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Terminal Iterative Learning Control with Multiple Intermediate Pass Points

Tong Duy Son, *Student Member, IEEE,* and Hyo-Sung Ahn, *Member, IEEE*

Abstract— **In this paper, we present iterative learning control (ILC) algorithms for terminal control in multi-input multi-output systems. The optimal ILC framework is investigated for the formulations of single-terminal point and multiple intermediate pass points tracking control. First, we consider an initial learning control technique for one final output, before sequentially exploring multiple terminal output-tracking via initial and changing inputs. The novel contribution of this work is in the analysis of the terminal ILC algorithms, regardless of the stability, monotonic convergence, and performance properties in both cases. Illustrative examples are then provided to verify the proposed approaches.**

I. INTRODUCTION

Iterative learning control (ILC) is a control scheme that refines the input sequences from trials in order to improve the performance of repetitive operation systems. The prime strategy of ILC algorithms is to update the control signal using the information measured in previous iterations. As such, the ILC controller achieves high tracking performance in the presence of systems with model uncertainty and repeatable disturbances. Another contribution of ILC theory is to investigate the repetitive nature of a system that operates repetitively. Since the ILC algorithm was initially proposed by Arimoto [1], there have been numerous publications and studies performed; a number of surveys [2]- [4] have effectively covered the novel ideas and development of ILC methodology.

Terminal iterative learning control (TILC) is a type of control technique that focuses solely on the terminal point of a system such that this point only tracks the given desired output. TILC was first presented for rapid thermal processing chemical vapor deposition in wafer fabrication industry applications [5], where the ultimate control objective is to control the deposition thickness at the end of the thermal processing cycle. In [6], the plastic sheet surface temperature control in thermoforming machine is controlled by tuning the temperature setpoint of the heater. In addition, there has been some research focused on the initial state learning for final state control in both theoretical [7] and practical applications [8]. The main approach of TILC in these studies has been to update control signals using the terminal tracking error alone, rather than the whole output trajectory tracking error. Moreover, these investigations have shown that the approach could achieve convergence in the iteration domain.

Authors are with the Distributed Control and Autonomous Systems Laboratory, Department of Mechatronics, Gwangju Institute of Science and Technology (GIST), Korea; Email: hyosung@gist.ac.kr

However, previous TILC research has typically only considered the final point; the existence of intermediate pass points has not been studied well in ILC theory although many systems require multiple target points to be tracked. For example, satellite antennas need to be maneuvered to point toward a desired ground location, which is determined from the desired azimuth and elevation angles at given sampling times [9]. For this problem, multiple point-to-point tracking control was considered by developing a frequency-domain framework in which the reference is updated between trials in [10]. However, since the control problem is focused on tracking multiple points during system operation—instead of the whole trajectory at all time instants—the ILC approach, which tracks a reference trajectory, is undesirable with regards to control effort and energy. Moreover, it requires further computational analysis to generate a reference trajectory from given terminal points. These reasons are the primary motivation for our development of a new TILC framework for tracking multiple intermediate pass points.

The objective of this work is to explore TILC in not only single but also multiple intermediate pass points tracking controls. We attempt to generate a control signal from information of the given pass points at given time instants rather than for the whole trajectory. Specifically, we apply an initial input learning technique to track one terminal point, because of the potential to increase the smooth motion of systems and reducing the effects of actuator limitations. Additionally, it is not necessary to continuously apply control signals that lead to more control efforts if the desired terminal output can be reached with an appropriate initial control input. After that, continuous control signals are generated to produce output curves that track multiple pass points. The technique investigates only essential information at terminal points. Note that our approach is based on the norm optimal ILC [11]–[13], in which the ILC update law is obtained by minimizing the cost function trial-by-trial. For these two problems, we show that stability and convergence in the iteration domain can be achieved and that the algorithms can produce superior performance by selecting suitable parameters.

The remainder of this paper is organized as follows. In Section II, we provide some background of terminal controls. Section III then considers a single terminal point with initial input learning. Multiple intermediate pass points control problems are subsequently examined in Section IV, by applying both an initial learning control input and a continuous control signal. Finally, simulation results are given in Section V, and Section VI concludes this work.

II. BACKGROUND

This section provides the problem setup of TILC control for a single terminal point and multiple pass points.

Consider a discrete time linear system

$$\begin{aligned} x_k(t+1) &= Ax_k(t) + Bu_k(t) \\ y_k(t) &= Cx_k(t) \end{aligned} \quad (1)$$

where k is the iteration index, and $t = 0, 1, 2 \ldots, N$ is the sampling time index. Matrices $A, B,$ and C are time invariant with appropriate dimensions; this system is a multi-input multi-output (MIMO) system that has $x_k(t) \in \mathbb{R}^p$,$u_k(t) \in \mathbb{R}^m$, and $y_k(t) \in \mathbb{R}^n$. Moreover, we assume that the system is both controllable and observable.

From linear control theory, the output of the system at the N-th sample time in the k-th iteration is given as

$$y_k(t_N) = CA^{t_N} x_k(0) + C \sum_{j=0}^{t_N-1} A^{t_N-j-1} Bu_k(j). \quad (2)$$

Also, in this paper, the initial state condition is assumed to be constant for all iterations; moreover, it is possible to assume that $x_k(0) = 0$ without loss of generality. The goal of TILC is to track terminal points during system operation by generating an optimal control signal through trials. After each trial, the outputs at the given terminal points are measured; consequently, the input is updated from an ILC learning law.

During the tracking of a single terminal point t_N that has its desired output is $y_d(t_N)$, the input signal is constant at all sampling times in the same iteration, i.e., $u_k(t_i) = u_k$ for $t_i = \{0, 1, 2, \ldots, N\}$. As a result, the output of the terminal point can be described by

$$y_k(t_N) = P_N u_k, \quad (3)$$

where P_N is the system matrix at the N-th terminal point, such that

$$P_N = C \sum_{j=0}^{t_N-1} A^{t_N-j-1} B. \quad (4)$$

Note that the system matrix P_N is full rank, because the system is both controllable and observable.

On the other hand, the terminal control task that has intermediate pass points between the initial point and the terminal point is relatively more challenging. In this case, a control signal is generated such that the produced output curves go through the given pass points iteratively. We define these points at each time instant as $t_1, t_2, \ldots, t_M$, where $0 \leq t_1 < t_2 < \ldots < t_M \leq t_N$, and the desired outputs at these points are

$$y_d(t_1), y_d(t_2), \ldots, y_d(t_M).$$

And in the k-th iteration, the output at the i-th intermediate point is calculated as

$$y_k(t_i) = C \sum_{j=0}^{t_i-1} A^{t_i-j-1} Bu_k(j). \quad (5)$$

III. INITIAL ITERATIVE LEARNING FOR SINGLE TERMINAL POINT

In this section, we consider the initial learning control strategy for tracking the N-th terminal point. To investigate the norm optimal ILC approach, we suggest a performance index with respect to the terminal point, such that

$$\begin{aligned} J(u_{k+1}) &= e_{k+1}^T(t_N) Q e_{k+1}(t_N) \\ &\quad + (u_{k+1} - u_k)^T R (u_{k+1} - u_k) + u_{k+1}^T S u_{k+1}, \end{aligned} \quad (6)$$

where $Q, R,$ and S are real symmetric positive definite matrices with appropriate dimensions, and the error at the terminal is

$$e_k(t_N) = y_d(t_N) - y_k(t_N). \quad (7)$$

Here, the control signal at the $(k+1)$-th trial can be attained by differentiating vector $J(u_{k+1})$ with respect to u_{k+1}; setting this derivative equal to zero yields

$$-P_N^T Q e_{k+1}(t_N) + R(u_{k+1} - u_k) + S u_{k+1} = 0. \quad (8)$$

Then, the control input is iteratively computed as

$$(P_N^T Q P_N + R + S) u_{k+1} = R u_k + P_N^T Q y_d(t_N). \quad (9)$$

Since $P_N^T Q P_N + R + S$ is nonsingular, (9) can be rewritten by the following ILC rule

$$u_{k+1} = L_u u_k + L_d y_d(t_N), \quad (10)$$

where

$$L_u = (P_N^T Q P_N + R + S)^{-1} R, \quad (11)$$

$$L_d = (P_N^T Q P_N + R + S)^{-1} P_N^T Q. \quad (12)$$

The linear iterative system (10) is asymptotically stable (AS) if it generates inputs through trials that are bounded for all iterations, and $u_\infty = \lim_{k \to \infty} u_k$ exists. Furthermore, the system is AS if

$$\rho(L_u) < 1, \quad (13)$$

where $\rho(L_u)$ is the spectral radius of L_u [3]. As a result, we obtain the following stability property of the ILC law (10).

Theorem 3.1: For the linear system (1), the ILC learning algorithm

$$u_{k+1} = L_u u_k + L_d y_d(t_N) \quad (14)$$

is asymptotically stable for all symmetric positive definite matrices $Q, R,$ and S.

Proof: Considering inverse of the nonsingular matrix L_u,

$$\begin{aligned} L_u^{-1} &= R^{-1}(P_N^T Q P_N + R + S) \\ &= I + R^{-1} P_N^T Q P_N + R^{-1} S. \end{aligned} \quad (15)$$

It is then obvious that $R^{-1} P_N^T Q P_N + R^{-1} S$ is positive definite; consequently, the matrix L_u^{-1} has its all eigenvalues greater than 1, as shown in the following.

Consider a positive definite matrix M and an identity matrix I. If we define the eigenvalues of the matrices M and $(I + M)$ as $\lambda(M)$ and $\lambda(I + M)$, respectively, then

$$\lambda(I + M) = \lambda(M) + 1. \quad (16)$$

Since $\lambda(M) > 0$, then $\lambda(I + M) > 1$. Therefore, L_u^{-1} has all eigenvalues greater than 1. Accordingly, it leads to the conclusion from the inverse eigenvalue theorem in linear algebra [14],

$$\rho(L_u) = \max |\lambda_i(L_u)| < 1. \tag{17}$$

■

However, the concept of AS is not strongly stated in ILC applications due to the inherently large transient growth possibility, which is one of main obstacles in ILC design. Hence, another strong concept in ILC is the monotonic convergence condition. Briefly, let us provide the background ideas and conditions for monotonic convergence.

The ILC algorithm is referred to as monotonically convergent if $\|u_\infty - u_{k+1}\| < \gamma \|u_\infty - u_k\|$ such that $0 < \gamma < 1$ [3]. In the learning algorithm (10), γ is defined as $\gamma = \|L_u\|$, which is the largest singular value of L_u. Accordingly, we consider another of the selected weighting matrices to guarantee the monotonic convergence of the law.

Lemma 3.1: For the linear system (1), the ILC learning algorithm

$$u_{k+1} = L_u u_k + L_d y_d(t_N) \tag{18}$$

guarantees monotonic convergence if the weighting matrices are chosen as $Q = qI, R = rI$, and $S = sI$, where q, r, and s are real positive parameters.

Proof: Applying the contraction mapping theorem with $u_{k+1} = f(u_k)$,

$$\begin{aligned} \|f(u_1) - f(u_2)\| &= \|L_u(u_1 - u_2)\| \\ &\leq \bar{\sigma}(L_u) \|u_1 - u_2\|. \end{aligned} \tag{19}$$

where the largest singular value of L_u is defined as

$$\bar{\sigma}(L_u) = \sqrt{\rho(L_u^T L_u)}. \tag{20}$$

Moreover, with $Q = qI, R = rI$, and $S = sI$, the learning matrix L_u is symmetric positive definite, $L_u^T = L_u$. Since the eigenvalues of L_u^2 are the squares of the eigenvalues of L_u, then

$$\begin{aligned} \sqrt{\rho(L_u^T L_u)} &= \sqrt{\rho(L_u)^2} \\ &= \rho(L_u). \end{aligned} \tag{21}$$

And the result of Theorem (3.1) leads to

$$\sigma(L_u) < 1, \tag{22}$$

and the final result is given. ■

As a consequence of the convergence property, the control signal at the steady state is calculated from (10) as

$$\begin{aligned} u_\infty &= (I - L_u)^{-1} L_d y_d(t_N) \\ &= \left(P_N^T Q P_N + S\right)^{-1} P_N^T Q y_d(t_N). \end{aligned} \tag{23}$$

Hence, the converged error $e_\infty = \lim_{k\to\infty} e_k$ is

$$\begin{aligned} e_\infty &= y_d(t_N) - P_N u_\infty \\ &= \left[I - P_N(P_N^T Q P_N + S)^{-1} P_N^T Q\right] y_d(t_N). \end{aligned} \tag{24}$$

From the steady state error, we can see that the steady state error depends on the relationship between matrices Q and S. Specifically, if Q is large compared to S, componentwise, then the error is small. Moreover, the smallest possible error, $e_\infty = 0$, requires $S = 0$.

IV. ILC FOR MULTIPLE INTERMEDIATE PASS POINTS

In the multiple intermediate pass points problem, there are given desired outputs $y_d(t_1), y_d(t_2), \ldots, y_d(t_M)$ at time instants $t_1, t_2, \ldots, t_M$ during system operation. The control task is to then construct a learning law that drives the outputs through, or at least close to, these points. In conventional control schemes, a reference trajectory y_{ref} is built such that y_{ref} passes the desired points at $t_1, t_2, \ldots, t_M$. In this case, we can design a controller that incorporates the system model to thereby track the given trajectory. However, in this work, we attempt to design a new ILC formulation that focuses on only information obtained from the given pass points rather than use a reference trajectory.

A. Initial Learning

As a initial attempt, we apply the same approach as the tracking single terminal point case. Hence, if the control signal is maintained constant as initial iterative learning, the output at the i-th pass points can be given as

$$y_k(t_i) = P_i u_k, \tag{25}$$

where

$$P_i = C \sum_{j=0}^{t_i - 1} A^{t_i - j - 1} B. \tag{26}$$

The error at the i-th point is then computed as

$$e_k(t_i) = y_d(t_i) - P_i u_k. \tag{27}$$

Next, we consider a norm optimal TILC performance index that incorporates multiple pass points $t_1, t_2, \ldots, t_M$, such that

$$\begin{aligned} J(u_{k+1}) = &\sum_{i=1}^{M} e_{k+1}^T(t_i) Q_i e_{k+1}(t_i) \\ &+ (u_{k+1} - u_k)^T R (u_{k+1} - u_k) + u_{k+1}^T S u_{k+1}, \end{aligned} \tag{28}$$

where Q_i is the weighting matrix at the i-th terminal point.

Note that the primary objective here is to generate control signals that produce an output that minimizes errors at all terminal points iteratively. The ILC algorithm is generated from (28) by differentiating $J(u_{k+1})$; setting this differentiation to zero then produces

$$\left(\sum_{i=1}^{M} P_i^T Q_i P_i + R + S\right) u_{k+1} = R u_k + \sum_{h=1}^{M} P_i^T Q_i y_d(t_i). \tag{29}$$

Next, since $\sum_{i=1}^{M} P_i^T Q_i P_i + R + S$ is nonsingular, (29) can be rewritten as

$$\begin{aligned} u_{k+1} = &\left(\sum_{i=1}^{M} P_i^T Q_i P_i + R + S\right)^{-1} R u_k \\ &+ \left(\sum_{i=1}^{M} P_i^T Q_i P_i + R + S\right)^{-1} \sum_{i=1}^{M} P_i^T Q_i y_d(t_i). \end{aligned} \quad (30)$$

Accordingly, a theorem can be formulated to analyze the stability of the update algorithm.

Theorem 4.1: For the linear system (1), the ILC learning algorithm (30) for tracking multiple pass points is asymptotically stable for symmetric positive definite matrices Q, R, and S.

Proof: Since $P_i^T Q_i P_i$ is positive definite with all pass points $t_1, t_2, \ldots, t_M$, the result can be shown in the same way as Theorem (3.1). ■

To show the effectiveness of investigating the initial learning for tracking multiple pass points, the performance at each point is subsequently analyzed. First, the steady state control signal achieved from (30) is

$$u_\infty = \left(\sum_{i=1}^{M} P_i^T Q_i P_i + S\right)^{-1} \sum_{i=1}^{M} P_i^T Q_i y_d(t_i). \quad (31)$$

After that, the converged error of the terminal points at the $i-$th sampling time is as follows:

$$e_\infty(t_i) = y_d(t_i) - P_i u_\infty. \quad (32)$$

Therefore, the steady state error of the terminal point depends on the other terminal pass points, in addition to the weighting matrices Q_i and S. Moreover, the weighting matrix Q_i in the relationship with other terminal weighting matrices describes how important it is that the output curve $y_\infty(t_i)$ goes close to the desired output $y_d(t_i)$. Thus, this technique only could achieve desirable performance at all points under certain conditions of given pass points.

B. Iterative Learning using Continuous Control Input

When there are a large number of given intermediate pass points, the initial learning approach has inherent drawbacks in achieving performance at all points. For this reason, we develop a new ILC framework in which the control signal is time continuous.

First, we formulate the N-sample sequence of inputs in a super-vector framework as

$$\mathbf{u}_k = \begin{bmatrix} u_k^T(0) & u_k^T(1) & \ldots & u_k^T(N-1) \end{bmatrix}^T. \quad (33)$$

Then, we define $p_i(t)$ as

$$p_i(t) = \begin{cases} CA^{t_i - t - 1}B & \text{if } t < t_i \\ 0 & \text{if } t \geq t_i \end{cases}.$$

By these formulations, the output at the i-th time instant is expressed as

$$\begin{aligned} y_k(t_i) &= C \sum_{j=0}^{t_i - 1} A^{t_i - j - 1} B u_k(j) \\ &= \sum_{t=0}^{N-1} p_i(t) u_k(t) \\ &= \mathbf{p}_i^T \mathbf{u}_k \end{aligned} \quad (34)$$

where $\mathbf{p}_i$ is expressed as

$$\mathbf{p}_i = \begin{bmatrix} p_i(0) & p_i(1) & \ldots & p_i(N-1) \end{bmatrix}^T. \quad (35)$$

Therefore, the cost function for the problem of tracking multiple intermediate pass points $t_1, t_2, ..., t_M$ can now be formulated as

$$\begin{aligned} J = &\sum_{i=1}^{M} e_k^T(t_i) Q_i e_k(t_i) + \mathbf{u}_{k+1}^T \mathbf{S} \mathbf{u}_{k+1} \\ &+ \left(\mathbf{u}_{k+1} - \mathbf{u}_k\right)^T \mathbf{R} \left(\mathbf{u}_{k+1} - \mathbf{u}_k\right) \end{aligned} \quad (36)$$

where

$$e_k(t_i) = y_d(t_i) - \mathbf{p}_i^T \mathbf{u}_{k+1} \quad (37)$$

and $\mathbf{R}, \mathbf{S}$, Q_i are symmetric positive definite matrices.

To work with multiple pass points, we define the super vector forms of system matrix $\mathbf{P}$ and the desired output at pass points $\mathbf{y}_d$ as

$$\mathbf{y}_d = \begin{bmatrix} y_d^T(t_1) & y_d^T(t_2) & \ldots & y_d^T(t_M) \end{bmatrix}^T \quad (38)$$

$$\mathbf{P} = \begin{bmatrix} \mathbf{p}_1^T & \mathbf{p}_2^T & \ldots & \mathbf{p}_M^T \end{bmatrix}^T. \quad (39)$$

Note that different $p_i(t)$ vanish at different times, thus the set of functions $p_i(t)$ with $i = 1, 2, \ldots, M$ are linearly independent. As such, the cost function (36) can be rewritten as

$$\begin{aligned} J = &\left[\mathbf{y}_d - \mathbf{P}\mathbf{u}_{k+1}\right]^T \mathbf{Q} \left[\mathbf{y}_d - \mathbf{P}\mathbf{u}_{k+1}\right] \\ &+ \mathbf{u}_{k+1}^T \mathbf{S} \mathbf{u}_{k+1} + \left(\mathbf{u}_{k+1} - \mathbf{u}_k\right)^T \mathbf{R} \left(\mathbf{u}_{k+1} - \mathbf{u}_k\right) \end{aligned} \quad (40)$$

where $\mathbf{Q} = \text{diag}(Q_1, Q_2, \ldots, Q_M)$. Consequently, the controller in the $(k+1)$-th trial is attained from the differential condition, leading to

$$-\mathbf{P}^T \mathbf{Q} \left(\mathbf{y}_d - \mathbf{P}\mathbf{u}_{k+1}\right) + \mathbf{R}\left(\mathbf{u}_{k+1} - \mathbf{u}_k\right) + \mathbf{S}\mathbf{u}_{k+1} = 0. \quad (41)$$

And the ILC algorithm is derived as

$$\left(\mathbf{P}^T \mathbf{Q} \mathbf{P} + \mathbf{R} + \mathbf{S}\right) \mathbf{u}_{k+1} = \mathbf{R}\mathbf{u}_k + \mathbf{P}^T \mathbf{Q} \mathbf{y}_d. \quad (42)$$

Also, since $\mathbf{P}^T \mathbf{Q} \mathbf{P} + \mathbf{R} + \mathbf{S}$ is positive definite,

$$\mathbf{L}_u = \left(\mathbf{P}^T \mathbf{Q} \mathbf{P} + \mathbf{R} + \mathbf{S}\right)^{-1} \mathbf{R} \quad (43)$$

$$\mathbf{L}_d = \left(\mathbf{P}^T \mathbf{Q} \mathbf{P} + \mathbf{R} + \mathbf{S}\right)^{-1} \mathbf{P}^T \mathbf{Q} \mathbf{y}_d. \quad (44)$$

The following theorem illustrates the results of this approach.

Theorem 4.2: For the linear system (1), the ILC learning algorithm

$$\mathbf{u}_{k+1} = \mathbf{L}_u \mathbf{u}_k + \mathbf{L}_d \quad (45)$$

is asymptotically stable for all symmetric positive definite matrices $\mathbf{Q}, \mathbf{R}$ and $\mathbf{S}$.

Moreover, if matrices $\mathbf{R}, \mathbf{S}$, and Q_i, where $i = 1, 2, \ldots, M$ are positive definite diagonal matrices, then the algorithm achieves monotonic convergence.

Proof: The learning algorithm is presented in the lift domain notation of the discrete system. However, the result is still based on the result of Theorem (3.1). ■

The steady state input and erorrs are given from (42) as

$$\mathbf{u}_\infty = \left(\mathbf{P}^T\mathbf{Q}\mathbf{P}_N + \mathbf{S}\right)^{-1}\mathbf{P}^T\mathbf{Q}\mathbf{y}_d. \tag{46}$$

The steady state error $\mathbf{e}_\infty = \mathbf{y}_d - \mathbf{P}\mathbf{u}_\infty$ is shown as

$$\mathbf{e}_\infty = \left[\mathbf{I} - \mathbf{P}\left(\mathbf{P}^T\mathbf{Q}\mathbf{P} + \mathbf{S}\right)^{-1}\mathbf{P}^T\mathbf{Q}\right]\mathbf{y}_d. \tag{47}$$

Hence, $\mathbf{Q}$ and $\mathbf{S}$ decide the performance of the tracking technique. In practical applications, there is always the case that the importances of particular points are different. As a result, the entries of the matrix $\mathbf{Q}$ decide how different performance the points are achieved.

Furthermore, the advantage of this approach can primarily be seen in the computational cost analysis, in which the system matrix $\mathbf{P}$ in the update law has the size $nM \times mM$ rather than $nN \times mN$ when tracking the whole trajectory. Note that in many applications, N could be much larger than M; thus, in comparison to operating the ILC controller for N time instants, it is expected that the control energy in tracking M terminal pass points is smaller.

V. SIMULATION

To illustrate the ideas presented in this paper, consider the discrete-time system

$$x_k(t+1) = \begin{pmatrix} 0.5 & 0.035 & 0.025 \\ 0.0255 & 0.6 & -0.99 \\ 0.75 & 0.03 & 0.025 \end{pmatrix} x_k(t) + \begin{pmatrix} 0.2 & 0.2 & 0.0 \end{pmatrix}^T u_k(t) \tag{48}$$

$$y_k(t) = \begin{pmatrix} 1.0 & 0.0 & 1.0 \end{pmatrix} x_k(t). \tag{49}$$

where the system operates on an interval $t \in [0, 20]$.

In the first simulation in Fig.1, the ILC law for tracking the single terminal point $y_d = 2$ at $t = 20$ is used with weighting parameters $Q = 15, R = 0.7$ and $S = 0.3$. The figure shows the superior performance and fast convergence of the errors in the iteration domain. As a second simulation example Fig.2 shows the output curves which are produced from initial learning algorithm to control 3 pass points. Although the errors achieve convergence, the performances are dependence at given points.

In the next simulation, we demonstrate multiple pass points TILC using continuous control signals in Fig.3. A set of 10 points in the interval $[0, 20]$ was selected. The ILC law is conducted with $\mathbf{Q} = q\mathbf{I}, \mathbf{R} = r\mathbf{I}$ and $\mathbf{S} = s\mathbf{I}$ where the scalar gains are chosen as $q = 50, r = 5$ and $s = 0.1$. It is shown in Fig.3 that the convergence is obtained after 8 iterations with the converged error is approximate to zero.

By comparison, the final simulation tests an ILC algorithm which tracks a reference trajectory instead of points. First, a trajectory which goes through given points is generated from an interpolation splines technique. Then, an ILC law is made with this trajectory and the same scalar gains of diagonal weighting matrices. Fig.4 shows the performance almost the same as in Fig.3. Hence, our approach which does not require generating trajectory could obtain a similar result of tracking a trajectory.

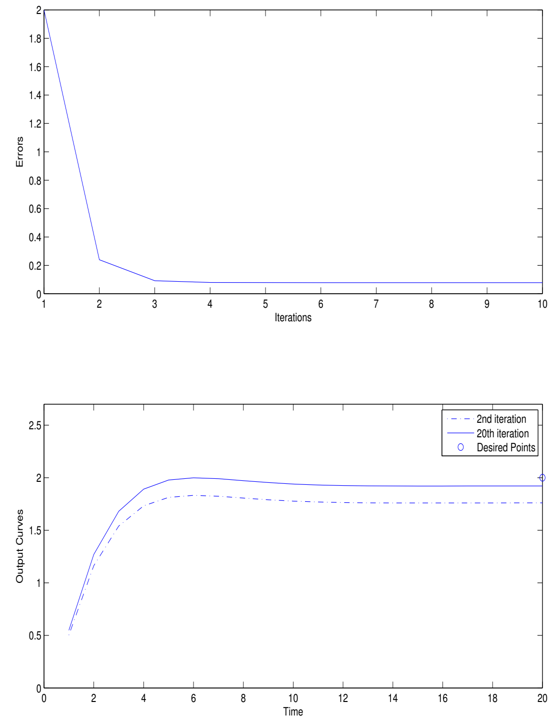

Fig. 1: Initial learning for single terminal point

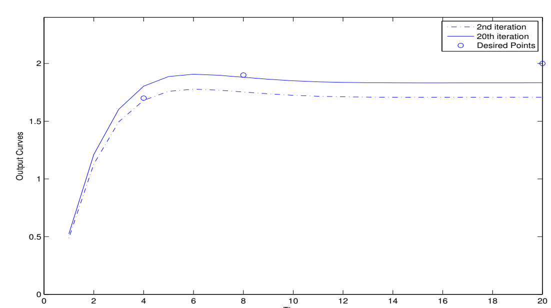

Fig. 2: Initial learning for multiple pass points

Moreover, we calculate the converged control energy in both cases. For our approach, the cost is calculated as

$$\frac{1}{2}\|u(t)\|^2 = 19.72, \tag{50}$$

while the trajectory tracking techinque requires the larger cost of 21.05.

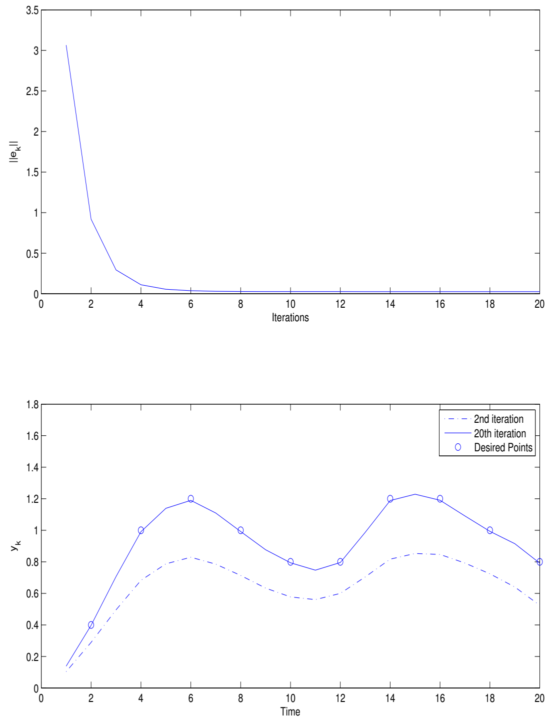

Fig. 3: Convergence of the first output's errors

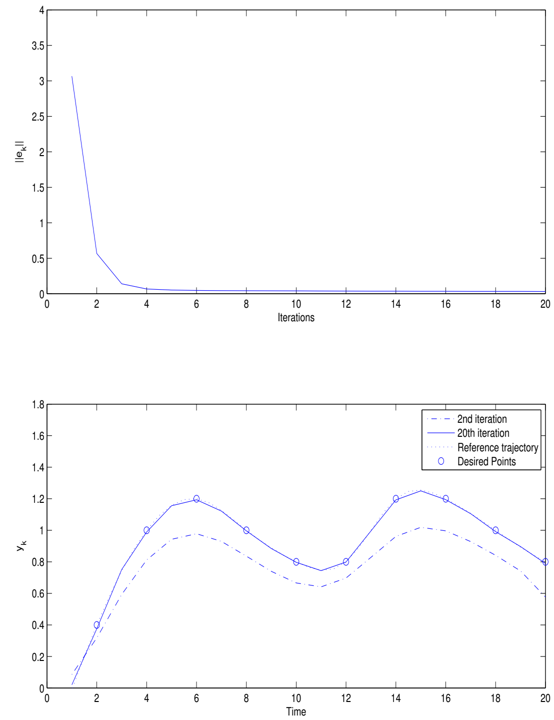

Fig. 4: Convergence of the second output's errors

VI. CONCLUSION

In this paper, we have formulated and analyzed the TILC problem in single and multiple pass points for MIMO systems. It was found here that the optimal TILC scheme provided a suitable framework for obtaining asymptotic stability and monotonic convergence properties. Moreover, the approach utilized only essential information at terminal points instead of the whole trajectory, which enabled us to improve the ILC controller, with respect to reducing the complexity and computational effort.

A notable future work will be to extend the tracking multiple pass points problem in nonlinear systems, which occurs in applications such as point-to-point controls in robotics.

VII. ACKNOWLEDGEMENTS

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REFERENCES

[1] S. Arimoto, S. Kawamura, and F. Miyazaki, "Bettering operation of robots by learning", *Journal of Robotic Systems*, vol. 1, no. 2, pp. 123-140, 1984.

[2] H.S. Ahn, Y.Q. Chen, and K.L. Moore, "Iterative learning control: Brief survey and categorization", *IEEE Transactions on Systems, Man and Cybernetics Part C: Applications and Reviews*, vol. 37, no. 6, pp. 1099-1121, 2007.

[3] D.A. Bristow, M. Tharayil, and A.G. Alleyne, "A survey of iterative learning control: A learning-based method for high-performance tracking control", *IEEE Control Systems Magazine*, vol. 26, no. 3, pp. 96-114, 2006.

[4] K.L. Moore, "Iterative Learning Control: An Expository Overview", *Applied and Computational Control, Signals, and Circuits*, vol. 1, no. 1, pp. 151-214, 1999.

[5] J.X. Xu, Y. Chen, L.Tong Heng, and S. Yamamoto, "Terminal iterative learning control with an application to RTPCVD thickness control", *Automatica*, vol. 35, no. 9, pp. 1535-1542, 1999.

[6] G. Gauthier and B. Boulet, "Terminal iterative learning control design with singular value decomposition decoupling for thermoforming ovens", *in Proc. American Control Conference*, pp. 1640-1645, 2009.

[7] J.X. Xu and D. Huang, "Initial state iterative learning for final state control in motion systems", *Automatica*, vol. 44, no. 12, pp. 3162-3169, 2008.

[8] Y. Wang and Z. Hou, "Terminal iterative learning control for station stop control of a train", Proc. Symp. Learning Control at IEEE CDC 2009, pp. 2009.

[9] H.S. Ahn, O. Jung, S. Choi, J.H. Son, D. Chung, G. Kim, "An optimal satellite antenna profile using reinforcement learning", *in IEEE Transactions on Systems, Man and Cybernetics Part C: Applications and Reviews*, pp. 1 - 14, 2010.

[10] C.T Freeman, Z. Cai, E. Rogers, and P.L. Lewin, "Iterative learning control for multiple point-to-point tracking application",*IEEE Trans. Control Syst. Technol.*, 2010.

[11] S. Gunnarsson and M. Norrlof, "On the design of ILC algorithms using optimization", *Automatica*, vol. 37, no. 12, pp. 2011-2016, 2001.

[12] J.X. Xu and Y. Tan, "Robust optimal design and convergence properties analysis of iterative learning control approaches", *Automatica*, vol. 38, no. 11, pp. 1867-1880, 2002.

[13] K.L. Barton and A.G. Alleyne, "A norm optimal approach to time-varying ILC with application to a multi-axis robotic testbed", *IEEE Trans. Control Syst. Technol.*, vol. 19, no. 1, pp. 166-180, 2011.

[14] R.A. Horn, C.R. Johnson, *Matrix Analysis*, Cambridge University Press, 1985.