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Velocity and Heading Tracking Control For Small-Scale Unmanned Helicopters

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***Abstract*— This paper presents a linear tracking control design technique for small-scale unmanned helicopters. The design objective is for the helicopter to track predefined velocity and heading reference trajectories. The controller design is based on a generic linear model which successfully describes the dynamic behavior of most small-scale helicopters. The flight controller is composed of two distinct output feedback loops of the helicopter's longitudinal/lateral and heading/heave dynamics tracking error. The tracking error is determined using a state generator based on the reference trajectories and their higher derivatives. The state generator can be systematically constructed using the backstepping approach and applying a physically meaningful simplification assumption over the helicopter dynamics. The controller performance was successfully evaluated using a realistic flight simulator.**

I. INTRODUCTION

Helicopters are underactuated, highly nonlinear systems with significant inter-axis dynamic coupling. They are considered to be much more unstable than fixed-wing aircraft, and constant control action must be sustained at all times.

In recent years there is considerable research related to the helicopter flight control problem. Early experimental results indicated that classical control techniques using Single-Input Single-Output feedback loops for each input exhibit moderate performance since they are unable to account for the coupled multivariable nature of the helicopter dynamics [1].

The majority of linear flight controllers that have been applied to autonomous helicopter platforms, are based on the $\mathcal{H}_\infty$ approach. An $\mathcal{H}_\infty$ static output feedback controller design was proposed in [2] for the stabilization of a small-scale helicopter at hover. Promising flight results have been obtained in the work reported in [3]. This approach applied a blending of multivariable $\mathcal{H}_\infty$ loop shaping control techniques and system identification for the development of the flight control system. An interesting comparative study between several controller designs is given in [4], [5]. The flight validation indicated that in the multivariable design case it is preferable to design multiple feedback loops that correspond to decoupled subsystems of the helicopter dynamics rather than designing the controller for the complete helicopter dynamics.

In most case studies that exist in the literature, the proposed designs are developed based on specific small-scale helicopter platforms. The dependence of the analysis to a particular platform is attributed to the lack of a generic nominal helicopter model that is capable of encapsulating the dynamic behavior of a large family of small scale helicopters. In addition, very few work has been done focusing to the tracking problem contrary to the hovering case. Tracking is typically achieved by simple feedback terms of the velocity error.

This paper presents a linear tracking controller for small scale helicopters. The objective is for the helicopter to track predefined translational velocity and yaw reference trajectories. The controller design is based on the structure of the helicopter linear dynamic model proposed in [6].

The main contribution of this design is the decoupling of the overall controller to two distinct feedback loops that pass the intuitive notion of helicopter manned piloting to the mathematical derivation of the controller. In addition, the controller includes a state generator, based on the reference trajectories, that guarantees the tracking objectives.

This is achieved by separating the helicopter dynamics into two interconnected subsystems representing the longitudinal/lateral and heading/heave motion, respectively. By disregarding the effect of the forces produced by the flapping motion of the main rotor, the approximated subsystems are in feedback form and, therefore, differentially flat. Due to the differential flatness of the system dynamics, a desired system state and input can be determined, composed of the components of the reference output and their higher derivatives. When the helicopter state is regulated to this desired state, the tracking error tends asymptotically to zero. The desired state can be easily and systematically determined by the backstepping approach. Similarly to [7], the desired state vector can be used for the design of meaningful system trajectories. The overall control law is a superposition of the desired input and an output feedback component of the state error. The output feedback component can be chosen by any design that exists in the literature. The design also allows for scheduling of multiple similar controllers based on linear models of the same structure.

The controller performance was successfully tested in *X-Plane*, a commercial flight simulator which is a very good indicator of the applicability of this approach to a real flight situation. The simulation results indicate that the combination of the trajectory generator and the separation of the control law to two distinct feedback loops achieved superior tracking performance for a wide range of operating modes.

This paper is organized as follows: Section II presents the linear helicopter model. Section III presents the outline of the controller design and the decomposition of the helicopter dynamics to two interconnected subsystems. Section IV gives a detailed derivation of the two feedback control laws for the longitudinal/lateral and heading/heave subsystems, as well as the stability of the complete helicopter dynamics. In Section V the simulation flight results obtained from *X-*

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Plane are presented. Finally, concluding remarks are given in Section VI.

II. Helicopter Linear Model

The controller design should be applicable to most small-scale helicopters. This claim requires the adoption of a nominal linear dynamic model structure, which is capable of capturing the dynamic behavior of a large family of small-scale helicopters. A suitable solution to this requirement is the adoption of the linear parametric model developed in [6].

The linearized dynamic model proposed by [6] has been successfully adopted for control applications in a large number of small-scale helicopters of different sizes and specifications [2], [8]–[11]. These experimental applications indicate that the linear model proposed by [6] provides a generalized and physically meaningful solution for developing practical linear models for small-scale helicopters.

The helicopter motion variables are expressed with respect to a body-fixed reference frame defined as $\mathcal{F}_B = \{O_B, \vec{i}_B, \vec{j}_B, \vec{k}_B\}$, where the center O_B is located at the Center of Gravity (CG) of the helicopter. The directions of the body-fixed frame orthonormal vectors $\{\vec{i}_B, \vec{j}_B, \vec{k}_B\}$ are shown in Fig. 1.

The helicopter's linear and angular velocity vectors, with respect to the body-fixed frame, are denoted by $v^B = [u\ v\ w]^T$ and $\omega^B = [p\ q\ r]^T$, respectively. The helicopter attitude is expressed by the roll (ϕ), pitch (θ) and yaw (ψ) angles. The helicopter motion variables are shown in Fig. 1.

The control input is defined as $u_c = [u_{lon}\ u_{lat}\ u_{col}\ u_{ped}]^T$ where u_{col} and u_{ped} are the collective controls of the main and tail rotor, respectively. The collective commands control the magnitude of the main and tail rotor thrust. The other two control commands u_{lon}, u_{lat} are the cyclic controls of the helicopter which control the inclination of the Tip-Path-Plane (TPP) on the longitudinal and lateral direction. The TPP is the plane in which the tips of the blades lie.

The TPP is characterized by two angles, a and b which represent the tilt of the TPP at the longitudinal and lateral axis respectively. The inclination of the TPP can be seen in Fig. 1. The TPP is itself a dynamic system.

The adopted linear model represents the dynamic response of the helicopter perturbed state vector from the reference flight condition. In this case, the reference operating condition is hover. The liner state space model is described by:

$$\dot{x} = Ax + Bu_c \tag{1}$$

where the state vector is given by:

$$x = [u\ \ v\ \ \theta\ \ \phi\ \ q\ \ p\ \ a\ \ b\ \ w\ \ r\ \ \psi]^T$$

The entries of the matrices A and B are given in Table I. These entries are also called stability and control derivatives, respectively. The term g denotes the gravitational constant while τ_f is the main rotor's time constant.

The overall dynamics constitute a coupled linear system of the helicopter motion variables and the main rotor flapping dynamics. The order of the above model can be increased by including the dynamics of the stabilizer bar and the yaw damping system. These two subsystems provide additional damping to the angular velocity dynamics. Since they constitute additional feedback sources of the angular dynamics, their presence in the state space system does not influence the controller design. Therefore, their effect has been omitted from the helicopter model.

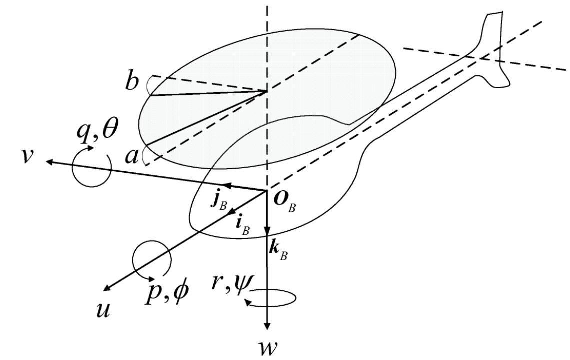

Fig. 1. The helicopter's body-fixed frame, the Tip-Path-Plane angles and linear/angular velocity components.

III. Controller Design Outline

The controller's objective is for the helicopter to autonomously track predefined bounded velocity and yaw reference trajectories. The output vector of interest of the helicopter is denoted by $y = [u\ \ v\ \ w\ \ \psi]^T$. The design task is for the helicopter to track the reference output $y_r = [u_r\ \ v_r\ \ w_r\ \ \psi_r]^T$. The measured states, available for feedback, are given by the vector:

$$y_m = [u\ \ v\ \ w\ \ p\ \ q\ \ r\ \ \theta\ \ \phi\ \ \psi]^T = C_m x$$

where C_m is a matrix of obvious entries and dimensions. In real life applications, only the helicopter motion state variables can be directly measured while the flapping angles are typically absent from the available measurements.

The first part of the design involves determining a desired state vector x_d that is composed only by the components of the reference output vector y_r and their higher derivatives. Denote $\varepsilon = x - x_d$ the error between the actual helicopter state and its desired value. The desired vector x_d should be chosen in such a way that, given:

$$\lim_{t\to\infty} \|\varepsilon(t)\| = 0 \qquad \text{then} \qquad \lim_{t\to\infty} \|y(t) - y_r(t)\| = 0 \tag{2}$$

The contribution of the proposed design is the development of a simple recursive procedure for deriving the pair (x_d, u_c^d) that satisfies (2) and also:

$$\dot{x}_d = Ax_d + Bu_c^d$$

The choice of the pair (x_d, u_c^d) is based on the backstepping design methodology. Backstepping provides a systematic methodology for the output tracking problem of systems in feedback form.

Due to the presence of the stability derivatives X_a and Y_b the system in (1), does not belong to this class of systems. A common simplification practice, presented in [7], is to neglect the effect of the lateral and longitudinal forces produced by the flapping angles. These parasitic forces have a minimal effect on the translational dynamics compared to the propulsion forces produced by the stability derivatives X_θ and Y_ϕ (in (1) are denoted by $-g$ and g, respectively). This assumption is physically meaningful and results into a linear system in feedback form.

TABLE I

LINEAR HELICOPTER MODEL STATE AND CONTROL MATRICES

$$A = \begin{bmatrix} X_u & 0 & -g & 0 & 0 & 0 & X_a & 0 & 0 & 0 & 0 \\ 0 & Y_v & 0 & g & 0 & 0 & 0 & Y_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ M_u & M_v & 0 & 0 & 0 & 0 & M_a & 0 & 0 & 0 & 0 \\ L_u & L_v & 0 & 0 & 0 & 0 & 0 & L_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & -1/\tau_f & A_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 & B_a & -1/\tau_f & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & Z_a & Z_b & Z_w & Z_r & 0 \\ 0 & N_v & 0 & 0 & 0 & N_p & 0 & 0 & N_w & N_r & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ A_{lon} & A_{lat} & 0 & 0 \\ B_{lon} & B_{lat} & 0 & 0 \\ 0 & 0 & Z_{col} & 0 \\ 0 & 0 & N_{col} & N_{ped} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

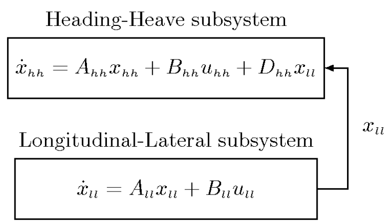

Fig. 2. Interconnection of the two helicopter subsystems.

Systems of strict feedback form are feedback linearizable and, therefore, differentially flat. A system is called differentially flat when all the state and input components may be expressed in terms of the system's outputs and their higher derivatives [12].

Having defined the desired state x_d and control vector u_c^d, the controller signal is constructed by the following superposition:

$$u_c = u_c^d + u_c^{fb} \qquad (3)$$

where u_c^{fb} is a feedback control law. Then, the error dynamics take the form:

$$\dot{\varepsilon} = A\varepsilon + Bu_c^{fb} \qquad (4)$$

The second control component, u_c^{fb}, may be chosen using a variety of output feedback techniques, such that the error ε is rendered globally asymptotically stable (GAS).

A close inspection of the model structure given in (1), indicates that the helicopter dynamics can be separated in two interconnected subsystems. The first subsystem accounts for the longitudinal and lateral motion. The second subsystem represents the coupled heading and heave dynamics. In particular, the longitudinal-lateral subsystem is given by:

$$\dot{x}_{ll} = A_{ll}x_{ll} + B_{ll}u_{ll} \qquad (5)$$

where:

$$x_{ll} = [u \;\; v \;\; \theta \;\; \phi \;\; q \;\; p \;\; a \;\; b]^T \quad \text{and} \quad u_{ll} = [u_{lon} \;\; u_{lat}]^T$$

The heading-heave dynamics subsystem is given by:

$$\dot{x}_{hh} = A_{hh}x_{hh} + B_{hh}u_{hh} + D_{hh}x_{ll} \qquad (6)$$

where:

$$x_{hh} = [\psi \;\; w \;\; r]^T \quad \text{and} \quad u_{hh} = [u_{ped} \;\; u_{col}]^T$$

The interconnection of the two subsystems is shown in Fig. 2. The controller design requires that the following assumptions associated with the helicopter linear model of (1) should hold:

Assumption 1. The matrix pairs (A_{ll}, B_{ll}) and (A_{hh}, B_{hh}) are controllable.

Assumption 2. The matrix $B \in \mathbb{R}^{11\times 4}$ has four linearly independent rows.

Assumption 3. The stability derivatives g, M_a and L_b are nonzero.

The above assumptions are necessary conditions required by the controller design. Regarding Assumption 1, lack of controllability indicates a helicopter that can not fly properly. In addition, each input must have a direct effect on the helicopter's motion, therefore, Assumption 2 should hold as well. Finally, if $M_a = 0$ or $L_b = 0$ this implies that no pitch and roll moments are transmitted to the helicopter.

At this stage, a preliminary control action is introduced for the input vectors u_{ll}, u_{hh} that normalizes the B_{ll} and B_{hh} matrices, respectively. Hence:

$$u_{ll} = (B_{ll}^n)^{-1}v_{ll} \qquad u_{hh} = (B_{hh}^n)^{-1}v_{hh}$$

where:

$$B_{ll}^n = \begin{bmatrix} A_{lon} & A_{lat} \\ B_{lon} & B_{lat} \end{bmatrix} \qquad B_{hh}^n = \begin{bmatrix} 0 & Z_{col} \\ N_{ped} & N_{col} \end{bmatrix}$$

and v_{ll}, v_{hh} are control vectors to be determined. Based on Assumption 3 the above inverse matrices are nonsingular. Substituting the above preliminary control actions the two subsystems of (5) and (6), become:

$$\begin{aligned} \dot{x}_{ll} &= A_{ll}x_{ll} + \bar{B}_{ll}v_{ll} \\ \dot{x}_{hh} &= A_{hh}x_{hh} + \bar{B}_{hh}v_{hh} + D_{hh}x_{ll} \end{aligned}$$

where $\bar{B}_{ll}^T = [0_{6\times 2} \; I_2]^T$ and $\bar{B}_{hh}^T = [0_{3\times 2} \; I_2]^T$. The initial system is now viewed as two interconnected subsystems in cascade form. The backstepping design is performed independently for each subsystem resulting in the cascaded error dynamics of the helicopter.

The controller structure requires designing of two independent feedback loops for each subsystem. This approach results in a mathematically consistent and rigorous methodology, which reflects the intuitive flight notion. The longitudinal/lateral motion is regulated independently from the heading and vertical motion of the helicopter. The same decomposition of the helicopter dynamics is also reported in [5]. The stability analysis of the controller design is given in detail in the following Section.

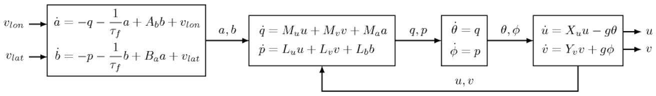

Fig. 3. Strict feedback interconnection of the longitudinal-lateral helicopter dynamics subsystem. The terms associated with the X_a and Y_b stability derivatives are disregarded.

IV. Velocity and Heading Tracking Controller Design

This Section provides details for designing the controller for velocity and heading tracking of the helicopter. The control problem is focused on the design of two feedback loops for each subsystem. After the introduction of the two feedback loops the stability analysis of the overall system dynamics is presented.

A. *Longitudinal-Lateral Dynamics*

As indicated in Section III, the effect of the translational forces produced by the flapping motion of the main rotor is parasitic and negligible compared to the main source of propulsion, which are the forces produced by the roll and pitch attitude change of the fuselage.

By neglecting the effect of the parameters X_a and Y_b, the longitudinal-lateral dynamics have a strict feedback form. The complete description of the longitudinal-lateral subsystem is given by:

$$\begin{aligned} \dot{x}_{ll} &= A_{ll}^{fb} x_{ll} + \bar{B}_{ll} v_{ll} \\ y_{ll} &= C_{ll} x_{ll} \\ y_{ll}^m &= C_{ll}^m x_{ll} \end{aligned} \tag{7}$$

where:

$$x_{ll} = [u \;\; v \;\; \theta \;\; \phi \;\; q \;\; p \;\; a \;\; b]^T \qquad v_{ll} = [v_{lon} \;\; v_{lat}]^T$$
$$y_{ll}^m = [u \;\; v \;\; \theta \;\; \phi \;\; q \;\; p]^T \qquad y_{ll} = [u \;\; v]^T$$

In the above equations y_{ll}^m is the measurement vector available for feedback, y_{ll} is the output of the subsystem and $C_{ll} \in \mathbb{R}2 \times 8$, $C_{ll}^m \in \mathbb{R}^{6\times 8}$ are matrices of obvious entries. The reference output vector is $y_{ll}^r = [u_r \;\; v_r]^T$. The matrix A_{ll}^{fb}, is identical to A_{ll} with the only difference that the stability derivatives X_a and Y_b are omitted. The interconnection of the approximated longitudinal-lateral subsystem is shown in Fig. 3.

The first goal of the controller design for this subsystem is to determine a desired state vector x_{ll}^d and a desired control input v_{ll}^d, with both of them being functions of the y_{ll}^r components and their higher derivatives, such that for the error $\varepsilon_{ll} = x_{ll} - x_{ll}^d$ given that:

$$\lim_{t\to\infty} \|\varepsilon_{ll}(t)\| = 0 \quad \text{then} \quad \lim_{t\to\infty} \|y_{ll}(t) - y_{ll}^r(t)\| = 0 \tag{8}$$

To do so, the control law of this subsystem is obtained by the following superposition:

$$v_{ll} = v_{ll}^d + v_{ll}^{fb} = \begin{bmatrix} v_{lon}^d \\ v_{lat}^d \end{bmatrix} + \begin{bmatrix} v_{lon}^{fb} \\ v_{lat}^{fb} \end{bmatrix}$$

where v_{ll}^{fb} is a feedback control law to be determined. The initial task is to select the pair (x_{ll}^d, v_{ll}^d) such that they satisfy the requirement of (8) and also:

$$\dot{x}_{ll}^d = A_{ll}^{fb} x_{ll}^d + \bar{B}_{ll} v_{ll}^d \tag{9}$$

For the derivation of the desired state vector x_{ll}^d and control input v_{ll}^d a recursive procedure based on the backstepping methodology is followed such that (8) and (9) are satisfied. The applicability of this approach is based on the fact that the longitudinal-lateral subsystem is differentially flat. Therefore, the derivation of the desired state and the nominal desired input based on the reference output is feasible.

Derivation of the error dynamics and the selection of the desired states and inputs occurs simultaneously. The basic idea of the recursive procedure is to start from the top state equations of the subsystem and gradually derive the desired state variables and the error dynamics of each level by moving downwards in each step. In each step the desired values of the state variables of lower levels are chosen in such a way that they cancel out the desired values of state variables of higher levels. Notation wise, from this point forward, denote by e_α the error of the variable α minus its desired value α_d.

The procedure begins by deriving the error dynamics of the translational velocity variables. Therefore:

$$\begin{aligned} \dot{e}_u &= -\dot{u}_r + X_u u_r - g\theta_d + X_u e_u - g e_\theta \\ \dot{e}_v &= -\dot{v}_r + Y_v v_r + g\phi_d + X_u e_v + g e_\phi \end{aligned}$$

The desired pitch and and roll angles are chosen such that they cancel out the values $\dot{u}_r$, u_r and $\dot{v}_r$, v_r, respectively. More precisely:

$$\theta_d = \frac{1}{-g}\left[\dot{u}_r - X_u u_r\right] \qquad \phi_d = \frac{1}{g}\left[\dot{v}_r - Y_v v_r\right] \tag{10}$$

It is apparent that the desired angles of (10) are functions of only the y_{ll}^r vector components and their first derivatives. With the above choice of the desired roll and pitch angles, the translational velocity error dynamics become:

$$\dot{e}_u = X_u e_u - g e_\theta \qquad \dot{e}_v = Y_v e_v + g e_\phi$$

The attitude angles error dynamics are:

$$\dot{e}_\theta = -\dot{\theta}_d + q_d + e_q \qquad \dot{e}_\phi = -\dot{\phi}_d + p_d + e_p$$

The desired values of the pitch and roll angular velocities are chosen such that they cancel out the effect of $\dot{\theta}_d$ and $\dot{\phi}_d$. Therefore:

$$q_d = \dot{\theta}_d \qquad p_d = \dot{\phi}_d$$

The roll and pitch attitude error dynamics become:

$$\dot{e}_\theta = e_q \qquad \dot{e}_\phi = e_p$$

Similarly, the angular velocity error dynamics are:

$$\begin{aligned} \dot{e}_q &= -\dot{q}_d + M_u u_d + M_v v_d + M_a a_d + M_u e_u + M_v e_v + M_a e_a \\ \dot{e}_p &= -\dot{p}_d + L_u u_d + L_v v_d + L_b b_d + L_u e_u + L_v e_v + L_b e_b \end{aligned}$$

The values of the desired flapping angles a_d and b_d are chosen as:

$$\begin{aligned} a_d &= \frac{1}{M_a}\left[\dot{q}_d - M_u u_d - M_v v_d\right] \\ b_d &= \frac{1}{L_b}\left[\dot{p}_d - L_u u_d - L_v v_d\right] \end{aligned}$$

Hence, the angular error velocity dynamics, become:

$$\begin{aligned} \dot{e}_q &= M_u e_u + M_v e_v + M_a e_a \\ \dot{e}_p &= L_u e_u + L_v e_v + L_b e_b \end{aligned}$$

Finally, the flapping angles error dynamics, are:

$$\begin{aligned}\dot{e}_a &= -\dot{a}_d - q_d - \frac{1}{\tau_f}a_d + A_b b_d \\ &\quad - e_q - \frac{1}{\tau_f}e_a + A_b e_b + v_{lon}^d + v_{lon}^{fb} \\ \dot{e}_b &= -\dot{b}_d - p_d - \frac{1}{\tau_f}b_d + B_a a_d \\ &\quad - e_p - \frac{1}{\tau_f}e_b + B_a e_a + v_{lat}^d + v_{lat}^{fb}\end{aligned}$$

The components of the control vector v_{ll}^d are chosen such that they cancel out the terms of all the desired state values and only the error state variables remain in the flapping error dynamic equations. Thus:

$$v_{lon}^{ds} = \dot{a}_d + q_d + \frac{1}{\tau_f}a_d - A_b b_d \qquad v_{lat}^{ds} = \dot{b}_d + p_d + \frac{1}{\tau_f}b_d - B_a a_d \tag{11}$$

It is easy to verify that the derived pair (x_{ll}^d, v_{ll}^d) satisfies the differential equation (9). The components of x_{ll}^d and v_{ll}^d are composed of the reference values u_r and v_r and their higher derivatives up to the fourth order. Therefore, the components of y_{ll}^r should belong to C^4. The final form of the longitudinal-lateral subsystem error dynamics is:

$$\begin{aligned}\dot{\varepsilon}_{ll} &= A_{ll}^{fb}\varepsilon_{ll} + \bar{B}_{ll}v_{ll}^{fb} \\ Y_{ll}^m &= C_{ll}^m \varepsilon_{ll}\end{aligned} \tag{12}$$

where:

$$\begin{aligned}\varepsilon_{ll} &= [e_u \quad e_v \quad e_\theta \quad e_\phi \quad e_q \quad e_p \quad e_a \quad e_b]^T \\ Y_{ll}^m &= [e_u \quad e_v \quad e_\theta \quad e_\phi \quad e_q \quad e_p]^T\end{aligned}$$

In the above equations Y_{ll} is measurement vector of the longitudinal-lateral error subsystem. The initial tracking problem of the longitudinal and lateral dynamics has been converted to the stabilization problem of the error vector e_{ll}. The measurement vector Y_{ll}^m does have available all the state variables of the system (12) since the flapping angles a and b can not be measured. A static output feedback control law is required of the form:

$$v_{ll} = -K_{ll}Y_{ll}^m \tag{13}$$

with K_{ll} being a gain matrix, such that for the closed loop system:

$$\dot{\varepsilon}_{ll} = (A_{ll}^{fb} - \bar{B}_{ll}K_{ll}C_{ll}^m)\varepsilon_{ll}$$

the closed loop matrix $A_{ll}^{cl} = A_{ll}^{fb} - \bar{B}_{ll}K_{ll}C_{ll}^m$ is Hurwitz.

Details about the output feedback problem are given in [13]. There are several iterative algorithms for calculating the output feedback gain K_{ll} of (13). However, the most practical convergent algorithm that results in a local minimum solution is given in [14].

B. Heading-Heave Dynamics

The goal of this Section is the design of the second control law responsible for the heading and vertical velocity tracking. The heading-heave dynamics subsystem, is summarized by the following equations:

$$\begin{aligned}\dot{x}_{hh} &= A_{hh}x_{hh} + \bar{B}_{hh}v_{hh} + D_{hh}x_{ll} \\ y_{hh} &= C_{hh}x_{hh} \\ y_{hh}^m &= x_{hh}\end{aligned} \tag{14}$$

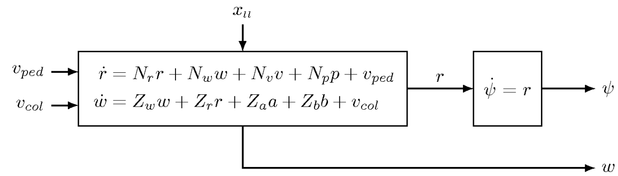

Fig. 4. Interconnection of the heading-heave helicopter dynamics subsystem. The heading-heave dynamics are additionally perturbed by the longitudinal-lateral dynamics state vector x_{ll}.

where:

$$\begin{aligned}x_{hh} &= [\psi \quad r \quad w]^T \qquad v_{hh} = [v_{ped} \quad v_{col}]^T \\ y_{hh} &= [\psi \quad w]^T\end{aligned}$$

In the above equations, y_{hh} is the output vector, y_{hh}^m is the measurement vector and $C_{hh} \in \mathbb{R}^{2\times 3}$ is a matrix of obvious entries. The reference output is denoted by $y_{hh}^r = [\psi_r \quad w_r]^T$. The heading-heave subsystem is in cascade connection with the longitudinal-lateral subsystem via the matrix D_{hh}. The interconnection of the heading-heave subsystem dynamics is shown in Fig. 4. The design procedure is similar with the one presented in Section IV-A. The controller design requires the determination of a desired state vector x_{hh}^d and a desired nominal control input v_{hh}^d, such that when the error $\varepsilon_{hh} = x_{hh} - x_{hh}^d$ is regulated to zero, then, the output y_{hh} of the yaw heave subsystem asymptotically tracks the reference output vector y_{hh}^r. The control law for the heading-heave subsystem, is obtained as the following superposition:

$$v_{hh} = v_{hh}^d + v_{hh}^{fb} = \begin{bmatrix} v_{ped}^d \\ v_{col}^d \end{bmatrix} + \begin{bmatrix} v_{ped}^{fb} \\ v_{col}^{fb} \end{bmatrix}$$

where v_{hh}^{fb} is a feedback control vector to be determined. The choice of the controller component v_{hh}^d and the desired state vector x_{hh}^d should satisfy:

$$\dot{x}_{hh}^d = A_{hh}x_{hh}^d + \bar{B}_{hh}v_{hh}^d + D_{hh}x_{ll}^d \tag{15}$$

where the state vector x_{ll}^d is defined in Section IV-A. The input v_{hh}^d and the desired state x_{hh}^d, are derived by using a similar recursive backstepping procedure with the one described in Section IV-A. The choice of v_{hh}^d and x_{hh}^d components emerge from the inspection of the error vector $\varepsilon_{hh} = x_{hh} - x_{hh}^d$ dynamics. The error dynamics of the heading-heave subsystem are given by:

$$\begin{aligned}\dot{e}_\psi &= -\dot{\psi}_r + r_d + e_r \\ \dot{e}_r &= -\dot{r}_r + N_v v_d + N_p p_d + N_w w_d + N_r r_d \\ &\quad + N_v e_v + N_p e_p + N_w e_w + N_r e_r + v_{ped}^d + v_{ped}^{fb} \\ \dot{e}_w &= -\dot{w}_r + Z_a a_d + Z_b b_d + Z_r r_d + Z_w w_d \\ &\quad + Z_a e_a + Z_b e_b + Z_r e_r + Z_w e_w + v_{col}^{ds} + v_{col}^{fb}\end{aligned}$$

The desired angular velocity r_d, and the components of v_{hh}^d, are chosen such that they cancel out all the terms associated with the rest of the desired state variables and only the error terms remain in the heading-heave subsystem error dynamics. Thus:

$$r_d = \dot{\psi}_r \quad (16)$$

$$v_{ped}^{ds} = \dot{r}_d - N_v v_d - N_p p_d - N_w w_d - N_p p_d \quad (17)$$

$$v_{col}^{ds} = \dot{w}_r - Z_a a_d - Z_b b_d - Z_r r_d - Z_w w_d \quad (18)$$

Based on the above choice, it is easy to verify that (15) is satisfied. The desired state vector x_{hh}^d and the control input v_{hh}^d are functions of the components of the y_{hh}^r, y_{ll}^r vectors and their higher derivatives. Moreover, ψ_r and w_r should belong to C^2 and C^1, respectively. The dependence of v_{hh}^d on the components of y_{ll}^r stems from the interconnection of the two subsystems through the matrix D_{hh}. Using the equations given in (16)-(18), the error dynamics of the heading-heave subsystem become:

$$\dot{\varepsilon}_{hh} = A_{hh}\varepsilon_{hh} + \bar{B}_{hh}v_{hh}^{fb} + D_{hh}\varepsilon_{ll} \quad (19)$$
$$Y_{hh}^m = \varepsilon_{hh}$$

where $\varepsilon_{hh} = [e_\psi \;\; e_r \;\; e_v]^T$. In the above equations Y_{hh}^m denotes the vector of available measurements. Similarly with the longitudinal-lateral subsystem, the tracking problem of y_{hh}^r is converted to the regulation of ε_{hh} to zero. However, in this particular case, the full state vector of the system in (19) is available for feedback. The design objective is to determine a static feedback law v_{hh}^{fb} of the form:

$$v_{hh}^{fb} = -K_{hh}\varepsilon_{hh} \quad (20)$$

where K_{hh} is a gain matrix, such that the closed loop stability matrix $A_{hh}^{cl} = A_{hh} - \bar{B}_{hh}K_{hh}$ of the heading-heave error subsystem is Hurwitz. As it will be illustrated later, if this condition is satisfied, the solution of the complete error dynamics is GAS given that A_{ll}^{cl} is Hurwitz as well. Since full state feedback is available, there are several options for determining the feedback gain K_{hh}.

C. Stability of the Complete System Error Dynamics

By applying the control laws v_{ll}^{fb} and v_{hh}^{fb}, the complete error system dynamics take the form:

$$\begin{bmatrix} \dot{\varepsilon}_{hh} \\ \hline \dot{\varepsilon}_{ll} \end{bmatrix} = \left[\begin{array}{c|c} (A_{hh} - \bar{B}_{hh}K_{hh}) & D_{hh} \\ \hline 0_{8\times 3} & (A_{ll}^{fb} - \bar{B}_{ll}K_{ll}C_{ll}^m) \end{array}\right] \begin{bmatrix} \varepsilon_{hh} \\ \hline \varepsilon_{ll} \end{bmatrix} \quad (21)$$

The stability of the complete error dynamics system given in (21), is specified by the following Theorem:

Theorem 1. *Given that the feedback gains K_{ll} and K_{hh} are selected such that the matrices $A_{ll}^{cl} = A_{ll}^{fb} - \bar{B}_{ll}K_{ll}C_{ll}^m$ and $A_{hh}^{cl} = A_{hh} - \bar{B}_{hh}K_{hh}$ are Hurwitz, then the solution $[\varepsilon_{hh}(t) \;\; \varepsilon_{ll}(t)]^T$ of the complete error dynamics system of* (21) *is GAS.*

Proof: Denote by λ the eigenvalues of the composite error dynamics system of (21). Since the state matrix of (21) is in block triangular form, its eigenvalues satisfy the following equality:

$$det\left(A_{hh}^{cl} - \lambda I_{3\times 3}\right) \cdot det\left(A_{ll}^{cl} - \lambda I_{8\times 8}\right) = 0$$

where $det(\cdot)$ denotes the determinant of a matrix. Therefore the eigenvalues of the composite error system are the union of the eigenvalues of A_{hh}^{cl} and A_{ll}^{cl}. Since both of those matrices are Hurwitz, then all the eigenvalues of (21) have strictly negative real parts. Therefore, the complete error dynamics system of (21) is GAS.

The stability analysis is based on the approximate helicopter model. Practical stability can be proven for the complete dynamics. This proof is omitted due to space limitations. Robustness analysis of the design with respect to disturbances and parametric uncertainty is also omitted. In robust control literature for linear system there is a wide variety of designs for choosing v_{ll}^{fb} and v_{hh}^{fb} such that the closed loop system meets certain performance specifications.

V. Experimental Results

The controller performance is evaluated using *X-Plane*, a realistic and commercially available flight simulator. Experiments are conducted in the *X-Plane* environment for a *Raptor 90 SE* radio controlled helicopter. The linear model parameters are extracted using the *CIFER*© (Comprehensive Identification from FrEquency Responses) [15] package which utilizes a frequency domain identification procedure. The linear model parameters are given in Table II. The dashed entries indicate parameters that where not included in the linear model.

The reference trajectories are specially designed in order to examine the performance of the controllers in multiple operating conditions that cover a wide portion of the flight envelope.

Controller performance was tested by executing two velocity tracking maneuvers. The first maneuver is a trapezoidal velocity profile in the lateral and longitudinal directions of the inertial space. The second maneuver under investigation requires cruising of the helicopter by tracking a simple forward flight routine. In both cases, the heading of the helicopter remains constant throughout the execution of the maneuver with $\psi_r = 0$. The maneuvers include an acceleration phase, a cruising phase with constant speed and a deccelaration phase which ends at hover.

The controller's feedback gains of (13)-(20) are shown in Table III. The controller responses versus the desired trajectory are illustrated in Fig. 5 and Fig. 6. The orientation angles for the two maneuvers are depicted in Figures 7 and 8. The position of the helicopter in the inertial space is shown in Figures 9 and 10. Finally, the control inputs for the two maneuvers are given in Figures 11 and 12. The control commands are bounded in the interval $[-1\;1]$. Based on the results, the performance of the controller design was deemed satisfactory. In both cases a single linear controller based only on the hover linear model, was adequate. To this extent, the identification of multiple models for different operating conditions was redundant.

VI. Conclusions

This paper presented a systematic velocity and heading tracking controller for small-scale unmanned helicopters. The design may be expanded such that the overall control law can be an interpolator of multiple controllers where each of them corresponds to a linear model around different operating condition of the helicopter. It is important, however, that all of the linearized models have the same structure and order with the base hover model and only their parameters may vary. The output feedback controllers v_{ll}^{fb} and v_{hh}^{fb} are not restricted only to the proposed designs of this paper, but they could be chosen from a wide variety of linear controller designs that exist in the literature. To this extent, the popular method of $\mathcal{H}_\infty$ may be also applied.

TABLE II

LINEAR STATE SPACE MODEL PARAMETERS

A matrix					
X_u	-0.03996	X_a	$-g$	Y_v	-0.05989
Y_b	g	M_u	0.2542	M_v	-0.06013
M_a	307.571	L_u	-0.0244	L_v	-0.1173
L_b	1172.4817	A_b	0.7713	B_a	0.6168
Z_a	–	Z_b	–	Z_w	-2.055
Z_r	–	N_v	2.982	N_p	–
N_w	-0.7076	N_r	-10.71	g	9.389
$1/\tau_f$	30.71				
B matrix					
A_{lon}	4.059	A_{lat}	-0.01610	B_{lon}	-0.01017
B_{lat}	4.085	N_{col}	3.749	N_{ped}	26.90
Z_{col}	-13.11				

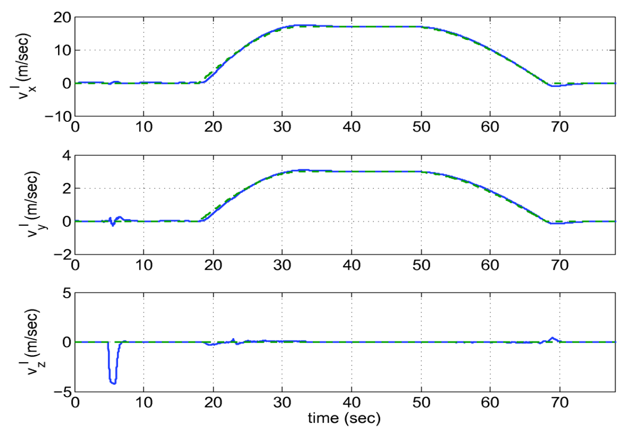

Fig. 5. *First maneuver*: Reference trajectory (dashed line) and actual velocity trajectory (solid line), expressed in inertial coordinates with respect to time.

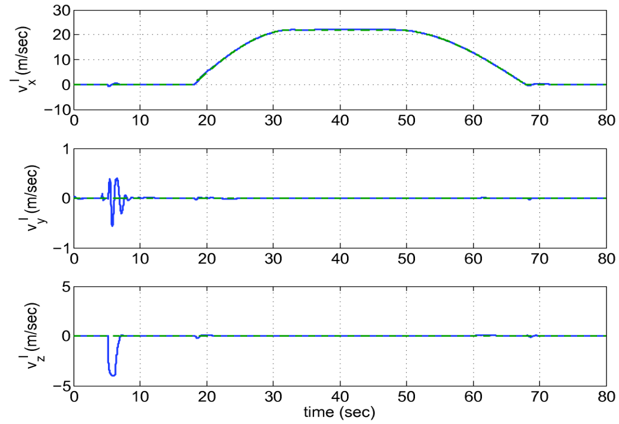

Fig. 6. *Second maneuver*: Reference trajectory (dashed line) and actual velocity trajectory (solid line), expressed in inertial coordinates with respect to time.

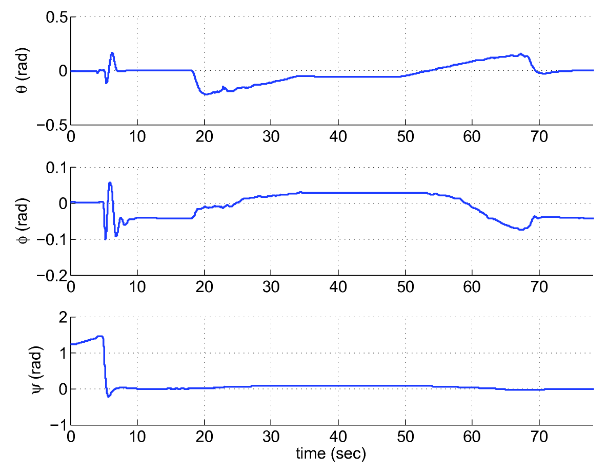

Fig. 7. *First maneuver*: Orientation angles.

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TABLE III

LINEAR TRACKING CONTROLLER FEEDBACK GAINS.

$$K_{ll} = \begin{bmatrix} -3.1353 & 0.6882 & 9.8054 & 1.9041 & 0.5662 & 0.2395 \\ -0.1847 & 0.9682 & 0.5038 & 2.9687 & 0.0632 & -0.5391 \end{bmatrix} \qquad K_{hh} = \begin{bmatrix} 0 & 10.9451 & 0 \\ 60 & 0 & 1 \end{bmatrix}$$

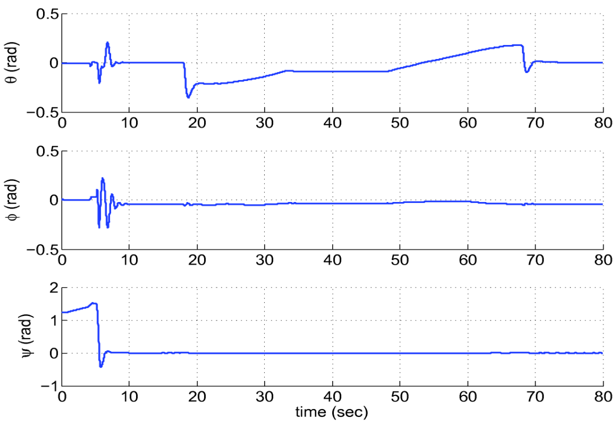

Fig. 8. *Second maneuver*: Orientation angles.

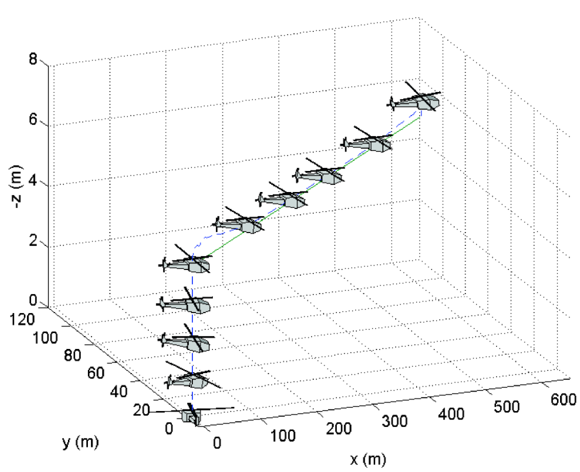

Fig. 9. *First maneuver*: Reference position trajectory (solid line) and the actual trajectory (dashed line) with respect to the inertial axis.

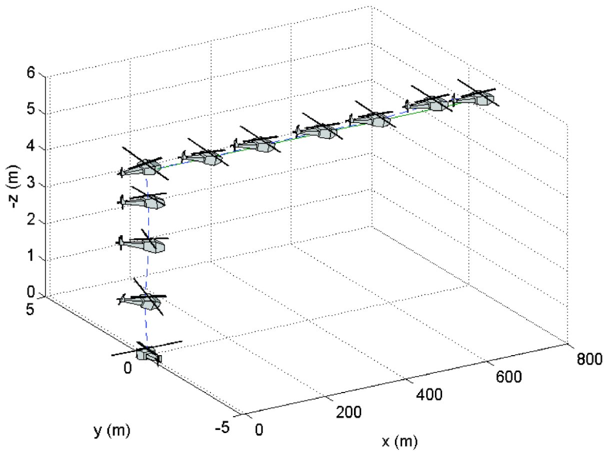

Fig. 10. *First maneuver*: Reference position trajectory (solid line) and the actual trajectory (dashed line) with respect to the inertial axis.

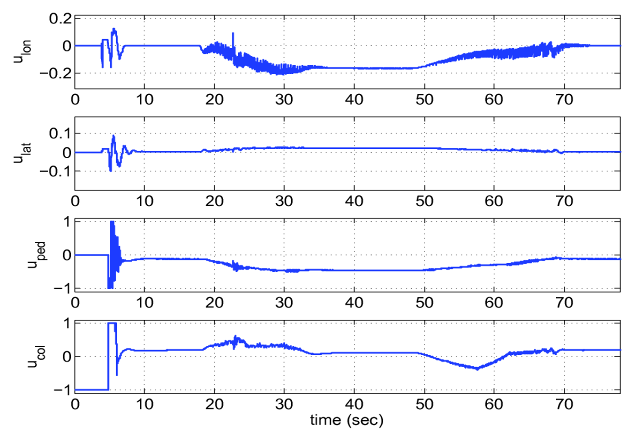

Fig. 11. *First maneuver*: Control inputs.

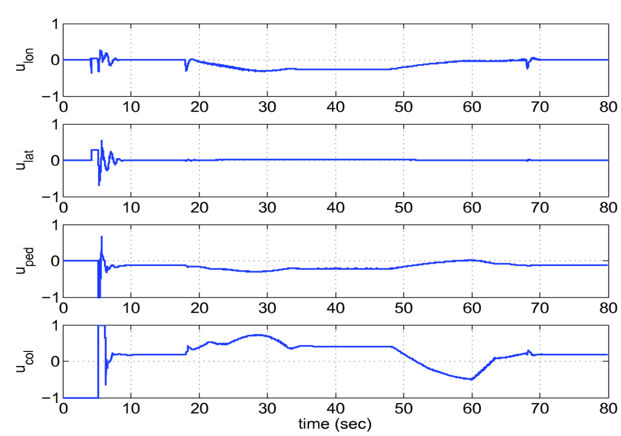

Fig. 12. *Second maneuver*: Control inputs.