nonlinear systems subject to decoupled constraints was proposed. In [8], a robust DMPC formulation was proposed for decoupled linear systems. In [9], it was proven that through multiple communications between distributed controllers and using system-wide control objective functions, the stability of the closed-loop system can be guaranteed for linear systems. In [10], DMPC of decoupled nonlinear systems coupled through cost functions was studied. In [11], a DMPC algorithm was proposed for a class of nonlinear discrete-time systems under the condition that no information is exchanged between local controllers. In [12], a game theory based DMPC scheme was proposed for linear systems coupled through the inputs. In our previous works [13], [14], two different DMPC architectures, namely, a sequential DMPC architecture and an iterative DMPC architecture, were designed for nonlinear systems via Lyapunov techniques. However, the results in [13], [14] were obtained under the assumption that each distributed controller has access to the full system state at every sampling time. In [15], [16], we considered the design of DMPC schemes for nonlinear systems with asynchronous and delayed measurements of the full system state.

In the present work, we consider the design of a DMPC scheme using multirate sampling for large-scale nonlinear

Financial support from the NSF and the European Commission, INFSOICT-223866, is gratefully acknowledged.

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978-1-4577-0079-8/11/$26.00 ©2011 AACC

systems composed of several coupled subsystems. Specifically, we assume that the states of each local subsystem can be divided into fast sampled states (which are available every sampling time) and slowly sampled states (which are available every several sampling times). The distributed model predictive controllers are connected through a shared communication network and cooperate in an iterative fashion to guarantee closed-loop stability when the network is closed at time instants in which full system state measurements (both fast and slow) are available. When the communication network is open, the distributed controllers operate in a decentralized fashion based only on local subsystem fast sampled state information to improve closed-loop performance. In the proposed design, the controllers are designed via Lyapunov-based MPC (LMPC). Sufficient conditions under which the state of the closed-loop system is ultimately bounded in an invariant region containing the origin are derived. The theoretical results are demonstrated through a nonlinear chemical process example. In [17], the results presented in this paper have been extended to account for measurement and communication noise as well as certain distributed convergence properties have been established.

II. Preliminaries

A. Notation and class of nonlinear systems

The operator $|\cdot|$ is used to denote Euclidean norm of a vector, and a continuous function $\alpha : [0,a) \to [0,\infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and satisfies $\alpha(0) = 0$. The symbol Ω_r is used to denote the set $\Omega_r := \{x \in R^{n_x} : V(x) \le r\}$ where V is a scalar function, and the operator '/' denotes set subtraction, that is, $A/B := \{x \in R^{n_x} : x \in A, x \notin B\}$. The symbol $diag(v)$ denotes a matrix whose diagonal elements are the elements of vector v and all the other elements are zeros. We consider a class of nonlinear systems which is composed of m subsystems where each of the subsystems can be described by the following state-space model:

$$\dot{x}_i(t) = f_i(x) + g_{si}(x)u_i(t) + k_i(x)w_i(t) \qquad (1)$$

where $i = 1,\dots,m$, $x_i(t) \in R^{n_{x_i}}$ denotes the vector of state variables of subsystem i, $u_i(t) \in R^{n_{u_i}}$ and $w(t) \in R^{n_w}$ denote the set of control (manipulated) inputs and disturbances associated with subsystem i, respectively. The variable $x \in R^{n_x}$ denotes the state of the whole system which is composed of the states of the m subsystems, that is $x = [x_1^T \cdots x_i^T \cdots x_m^T]^T$.

The dynamics of x can be described in a compact form as follows:

$$\dot{x}(t) = f(x) + \textstyle\sum_{i=1}^{m} g_i(x)u_i(t) + k(x)w(t) \qquad (2)$$

where $f = [f_1^T \cdots f_i^T \cdots f_m^T]^T$, $g_i = [\mathbf{0}^T \cdots g_{si}^T \cdots \mathbf{0}^T]^T$ with $\mathbf{0}$ being the zero matrix of appropriate dimensions, k is a matrix composed of k_i $(i = 1,\dots,m)$ and zeros whose explicit expression is omitted for brevity, and $w = [w_1^T \ \cdots \ w_i^T \cdots w_m^T]^T$ which is assumed to be bounded, that is, $w(t) \in W$ with $W := \{w \in R^{n_w} : |w| \le \theta, \theta > 0\}$.

The m sets of inputs are restricted to be in m nonempty convex sets $U_i \subseteq R^{m_{u_i}}$, $i = 1,\dots,m$, which are defined as $U_i := \{u_i \in R^{n_{u_i}} : |u_i| \le u_i^{\max}\}$ where $u_i^{\max}$, $i = 1,\dots,m$, are the magnitudes of the input constraints. We will design m controllers to compute the m sets of control inputs u_i, $i = 1,\dots,m$, respectively. We will refer to the controller computing u_i associated with subsystem i as controller i.

We assume that f, g_i, $i = 1,\dots,m$, and k are locally Lipschitz vector functions and that the origin is an equilibrium point of the unforced nominal system (i.e., system of Eq. 2 with $u_i(t) = 0$, $i = 1,\dots,m$, $w(t) = 0$ for all t) which implies that $f(0) = 0$.

B. Modeling of measurements and communication networks

We assume that the states of each of the m subsystems, x_i $(i = 1,\dots,m)$, are divided into two parts: $x_{f,i}$, states that can be measured at each sampling time (e.g., temperatures and pressures) and $x_{s,i}$, states which are sampled at a relatively slow rate (e.g., species concentrations). Specifically, we assume that $x_{f,i}$, are available at synchronous time instants $t_m = t_0 + m\Delta$, $m = 0,1,\dots$, where t_0 is the initial time and Δ is the sampling time; and assume that $x_{s,i}$, are available every T sampling times (i.e., $x_{s,i}$, are available at t_k with $k = 0, T, 2T, \dots$). Note that, in order to simplify the development, we assume that the slowly sampled states of different subsystems are available at the same time instants. This modeling of measurements is relevant to systems involving heterogeneous measurements which have different sampling rates; please see example in Section IV.

We also assume that each subsystem is connected to its local sensors, actuators and controller using point-to-point links, which implies that $x_{f,i}$ and $x_{s,i}$ are available without delay to controller i once they are measured. We further assume that the controllers for different subsystems are connected through a shared communication network which is closed when the full system state is available (i.e., at time instants t_k with $k = 0, T, 2T, \dots$). When the network is closed, each controller communicates with the rest of the controllers to share state and future input trajectories information.

This class of systems is relevant to the case of large-scale chemical processes that are controlled by distributed control systems that exchange information over a shared communication network through which it is not cost-effective to communicate at every sampling time. Instead, in order to achieve closed-loop stability and good closed-loop performance, the controllers communicate every several sampling times. Please see Fig. 1 in Section III for a schematic of such type of DMPC system with the local controllers designed via Lyapunov-based MPC techniques.

C. Lyapunov-based controller

We assume that there exists a Lyapunov-based controller $h(x) = [h_1^T(x) \ \cdots \ h_m^T(x)]^T$ with $u_i = h_i(x)$, $i = 1,\dots,m$, which renders the origin of the nominal closed-loop system asymptotically stable while satisfying the input constraints

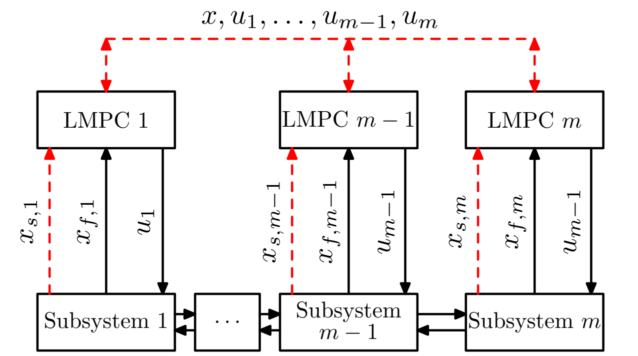

Fig. 1. Distributed LMPC control architecture (solid line denotes fast state sampling and point-to-point links; dashed line denotes slow state sampling and shared communication networks).

for all the states x inside a given stability region. We note that this assumption is essentially similar to the assumption that the process is stabilizable or that the pair (A, B) in the case of linear systems is stabilizable. Using converse Lyapunov theorems [18], [19], this assumption implies that there exist class $\mathcal{K}$ functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$ and a continuously differentiable Lyapunov function $V(x)$ for the nominal closed-loop system, that satisfy the following inequalities:

$$\begin{aligned} &\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|) \\ &\frac{\partial V(x)}{\partial x}\left(f(x) + \textstyle\sum_{i=1}^{m} g_i(x) h_i(x)\right) \leq -\alpha_3(|x|) \\ &\left|\frac{\partial V(x)}{\partial x}\right| \leq \alpha_4(|x|),\ h_i(x) \in U_i,\ i = 1, \ldots, m \end{aligned} \tag{3}$$

for all $x \in D \subseteq R^{n_x}$ where D is an open neighborhood of the origin. We denote the region $\Omega_\rho \subseteq D$ as the stability region of the closed-loop system under the Lyapunov-based controller $h(x)$. By continuity, the local Lipschitz property assumed for the vector fields f and g_i, $i = 1, \ldots, m$, and taking into account that the manipulated inputs u_i, $i = 1, \ldots, m$, and the disturbance w are bounded in a convex set, there exists a positive constant M such that

$$|f(x) + \textstyle\sum_{i=1}^{m} g_i(x) u_i + k(x) w| \leq M \tag{4}$$

for all $x \in \Omega_\rho$, $u_i \in U_i$, $i = 1, \ldots, m$, and $w \in W$. In addition, by the continuous differentiable property of the Lyapunov function $V(x)$ and the Lipschitz property assumed for the vector field f, there exist positive constants L_x, L_{u_i}, $i = 1, \ldots, m$, and L_w such that

$$\begin{aligned} &\left|\frac{\partial V}{\partial x} f(x) - \frac{\partial V}{\partial x} f(x')\right| \leq L_x |x - x'| \\ &\left|\frac{\partial V}{\partial x} g_i(x) - \frac{\partial V}{\partial x} g_i(x')\right| \leq L_{u_i} |x - x'|,\ i = 1, \ldots, m \\ &\left|\frac{\partial V}{\partial x} k(x)\right| \leq L_w,\ \left|\frac{\partial V}{\partial x} g_i(x)\right| \leq C_{gi},\ i = 1, \ldots, m \end{aligned} \tag{5}$$

for all $x, x' \in \Omega_\rho$, $u_i \in U_i$, $i = 1, \ldots, m$, and $w \in W$.

III. Multirate DMPC

A. Multirate DMPC implementation strategy

In this work, the m controllers manipulating the m sets of inputs will be designed through LMPC technique [20]. For the LMPC associated with controller i, $i = 1, \ldots, m$, we will refer to it as LMPC i. A schematic of the control system is shown in Fig. 1.

At a sampling time in which the communication network is closed, the distributed controllers coordinate their actions and predict future input trajectories which, if applied until the next instant the network is closed, guarantee the closed-loop stability. At a sampling time in which the network is open, each distributed controller tries to further optimize the input trajectories calculated at the latest network-closed instant within a constrained set of values to improve the closed-loop performance with the help of the available fast sampled states of its subsystem.

The proposed implementation strategy of the DMPC architecture at time instants when the communication network is closed is as follows:

1. At a sampling time t_k with $k = 0, T, 2T, \ldots$, all the controllers first broadcast their local subsystem states to other controllers, so each controller has the current full system state $x(t_k)$; and then all controllers iterate to evaluate their future input trajectories in an iterative fashion.
2. At iteration c ($c \geq 1$)
   - 2.1. All the distributed controllers exchange their latest future input trajectories.
   - 2.2. Each controller evaluates its own future input trajectory based on $x(t_k)$ and the latest received input trajectories of others.
   - 2.3. If a termination condition is satisfied, each controller sends its entire future input trajectory to its actuators and the other controllers; if the termination condition is not satisfied, go to step 2.1 ($c \leftarrow c + 1$).

The proposed implementation strategy of the DMPC architecture at time instants when the communication network is open and only local fast sampled states are available is as follows:

1. At a sampling time t_l with $l \neq 0, T, 2T, \ldots$, the shared communication network is open. Controller i, $i = 1, \ldots, m$, receives its local fast sampled states, $x_{f,i}$.
2. Each controller i estimates the current full system state and evaluates its future input trajectory and sends the first step input value to its actuators.

In the sequel, we describe these steps in detail.

B. Multirate DMPC formulation

In this subsection, we describe the design of the distributed controllers in the DMPC architecture. We first describe the design of the LMPCs at time instants when the network is closed. Before presenting the design of the LMPCs, we need to define a nominal sampled trajectory $x_h(\tau|t_k)$, $k = 0, T, 2T, \ldots$, which will be employed in the construction of the stability constraints. This nominal sampled trajectory is obtained by integrating recursively, for $t \in [t_k, t_{k+T})$ and $k = 0, T, 2T, \ldots$, the following equation:

$$\begin{aligned} \dot{x}_h(\tau|t_k) &= f(x_h(\tau|t_k)) + \textstyle\sum_{i=1}^{m} g_i(x_h(\tau|t_k)) h_i(x_h(l\Delta|t_k)), \\ &\qquad \forall \tau \in [l\Delta, (l+1)\Delta) \\ x_h(0|t_k) &= x(t_k) \end{aligned} \tag{6}$$

where $l = 0, \dots, T-1$, $x(t_k)$ is the actual system state at t_k. Based on this sampled trajectory, we can define the following input trajectories:

$$u_{n,j}(\tau|t_k) = h_j(x_h(l\Delta|t_k)),\ j = 1, \dots, m, \quad \forall \tau \in [l\Delta, (l+1)\Delta), l = 0, \dots, T-1. \tag{7}$$

From the definition of x_h, we see that this trajectory is a prediction of the evolution of the system of Eq. 2 under the Lyapunov-based controller $h(x)$ applied in a sample-and-hold fashion. x_h is re-set to the actual system state every T sampling times. This sampled trajectory, $x_h(\tau|t_k)$, will be used in the LMPCs.

At time t_k, $k = 0, T, 2T, \dots$, the LMPCs are evaluated in an iterative fashion to obtain the future input trajectories. Specifically, the optimization problem of LMPC j at iteration c is as follows:

$$\min_{u_j \in S(\Delta)} \int_0^{N\Delta} [\tilde{x}^j(\tau)^T Q_c \tilde{x}^j(\tau) + \textstyle\sum_{i=1}^m u_i(\tau)^T R_{ci} u_i(\tau)] d\tau \tag{8a}$$

$$\text{s.t. } \dot{\tilde{x}}^j(\tau) = f(\tilde{x}^j(\tau)) + \textstyle\sum_{i=1}^m g_i(\tilde{x}^j(\tau)) u_i \tag{8b}$$

$$u_i(\tau) = u_i^{*,c-1}(\tau|t_k),\ \forall i \neq j \tag{8c}$$

$$\left|u_j(\tau) - u_j^{*,c-1}(\tau|t_k)\right| \leq \Delta u_j,\ \forall \tau \in [0, T\Delta] \tag{8d}$$

$$u_j(\tau) \in U_j \tag{8e}$$

$$\tilde{x}^j(0) = x(t_k) \tag{8f}$$

$$\frac{\partial V(\tilde{x}^j)}{\partial \tilde{x}^j} \left(\frac{1}{m} f(\tilde{x}^j(\tau)) + g_j(\tilde{x}^j(\tau)) u_j(\tau) \right) \leq \frac{\partial V(x_h)}{\partial x_h} \left(\frac{1}{m} f(x_h(\tau|t_k)) + g_j(x_h(\tau|t_k)) u_{n,j}(\tau|t_k) \right), \quad \forall \tau \in [0, T\Delta] \tag{8g}$$

where $S(\Delta)$ is the family of piece-wise constant functions with sampling period Δ, N is the prediction horizon, Q_c and R_{ci}, $i = 1, \dots, m$, are positive definite weight matrices that define the cost, the state $\tilde{x}^j$ is the predicted trajectory of the nominal system with u_j computed by the LMPC of Eq. 8 and all the other inputs are the optimal input trajectories at iteration $c-1$ of the rest of the distributed controllers. The optimal solution to this optimization problem is denoted by $u_j^{*,c}(\tau|t_k)$ which is defined for $\tau \in [0, N\Delta)$. Accordingly, we define the final optimal input trajectory of LMPC j (that is, the optimal trajectories computed at the last iteration) as $u_j^{*,f}(\tau|t_k)$ which is also defined for $\tau \in [0, N\Delta)$.

Note that for the first iteration of each distributed LMPC, the input trajectories defined in Eq. 7 are used as the initial input trajectory guesses; that is, $u_i^{*,0} = u_{n,i}$ with $i = 1, \dots, m$.

The constraint of Eq. 8d imposes a limit on the input change between two consecutive iterations. This constraint allows LMPC j to take advantage of the input trajectories received in the last iteration (i.e., $u_i^{*,c-1}$, $\forall i \neq j$) to predict the future evolution of the system state without introducing significant errors. For LMPC j (i.e., u_j), the magnitude of input change between two consecutive iterations is restricted to be smaller than a positive constant Δu_j. The constraint of Eq. 8g is used to guarantee the closed-loop stability.

The manipulated inputs of the proposed control design from time t_k to t_{k+1} ($k = 0, T, 2T, \dots$) are defined as follows:

$$u_i(t) = u_i^{*,f}(t - t_k|t_k),\ i = 1, \dots, m,\ \forall t \in [t_k, t_{k+1}). \tag{9}$$

For the iterations in the design of Eq. 8, there are different choices of the termination condition. For example, the number of iterations c may be restricted to be smaller than a maximum iteration number $c_{\max}$ (i.e., $c \leq c_{\max}$) or the iterations may be terminated when the difference of the performance or the solution between two consecutive iterations is smaller than a threshold value or the iterations maybe terminated when a maximum computational time is reached.

Next, we describe the design of the distributed controllers at the time instants in which the network is open. In order to improve the performance, between two slow sampling times, each controller uses the available local fast sampled measurements to adjust its control input based on the calculated optimal input trajectory for the current time obtained at the last time instant in which the network was closed. In order to guarantee closed-loop stability, the maximum deviation of the adjusted inputs from the optimal input trajectory at each time step is bounded.

Between two slow sampling times, each controller estimates the current full system state using an observer based on the system model and the available information. Specifically, the observer for controller i takes the following form for $t \in [t_{l-1}, t_l)$:

$$\begin{aligned} \dot{\hat{x}}^i(t) &= f(\hat{x}^i(t)) + g_i(\hat{x}^i(t)) u_i^*(t) \\ &\quad + \textstyle\sum_{j=1}^{m, j\neq i} g_j(\hat{x}^i(t)) u_j^{*,f}(t - t_k|t_k) \\ \hat{x}^i(t_{l-1}) &= x_e^i(t_{l-1}) \end{aligned} \tag{10}$$

where $\hat{x}^i$ is the state of this observer, $u_j^{*,f}(\tau|t_k)$ is the predicted optimal input trajectories at time instant t_k (in which the network was closed), $u_i^*(t)$ is the actual input that has been applied to subsystem i, and $x_e^i(t_{l-1})$ is the full state estimate obtained at t_{l-1}. The state estimate $x_e^i(t_l)$, $l \neq 0, T, 2T, \dots$, is a combination of the state of the observer of Eq. 10 and of the available local state information $x_{f,i}(t_l)$ as follows:

$$x_e^i(t_l) = [\hat{x}_1^i(t_l)^T \cdots \check{x}_i(t_l)^T \cdots \hat{x}_m^i(t_l)^T]^T$$

where $\check{x}_i(t_l)^T = [x_{f,i}^T\ \hat{x}_{s,i}^T]$.

Specifically, the optimization problem of LMPC j for a time instant t_l, $l \neq 0, T, 2T, \dots$ is as follows:

$$\min_{u_j \in S(\Delta)} \int_0^{N\Delta} [\check{x}^j(\tau)^T Q_c \check{x}^j(\tau) + \textstyle\sum_{i=1}^m u_i(\tau)^T R_{ci} u_i(\tau)] d\tau \tag{11a}$$

$$\text{s.t. } \dot{\check{x}}^j(\tau) = f(\check{x}^j(\tau)) + \textstyle\sum_{i=1}^m g_i(\check{x}^j(\tau)) u_i \tag{11b}$$

$$u_i(\tau) = u_i^{*,f}(t_l - t_k + \tau|t_k), \quad \forall i \neq j,\ \tau \in [0, t_k + N\Delta - t_l) \tag{11c}$$

$$u_i(\tau) = h_i(\check{x}^j(\tau)),\ \forall i \neq j,\ \tau \in [t_k + N\Delta - t_l, N\Delta) \tag{11d}$$

$$\left|u_j(\tau) - u_j^{*,f}(t_l - t_k + \tau|t_k)\right| \le \Delta u_j,$$
$$\tau \in [0, t_k + N\Delta - t_l) \quad (11e)$$
$$u_j(\tau) \in U_j \quad (11f)$$
$$\check{x}^j(0) = x_e^j(t_l) \quad (11g)$$

where t_k is the last time instant in which the communication network is closed, the state $\check{x}^j$ is the predicted trajectory of the nominal system with u_j computed by the LMPC of Eq. 11 and all the other inputs are the optimal input trajectories determined by the constraints of Eqs. 8c and 11d. In this optimization problem, the input u_j is restricted to be within a bounded region around the reference input trajectories given by $u_j^{*,f}(\tau|t_k)$ and $h(x)$. The optimal solution to this optimization problem is denoted by $u_j^{*,l}(\tau|t_l)$ which is defined for $\tau \in [0, N\Delta)$.

The manipulated inputs of the control design of Eq. 11 from t_l to t_{l+1} ($l \neq 0, T, 2T, \ldots$) are defined as follows:

$$u_i(t) = u_i^{*,l}(t - t_l|t_l),\ i = 1, \ldots, m,\ \forall t \in [t_l, t_{l+1}). \quad (12)$$

In the design of Eqs. 8-9 and 11-12, the closed-loop stability of the system of Eq. 2 is guaranteed by the design of Eqs. 8-9 at each sampling time t_k, $k = 0, T, 2T, \ldots$, when the full state measurements are available and the communication network is closed. The design of Eqs. 11-12 takes advantage of the predicted input trajectories $u_i^{*,f}$, $i = 1, \ldots, m$, at sampling times t_k, $k = 0, T, 2T, \ldots$, and the additional available fast-sampling state measurements to adjust the predicted inputs, $u_i^{*,f}$, to improve the closed-loop performance.

C. Stability analysis

The proposed DMPC of Eqs. 8-9 and 11-12 computes the inputs u_i, $i = 1, \ldots, m$, applied to the system of Eq. 2 in a way such that in the closed-loop system, the value of the Lyapunov function at time instant t_k (i.e., $V(x(t_k))$) is a decreasing sequence of values with a lower bound. Following Lyapunov arguments, this property guarantees practical stability of the closed-loop system. This is achieved due to the constraints of Eq. 8g incorporated in each LMPC. This property is presented in Theorem 1 below. To prove this theorem, we need the following propositions.

Proposition 1 (c.f. [14]): Consider the nominal sampled trajectory x_h of the system of Eq. 2 in closed-loop with the Lyapunov-based controller $h(x)$ applied in a sample-and-hold fashion and with open-loop state estimation. Let $\Delta, \epsilon_s > 0$ and $\rho > \rho_s > 0$ satisfy

$$-\alpha_3(\alpha_2^{-1}(\rho_s)) + L^* M \le -\epsilon_s/\Delta \quad (13)$$

with $L^* = L_x + \sum_{i=1}^m L_{u_i} u_i^{\max}$. Then, if $\rho_{\min} < \rho$ where

$$\rho_{\min} = \max\{V(x_h(t+\Delta)) : V(x_h(t)) \le \rho_s\} \quad (14)$$

and $x_h(0) \in \Omega_\rho$, the following inequality holds:

$$V(x_h(k\Delta)) \le \max\{V(x_h(0)) - k\epsilon_s, \rho_{\min}\}. \quad (15)$$

Proposition 1 ensures that if the nominal system under the control $h(x)$ implemented in a sample-and-hold fashion and with open-loop state estimation starts in Ω_ρ, then it is ultimately bounded in $\Omega_{\rho_{\min}}$. The following proposition provides an upper bound on the deviation of the state trajectory obtained using the nominal model, from the actual state trajectory when the same control actions are applied.

Proposition 2 (c.f. [14]): Consider the systems

$$\dot{x}_a(t) = f(x_a(t)) + \textstyle\sum_{i=1}^m g_i(x_a(t))u_i(t) + k(x_a(t))w(t))$$
$$\dot{x}_b(t) = f(x_b(t)) + \textstyle\sum_{i=1}^m g_i(x_b(t))u_i(t)$$

with initial states $x_a(t_0) = x_b(t_0) \in \Omega_\rho$. There exists a class $\mathcal{K}$ function $f_W(\cdot)$ such that

$$|x_a(t) - x_b(t)| \le f_W(t - t_0), \quad (16)$$

for all $x_a(t), x_b(t) \in \Omega_\rho$ and all $w(t) \in W$ with $f_W(\tau) = R_w\theta(e^{R_x\tau} - 1)/R_x$ and R_w, R_x are positive numbers.

Proposition 3 bounds the difference between the magnitudes of the Lyapunov function of two states in Ω_ρ.

Proposition 3 (c.f. [14]): Consider the Lyapunov function $V(\cdot)$ of the system of Eq. 2. There exists a quadratic function $f_V(\cdot)$ such that

$$V(x) \le V(\hat{x}) + f_V(|x - \hat{x}|) \quad (17)$$

for all $x, \hat{x} \in \Omega_\rho$ with $f_V(s) = \alpha_4(\alpha_1^{-1}(\rho))s + M_v s^2$ and $M_v > 0$.

Proposition 4: (c.f. [16]) Consider the systems

$$\begin{aligned} \dot{x}_a(t) &= f(x_a(t)) + \textstyle\sum_{i=1}^m g_i(x_a(t))u_i^c(t) \\ \dot{x}_b(t) &= f(x_b(t)) + \textstyle\sum_{i=1}^{m,\ i\neq j} g_i(x_b(t))u_i^{c-1}(t) \\ &\quad + g_j(x_b(t))u_j^c(t) \end{aligned}$$

with initial states $x_a(t_0) = x_b(t_0) \in \Omega_\rho$. There exists a class $\mathcal{K}$ function $f_{X,j}(\cdot)$ such that

$$|x_a(t) - x_b(t)| \le f_{X,j}(t - t_0) \quad (18)$$

for all $x_a(t)$, $x_b(t) \in \Omega_\rho$, and $u_i^c(t)$, $u_i^{c-1} \in U_i$ and $|u_i^c(t) - u_i^{c-1}(t)| \le \Delta u_i$, $i = 1, \ldots, m$ and $f_{X,j}(\tau) = \frac{C_{2,j}}{C_{1,j}}(e^{C_{1,j}\tau} - 1)$ with $C_{2,j}$ and $C_{1,j}$ are positive constants.

Proposition 4 bounds the difference between the nominal state trajectory under the optimized control inputs and the predicted nominal state trajectory generated in each LMPC optimization problem. To simplify the proof of Theorem 1, we define a new function $f_X(\tau)$ based on $f_{X,i}$, $i = 1, \ldots, m$, as follows:

$$f_X(\tau) = \textstyle\sum_{i=1}^m \left(\frac{1}{m}L_x + L_{u_i}u_i^{\max}\right)\left(\frac{1}{C_{1,i}}f_{X,i}(\tau) - \frac{C_{2,i}}{C_{1,i}}\tau\right).$$

It is easy to verify that $f_X(\tau)$ is a strictly increasing and convex function of its argument. In Theorem 1 below, we provide sufficient conditions under which the DMPC of Eqs. 8-9 and 11-12 guarantees that the state of the closed-loop system is ultimately bounded in a region that contains the origin.

Theorem 1: Consider the system of Eq. 2 in closed-loop with the DMPC design of Eqs. 8-9 and 11-12 based on the controller $h(x)$ that satisfies the conditions of Eq. 3 with class $\mathcal{K}$ functions $\alpha_i(\cdot)$, $i = 1, 2, 3, 4$. If there exist $\Delta > 0$, $\epsilon_s > 0$, $\rho > \rho_{\min} > 0$, $\rho > \rho_s > 0$ and $N \ge T \ge 1$ that *satisfy the conditions of Eqs. 13 and 14 and the following inequality:*

$$-T\epsilon_s + f_X(T\Delta) + f_V(f_W(T\Delta)) + \sum_{i=1}^{m} C_{gi}\Delta u_i(T-1)\Delta < 0, \tag{19}$$

and if the initial state of the closed-loop system $x(t_0) \in \Omega_\rho$, then $x(t)$ is ultimately bounded in $\Omega_{\rho_b} \subseteq \Omega_\rho$ where

$$\rho_b = \rho_{\min} + f_X(T\Delta) + f_V(f_W(T\Delta)) + \sum_{i=1}^{m} C_{gi}\Delta u_i(T-1)\Delta.$$

Proof: We first consider two consecutive time instants in which the network is closed and we have full system state measurements: t_k and t_{k+T} ($k = 0, T, 2T, \ldots$). We prove that the Lyapunov function of the system is decreasing from t_k to t_{k+T}. In the following, we will denote the trajectory of the nominal system of Eq. 2 under the DMPC of Eqs. 8-9 and 11-12 as $\tilde{x}$ and denote the predicted nominal system trajectory in the evaluation of the LMPC of Eq. 8 at the final iteration as $\tilde{x}^j$ with $j = 1, \ldots, m$.

The derivative of the Lyapunov function of the nominal system of Eq. 2 under the DMPC of Eqs. 8-9 and 11-12 from t_k to t_{k+T} is expressed as follows:

$$\dot{V}(\tilde{x}(\tau)) = \frac{\partial V}{\partial x}\left(f(\tilde{x}(\tau)) + \sum_{i=1}^{m} g_i(\tilde{x}(\tau))u_i^*(\tau)\right). \tag{20}$$

where $u_i^*(\tau)$ is the actual input applied to the system and defined as follows:

$$u_i^*(\tau) = \begin{cases} u_i^{*,f}(\tau|t_k), \tau \in [0,\Delta) \\ u_i^{*,l}(\tau|t_l),\ \tau \in [0,\Delta), l = k+1, \ldots, k+T-1. \end{cases}$$

Adding the equality of Eq. 20 and the constraints of Eq. 8g together, we can obtain the following inequality for all $\tau \in [0, T\Delta]$:

$$\begin{aligned}\dot{V}(\tilde{x}(\tau)) \leq\ & \frac{\partial V}{\partial x}\left(f(\tilde{x}(\tau)) + \sum_{i=1}^{m} g_i(\tilde{x}(\tau))u_i^*(\tau)\right) \\ & + \frac{\partial V}{\partial x}\left(f(x_h(\tau|t_k)) + \sum_{i=1}^{m} g_i(x_h(\tau|t_k))u_{n,i}(\tau|t_k)\right) \\ & - \frac{\partial V}{\partial x}\left(\frac{1}{m}f(\tilde{x}^1(\tau)) + g_1(\tilde{x}^1(\tau))u_1^{*,f}(\tau|t_k)\right) - \ldots \\ & - \frac{\partial V}{\partial x}\left(\frac{1}{m}f(\tilde{x}^m(\tau)) + g_m(\tilde{x}^m(\tau))u_m^{*,f}(\tau|t_k)\right).\end{aligned} \tag{21}$$

Reworking the inequality of Eq. 21, the following inequality can be obtained for $\tau \in [0, T\Delta]$:

$$\begin{aligned}\dot{V}(\tilde{x}(\tau)) \leq\ & \frac{\partial V}{\partial x}\left(f(x_h(\tau|t_k)) + \sum_{i=1}^{m} g_i(x_h(\tau|t_k))u_{n,i}(\tau|t_k)\right) \\ & + \frac{\partial V}{\partial x}\left(\frac{1}{m}f(\tilde{x}(\tau)) + g_1(\tilde{x})u_1^{*,f}(\tau|t_k)\right) \\ & - \frac{\partial V}{\partial x}\left(\frac{1}{m}f(\tilde{x}^1(\tau)) + g_1(\tilde{x}^1(\tau))u_1^{*,f}(\tau|t_k)\right) \\ & + \ldots \\ & + \frac{\partial V}{\partial x}\left(\frac{1}{m}f(\tilde{x}(\tau)) + g_m(\tilde{x})u_m^{*,f}(\tau|t_k)\right) \\ & - \frac{\partial V}{\partial x}\left(\frac{1}{m}f(\tilde{x}^m(\tau)) + g_m(\tilde{x}^m(\tau))u_m^{*,f}(\tau|t_k)\right) \\ & + \sum_{i=1}^{m} \frac{\partial V}{\partial x}g_i(\tilde{x})\left(u_i^*(\tau) - u_i^{*,f}(\tau|t_k)\right)\end{aligned} \tag{22}$$

By the continuity and locally Lipschitz properties assumed for the vector fields $f(\cdot)$, $g_i(\cdot)$, $i = 1, \ldots, m$, and using the constants defined in Eq. 5, the following inequality can be obtained for $\tau \in [0, T\Delta]$ from the inequality of Eq. 22:

$$\begin{aligned}\dot{V}(\tilde{x}(\tau)) \leq\ & \dot{V}(x_h(\tau|t_k)) \\ & + \left(\frac{1}{m}L_x + L_{u_1}u_1^{*,f}(\tau|t_k)\right)|\tilde{x}(\tau) - \tilde{x}^1(\tau)| + \ldots \\ & + \left(\frac{1}{m}L_x + L_{u_m}u_m^{*,f}(\tau|t_k)\right)|\tilde{x}(\tau) - \tilde{x}^m(\tau)| \\ & + \sum_{i=1}^{m} C_{g_i}\left(u_i^*(\tau) - u_i^{*,f}(\tau|t_k)\right).\end{aligned} \tag{23}$$

Using Proposition 4 and the inequality of Eq. 23, we have:

$$\begin{aligned}\dot{V}(\tilde{x}(\tau)) \leq\ & \dot{V}(x_h(\tau|t_k)) + \left(\frac{1}{m}L_x + L_{u_1}u_1^{\max}\right)f_{X,1}(\tau) + \ldots \\ & + \left(\frac{1}{m}L_x + L_{u_m}u_m^{\max}\right)f_{X,m}(\tau) \\ & + \sum_{i=1}^{m} C_{g_i}\left(u_i^*(\tau) - u_i^{*,f}(\tau|t_k)\right)\end{aligned} \tag{24}$$

Integrating the inequality of Eq. 24 from $\tau = 0$ to $\tau = T\Delta$ and taking into account that $\tilde{x}(t_k) = x_h(t_k)$, the constraints of Eqs. 8d and 11e and the definitions of $f_X(\cdot)$ and $u^*(\tau)$, the following inequality can be obtained:

$$V(\tilde{x}(t_{k+T})) \leq V(x_h(t_{k+T})) + f_X(T\Delta) + \sum_{i=1}^{m} C_{gi}\Delta u_i(T-1)\Delta.$$

Using the above inequality and Propositions 1 and 3, we can obtain the following inequality

$$\begin{aligned}V(x(t_{k+T})) \leq\ & \max\{V(x(t_k)) - T\epsilon_s, \rho_{\min}\} \\ & + f_X(T\Delta) + f_V(f_W(T\Delta)) \\ & + \sum_{i=1}^{m} C_{gi}\Delta u_i(T-1)\Delta.\end{aligned} \tag{25}$$

If the condition of Eq. 19 is satisfied, we know that there exists $\epsilon_w > 0$ such that the following inequality holds

$$V(x(t_{k+T})) \leq \max\{V(x(t_k)) - \epsilon_w, \rho_b\} \tag{26}$$

which implies that if $x(t_k) \in \Omega_\rho/\Omega_{\rho_b}$, then $V(x(t_{k+T})) < V(x(t_k))$, and if $x(t_k) \in \Omega_{\rho_b}$, then $V(x(t_{k+T})) \leq \rho_b$.

Because the upper bound on the difference between the Lyapunov function of the actual trajectory x and the nominal trajectory $\tilde{x}$ (the term $f_X(T\Delta) + f_V(f_W(T\Delta)) + \sum_{i=1}^{m} C_{gi}\Delta u_i(T-1)\Delta$) is a strictly increasing function of time, the inequality of Eq. 26 also implies that

$$V(x(t)) \leq \max\{V(x(t_k)) - \epsilon_w, \rho_b\},\ \forall t \in [t_k, t_{k+T}]. \tag{27}$$

Using the inequality of Eq. 27 recursively, it can be proved that if $x(t_0) \in \Omega_\rho$, then the closed-loop trajectories of the system of Eq. 2 under the proposed DMPC design stay in Ω_ρ for all times (i.e., $x(t) \in \Omega_\rho$ for all t). Moreover, if $x(t_0) \in \Omega_\rho$, the closed-loop trajectories of the system of Eq. 2 under the proposed iterative DMPC design satisfy

$$\limsup_{t \to \infty} V(x(t)) \leq \rho_b.$$

This proves that the results stated in Theorem 1 hold. ■

IV. APPLICATION TO A CHEMICAL PLANT

The process considered in this study is a three vessel, reactor-separator system consisting of two continuously stirred tank reactors (CSTRs) and a flash tank separator. The process description, a detailed version of which can be found in [21], is briefly reviewed below: a feed stream to the first CSTR F_{10} contains the reactant A which is converted into the desired product B. The desired product B can then further react into an undesired side-product C. The effluent of the first CSTR along with additional fresh feed F_{20} makes up the inlet to the second CSTR. The reactions $A \to B$ and $B \to C$ (referred to as 1 and 2, respectively) take place in the two CSTRs in series before the effluent from CSTR 2 is fed to a flash tank. The overhead vapor from the flash tank is condensed and recycled to the first CSTR, and the bottom product stream is removed. Each of the tanks has an external heat input/removal actuator. The process model, consisting of twelve nonlinear ordinary differential equations (ODEs), is numerically simulated using an explicit Euler integration method. Bounded random noise was added to the right-hand side of the ODEs to simulate disturbances/model uncertainty. Please refer to [21] for the detailed modeling of the process and the process parameters used in the simulations.

This process is divided into three subsystems corresponding to the first CSTR, the second CSTR and the separator, respectively. For the three subsystems, we will refer to them as subsystem 1, subsystem 2 and subsystem 3, respectively. The state of subsystem 1 is defined as the deviations of the temperature and species concentrations in the first CSTR from their desired steady-state; that is, $x_1^T = [x_{f,1}^T, x_{s,1}^T]$ where $x_{f,1} = T_1 - T_{1s}$ and $x_{s,1}^T = [C_{A1} - C_{A1s}\ C_{B1} - C_{B1s}\ C_{C1} - C_{Cs}]$ denote fast sampled and slowly sampled measurements of subsystem 1, respectively. The states of subsystems 2 and 3 are defined similarly; they are $x_2^T = [T_2 - T_{2s}\ C_{A2} - C_{A2s}\ C_{B2} - C_{B2s}\ C_{C2} - C_{C2s}]$ and $x_3^T = [T_3 - T_{3s}\ C_{A3} - C_{A3s}\ C_{B3} - C_{B3s}\ C_{C3} - C_{C3s}]$. Accordingly, the state of the whole process is defined as a combination of the states of the three subsystems; that is, $x^T = [x_1^T\ x_2^T\ x_3^T]$. Due to the simplicity of temperature measurement at each sampling time, we denote the temperature as the fast sampled measurement of each subsystem.

Each of the tanks has an external heat input which is the control input associated with each subsystem, that is, $u_1 = Q_1 - Q_{1s}$, $u_2 = Q_2 - Q_{2s}$ and $u_3 = Q_3 - Q_{3s}$ where Q_{1s}, Q_{2s} and Q_{3s} are the steady-state input values corresponding to a desired unstable operating steady-state (set-point): $x_s^T = [369.53\ 3.31\ 0.17\ 0.04\ 435.25\ 2.75\ 0.44\ 0.11\ 435.25\ 2.88\ 0.49\ 0.12]$. The inputs are subject to constraints as follows: $|u_1| \leq 5 \times 10^4\ KJ/hr$, $|u_2| \leq 1.5 \times 10^5\ KJ/hr$, and $|u_3| \leq 2 \times 10^5\ KJ/hr$. Three distributed MPC controllers (controller 1, controller 2 and controller 3) will be designed to manipulate the three inputs in the three subsystems, respectively.

We assume that $x_{f,1}$, $x_{f,2}$, $x_{f,3}$ are measured and sent to controller 1, controller 2 and controller 3, respectively, at synchronous time instants $t_l = l\Delta$, $l = 0, 1, \ldots$, with $\Delta = 0.01\ hr = 36\ sec$ while we assume that each controller receives $x_{s,i}$ every $T = 4$ sampling times. The three subsystems exchange their states at $t_k = kT\Delta$, $k = 0, 1, \ldots$; that is, the full system state x is sent to all the controllers every $T = 4$ sampling times.

In the simulations, we consider a quadratic Lyapunov function $V(x) = x^T P x$ with $P = diag([20\ 10^3\ 10^3\ 10^3\ 20\ 10^3\ 10^3\ 10^3\ 20\ 10^3\ 10^3\ 10^3])$.

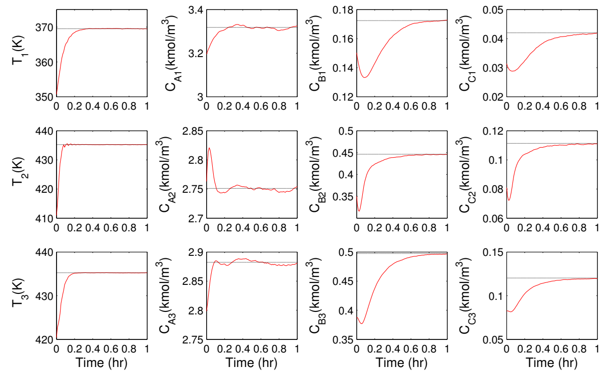

Fig. 2. State trajectories of the process under the DMPC design of Eqs. 8-9 and 11-12.

We first design three P controllers with proportional gains $K_{p1} = K_{p3} = 5000$, $K_{p2} = 5100$ based on the measurements of T_1, T_2 and T_3, respectively. Through extensive simulations, it has been verified that the three P controllers can stabilize the process at the open-loop unstable operating steady-state x_s and satisfy the input constraints in the state region $V(x) \leq 2000$ (i.e., $\rho \leq 2000$). After designing the P controllers, we add very small integral terms (with integral time constants $\tau_{I1} = \tau_{I2} = \tau_{I3} = 10^6$, respectively) to the three P controllers to form three new PI controllers to achieve offset-less tracking. Because of the small integral terms, the closed-loop system stability region estimate under P control is nearly preserved under PI control. Therefore, in the design of the DMPC, the PI controllers will be used to approximate a Lyapunov-based controller; that is $h(x)$.

Based on the PI controllers and $V(x)$, we design the three LMPCs following Eqs. 8-9 and 11-12 and refer to them as LMPC 1, LMPC 2 and LMPC 3. For each LMPC, we also design a state observer following Eq. 10. In the design of the LMPC controllers, the weighting matrices are chosen to be $Q_c = diag([20\ 10^3\ 10^3\ 10^3\ 20\ 10^3\ 10^3\ 10^3\ 20\ 10^3\ 10^3\ 10^3])$, $R_1 = R_2 = R_3 = 10^{-6}$. The prediction horizon for the optimization problem is $N = 5$ with a time step of $\Delta = 0.01\ hr$. In the simulations, we put a maximum iteration number $c_{\max}$ on the DMPC evaluation and the maximum iteration number is chosen to be $c_{\max} = 2$. Also, we set Δu_i as 10% percent of $u_i^{\max}$ ($i = 1, 2, 3$). The optimization problems are solved by the open source interior point optimizer Ipopt. The initial condition which is utilized to carry out simulations is $x(0)^T = [350.69\ 3.19\ 0.15\ 0.03\ 410.91\ 2.76\ 0.34\ 0.08\ 420.42\ 2.79\ 0.38\ 0.08]$.

Figures 2 and 3 show the temperature, concentration and input trajectories of the process under the DMPC design of Eqs. 8-9 and 11-12. As it can be seen from these figures, the proposed DMPC system can steer the system state to the desired steady-state. We have also carried out a set of simulations to compare four different control schemes from a performance point of view. The four control schemes considered are as follows: (1) the proposed DMPC design of Eqs. 8-9 and 11-12; (2) a DMPC design with LMPCs formulated

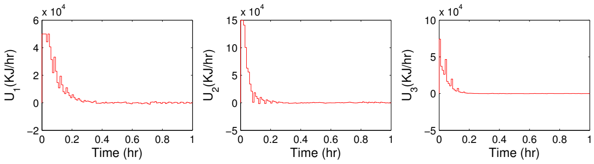

Fig. 3. Input trajectories of the process under the DMPC design of Eqs. 8-9 and 11-12.

TABLE I

TOTAL PERFORMANCE COST COMPARISON ALONG THE CLOSED-LOOP SYSTEM TRAJECTORIES UNDER DIFFERENT CONTROL SCHEMES.

sim.	(1)	(2)	(3)	(4)
1	7650	8158	9034	10182
2	7722	8388	9028	9033
3	7824	8570	9404	10020
4	7918	9286	9744	11863
5	7947	8815	9631	10048
6	8089	9803	9917	12868
7	8126	9051	9207	10056
8	8158	9151	9510	10463
9	8278	9499	9797	11154
10	11189	12036	13794	13988
11	20459	21843	22201	22882
12	33637	36814	36818	40737
13	40913	45335	45149	53374

as in Eq. 8 which are only evaluated at instants when full system states are available and the inputs are implemented in open-loop between two full system state measurements; in this case, the additional fast sampled measurements are not used to improve the closed-loop performance; (3) the proposed DMPC but without communication between distributed controllers and each controller estimates the full system states and the actions of the other controllers based on the process model and $h(x)$; in this case, a distributed LMPC in the DMPC design takes advantage of both fast and slowly sampled measurements of its own local subsystem but does not receive any input or state information from the other subsystems; (4) the DMPC design as in (2) but without communication between distributed controllers and each controller estimates the full system states and actions of the other controllers based on the process model and $h(x)$. We perform these simulations under different initial conditions and different process noise/disturbances. To carry out this comparison, we have computed the total cost of each simulation based on the index of the following form:

$$J = \sum_{i=0}^{M}[x(t_i)^T Q_c x(t_i) + \sum_{j=1}^{3} u_j(t_i)^T R_{cj} u_j(t_i)]$$

where $t_0 = 0$ is the initial time of the simulations and $t_M = 1\ hr$ is the end of the simulations. Table I shows the total cost computed for 13 different closed-loop simulations under the four different control schemes. From Table I, we see that the proposed DMPC design gives the best performance in all the simulations.

In the final set of simulations, we compared the ratio of the evaluation times of the LMPC problems in the centralized LMPC [20] and the proposed DMPC design. We consider the case where each controller evaluates the input trajectories every $T = 4$ sampling times and evaluate the computational time of the LMPC optimization problems for 1000 independent closed-loop runs. We found that the ratio of the mean evaluation time of the LMPC that requires the largest evaluation time in the DMPC architecture versus the mean evaluation time of the centralized LMPC is 0.277. The reduction in real time units depends on the type of processor used to carry out the calculations. From this set of simulations, we see that the proposed DMPC design leads to about 70% reduction in the controller evaluation time.

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