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Adaptive Control Schemes for Discrete-Time T-S Fuzzy Systems with Unknown Parameters and Actuator Failures

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Abstract— **This paper develops a new solution framework with detailed system modeling, and control design, analysis and evaluation, for adaptive control of discrete-time input-output multiple-delay T-S fuzzy systems with unknown parameters and uncertain actuator failures. A multiple-delay prediction fuzzy system model is derived and its minimum phase property is clarified. Based on a model-based approach, the design and analysis are presented for an adaptive control scheme for multiple-delay T-S fuzzy systems, and an adaptive actuator failure compensation for systems with redundant actuators subject to uncertain failures, for which new system parametrizations and controller structures are developed. Illustrative examples and simulation results are presented to demonstrate the studied new concepts and to verify the desired performance of the new types of adaptive fuzzy control systems.**

Keywords: Actuator failure, adaptive control, fuzzy systems, output tracking, system uncertainties.

I. Introduction

Fuzzy control techniques have emerged in recent years as a powerful tool to deal with uncertain nonlinear systems. Takagi-Sugeno (T-S) fuzzy models are derived based on the idea to decompose complex nonlinear systems into a group of local linear models. By fuzzily blending local linear models, the global T-S fuzzy models are essentially nonlinear models. It has been proven that T-S fuzzy models are universal approximators [12], which makes them attract more and more attentions in the development of model-based control approaches. Model-based approaches for fuzzy control systems design and analysis make use of well-developed control theory tools and have solid technical foundations, as demonstrated in [1] where a comprehensive overview is given on the basic theory, fundamental design techniques and popular algorithms for model-based fuzzy system modeling and control, including rigorously designed adaptive control schemes single-input single-output single-delay fuzzy systems with parameter uncertainties.

Considerable research work has been done in adaptive fuzzy control field. In [11], various adaptive fuzzy logic controllers have been proposed and analyzed for some classes of nonlinear systems. Recent adaptive fuzzy control approaches for continuous-time nonlinear systems can be found in [4], [13]. Since in applications the fuzzy controllers are actually realized through computer-generated digital control signals and not all the continuous-time designs can be directly applied to discrete-time cases, it is of both theoretical and practical significance to carry out research work on adaptive fuzzy control design for discrete-time fuzzy systems [3], [5], [7].

Although many progresses have been made in model-based fuzzy adaptive control, to the best of the authors' knowledge, there are still open issues, including system characterizations under fuzzy modeling, large parameter variations due to membership functions and related system stability, convergence of tracking errors of the baseline fuzzy control systems, robustness with respect to modeling errors, as well as uncertain actuator failures, an issue that is also open for regular systems. Adaptive control techniques need to be further developed for fuzzy dynamic systems, in order to deal with the above mentioned issues.

In this paper, we address adaptive fuzzy control of systems with uncertain actuator failures and develop a solution framework with detailed design, analysis and evaluation, for discrete-time input-output multiple-delay T-S fuzzy systems. Our solution consists of the derivation of an input-output T-S fuzzy system model with multiple delays, clarification of the minimum phase property of such T-S fuzzy systems, the design and analysis of a new adaptive control scheme for multiple-delay T-S fuzzy systems, and the design and analysis of adaptive actuator failure compensation for systems with redundant actuators subject to uncertain failures. We prove closed-loop stability and asymptotic tracking for systems with multiple delays and with uncertain actuator failures.

II. System Modeling and Problem Formulation

We first derive a discrete-time input-output multiple-delay T-S fuzzy system model and its d-step prediction form, and then formulate the new adaptive fuzzy control problems.

A. Input-Output Multiple-Delay T-S Fuzzy System Models

Consider a single-input single-output nonlinear system in its discrete-time input-output form

$$y(t) = f(y(t-1), \ldots, y(t-n), u(t-d), \ldots, u(t-n)) \quad (1)$$

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where $f(\cdot,\ldots,\cdot)$ is some nonlinear function, $y(\cdot)$ is the system output signal, $u(\cdot)$ is the system input signal, $t = 0, 1, 2, \ldots,$ is the discrete-time time variable, n is the system order, and d is the number of system input-output delays. In this paper, we are interested in the general case with $1 \leq d \leq n$ (especially, $d > 1$). To employ fuzzy control techniques, we will first look for a prediction-based fuzzy model for (1):

$$y(t+d) = f_d(y(t), y(t-1), \ldots, y(t-n+1), u(t), u(t-1), \ldots, u(t-n+1)), \quad (2)$$

for some function $f_d(\cdot,\ldots,\cdot)$, as used in the literature.

Let us consider the following discrete-time T-S fuzzy system model with the ith fuzzy rule:

$$\begin{aligned} &\text{IF } \xi_1 \text{ is } \mathcal{F}_1^i \text{ and } \ldots \text{ and } \xi_L \text{ is } \mathcal{F}_L^i, \quad \text{THEN} \\ &y(t) + a_{i1}y(t-1) + \cdots + a_{in}y(t-n) = b_{i0}u(t-d) \\ &\quad + b_{i1}u(t-d-1) + \cdots + b_{i,\,n-d}u(t-n), \end{aligned} \quad (3)$$

for $i = 1, 2, \ldots, N$, $b_{i0} \neq 0$, where N is the number of fuzzy rules, $u(t) \in R$ and $y(t) \in R$ are the input and output variables, $d > 0$ denotes the system delay, and $\mathcal{F}_j^i$ being typically an interval of real numbers, called a fuzzy set associated with which there is a membership function $F_j^i(\xi_j(t))$ to indicate the degree of membership of $\xi_j(t)$ in $\mathcal{F}_j^i$.

Such local systems have transfer functions:

$$T_i(z) = \frac{b_{i0}z^{n-d} + \cdots + b_{i,n-d-1}z + b_{i,\,n-d}}{z^n + a_{i1}z^{n-1} + \cdots + a_{i,n-1}z + a_{in}}. \quad (4)$$

Each fuzzy system model (3) defines a local linear model for the original nonlinear system (1). Introducing the polynomials in z^{-1}:

$$\begin{aligned} A_i(z^{-1}) &= 1 + a_{i1}z^{-1} + \cdots + a_{in}z^{-n} & (5) \\ \bar{B}_i(z^{-1}) &= b_{i0} + b_{i1}z^{-1} + \cdots + b_{i,n-d}z^{-n+d}, & (6) \end{aligned}$$

we express the local linear system (3) as

$$A_i(z^{-1})[y](t) = z^{-d}\bar{B}_i(z^{-1})[u](t). \quad (7)$$

Following [2], solving the polynomial equation

$$z^d = z^d F_i(z^{-1})A_i(z^{-1}) + G_i(z^{-1}) \quad (8)$$

we obtain the unique polynomials

$$\begin{aligned} F_i(z^{-1}) &= 1 + f_{i1}z^{-1} + \cdots + f_{i,d-1}z^{-d+1} & (9) \\ G_i(z^{-1}) &= g_{i0} + g_{i1}z^{-1} + \cdots + g_{i,n-1}z^{-n+1}. & (10) \end{aligned}$$

Then, operating both sides of (8) on $y(t)$ and substituting (7), we obtain the local d-step prediction equation

$$y(t+d) = \alpha_i(z^{-1})[y](t) + \beta_i(z^{-1})[u](t), \quad (11)$$

where $\alpha_i(z^{-1}) = G_i(z^{-1})$ and $\beta_i(z^{-1}) = F_i(z^{-1})\bar{B}_i(z^{-1})$:

$$\begin{aligned} \alpha_i(z^{-1}) &= \alpha_{i0} + \alpha_{i1}z^{-1} + \cdots + \alpha_{i,n-1}z^{-n+1} \\ \beta_i(z^{-1}) &= \beta_{i0} + \beta_{i1}z^{-1} + \cdots + \beta_{i,n-1}z^{-n+1}, \end{aligned} \quad (12)$$

with $\beta_{i0} = b_{i0} \neq 0$, for $i = 1, 2, \ldots, N$.

In our study, we will use the local models in (11) to form a global d-step prediction fuzzy system model.

Proposition 1: *Following a standard fuzzy modeling procedure, a nonlinear dynamic system (1), via the local fuzzy system model (3), can be approximated by a global d-step prediction fuzzy system model:*

$$y(t+d) = \sum_{i=1}^{N} \mu_i\alpha_i(z^{-1})[y](t) + \sum_{i=1}^{N} \mu_i\beta_i(z^{-1})[u](t), \quad (13)$$

where μ_i is the normalized membership function: $\mu_i(\xi) = \frac{\lambda_i(\xi)}{\sum_{i=1}^{N}\lambda_i(\xi)}$, $\lambda_i(\xi) = \prod_{j=1}^{L} F_j^i(\xi_j)$, such that $\mu_i(\xi) \geq 0$ and $\sum_{i=1}^{N}\mu_i(\xi) = 1$.

The fuzzy system model (13) is an approximate model for the original nonlinear system model (1), and the approximation errors from such a standard fuzzy system modeling technique can be made small by increasing the number of membership base functions [12].

B. Control Problems

We study two adaptive fuzzy control problems in this paper: one is adaptive control of the global fuzzy system (13), and the other is adaptive control of the fuzzy system with redundant actuators which are subject to uncertain failures.

Adaptive Control Problem I: For this adaptive control problem, the control objective is to find an adaptive control scheme for the system (13) with unknown parameters $\alpha_{i0}, \alpha_{i1}, \ldots, \alpha_{in}$, $\beta_{i0}, \beta_{i1}, \ldots, \beta_{i,\,n-d}$, $i = 1, \ldots, N$, to ensure closed-loop signal boundedness and asymptotic tracking of a bounded reference output $y_m(t)$ by the system output $y(t)$, under the following assumptions:

(A.1): The fuzzy system (13) is minimum phase.

(A.2): $\sum_{i=1}^{N}\mu_i(\xi(t))\beta_{i0} \neq 0$, for all $t \geq 0$.

Without loss of generality, we assume $\beta_{i0} = b_{i0} > 0$. Under this practical assumption, due to the properties of μ_i, in particular, $\mu_i(\xi) \geq 0$ and $\sum_{i=1}^{N}\mu_i(\xi) = 1$, we actually do have Assumption (A.2) satisfied.

For adaptive control, we will need the condition:$\sum_{i=1}^{N}\mu_i(\xi(t))\beta_{i0} \neq 0$, where $\hat{\beta}_{i0}$, $i = 1, 2, \ldots, N$, are the estimates of β_{i0}. This condition can be ensured by using parameter projection [9] on the parameter estimates $\hat{\beta}_{i0}$, $i = 1, 2, \ldots, N$, using the knowledge of the positive upper and lower bounds of β_{i0}.

Minimum phase fuzzy system definition. We now clarify the conditions for Assumption (A.1).

For a regular linear time-invariant system

$$A(z^{-1})[y](t) = z^{-d}\bar{B}(z^{-1})[u](t), \quad (14)$$

recall that it is minimum phase if all zero of $\bar{B}(z^{-1})$ are in $|z| < 1$. This condition implies that

$$|u(t-d)| \leq c_1|y(t)| + c_2\sum_{\tau=0}^{t-1}\lambda^{t-\tau-1}|y(\tau)|, \; t \geq d, \quad (15)$$

for some constants $c_1 > 0$, $c_2 > 0$ and $\lambda \in (0, 1)$.

Based on this reasoning, we use the following minimum phase definition for the global fuzzy system model (13).

Definition 1: The fuzzy system (13) is minimum phase if the condition (15) is satisfied.

Unlike the case with a regular LTI system (14) whose minimum phase property can be checked using the knowledge of the zeros of $\bar{B}(z^{-1})$, the fuzzy system (13) is nonlinear and time-varying and its zeros can not be simply defined (they are only partially related to the zeros of each $\bar{B}_i(z^{-1})$ but largely related to $\mu_i(t)$ and their combined effect).

Adaptive Control Problem II: The second adaptive control problem deals with the compensation of uncertain failures of redundant actuators in a fuzzy dynamic system. To formulate such a problem, consider a multiple-input single-output nonlinear system in its discrete-time input-output form

$$\begin{aligned} y(t) = \ & f(y(t-1),\dots,y(t-n),u_1(t-d),\dots, \\ & u_1(t-n),\dots,u_m(t-d),\dots,u_m(t-n)), \end{aligned} \quad (16)$$

where $u_i(\cdot)$, $i = 1, 2, \dots, m$, are the input signals whose actuators may fail during the system operation. Similar to (3) for the case when $m = 1$ (that is, the non-redundant actuator case), the following discrete-time T-S fuzzy system model can be used for the case when $m > 1$:

$$\begin{aligned} & \text{IF} \quad \xi_1 \quad \text{is} \quad \mathcal{F}_1^i \quad \text{and} \ \dots \ \text{and} \quad \xi_L \quad \text{is} \quad \mathcal{F}_L^i \\ & \text{THEN} \quad y(t) + a_{i1}y(t-1) + \dots + a_{in}y(t-n) \\ & = b_{1i0}u_1(t-d) + b_{1i1}u_1(t-d-1) + \cdots \\ & \quad + b_{1i,\,n-d}u_1(t-n) + \dots + b_{mi0}u_m(t-d) \\ & \quad + b_{mi1}u_m(t-d-1) + \dots + b_{mi,\,n-d}u_m(t-n), \end{aligned} \quad (17)$$

with $b_{ji0} \neq 0$, $j = 1, 2, \dots, m$, $i = 1, 2, \dots, N$.

The actuator failures can be described by

$$u_j(t) = \bar{u}_j(t) = \bar{u}_{j0} + \sum_{l=1}^{n_j} \bar{u}_{jl} f_{jl}(t), \ t \geq t_j \quad (18)$$

for some unknown constants $\bar{u}_{j0}$ and $\bar{u}_{jl}$ and known bounded signals $f_{jl}(t)$, $l = 1, \dots, n_j$, and $n_j \geq 1$ [10]. This parametrized actuator failure model can be used to closely approximate a large class of practical failures, by a proper selection of these "basis" functions $f_{jl}(t)$.

The control objective is to find an adaptive control scheme for the global version of the fuzzy system (18) with unknown parameters and subject to failures belonging to a failure set (e.g., for up to $m-1$ actuators), with unknown failure patterns, values and time instants, to ensure closed-loop signal boundedness and asymptotic tracking of a bounded reference output $y_m(t)$ by the system output $y(t)$.

Remark 1: For a realistic system, the system model (13) is subject to certain modeling error $\Delta(y(\cdot), u(\cdot), t)$:

$$\begin{aligned} y(t+d) = & \sum_{i=1}^{N} \mu_i \alpha_i(z^{-1})[y](t) + \sum_{i=1}^{N} \mu_i \beta_i(z^{-1})[u](t) \\ & + \Delta(y(t), y(t-1), \dots, u(t-1), u(t-2), \dots, t). \end{aligned}$$

For adaptive control of the fuzzy system model (13), the robustness issue can be similarly addressed by using robust adaptive control designs [1], [9]. Due to space limit, this issue is not addressed in this paper.

III. Adaptive Control Design and Analysis for T-S Fuzzy Systems

In this section, we design and analyze an adaptive control scheme for the T-S fuzzy system (13), to solve the first adaptive control problem stated in Section 2.2. We first give a nominal control scheme for the system (13), assuming all system parameters are known. We then derive a parametrization of the system (13) with unknown parameters, design an adaptive parameter estimation algorithm to estimate the unknown system parameters, and develop an adaptive control law and analyze the closed-loop system performance.

A. Parameter Estimation

To estimate the system parameters, we need to develop a parametrized model. With the knowledge of n and d, the fuzzy system (13) can be expressed as

$$y(t+d) = \theta^T \phi(t), \quad (19)$$

where $\phi(t) = [\phi_1^T(t), \dots, \phi_N^T(t)]^T$,$\theta = [\theta_1^T, \dots, \theta_N^T]^T$,

$$\begin{aligned} \phi_i(t) \ &= \ [\mu_i y(t), \mu_i y(t-1), \dots, \mu_i y(t-n+1), && (20) \\ & \qquad \mu_i u(t), \mu_i u(t-1), \dots, \mu_i u(t-n+1)]^T && (21) \\ \theta_i \ &= \ [\alpha_{i0}, \alpha_{i1}, \dots, \alpha_{i,n-1}, \beta_{i0}, \beta_{i1}, \dots, \beta_{i,n-1}]^T. && (22) \end{aligned}$$

The expression (19), with θ unknown and $\phi(t)$ known, is a regression form with a linear parametrization for which many parameter adaptation algorithms can be adopted to estimate these unknown parameters in θ. As a choice, the following adaptive law is employed to obtain the estimate $\hat{\theta}(t)$ of θ:

$$\hat{\theta}(t) = \hat{\theta}(t-1) + \frac{\gamma(t)\phi(t-d)\varepsilon(t)}{c + \phi^T(t-d)\phi(t-d)}, \quad (23)$$

where $\gamma(t) \in (\gamma_0, 2-\gamma_0)$ is an adaptation gain for some constant $\gamma_0 \in (0, 1)$, $c > 0$ is a small design parameter, and

$$\varepsilon(t) = y(t) - \hat{\theta}^T(t-1)\phi(t-d). \quad (24)$$

For this parameter estimation algorithm, we have:

Lemma 1: *The parameter adaptation law (23), when applied to the fuzzy system (19), has the properties:*

(i) $\|\hat{\theta}(t) - \theta\| \leq \|\hat{\theta}(t-1) - \theta\| \leq \|\hat{\theta}(0) - \theta\|$*, for the* l^2*-vector norm* $\|\cdot\|$*;*
(ii) $\frac{\varepsilon(t)}{\sqrt{c+\phi^T(t-d)\phi(t-d)}} \in L^2$*;*
(iii) $\lim_{t\to\infty} \frac{\varepsilon(t)}{\sqrt{c+\phi^T(t-d)\phi(t-d)}} = 0$*;*
(iv) $\|\hat{\theta}(t) - \hat{\theta}(t-t_1)\| \in L^2$*; and*
(vi) $\lim_{t\to\infty} \|\hat{\theta}(t) - \hat{\theta}(t-t_1)\| = 0$*, for any finite* $t_1 > 0$.

The proof of this lemma is standard [2]. The adaptive law generates online estimates $\hat{\theta}(t)$ of the unknown parameter θ, with desired stability and L^2 properties.

B. Adaptive Control Law

Let $\hat{\alpha}_i(z^{-1})$ and $\widehat{\beta}_i(z^{-1})$ be the estimates of $\alpha_i(z^{-1})$ and $\beta_i(z^{-1})$ in (12) and (12), with parameters estimates $\hat{\alpha}_{ij}$ and $\hat{\beta}_{ij}$. We choose the global fuzzy control law as

$$u(t) = \frac{1}{\sum_{i=1}^N \mu_i \hat{\beta}_{i0}}[-\sum_{i=1}^N \mu_i \hat{\alpha}_i(z^{-1})[y](t) - \sum_{i=1}^N \mu_i \widehat{\bar{\beta}}_i(z^{-1})[u](t) + y_m(t+d)]. \quad (25)$$

For this adaptive control law, parameter projection [9] may be used for the parameter estimation algorithm (23) to ensure that $\sum_{i=1}^N \mu_i \hat{\beta}_{i0} > \beta_0$ for some constant $\beta_0 > 0$.

Stability analysis. We now show that the adaptive control system has desired stability and tracking properties. Substituting (25) into (13), we obtain the closed-loop system as

$$y(t+d) = \theta^T \phi(t) - \hat{\theta}^T(t)\phi(t) + y_m(t+d). \quad (26)$$

With $e(t) = y(t) - y_m(t)$ and $\tilde{\theta}(t) = \hat{\theta}(t) - \theta$, we obtain

$$e(t+d) = -\tilde{\theta}^T(t)\phi(t). \quad (27)$$

We first present a desired property for $\phi(t)$.

Lemma 2: *Under Assumption (A.1), the regressor $\phi(t)$ defined in (20) satisfies*

$$\|\phi(t-d)\| \leq \rho_1 + \rho_2 \max_{\tau=0,1,\ldots,t} |e(\tau)| \quad (28)$$

for some positive constants ρ_1 and ρ_2.

The proof of Lemma 2 is based on Assumption (A.1) and Definition 1 and details can be found in [6].

We now show the desired closed-loop system properties.

Theorem 1: *All signals in the closed-loop system, with the plant (13) satisfying Assumptions (A.1) and (A.2), the controller (25) and the adaptive law (23), are bounded, and* $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$.

Thus far, we have solved the first adaptive control problem of Section 2.2, for the system (13) with uncertain parameters.

IV. Adaptive Actuator Failure Compensation

In this section, we develop the solution to the adaptive actuator failure compensation problem. For simplicity of presentation and without loss of generality, we consider the fuzzy system model (18) with two actuators.

A. Nominal Failure Compensation Design

Based on the fuzzy system model (18) with two actuators ($m = 2$): u_1 and u_2, using a derivation procedure similar to that in Section II.A, we can derive the following global d-step prediction fuzzy system model (as similar to that described in Proposition 1 for the one-actuator system):

$$y(t+d) = \sum_{i=1}^N \mu_i \alpha_i(z^{-1})[y](t) + \sum_{i=1}^N \mu_i \beta_{1i}(z^{-1})[u_1](t) + \sum_{i=1}^N \mu_i \beta_{2i}(z^{-1})[u_2](t). \quad (29)$$

For this system, there are three possible situations which we need to deal with: (i) both actuators u_1 and u_2 are healthy, (ii) the actuator u_1 is healthy while u_2 is failed (that is, $u_2 = \bar{u}_2$ in (18)), and (iii) the actuator u_2 is healthy while u_1 is failed (that is, $u_1 = \bar{u}_1$ in (18)). Our goal is to develop one controller structure which is suitable for all three situations.

For either case (ii) or case (iii), in order for the healthy actuator to meet the control objective (stability, tracking and faliure compensation), we need to assume

> **(A.1a)**: Both individual subsystems (u_1, y) and (u_2, y) are minimum phase.

For the first case with no failure, that is, when both actuators u_1 and u_2 are healthy, there is a need of actuator coordination to meet a desired system output performance (otherwise, for example, $u_1(t)$ and $u_2(t)$ are against to each other, there would be a problem). Such an actuator coordination is characterized by an actuation scheme

$$u_i(t) = \delta_i v_0(t), \ \delta_i > 0, \ i = 1, 2 \quad (30)$$

for some applied input signal $v_0(t)$ to be designed. With this actuation scheme, the system (29) becomes

$$y(t+d) = \sum_{i=1}^N \mu_i \alpha_i(z^{-1})[y](t) + \sum_{i=1}^N \mu_i \beta_i(z^{-1})[v_0](t), \quad (31)$$

where $\beta_i(z^{-1}) = \delta_1\beta_{1i}(z^{-1}) + \delta_2\beta_{2i}(z^{-1}) \stackrel{\triangle}{=} \beta_{i0} + \beta_{i1}z^{-1} + \cdots + \beta_{i,n-1}z^{-n+1}$. The control signal $v_0(t)$ can be designed as $u(t)$ in (25), under Assumptions (A.1) and (A.2) for the system (31). This motivates the following assumption for the system (29):

> **(A.1b)**: The coordinated system (31) is minimum phase for a set of chosen $\delta_1 > 0$ and $\delta_2 > 0$.

Then, under a chosen actuation scheme (30), we propose the following nominal controller structure for $v_0(t)$ to accommodate the above all three cases:

$$\begin{aligned} v_0(t) \ = \ & \frac{1}{\sum_{i=1}^N \mu_i \beta_{i0}^*}[-\sum_{i=1}^N \mu_i \alpha_i(z^{-1})[y](t) \\ & - \sum_{i=1}^N \mu_i \bar{\beta}_i^*(z^{-1})[v_0](t) - \sum_{i=1}^N \mu_i \beta_{1i}^*(z^{-1})[\bar{u}_1](t) \\ & - \sum_{i=1}^N \mu_i \beta_{2i}^*(z^{-1})[\bar{u}_2](t) + y_m(t+d)], \end{aligned} \quad (32)$$

where $\beta_i^*(z^{-1}) = \delta_1\beta_{1i}(z^{-1}) + \delta_2\beta_{2i}(z^{-1})$, and $\beta_{1i}^*(z^{-1}) = \beta_{2i}^*(z^{-1}) = 0$ for case (i); $\beta_i^*(z^{-1}) = \delta_1\beta_{1i}(z^{-1})$, $\beta_{1i}^*(z^{-1}) = 0$ and $\beta_{2i}^*(z^{-1}) = \beta_{2i}(z^{-1})$ for case (ii); and $\beta_i^*(z^{-1}) = \delta_2\beta_{2i}(z^{-1})$, $\beta_{1i}^*(z^{-1}) = \beta_{1i}(z^{-1})$, and $\beta_{2i}^*(z^{-1}) = 0$ for case (iii). They are piecewise polynomials which change when an actuator failure occurs.

Although in the nominal control $\beta_{ji}^*(z^{-1})$ may be zero for some $j = 1, 2$, for adaptive control when the actuator failure pattern (which actuator fails) is uncertain, they both are treated as parametrized polynomials whose parameters are to be adaptively estimated by some adaptive laws.

B. Adaptive Failure Compensation Design

To develop an adaptive actuator failure compensation scheme, we need to obtain the estimates of $\alpha_i(z^{-1})$, $\beta_i^*(z^{-1})$, $\beta_{1i}^*(z^{-1})$ and $\beta_{2i}^*(z^{-1})$. In view of their definitions, we express the system (29) as

$$y(t+d) = \sum_{i=1}^{N} \mu_i \alpha_i(z^{-1})[y](t) + \sum_{i=1}^{N} \mu_i \beta_i^*(z^{-1})[v_0](t) + \sum_{i=1}^{N} \mu_i \beta_{1i}^*(z^{-1})[\bar{u}_1](t) + \sum_{i=1}^{N} \mu_i \beta_{2i}^*(z^{-1})[\bar{u}_2](t). \quad (33)$$

In this model, $\beta_i^*(z^{-1})$ is parametrized in the same structure as that of $\delta_1\beta_{1i}(z^{-1}) + \delta_2\beta_{2i}(z^{-1})$, $\delta_1\beta_{1i}(z^{-1})$ and $\delta_2\beta_{2i}(z^{-1})$ (they all have the same structure), $\beta_{1i}^*(z^{-1})$ and $\beta_{2i}^*(z^{-1})$ are parametrized in the structures of $\beta_{1i}(z^{-1})$ and $\beta_{2i}(z^{-1})$, respectively. Moreover, the parametrization of $\sum_{i=1}^{N}\mu_i\beta_{1i}^*(z^{-1})[\bar{u}_1](t)$ (or $\sum_{i=1}^{N}\mu_i\beta_{2i}^*(z^{-1})[\bar{u}_2](t)$) is based on combining the known signals μ_i with the known basis functions $f_{jl}(t)$, $j = 1, 2$, $l = 0, 1, \ldots, n_j$, of the actuator failure model (18) (with $f_{j0} = 1$, $j = 1, 2$).

1) Parameter Estimation: Express the system (33) as

$$y(t+d) = \theta_a^T \phi_a(t) + \sum_{j=1}^{2}\sum_{i=1}^{N} \mu_i \beta_{ji}^*(z^{-1})[\bar{u}_j](t), \quad (34)$$

where $\theta_a = [\theta_{a1}^T, \ldots, \theta_{aN}^T]^T$, $\phi_a(t) = [\phi_{a1}^T, \ldots, \phi_{aN}^T]^T$

$$\phi_{ai}(t) = [\mu_i y(t), \mu_i y(t-1), \ldots, \mu_i y(t-n+1), \mu_i v_0(t), \mu_i v_0(t-1), \ldots, \mu_i v_0(t-n+1)]^T \quad (35)$$

$$\theta_{ai} = [\alpha_{i0}, \alpha_{i1}, \ldots, \alpha_{i,n-1}, \beta_{i0}^*, \beta_{i1}^*, \ldots, \beta_{i,n-1}^*]^T. \quad (36)$$

Parametrize the actuator fault $\bar{u}_j(t)$ in (18) as

$$\bar{u}_j(t) = p_{j0} + p_j^T f_j(t), \quad j = 1, 2, \quad (37)$$

where $p_{j0} = \bar{u}_{j0}$, $p_j = [\bar{u}_{j1}, \bar{u}_{j2}, \ldots, \bar{u}_{jn_j}]^T \in R^{n_j}$ and $f_j(t) = [f_{j1}(t), f_{j2}(t), \ldots, f_{jn_j}(t)]^T \in R^{n_j}$.

From (37), the second term in (34) can be written as

$$\sum_{i=1}^{N} \mu_i \beta_{ji}^*(z^{-1})[\bar{u}_j](t) = \bar{p}_{j0}^T \bar{\mu}(t) + \bar{p}_j^T \phi_{f_j}(t), \quad (38)$$

where, with $\otimes$ denoting the Kronecker product,

$$\bar{\mu}(t) = [\mu_1(t), \mu_2(t), \ldots, \mu_N(t)]^T \in R^N$$

$$\bar{f}_j(t) = M_j(z^{-1})[f_j](t) \in R^{n \times n_j}$$

$$\bar{p}_{j0} = [\sum_{k=0}^{n-1}\beta_{j1k}^* p_{j0}, \sum_{k=0}^{n-1}\beta_{j2k}^* p_{j0}, \ldots, \sum_{k=0}^{n-1}\beta_{jNk}^* p_{j0}]^T \in R^N$$

$$\bar{p}_{j0} = [\sum_{k=0}^{n-1}\beta_{j1k}^* p_{j0}, \sum_{k=0}^{n-1}\beta_{j2k}^* p_{j0}, \ldots, \sum_{k=0}^{n-1}\beta_{jNk}^* p_{j0}]^T \in R^N$$

$$\bar{p}_j = [\beta_{j1}^{*T} \otimes p_j^T, \beta_{j2}^{*T} \otimes p_j^T, \ldots, \beta_{jN}^{*T} \otimes p_j^T]^T \in R^{n \times n_j \times N}$$

$$\phi_{f_j}(t) = [\mu_1 \bar{f}_j^{\,T}(t), \mu_2 \bar{f}_j^{\,T}(t), \ldots, \mu_N \bar{f}_j^{\,T}(t)]^T \in R^{n \times n_j \times N}$$

$$M_j(z^{-1}) = [I_{n_j}, z^{-1}I_{n_j}, \ldots, z^{-n+1}I_{n_j}]^T \in R^{(n \times n_j) \times n_j}.$$

Substituting (38) into (34) yields

$$\begin{aligned} y(t+d) &= \theta_a^T \phi_a(t) + \sum_{j=1}^{2} (\bar{p}_{j0}^T \bar{\mu}(t) + \bar{p}_j^T \phi_{f_j}(t)) \\ &= \theta^T \phi(t), \end{aligned} \quad (39)$$

where, with $\bar{p}_0 = \bar{p}_{10} + \bar{p}_{20}$, $\theta = [\theta_a^T, \bar{p}_0^T, \bar{p}_1^T, \bar{p}_2^T]^T$ and $\phi(t) = [\phi_a^T(t), \bar{\mu}^T(t), \phi_{f_1}^T(t), \phi_{f_2}^T(t)]^T$.

With the closed-loop system in the form (39), the estimation of θ can be obtained by using (23), which has the desired stability properties summarized in Lemma 1.

2) Adaptive Control Law: With the parameter estimation, the nominal control law (32) can be implemented with the parameter estimates. $\hat{\alpha}_i(z^{-1})$ and $\hat{\beta}_i^*(z^{-1})$ can be obtained directly from corresponding terms in $\hat{\theta}$ and the remaining failure-related parts can be calculated as

$$\sum_{i=1}^{N} \mu_i \hat{\beta}_{1i}^*(z^{-1})[\bar{u}_1](t) + \sum_{i=1}^{N} \mu_i \hat{\beta}_{2i}^*(z^{-1})[\bar{u}_2](t) = \widehat{\bar{p}}_0^T \bar{\mu}(t) + \widehat{\bar{p}}_1^T \phi_{f_1}(t) + \widehat{\bar{p}}_2^T \phi_{f_2}(t) \quad (40)$$

with $\widehat{\bar{p}}_0$, $\widehat{\bar{p}}_1$ and $\widehat{\bar{p}}_2$ being the corresponding elements in the parameter estimate vector $\hat{\theta} = [\hat{\theta}_a^T, \widehat{\bar{p}}_0^T, \widehat{\bar{p}}_1^T, \widehat{\bar{p}}_2^T]^T$.

Based on the desired properties of the parameter adaptation law and the regressor, the following closed-loop stability and asymptotic tracking results can be proved [6].

Theorem 2: *The controller (32) with the parameters estimated by the adaptive law (23), applied to the system (33) under Assumptions (A.1a) and (A.1b) and with actuator failures (18), guarantees that all closed-loop system signals are bounded and* $\lim_{t\to\infty}(y(t) - y_m(t)) = 0$.

The above design and analysis can be extended to systems with more than two actuators subject to uncertain failures.

V. Simulation Study

In this section, we present an illustrative example with simulation results to show the control design and evaluation, based on a mass-spring-damper mechanical system [8]:

$$M\ddot{x} + c_1\dot{x} + c_2 x = (1 + c_3\dot{x}^3)u, \quad (41)$$

where M denotes the mass, x is the displacement(in meters) of the mass , u is the force (in Newtons) applied to the spring, c_1 is the damping constant, c_2 is the spring constant, c_3 is a constant related to the nonlinear term $\dot{x}^3$. For simulation, the parameters are set as $M = 1kg, \quad c_1 = 150N \cdot s/m, c_2 = 200N/m, \quad c_3 = 0.13N/(m/s)^3$.

Choose the output $y = x$. Assuming $\dot{y} \in [-1.5, 1.5]$ and using the same approach as that in [8], a two-rule continuous-time Takagi-Sugeno fuzzy model to approximate (41) is given as

IF $\dot{y}$ is $\mathcal{F}_1^1$, THEN $\ddot{y} = -150\dot{y} - 200y + 1.4387u$,

IF $\dot{y}$ is $\mathcal{F}_1^2$, THEN $\ddot{y} = -150\dot{y} - 200y + 0.5613u$

with the membership functions describing "$\mathcal{F}_1^1$" and "$\mathcal{F}_1^2$" chosen as $F_1^1(\dot{y}) = 0.5 + \dot{y}^3/6.75$ and $F_1^2(\dot{y}) = 0.5 - \dot{y}^3/6.75$.

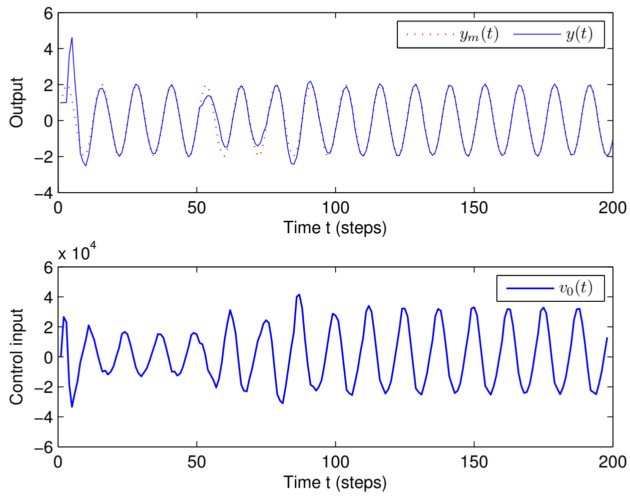

Fig. 1. Adaptive system response with $\bar{u}_2(t) = 100\sin(0.2t)(t > 50)$.

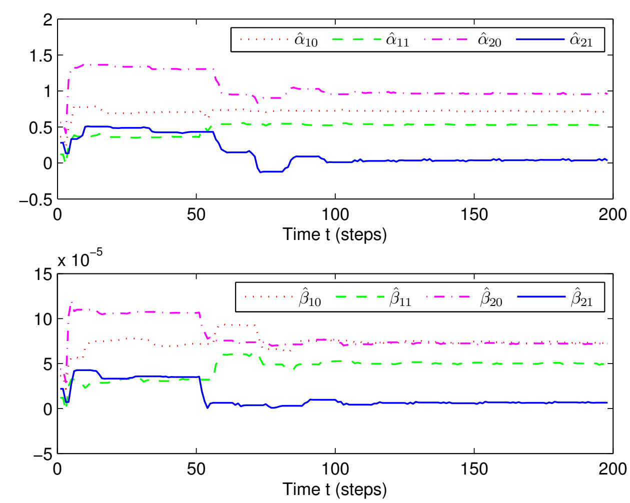

Fig. 2. Parameter adaptation of Rule 1 and Rule 2 ($\bar{u}_2 = 100\sin(0.2t)$).

If the sampling time T is chosen small enough, we can approximate $\dot{y}$ and $\ddot{y}$ with $\dot{y} = [y(t+1) - y(t)]/T$ and $\ddot{y} = [y(t+2) - 2y(t+1) + y(t)]/T^2$. Then a discrete-time model can be obtained as

$$R^i: \quad \text{IF} \quad \xi_1(t) \quad \text{is} \quad \mathcal{F}_1^i, \quad \text{THEN}$$
$$y(t+2) + a_{i1}y(t+1) + a_{i2}y(t) = b_{i0}u(t), \quad (42)$$

where $\xi_1(t) = [y(t+1) - y(t)]/T$, $a_{i1} = 150T - 2$, $a_{i2} = 1 - 150T + 200T^2, i = 1, 2$ and $b_{10} = 1.4387T^2$, $b_{20} = 0.5613T^2$.

The discrete-time T-S fuzzy system model (42) can be equivalently written into the form of (3) with $n = 2$ and the system delay $d = 2$. In this simulation, a redundant actuator is added to the fuzzy model (42) so that when one fails, the other can adaptively compensate the effect of the failed actuator. We start our design from the following model:

$$R^i: \quad \text{IF} \quad \xi_1(t-2) \quad \text{is} \quad \mathcal{F}_1^i, \quad \text{THEN}$$
$$y(t) + a_{i1}y(t-1) + a_{i2}y(t-2)$$
$$= b_{1i0}u_1(t-2) + b_{2i0}u_2(t-2), \quad i = 1, 2$$

with $a_{i1} = 150T - 2$, $a_{i2} = 1 - 150T + 200T^2$, $b_{110} = b_{210} = 1.4387T^2$ and $b_{120} = b_{220} = 0.5613T^2$.

We then obtain the global fuzzy system model (29) for the two actuator case ($m = 2$) with $\alpha_i(z^{-1}) = \alpha_{i0} + \alpha_{i1}z^{-1} = a_{i1}^2 - a_{i2} + a_{i1}a_{i2}z^{-1}$ and $\beta_{ji}(z^{-1}) = \beta_{ji0} + \beta_{ji1}z^{-1} = b_{ji0} - a_{j1}b_{ji0}z^{-1}, i, j = 1, 2$.

In the simulation, we assume $u_2(t) = \bar{u}_2(t)$ and consider a sinusoidal failure $\bar{u}_2(t) = 100\sin(0.2t)$. The initial parameter values are set as 50% of their true values. Other parameters are chosen as $T = 0.01s$, $\gamma(t) = 1$ and $c = 0.01$. The failure is added at $t = 50$. We consider output tracking of a sinusoidal signal $y_m(t) = 2\sin(0.5t)$ under failures. Simulation results are given in Fig. 1 and Fig. 2. More simulation results can be found in [6].

VI. Concluding Remarks

In this paper, we have formulated an adaptive fuzzy control problem: adaptive fuzzy control of systems with uncertain actuator failures, and developed a detailed solution for discrete-time single-input single-output multiple-delay T-S fuzzy systems in an input-output form. A multiple-delay fuzzy prediction model has been developed, based on which an adaptive control scheme and an adaptive actuator failure compensation scheme have been developed and analyzed, which have the desired system performance in the presence of parameter and failure uncertainties. Simulation results have also verified the desired performance of the developed adaptive fuzzy control systems.

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