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Sliding mode observers for sensorless control of current-fed induction motors

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***Abstract*— This paper presents the use of a higher order sliding mode scheme for sensorless control of induction motors. The second order sub-optimal control law is based on a reduced-order model of the motor, and produces the references for a current regulated PWM inverter. A nonlinear observer structure, based on Lyapunov theory and on different sliding mode techniques (first order, sub-optimal and super-twisting) generates the velocity and rotor flux estimates necessary for the controller, based only on the measurements of phase voltages and currents. The proposed control scheme and observers are tested on an experimental setup, showing a satisfactory performance.**

I. INTRODUCTION

Most of the research in induction motors control during the last years has been on sensorless solutions. In such control schemes, the motor is controlled without measuring its speed, making the system less expensive and unaffected by sensor failures, maintaining at the same time the advantages that the induction motor has with respect to other electric machines [1]. Nonetheless, controlling an induction motor presents some difficulties, related with the strong nonlinearities and couplings of the system, and with the fact that some parameters can vary in a wide range (e.g. the rotor resistance varies with temperature up to 200% of the nominal value). Also, in a sensorless scheme it is necessary to observe both the rotor flux and the velocity, together with the unknown parameters, using only measurements of the electrical variables (phase voltages and currents). For all these reasons, control and observation methodologies with strong robustness properties are required [2].

During the last years, many proposals for sensorless schemes have appeared in the literature (see, for instance [3], [4], [5], and the references therein). Some of the proposed strategies (e.g. [6], [7], [8], and [9]) rely on sliding mode control, which guarantees robustness with respect to matched disturbances and parameter variations [10]. The main drawback associated with the use of sliding mode techniques is the generation of the so-called 'chattering' phenomenon [11], [12], which consists in a high-frequency oscillation of the controlled or observed variables when the sliding variables [10] are steered to zero. To reduce the chattering, higher order sliding mode techniques (like the super-twisting and sub-optimal algorithms) have been introduced in recent years [13].

In this work, the sensorless control scheme (introduced in [14] for induction motors with speed measurement) consists in a speed and rotor flux controller based on the so-called sub-optimal control law [15]. The rotor flux and speed, together with a parameter that takes into account the rotor resistance, are estimated using a nonlinear observer which can be based on first order sliding mode, similarly to [16], or on the super-twisting algorithm, like in [17], or on the sub-optimal algorithm. The control scheme and the observers are presented, and the stability properties of the whole system are discussed. Finally, some experimental results confirm the convergence properties of the sliding mode based observers.

The contribution of this paper consists mainly in the proposal of the sub-optimal algorithm as an alternative method for the design of speed and rotor flux observers, comparing its performance with other similar approaches presented in the literature.

The paper is organized as follows: Section II introduces the full-order and reduced-order models of the electric machine, together with the control and observation objectives. The control strategy is described in Section III, while Section IV introduces the rotor flux and velocity observer structure. Section V analyzes the stability properties of the overall sensorless control scheme, while some experimental results on the observer performances are reported in Section VI. Finally, some conclusions are gathered in Section VII.

II. PROBLEM FORMULATION

In the fixed reference frame a-b, the fifth order induction motor model is defined by [1]

$$\left\{\begin{array}{rcl} \dot{\omega} & = & \mu(\psi_a i_b - \psi_b i_a) - \frac{K_f}{J}\omega - \frac{\Gamma_l}{J} \\ \dot{\psi}_a & = & -\alpha\psi_a - n_p\omega\psi_b + M\alpha i_a \\ \dot{\psi}_b & = & -\alpha\psi_b + n_p\omega\psi_a + M\alpha i_b \\ \dot{i}_a & = & -\gamma i_a + \alpha\beta\psi_a + \beta n_p\omega\psi_b + \frac{u_a}{\sigma L_s} \\ \dot{i}_b & = & -\gamma i_b + \alpha\beta\psi_b - \beta n_p\omega\psi_a + \frac{u_b}{\sigma L_s} \end{array}\right. \quad (1)$$

where $\alpha = \frac{R_r}{L_r}$, $\sigma = 1 - \frac{M^2}{L_s L_r}$, $\beta = \frac{M}{\sigma L_s L_r}$, $\gamma = \frac{M^2 R_r}{\sigma L_r^2 L_s} + \frac{R_s}{\sigma L_s}$, $\mu = \frac{n_p M}{L_r J}$, ω is the rotor speed, ψ_a and ψ_b are the rotor fluxes, u_a and u_b are the stator voltages, Γ_l is the load torque (which is considered known in this work), n_p is the number of pole-pairs, J is the moment of inertia, K_f is the friction coefficient, R_r, R_s, L_r, L_s, and M are the rotor and stator windings resistances and inductances, and the mutual inductance, respectively.

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Assuming $\sqrt{\psi_a^2+\psi_b^2} \geq \tilde{\psi}_d > 0$, defining the new inputs

$$\begin{array}{rcl} v_d & = & \cos(\rho)u_a + \sin(\rho)u_b \\ v_q & = & -\sin(\rho)u_a + \cos(\rho)u_b \end{array} \quad (2)$$

and applying the nonlinear coordinate transformation

$$\left\{ \begin{array}{rcl} \omega & = & \omega \\ \rho & = & \operatorname{atan}\left(\frac{\psi_b}{\psi_a}\right) \\ i_d & = & \cos(\rho)i_a + \sin(\rho)i_b \\ i_q & = & -\sin(\rho)i_a + \cos(\rho)i_b \\ \psi_d & = & \sqrt{\psi_a^2+\psi_b^2} \end{array} \right. \quad (3)$$

to (1), one obtains the direct-quadrature model (d-q)

$$\left\{ \begin{array}{rcl} \dot{\omega} & = & \mu\psi_d i_q - \frac{K_f}{J}\omega - \frac{\Gamma_l}{J} \\ \dot{\psi}_d & = & -\alpha\psi_d + M\alpha i_d \\ \dot{\rho} & = & n_p\omega + M\alpha\frac{i_q}{\psi_d} \\ \dot{i}_d & = & -\gamma i_d + \frac{M\alpha\psi_d}{\sigma L_r L_s} + n_p\omega i_q + M\alpha\frac{i_q^2}{\psi_d} + \frac{v_d}{\sigma L_s} \\ \dot{i}_q & = & -\gamma i_q - \frac{M n_p\omega\psi_d}{\sigma L_r L_s} - n_p\omega i_d - \frac{M\alpha i_d i_q}{\psi_d} + \frac{v_q}{\sigma L_s} \end{array} \right. \quad (4)$$

If the currents are considered as inputs, the model of the motor can be simplified, reducing its order. In this way, (1) becomes

$$\left\{ \begin{array}{rcl} \dot{\omega} & = & \mu(\psi_a i_b - \psi_b i_a) - \frac{K_f}{J}\omega - \frac{\Gamma_l}{J} \\ \dot{\psi}_a & = & -\alpha\psi_a - n_p\omega\psi_b + M\alpha i_a \\ \dot{\psi}_b & = & -\alpha\psi_b + n_p\omega\psi_a + M\alpha i_b \end{array} \right. \quad (5)$$

where i_a and i_b are now considered as inputs. Analogously, (4) can be replaced by

$$\left\{ \begin{array}{rcl} \dot{\omega} & = & \mu\psi_d i_q - \frac{K_f}{J}\omega - \frac{\Gamma_l}{J} \\ \dot{\psi}_d & = & -\alpha\psi_d + M\alpha i_d \\ \dot{\rho} & = & n_p\omega + M\alpha\frac{i_q}{\psi_d} \end{array} \right. \quad (6)$$

The standard approach to simplify the fifth order model (e.g. (4)) is to use an inner current loop. In this paper, a standard current regulated pulse width modulation (CRPWM) inverter is used. The control is performed using relays, which compare the currents of each phase (i_1, i_2, i_3) with the reference currents (i_1^*, i_2^*, i_3^*) generated by the speed and rotor flux controllers (which generate two control variables i_q^* and i_d^*), and force the voltages of the phases (v_1, v_2, v_3) to be one of the two extreme values $\pm\bar{V}$. The commutation of the power electronics switches takes place when the difference between the reference current and the actual one exceeds a tolerance (hysteresis) value. The reference phase current is transformed from the a-b or d-q axes to the 1-2-3 reference system of the three phases and vice-versa using the Park transformation matrix [1]. Using this voltage generation system, the current dynamics are neglected with respect to the flux and velocity ones, so one can consider $i_1^* \simeq i_1$, $i_2^* \simeq i_2$, $i_3^* \simeq i_3$, and then $i_d^* \simeq i_d$ and $i_q^* \simeq i_q$.

The objective of the designed control strategy is to make the induction motor follow the generated references of rotor flux and velocity, defined as ψ_d^* and ω_r^*. The controller is designed in the $d-q$ reference system on system (6). Only the measurements of phase currents (output variable) and voltages (input variable) are available. Then, the observer, designed in the $a-b$ reference system on system (1), must provide a good estimate of ψ_a, ψ_b, ω and α.

III. The proposed speed and flux controller

The design of the controller is based on the method described in [14] for a control scheme with both velocity and rotor flux measurements. The reduced third-order model in the d-q reference frame reported in (6) is considered, because, in this reference frame, a natural decoupling is realized, i.e. the control variable i_d acts on the flux ψ_d, while the control variable i_q acts on the velocity ω. Then, the following sliding variables are defined

$$s_{\omega 1} = \omega - \omega^* \quad (7)$$

$$s_{\psi 1} = \psi_d - \psi_d^* \quad (8)$$

where ω^* and ψ_d^* (generated such that their first and second order time derivatives $\dot{\omega}^*, \ddot{\omega}^*, \dot{\psi}_d^*, \ddot{\psi}_d^*$ are bounded) are the desired values of ω and ψ_d, respectively. To solve the control problem, the idea is to suitably design $\dot{i}_q$ and $\dot{i}_d$ as discontinuous control variables, the values of i_d and i_q being kept bounded, so that $|i_d| < \mathcal{I}_d$ and $|i_q| < \mathcal{I}_q$, where $\mathcal{I}_d$ and $\mathcal{I}_q$ are positive values. More precisely, the method proposed in [14] relies on the so-called modified sub-optimal second order sliding mode approach (the theoretical development of which has been presented in [15]). To this end, the values of f_ω, f_ψ, d_ω, h_ω, d_ψ and h_ψ must be bounded, that is, the boundedness of the variables on which they depend must be assured.

A multi-input auxiliary control problem can then be formulated. The control objective is to steer to zero in a finite time the state variables $s_{\omega 1}$, $s_{\psi 1}$ and their first order derivatives in spite of the presence of the uncertain terms. This can be accomplished by designing the auxiliary control laws

$$\begin{array}{rcl} i_q & = & \operatorname{sat}_{[-\mathcal{I}_q;+\mathcal{I}_q]}\left\{i_q(t_0) + \int_{t_0}^{t}\dot{i}_q\right\} \\ \dot{i}_q & = & -W_q\operatorname{sign}\left(s_{\omega 1} - \frac{1}{2}s_{\omega 1_M}\right) \end{array} \quad (9)$$

$$\begin{array}{rcl} i_d & = & \operatorname{sat}_{[-\mathcal{I}_d;+\mathcal{I}_d]}\left\{i_d(t_0) + \int_{t_0}^{t}\dot{i}_d\right\} \\ \dot{i}_d & = & -W_d\operatorname{sign}\left(s_{\psi 1} - \frac{1}{2}s_{\psi 1_M}\right) \end{array} \quad (10)$$

where $i_q(t_0)$ and $i_q(t_0)$ are the values of the currents i_q and i_d at the initial time instant t_0, W_q and W_d are positive constant values, $s_{\omega 1_M}$ and $s_{\psi 1_M}$ are the last extremal values of $s_{\omega 1}$ and $s_{\psi 1}$, respectively. The terms $\operatorname{sat}_{[-\mathcal{I}_q;+\mathcal{I}_q]}$ and $\operatorname{sat}_{[-\mathcal{I}_d;+\mathcal{I}_d]}$ mean that the values of i_q and i_d can increase until $|i_q| \leq \mathcal{I}_q$ and $|i_d| \leq \mathcal{I}_d$, then their values remain constant until a new switching in the sign of $\dot{i}_q$ (or $\dot{i}_q$) makes their absolute value decrease.

IV. The nonlinear observer

Until this point, the whole state of the system was considered accessible for measurement. Actually, the rotor flux cannot be easily measured in practical applications, the coefficient α is unknown, and the velocity sensor can be unavailable in some applications. Then, a particular nonlinear observer is introduced, in order to obtain estimates of ψ_a, ψ_b, ω and α. The complete scheme of the control system is shown in Fig. 1. The whole structure of the observer is based on a sliding mode current observer, defined as

$$\begin{array}{rcl} \dot{\hat{i}}_a & = & -\beta\dot{\hat{\psi}}_a + \frac{1}{\sigma L_s}(u_a - R_s i_a) + \chi_a \\ \dot{\hat{i}}_b & = & -\beta\dot{\hat{\psi}}_b + \frac{1}{\sigma L_s}(u_b - R_s i_b) + \chi_b \end{array} \qquad (11)$$

where $\hat{\psi}_a$, $\hat{\psi}_b$, $\hat{i}_a$, and $\hat{i}_b$ are the observed components of the rotor flux and stator currents, respectively. The terms χ_a and χ_b can be defined, depending on the type of approach adopted, among First Order Sliding Mode (SM), Super Twisting and Sub Optimal, as follows

$$\begin{array}{ll} \chi_a = -K_a \mathrm{sign}(\tilde{i}_a) & \text{First Order SM} \\ \left\{\begin{array}{rcl} \chi_a & = & z_a - k_{\lambda_a}|\tilde{i}_a|^{\frac{1}{2}}\mathrm{sign}(\tilde{i}_a) \\ \dot{z}_a & = & -k_{\alpha_a}\mathrm{sign}(\tilde{i}_a) \end{array}\right. & \text{Super Twisting} \\ \left\{\begin{array}{rcl} \chi_a & = & \chi'_a \\ \dot{\chi}'_a & = & -\mu_a \mathrm{sign}(\tilde{i}_a - \frac{1}{2}\tilde{i}_{a_M}) \end{array}\right. & \text{Sub Optimal} \end{array} \qquad (12)$$

$$\begin{array}{ll} \chi_b = -K_b \mathrm{sign}(\tilde{i}_b) & \text{First Order SM} \\ \left\{\begin{array}{rcl} \chi_a & = & z_b - k_{\lambda_b}|\tilde{i}_b|^{\frac{1}{2}}\mathrm{sign}(\tilde{i}_b) \\ \dot{z}_b & = & -k_{\alpha_b}\mathrm{sign}(\tilde{i}_b) \end{array}\right. & \text{Super Twisting} \\ \left\{\begin{array}{rcl} \chi_b & = & \chi'_b \\ \dot{\chi}'_b & = & -\mu_b \mathrm{sign}(\tilde{i}_b - \frac{1}{2}\tilde{i}_{b_M}) \end{array}\right. & \text{Sub Optimal} \end{array} \qquad (13)$$

where K_a, K_b, k_{α_a}, k_{α_b}, k_{λ_a}, k_{λ_b}, μ_a, μ_b are positive constants. The variables $\tilde{i}_a = \hat{i}_a - i_a$ and $\tilde{i}_b = \hat{i}_b - i_b$ are the estimation errors of the currents, while the extremal values i_{a_M} and i_{b_M} are defined according to the sub-optimal algorithm in [18].

Now, take into account the dynamics of the estimation errors

$$\begin{array}{rcl} \dot{\tilde{i}}_a & = & -\beta\dot{\tilde{\psi}}_a + \chi_a \\ \dot{\tilde{i}}_b & = & -\beta\dot{\tilde{\psi}}_b + \chi_b \end{array} \qquad (14)$$

If the first order SM observer is used, it can be proved that, according to [10], a sufficient large value of K_a and K_b can steer the estimation error of the currents to zero in a finite time. As for the Super-Twisting strategy, choosing the terms k_{α_a}, k_{α_b}, k_{λ_a}, k_{λ_b} large enough, one has that the convergence to zero of k_{α_a}, k_{α_b}, k_{λ_a}, k_{λ_b} in a finite time is guaranteed [13]. As for the last observer, the one designed according to a Sub Optimal approach, it guarantees the convergence if suitable values of μ_a and μ_b are chosen in order to dominate the system uncertainties (that is, the rotor flux errors). Of course, $\tilde{i}_a$ and $\tilde{i}_b$ can be obtained from measurements, while the estimation errors on the flux components is now assumed to be bounded, together with their first derivatives. This assumption will be verified in the sequel.

Once guaranteed that the current estimation errors go to zero in a finite time (which is obtained with all these sliding mode strategies) a nonlinear observer based on the design of a suitable Lyapunov function design can be introduced, analogously to [2].

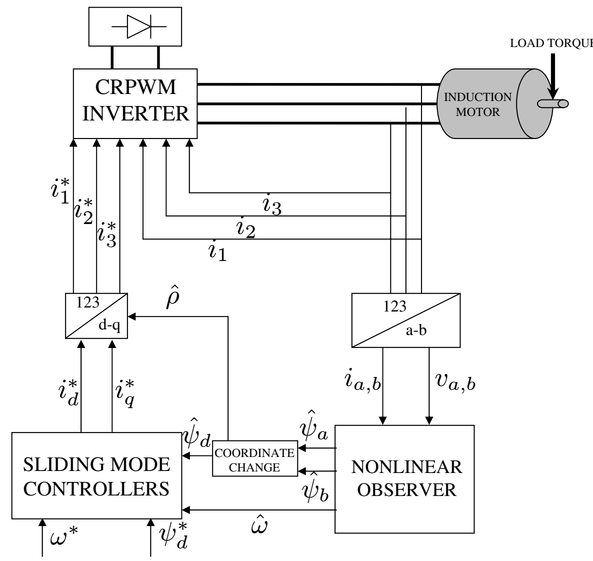

Fig. 1. Scheme of the induction motor control system

Two auxiliary quantities are introduced as

$$\begin{array}{rcl} z_a & = & \tilde{i}_a + \beta\tilde{\psi}_a \\ z_b & = & \tilde{i}_b + \beta\tilde{\psi}_b \end{array} \qquad (15)$$

and their dynamics is

$$\begin{array}{rcl} \dot{z}_a & = & \chi_a \\ \dot{z}_b & = & \chi_b \end{array} \qquad (16)$$

and then, as in [16], it is possible to state that

$$\begin{array}{rcl} \tilde{\psi}_a & = & \frac{z_a - \tilde{i}_a}{\beta} \\ \tilde{\psi}_b & = & \frac{z_b - \tilde{i}_b}{\beta} \end{array} \qquad (17)$$

to reconstruct the flux error estimates. Define now the nonlinear observer as

$$\begin{array}{rcl} \dot{\hat{\psi}}_a & = & -\hat{\alpha}\hat{\psi}_a - \hat{\omega}\hat{\psi}_b + M\hat{\alpha}i_a + f_{\psi_a} \\ \dot{\hat{\psi}}_b & = & -\hat{\alpha}\hat{\psi}_b - \hat{\omega}\hat{\psi}_a + M\hat{\alpha}i_b + f_{\psi_b} \\ \dot{\hat{\omega}} & = & \frac{M}{JL_r}(\hat{\psi}_a i_b - \hat{\psi}_b i_a) - \frac{K_f}{J}\hat{\omega} - \frac{\Gamma_l}{J} + f_\omega \\ \dot{\hat{\alpha}} & = & f_\alpha \end{array} \qquad (18)$$

Choosing the functions f_{ψ_a}, f_{ψ_b}, f_ω, f_α as follows

$$\begin{array}{rcl} f_{\psi_a} & = & k_\psi\tilde{\psi}_a \\ f_{\psi_b} & = & k_\psi\tilde{\psi}_b \\ f_\omega & = & \gamma_\omega\left(\tilde{\psi}_a\hat{\psi}_b - \tilde{\psi}_b\hat{\psi}_a\right) - \frac{M}{JL_r}\left(\tilde{\psi}_a i_b - \tilde{\psi}_b i_a\right) \\ f_\alpha & = & \gamma_\alpha\left(\tilde{\psi}_a\left(\hat{\psi}_a - Mi_a\right) + \tilde{\psi}_b\left(\hat{\psi}_b - Mi_b\right)\right) \end{array} \qquad (19)$$

where k_ψ, γ_ω and γ_α are strictly positive constants, it can be proven analogously to [16] that the asymptotic convergence of the estimation errors to zero is guaranteed. This is done introducing the Lyapunov function candidate

$$V = \frac{1}{2}\left(\tilde{\psi}_a^2 + \tilde{\psi}_b^2 + \frac{\tilde{\omega}^2}{\gamma_\omega} + \frac{\tilde{\alpha}^2}{\gamma_\alpha}\right) \qquad (20)$$

and proving that

$$
\begin{aligned}
\dot{V} &= \tilde{\psi}_a\dot{\tilde{\psi}}_a + \tilde{\psi}_b\dot{\tilde{\psi}}_b + \frac{\tilde{\omega}\dot{\tilde{\omega}}}{\gamma_\omega} + \frac{\tilde{\alpha}\dot{\tilde{\alpha}}}{\gamma_\alpha} \\
&= -(\alpha + k_\psi)\left(\tilde{\psi}_a^2 + \tilde{\psi}_b^2\right) - \frac{K_f\tilde{\omega}^2}{J\gamma_\omega} \\
&< 0
\end{aligned}
\tag{21}
$$

V. Analysis of the sensorless scheme

Now the stability properties of the overall sensorless scheme will be analyzed. The sliding mode based current observer, independently from the used technique, converges if the variables $\hat{\psi}_a$, $\hat{\psi}_b$, $\dot{\hat{\psi}}_a$ and $\dot{\hat{\psi}}_b$ are bounded, and this happens because it has been proved that the dynamics of the observation errors $\tilde{\psi}_a$. $\tilde{\psi}_b$, $\tilde{\omega}$ and $\tilde{\alpha}$ is asymptotically stable, and then the values of their time derivatives are bounded. Based on the same arguments used in [19], the persistency of excitation can be guaranteed for variables $\hat{\alpha}$ and $\hat{\omega}$, making the origin a globally exponentially stable equilibrium point for the estimation errors $\tilde{\psi}_a$. $\tilde{\psi}_b$, $\tilde{\omega}$ and $\tilde{\alpha}$. In particular, as for the estimation of α, it is possible to write that

$$
\begin{aligned}
\dot{\tilde{\alpha}} &= \gamma_\alpha \begin{bmatrix} \hat{\psi}_a - Mi_a & \hat{\psi}_b - Mi_b \end{bmatrix} \begin{bmatrix} \tilde{\psi}_a \\ \tilde{\psi}_b \end{bmatrix} \\
&= \gamma_\alpha \Gamma(t) \begin{bmatrix} \tilde{\psi}_a \\ \tilde{\psi}_b \end{bmatrix}
\end{aligned}
\tag{22}
$$

and persistency of excitation is given if

$$
\int_t^{t+T} \Gamma(\tau)\Gamma^T(\tau)d\tau \tag{23}
$$

is positive definite for any $t \geq 0$ and $T > 0$, which is exactly our case. Analogous considerations can be done for the velocity estimation.

When using the observed values instead of the actual ones in the control system described in Section III, the sliding variables (7) and (8) now become

$$
s_{\omega 1} = \hat{\omega} - \omega^* \tag{24}
$$

$$
s_{\psi 1} = \hat{\psi}_d - \psi_d^* \tag{25}
$$

where $\hat{\psi}_d = \sqrt{\hat{\psi}_a^2 + \hat{\psi}_b^2}$. Considering that the observation errors $\dot{\tilde{\psi}}_a$. $\dot{\tilde{\psi}}_b$, $\dot{\tilde{\omega}}$, $\dot{\tilde{\alpha}}$ and their derivatives are bounded, sufficiently large values of $\mathcal{I}_d$ and $\mathcal{I}_q$ can guarantee the convergence to zero in a finite time of the sliding variables (24) and (25). Note that these variables will coincide asymptotically with (7) and (8) for the convergence properties of the nonlinear observer.

VI. Experimental results

Though the proposed control scheme was validated in simulation also, we show here directly some results obtained on an experimental setup. An induction motor with a nominal power of $0.75\ kW$ has been directly coupled with a direct current machine, which acts as a time-varying load. The rotor speed is measured using an incremental optic encoder, while the voltages and currents of the different phases are measured by Hall-type sensors. The acquired analog signals are converted to 12-bit digital with a sampling time of $10^{-4}\ s$. A Pentium IV personal computer equipped with Simulink Real-Time Workshop is used to implement the proposed schemes. The simulation results obtained in [14] were based on the data of this motor, and the results relative to the tracking of the velocity reference here obtained are not very different from them. In the sequel the performance of the observer, which of course was not present in [14], is shown.

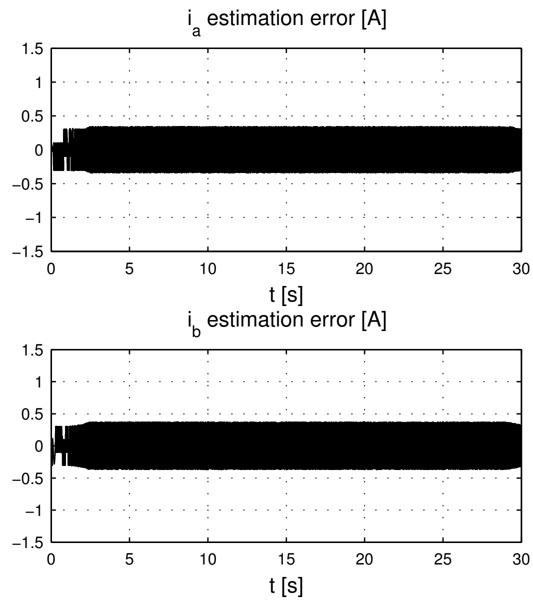

Fig. 2. Estimation errors of the currents $\tilde{i}_a$ and $\tilde{i}_b$ for the first order sliding mode scheme with equivalent control

The first implemented observer made use of the first order sliding mode control strategy, the signal generated by which has been filtered, according to the equivalent control method introduced in [10]. In Fig. 2 the time evolutions of the current estimation errors is shown, while Fig. 3 shows the time evolutions of the estimated rotor flux $\hat{\psi}_d$, the velocity estimate $\hat{\omega}$ and the velocity estimation error $\tilde{\omega}$. The super-twisting and the sub-optimal controllers have also been implemented, and analogous results to those shown for the first order controller are depicted in Figs. 4, 5, 6 and 7. All the methods seem to have similar performances, in particular the sub-optimal controller here proposed can significantly reduce the chattering amplitude on the current estimation errors.

VII. Conclusions

In this work a sensorless control scheme for induction motors is proposed. The flux and velocity controller are based on the sub-optimal second order sliding mode strategy, acting on a reduced order of the system. While the nonlinear observer, based on the full order model of the motor, exploit sliding mode techniques: all of them are shown to give appreciable results in order to estimate the rotor flux and the velocity. The proposed scheme is tested on an experimental setup, confirming its convergence properties.

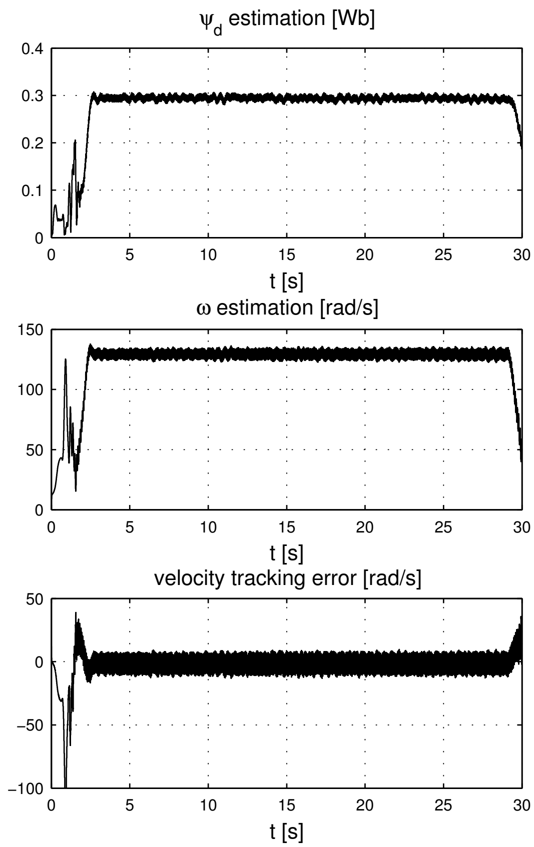

Fig. 3. Estimated rotor flux $\hat{\psi}_d$, velocity estimate $\hat{\omega}$ and velocity estimation error $\tilde{\omega}$ for the first order sliding mode scheme with equivalent control

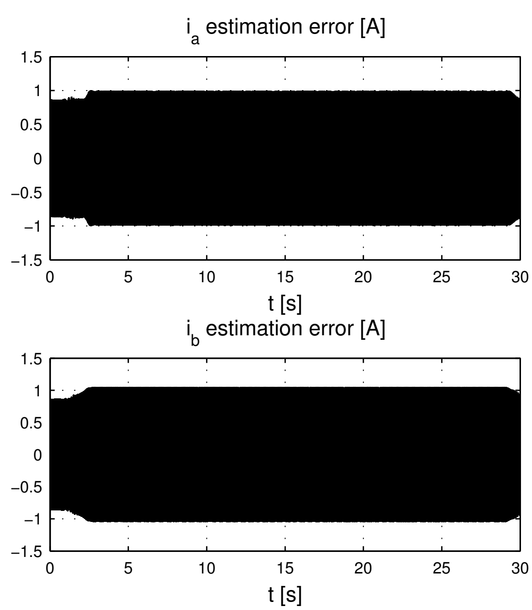

Fig. 4. Estimation errors of the currents $\tilde{i}_a$ and $\tilde{i}_b$ for the super twisting scheme

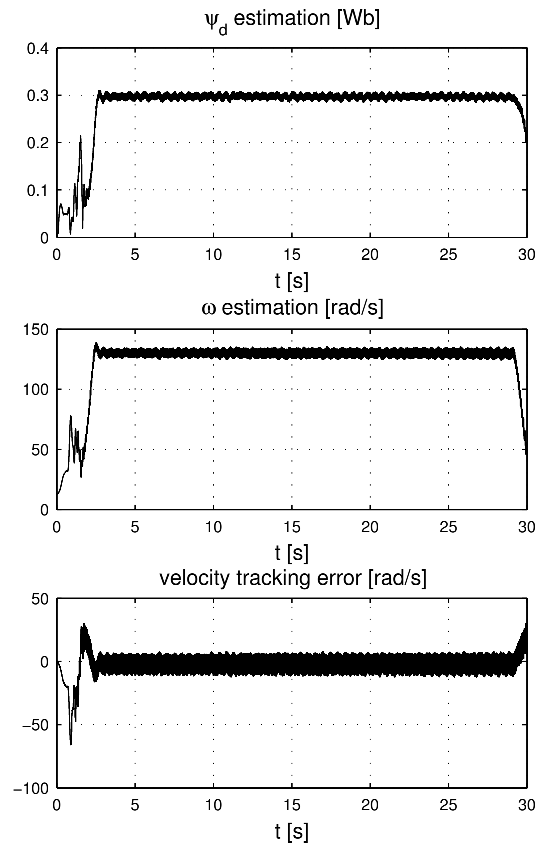

Fig. 5. Estimated rotor flux $\hat{\psi}_d$, velocity estimate $\hat{\omega}$ and velocity estimation error $\tilde{\omega}$ for the super twisting scheme

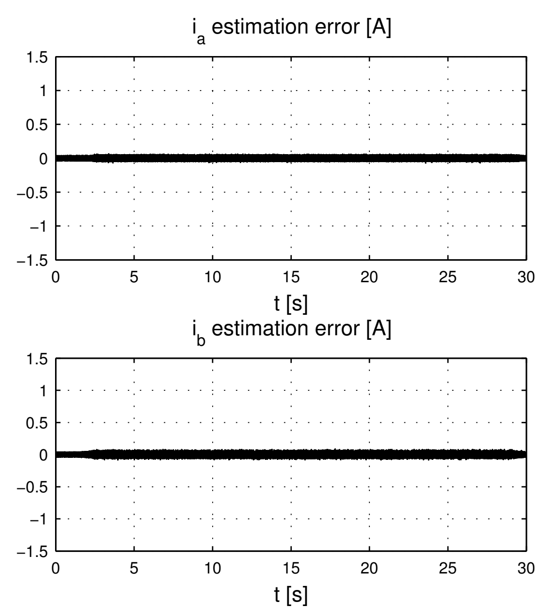

Fig. 6. Estimation errors of the currents $\tilde{i}_a$ and $\tilde{i}_b$ for the sub-optimal scheme

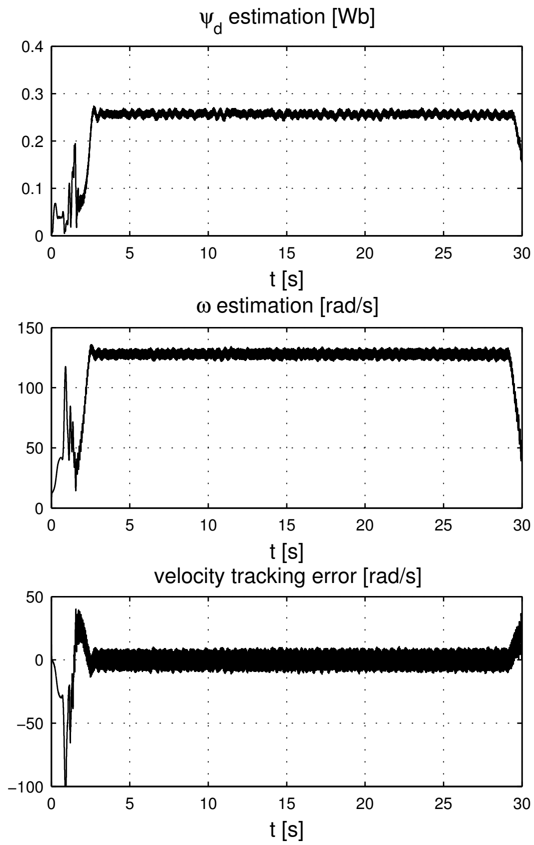

Fig. 7. Estimated rotor flux $\hat{\psi}_d$, velocity estimate $\hat{\omega}$ and velocity estimation error $\tilde{\omega}$ for the sub-optimal scheme

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