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Decentralized Cooperative Control of Autonomous Surface Vehicles with Uncertain Dynamics: A Dynamic Surface Approach

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***Abstract*— We study the cooperative control problem for a group of autonomous surface vehicles (ASV) with uncertain dynamics. A new decentralized cooperative controller is developed for a group of underactuated surface vehicles by employing the neural network-based dynamic surface approach, graph theory and Lyapunov stability theory. Using this design, it does not require to calculate the numerical derivatives of the virtual control signals as in traditional backstepping-based design. The advantages of the proposed cooperative controller are that, in addition to achieve a desired formation, the uncertain dynamics such as coriolis and centripetal force, hydrodynamic damping, unmodelled hydrodynamics, disturbances from environment can be compensated by on-line learning. An illustrative example is provided to demonstrate the effectiveness of the proposed approach.**

I. Introduction

Cooperative and coordinated control of the multi-agent (vehicle) systems have been an active subject in system and control, motivated by different applications in engineering, such as cooperative disaster search and rescue, coordinated resource exploration and exploitation, distributed environmental monitoring, situation awareness for military purpose and so on. To achieve a desired formation, several methods have been proposed, which include leader-follower strategy [1], virtual structure method [2], behavioral approach [3], graph theory-based method [4], [5], artificial potential mechanism [16] and so on.

Due to the limited sensing capabilities and communication bandwidth of autonomous agents, one of the most challenging problem in cooperative control is to design a decentralized control law to achieve global behavior using only local information. Many feedback control schemes have been suggested in the literature to achieve the desired formation [4], [5], [6], [7], [8], [9], [10]. In terms agent dynamics, these results are developed using identical single integrators [6], [10], double integrators [9], linear dynamics [4], [7] and fully-actuated nonholonomic integrators [8], [10]. In these studies, the emphasis is on the communication constraints rather than on the individual dynamics. The topology of the communication network plays a key role in the formation stabilization. These results are clean and elegant. However, most vehicles in the real world may have more complicated nonlinear dynamics as they undergo maneuvers in the hazardous environment.

In most cases, the cooperative control problem is reduced to an agreement or consensus problem of some variables of interest. This method has been wildly applied to cooperative control of ariel vehicles [14], ground vehicles [15] and surface vehicles. Decentralized cooperative control of surface vehicles has been studied in [17], [18], [19]. In [17], a decentralized coordination control scheme is proposed for a group of underactuated marine vehicles with communication constraints. In [18], a path following formation control scheme is proposed for surface vehicles where the path variables are synchronized using the passivity-based synchronization algorithm. In [19], the authors considered the coordinated path following problem of networked autonomous vehicles with discrete time periodic communications. One characteristics of these studies is that most of them typically use some variants of the model in [25], assuming that the model parameters are perfectly known or known with a small degree of uncertainty. In practice, it is quite hard to acquire the model parameters accurately, especially with hydrodynamic damping matrix. The presence of uncertain dynamics, in the form of functional uncertainties, unmodelled hydrodynamics and disturbances from environment, is a common problem. Therefore, how to achieve the cooperative control in the presence of uncertain dynamics needs to be further investigated.

Recently, in order to achieve cooperative control of multi-agent systems with uncertainties, some decentralized adaptive control algorithms have been proposed. In [13], the consensus problem of multi-agent systems with second-order nonlinear dynamics is solved by employing the backstepping technique and neural network (NN). In [12], a decentralized cooperative controller is proposed for multiple nonholonomic mobile robots with the aid of the Lyapunov stability theory, graph theory and adaptive backstepping. However, the traditional backstepping-based design suffers from the ”explosion of complexity” problem. It is noticed that these controllers are complicated for the sake of the needs to calculate the derivatives of virtual control signals. Especially, when the state information of the neighbors enter into the virtual control law, the ”explosion of complexity” problem is more serious though the system order is two.

To solve the above problems, in this paper we propose a new decentralized cooperative controller for underactuated ASVs, using NN-based dynamic surface control (DSC) approach [23], [24]. The DSC approach simplifies the controller

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design by introducing first-order filters, and is incorporated into neural network-based adaptive control design framework for systems in strict-feedback form in [24]. It is shown that our proposed controller can make a group of ASVs converge to a trajectory relative to a time-varying reference signal. Since the proposed algorithm only depends on the local information of its neighbors, it works in a distributed manner. In addition, by employing the NN approximation, the unknown dynamics such as the coriolis and centripetal force, hydrodynamic damping, unmodelled hydrodynamics and disturbances from environment are compensated by on-line learning. Compared with the existing results, the main contributions of this paper are as follows: First, the cooperative control problem for ASVs with uncertain dynamics is first considered and solved. Second, our NN-based DSC approach leads to a much simpler cooperative controller than traditional backstepping-based design. Third, the cooperative controller requires a minimum of system identification of vehicle model and shows robustness to uncertain dynamics.

This paper is organized as follows: Section II gives the problem formulation. Section III presents the NN-based DSC control design. Section IV gives the main stability results. An example to illustrate the proposed method is presented in Section V. Concluding remarks are given in section VI.

Notations: $\mathbb{R}^n$ denotes the n-dimensional Euclidean Space. $||\cdot||$ denotes the Euclidean norm. $||\cdot||_F$ denotes the Frobenius norm. $(\cdot)_{ij}$ denotes the element of $|\cdot|$ in the row i, column j. $\lambda_{min}(\cdot)$ denotes the smallest eigenvalue of a square matrix $(\cdot)$. $\otimes$ denotes the Kronecker product. Let $X = [x_1, ..., x_N]^T, Y = [y_1, ..., y_N]^T$. Then, we say $X \leq Y$ if and only if $x_i \leq y_i$, for all $1 \leq i \leq N$. $|X|$ Denotes $[|x_1|, ..., |x_N|]^T$.

II. Problem formulation

Consider a group of N underactuated ASVs, each of which has the following dynamics found in [25] with kinematics

$$\dot{\eta}_i = \begin{bmatrix} \cos\psi_i & -\sin\psi_i & 0 \\ \sin\psi_i & \cos\psi_i & 0 \\ 0 & 0 & 1 \end{bmatrix} \nu_i \tag{1}$$

and kinetics

$$M_i\dot{\nu}_i + C_i(\nu_i)\nu_i + D_i(\nu_i)\nu_i + g_i(\nu_i) = \tau_i + \tau_{iw} \tag{2}$$

where $\eta_i = [x_i, y_i, \psi_i]^T \in \mathbb{R}^3$ is the position vector in the earth-fixed reference frame; $\nu_i = [u_i, v_i, r_i]^T \in \mathbb{R}^3$ is the velocity vector in the body-fixed reference frame; $M_i \in \mathbb{R}^{3\times3}, C_i(\nu_i) \in \mathbb{R}^{3\times3}, D_i(\nu_i) \in \mathbb{R}^{3\times3}, g_i(\nu_i) \in \mathbb{R}^3$ denote the inertia matrix , coriolis/centripetal matrix, damping matrix and model uncertainties, respectively; $\tau_{iw} \in \mathbb{R}^3$ denotes the disturbances from environment ; $\tau_i = [\tau_{iu}, 0, \tau_{ir}]^T \in \mathbb{R}^3$ is the control vector with τ_{iu} the surge force and τ_{ir} the yaw moment.

In this paper, each ASV is assumed to know its own states and have access to the state information from a subset of vehicle group called neighbor set denoted by $\mathcal{N}_i \subseteq \{1, ..., N\} \setminus \{i\}$. If each ASV is considered as a node, the neighbor relation can be described by a graph $\mathcal{G} = \{\mathcal{V}, \mathcal{E}\}$, where $\mathcal{V} = \{n_1, ..., n_N\}$ is a node set and $\mathcal{E} = \{(n_i, n_j) \in v \times v\}$ is an edge set with element (n_i, n_j) that describes the communication from node i to node j. Further, the adjacency matrix $\mathcal{A} = (a_{ij})$ is a $N \times N$ matrix given by $a_{ij} = 1$, if $(n_i, n_j) \in \mathcal{E}$, and $a_{ij} = 0$, otherwise. For simplicity, we assume that the communications between ASVs are bidirectional, which means $a_{ij} = a_{ji}$. The Laplacian matrix $L = (l_{ij})$ associated to the graph $\mathcal{G}$ is defined as $l_{ij} = -a_{ij}$, if $j \neq i$, and $l_{ij} = \sum_{k=1}^N a_{ik}$, otherwise.

Let $q_i = [x_i, y_i]^T$ and given a desired relative position $\mathcal{P}$ described by $\rho_{ij} = \rho_i - \rho_j$ $(1 \leq i, j \leq N)$ and a time-varying reference trajectory q^r, the cooperative control problem is stated as follows:

Cooperative Control Problem: Given a team of N ASVs described by the model in (1) (2), design a control law τ_i for the ith ASV such that $q_i - \rho_i \to q_j - \rho_j \to q^r, 1 \leq i, j \leq N$, in the presence of uncertain dynamics. Moreover, each ASV i only has access to the local information from its neighbors.

Remark 1: In the literature, the vehicles are usually modeled as single integrators, double integrators or fully-actuated nonholonomic integrators. However, these simplified models may not be adequate to describe the practical dynamics of ASVs as they undergo the maneuvers on the widely-changing sea condition. This paper considers vehicles in the real world with practical model of underactuated ASV. To the best of our knowledge, there are very few studies that consider the cooperative problem of underactuated surface vehicles with uncertain dynamics.

In this paper, we make use of the Lemma 1 and the following assumptions:

Lemma 1: $\forall\delta > 0$, there exists a smooth function $\varphi(.)$ such that $\varphi(0) = 0$ and $|\chi| \leq \chi\varphi(\chi) + \delta, \forall\chi \in \mathbb{R}$.

Assumption 1: Assume that the communication graph $\mathcal{G}$ is undirected, fixed and connected.

Assumption 2: Assume that $|\tau_{iw}| \leq \tau_{iwd}$, where $\tau_{iwd} \in \mathbb{R}^3$ is a positive constant vector.

Assumption 3: The time-varying reference trajectory q_r is continuous, and $[q^r, \dot{q}^r, \ddot{q}^r]^T \in \Omega_d$ with a known compact set $\Omega_d = \{(q^r, \dot{q}^r, \ddot{q}^r) : \|q^r\|^2 + \|\dot{q}^r\|^2 + \|\ddot{q}^r\|^2 \leq B_0\} \subset \mathbb{R}^6$ whose size B_0 is a positive constant.

Definition 1: [21]Consider a system $\dot{x}_i = f(X) + d$, where $X = [x_1, ..., x_i, ..., x_n]^T, f : \mathbb{R}^n \to \mathbb{R}$ is a function and d is a disturbance term. For all bounded $x_j, j \neq i$ and d, if there exists a scalar function $V(x_i) \in C^1$ such that

1) $V(x_i)$ is globally positive definite and radially unbounded;
2) $\dot{V}(x_i) < 0$ if $|x_i| > b$, where b is a positive constant and its magnitude is related to the bounds of $x_j, j \neq i$ and d;

then, the variable x_i is passive-bounded.

Assumption 4: Assume that the sway velocity v_i is passive-bounded and satisfies $|\dot{v}_i| \leq v_d$ where v_d is a positive constant.

Remark 2: Passive-boundedness of the sway dynamics has been systematically analyzed considering different cases in [21]. This assumption is highly realistic in practice since the hydrodynamics damping force dominates in the sway direction and the sway speed is damped out by this force.

III. Cooperative Controller Design

In this section, we present the cooperative control design using the NN-based DSC approach. Before the cooperative controller design, we make the following state transformation. By assumption 4, the vehicle dynamics in (1) and (2) can be rewritten as

$$\begin{aligned} \dot{q}_i &= \omega_i + w_i && (3)\\ \psi_i &= r && (4)\\ \bar{M}_i\dot{\bar{\nu}}_i &= f_i(\nu_i) + \bar{\tau}_i + \bar{\tau}_{iw} && (5) \end{aligned}$$

where $q_i = [x_i, y_i]^T$, $\omega_i = [\omega_{i1}, \omega_{i2}]^T = [u_i\cos\psi_i, u_i\sin\psi_i]^T$, $\bar{\nu}_i = [u_i, r_i]^T$, $\bar{\tau}_i = [\tau_{iu}, \tau_{ir}]^T$, $w_i = [-v_i\sin\psi_i, v_i\cos\psi_i]^T$ and the function $f_i(\nu_i) = -\bar{C}_i(\nu_i)\nu_i - \bar{D}_i(\nu_i)\nu_i - \bar{g}(\nu_i)$ with the matrix $\bar{M}_i \in \mathbb{R}^{2\times 2}, \bar{C}_i \in \mathbb{R}^{2\times 3}, \bar{D}_i \in \mathbb{R}^{2\times 3}$. The cooperative controller design follows two steps.

A. kinematic control design

First, let $S_{i1} = q_i - \rho_i - q^r$ where q_r represents the desired trajectory. Recalling (3), we have

$$\dot{S}_{i1} = \omega_i + w_i - \dot{q}^r \tag{6}$$

Consider the virtual control $\bar{\omega}_i = [\bar{\omega}_{i1}, \bar{\omega}_{i2}]^T$ as

$$\bar{\omega}_i = -K_{i1}\sum_{j\in\mathcal{N}_i}(S_{i1} - S_{j1}) + \dot{q}^r - K_{i2}S_{i1} - w_i \tag{7}$$

where $K_{i1} > 0, K_{i2} > 0$. Let $\bar{\omega}_i = [\bar{u}_i\cos\bar{\psi}_i, \bar{u}_i\sin\bar{\psi}_i]^T$, then, the signals $\bar{u}_i$ and $\bar{\psi}_i$ can be solved by

$$\begin{aligned} \bar{\psi}_i &= atan2(\bar{\omega}_{i2}, \bar{\omega}_{i1}) + 2\beta\pi && (8)\\ \bar{u}_i &= \bar{\omega}_{i1}\cos\bar{\psi}_i + \bar{\omega}_{i2}\sin\bar{\psi}_i \end{aligned}$$

where $atan2(y, x)$ returns the arc tangent of y/x with a continuous range of $(-\pi, \pi]$. The integer β is chosen such that $\bar{\psi}_i$ is continuous. Introduce two new states ψ_{id} and u_{id} and let $\bar{\psi}_i$ and $\bar{u}_i$ pass through two first-order filters with time constants γ_{i1} and γ_{i2} to obtain ψ_{id} and u_{id}, respectively

$$\gamma_{i1}\dot{\psi}_{id} = \bar{\psi}_i - \psi_{id}, \gamma_{i2}\dot{u}_{id} = \bar{u}_i - u_{id} \tag{9}$$

Next, let $S_{i2} = \psi_i - \psi_{id}$, and from (4), it follows that

$$\dot{S}_{i2} = r_i - \dot{\psi}_{id} \tag{10}$$

Choose the virtual control law $\bar{r}_i$ as

$$\bar{r}_i = \dot{\psi}_{id} - k_{i2}S_{i2} \tag{11}$$

where $k_{i2} > 0$. Similarly, introduce a new state r_{id} and let $\bar{r}_i$ pass through a first-order filter with a time constant γ_{i3} to obtain r_{id}

$$\gamma_{i3}\dot{r}_{id} = \bar{r}_i - r_{id} \tag{12}$$

B. kinetic control design

Define $S_{i3} = [u_i - u_{id}, r_i - r_{id}]^T$ and from (5), we have

$$\bar{M}_i\dot{S}_{i3} = f_i(\nu_i) + \bar{\tau}_i + \bar{\tau}_{iw} - \bar{M}_i[\dot{u}_{id}, \dot{r}_{id}]^T \tag{13}$$

In (13), without the explicit knowledge of $\bar{C}_i, \bar{D}_i, \bar{g}$, $f_i(\nu)$ is an unknown function. Hence, we can use NN to approximate it as follows [22]:

$$f_i(\nu_i) = W_i^T\sigma(V_i^T\vec{\nu}_i) + \epsilon_i \tag{14}$$

where $\vec{\nu}_i = [\nu_i^T, 1]^T \in \mathbb{R}^4$, W_i, V_i are the weights of the NN and ϵ_i is the approximation error satisfying $\|\epsilon_i\| \le \epsilon_i^*$. By assumption 2, there exists a positive constant vector ϱ_i such that $|\bar{\tau}_{iw}| \le \varrho_i$ where $\varrho_i \in \mathbb{R}^2$ is a constant vector. Consider the kinetic control law as

$$\bar{\tau}_i = \bar{M}_i[\dot{u}_{id}, \dot{r}_{id}]^T - \hat{W}_i^T\sigma(\hat{V}_i^T\vec{\nu}_i) - K_{i3}S_{i3} - \varphi(S_{i3})\hat{\varrho}_i \tag{15}$$

where $\hat{W}_i, \hat{V}_i, \hat{\varrho}_i$ are the estimates of W_i, V_i, ϱ_i and are updated as

$$\begin{aligned} \dot{\hat{W}}_i &= \Gamma_{W_i}[(\hat{\sigma} - \hat{\sigma}'\hat{V}_i^T\vec{\nu}_i)S_{i3}^T - k_{W_i}\hat{W}_i]\\ \dot{\hat{V}}_i &= \Gamma_{V_i}[\vec{\nu}_i S_{i3}^T\hat{W}_i^T\hat{\sigma}' - k_{V_i}\hat{V}_i] && (16)\\ \dot{\hat{\varrho}}_i &= \Gamma_{\varrho_i}[\varphi(S_{i3})S_{i3} - k_{\varrho_i}\hat{\varrho}_i] \end{aligned}$$

where $K_{i3} > 0, k_{W_i} > 0, k_{V_i} > 0, k_{\varrho_i} > 0, \Gamma_{W_i} > 0, \Gamma_{V_i} > 0, \Gamma_{\varrho_i} > 0$.

Remark 3: By observing the form of the controller in (7) (11) (15), we can see that each ASV i only uses the information of its neighbors according to the communication graph. Therefore, the proposed control algorithm is decentralized.

Remark 4: By incorporating the DSC technique, our design leads to a much simpler cooperative controller than the traditional backstepping-based design. In fact, using the traditional backstepping-based method, the higher order derivatives of $\bar{\psi}_i, \bar{u}_i, \bar{r}_i$ have to appear in the kinetic control law $\bar{\tau}_i$. As a result, the expression of $\bar{\tau}_i$ would be much more complicated. Due to the multi-input multi-output characteristics of the system and the entering states of the neighbors into the virtual control (7), the "explosion of complexity" problem is serious in this case though the system order is two.

Remark 5: The vehicle kinetics in [20] only contains the linearly parameterized uncertainty, i.e, the uncertain parts of the kinetics are in form of $\theta^T f(.)$ where θ is an unknown constant and $f(.)$ is a known smooth function. Therefore, the adaptive control method given in [20] can not be applied to our case where the uncertain part $f_i(\nu_i)$ is totally unknown.

IV. Stability Analysis

In this section, we will analyze the stability of the closed-loop system. First, recalling the control law and the adaptive law in Section III, the closed-loop system can be rewritten as

$$
\begin{aligned}
\dot{S}_{i1} &= -K_{i1}\sum_{j\in\mathcal{N}_i}(S_{i1}-S_{j1})-K_{i2}S_{i1} \qquad (17)\\
&\quad +\omega_i-\bar{\omega}_i\\
\dot{S}_{i2} &= -k_{i2}S_{i2}+r_i-\bar{r}_i \qquad (18)\\
\bar{M}_i\dot{S}_{i3} &= -K_{i3}S_{i3}-\tilde{W}_i^T(\hat{\sigma}-\hat{\sigma}'\hat{V}_i^T\vec{\nu}_i)- \qquad (19)\\
&\quad \hat{W}_i^T\hat{\sigma}'\tilde{V}_i^T\vec{\nu}_i+\epsilon_i-d_i+\bar{\tau}_{iw}-\varphi(S_{i3})\hat{\varrho}_i
\end{aligned}
$$

In addition, define three new variables z_{i1}, z_{i2}, z_{i3} as

$$z_{i1}=\psi_{id}-\bar{\psi}_i, z_{i2}=u_{id}-\bar{u}_i, z_{i3}=r_{id}-\bar{r}_i \qquad (20)$$

Then

$$
\begin{aligned}
\dot{z}_{i1} &= \dot{\psi}_{id}-\dot{\bar{\psi}}_i=-\frac{z_{i1}}{\gamma_{i1}} \qquad (21)\\
&\quad +\zeta_{i1}(S_{k1},S_{l2},S_{l3},z_{l1},z_{l2},q^r,\dot{q}^r,\ddot{q}^r,\dot{w}_i)\\
\dot{z}_{i2} &= \dot{u}_{id}-\dot{\bar{u}}_i=-\frac{z_2}{\gamma_{i2}} \qquad (22)\\
&\quad +\zeta_{i2}(S_{k1},S_{l2},S_{l3},z_{l1},z_{l2},q^r,\dot{q}^r,\ddot{q}^r,\dot{w}_i)\\
\dot{z}_{i3} &= \dot{r}_{id}-\dot{\bar{r}}_i=-\frac{z_{i3}}{\gamma_{i3}} \qquad (23)\\
&\quad +\zeta_{i3}(S_{k1},S_{l2},S_{l3},z_{l1},z_{l2},z_{j3},q^r,\dot{q}^r,\ddot{q}^r,\dot{w}_i)
\end{aligned}
$$

where $k \in \mathcal{N}_i \cup (\cup_{k\in\mathcal{N}_i}\mathcal{N}_k), l \in \{i\}\cup\mathcal{N}_i$ and $\zeta_{i1}(\cdot),\zeta_{i2}(\cdot),\zeta_{i3}(\cdot)$ are continuous functions. The next theorem gives our main result.

Theorem 1: Consider the closed-loop system consisting of the vehicle dynamics in (1) (2), under the assumptions 1-4, and the control laws in (7) (11) (15), the adaptive laws in (16) and the first-order filters in (9) (12). For bounded initial conditions, there exist $K_{i1}>0, K_{i2}>0, k_{i2}>0, K_{i3}>0, \Gamma_{\varrho_i}>0, \Gamma_{W_i}>0, \Gamma_{V_i}>0, k_{\varrho_i}>0, k_{W_i}>0, k_{V_i}>0$, and $p>0$ satisfying $V\le p$, such that all the signals in the closed-loop system are bounded, and the steady-state tracking errors are smaller than the prescribed bounds.

Proof: Consider the following Lyapunov function candidate

$$
\begin{aligned}
V=&\frac{1}{2}\sum_{i=1}^{N}(\sum_{j=1}^{2}(S_{ij}^TS_{ij})+S_{i3}^T\bar{M}_iS_{i3}+\sum_{j=1}^{3}z_{ij}^2+\tilde{\varrho}_i^T\Gamma_{\varrho_i}^{-1}\tilde{\varrho}_i\\
&+tr\{\tilde{W}_i^T\Gamma_{W_i}^{-1}\tilde{W}_i\}+tr\{\tilde{V}_i^T\Gamma_{V_i}^{-1}\tilde{V}_i\})
\end{aligned}
$$

Recalling (17)(18) and (19), the time derivative of V satisfies

$$
\begin{aligned}
\dot{V}=&\sum_{i=1}^{N}(S_{i1}(-K_{i1}\sum_{j\in\mathcal{N}_i}(S_{i1}-S_{j1})-K_{i2}S_{i1}+\omega_i-\bar{\omega}_i)\\
&+S_{i2}^T(-k_{i2}S_{i2}+r_i-\bar{r}_i)+S_3^T(-K_{i3}S_{i3}\\
&-\tilde{W}_i^T(\hat{\sigma}-\hat{\sigma}'\hat{V}_i^T\vec{\nu}_i)-\hat{W}_i^T\hat{\sigma}'\tilde{V}_i^T\vec{\nu}_i+\epsilon_i-d_i+\bar{\tau}_{iw}\\
&-\varphi(S_{i3})\hat{\varrho}_i)+\sum_{j=1}^{3}z_j\dot{z}_j+\tilde{\varrho}_i^T\Gamma_{\varrho_i}^{-1}\dot{\hat{\varrho}}_i\\
&+tr\{\tilde{W}_i^T\Gamma_{W_i}^{-1}\dot{\hat{W}}_i\}+tr\{\tilde{V}_i^T\Gamma_{V_i}^{-1}\dot{\hat{V}}_i\})
\end{aligned}
$$

Since $u_i=(S_{i3})_{11}+\bar{u}_i+z_{i2}$ and $\psi_i=S_{i2}+\bar{\psi}_i+z_{i1}$, we obtain

$$
\begin{aligned}
(\omega_i-\bar{\omega}_i) &= \begin{bmatrix}((S_{i3})_{11}+\bar{u}_i+z_{i2})\cos(S_{i2}+\bar{\psi}_i+z_{i1})\\((S_{i3})_{11}+\bar{u}_i+z_{i2})\sin(S_{i2}+\bar{\psi}_i+z_{i1})\end{bmatrix}\\
&\quad -\begin{bmatrix}\bar{u}_i\cos(\bar{\psi}_i)\\ \bar{u}_i\sin(\bar{\psi}_i)\end{bmatrix} \qquad (24)
\end{aligned}
$$

Thus,

$$\|(\omega_i-\bar{\omega}_i)\|\le\zeta_{i4}(S_{l1},S_{i2},S_{i3},z_{i1},z_{i2},q_i^r,\dot{q}^r,w_i)$$

where $l\in\{i\}\cup\mathcal{N}_i$ and ζ_{i4} is a continuous function.

Using the Young's inequality, the following inequalities hold

$$
\begin{aligned}
S_{i2}(r_i-r_{id}) &\le S_{i2}^2+\frac{1}{4}S_{i3}^2 \qquad (25)\\
S_{i2}z_{i3} &\le S_{i2}^2+\frac{1}{4}z_{i3}^2 \qquad (26)\\
z_i\dot{z}_i &\le -\frac{z_i^2}{\gamma_i}+|z_i|\zeta_i\le-\frac{z_i^2}{\gamma_i}+z_i^2+\frac{1}{4}\zeta_i^2 \qquad (27)\\
S_{i3}^Td_i &\le \|S_{i3}\|^2+\frac{\|d_i^*\|^2}{4} \qquad (28)\\
S_{i3}^T\epsilon_i &\le \|S_{i3}\|^2+\frac{\|\epsilon_i^*\|^2}{4} \qquad (29)
\end{aligned}
$$

and by completion of squares, we have

$$
\begin{aligned}
-k_{W_i}tr\{\tilde{W}_i^T\hat{W}_i\} &\le -k_{W_i}\|\tilde{W}_i\|_F^2+\frac{k_{W_i}}{4}\|W_i\|_F^2 \qquad (30)\\
-k_{V_i}tr\{\tilde{V}_i^T\hat{V}_i\} &\le -k_{V_i}\|\tilde{V}_i\|_F^2+\frac{k_{V_i}}{4}\|V_i\|_F^2 \qquad (31)\\
-k_{\varrho_i}\tilde{\varrho}_i\hat{\varrho}_i &\le -k_{\varrho_i}\|\tilde{\varrho}_i\|^2+\frac{k_{\varrho_i}}{4}\|\varrho_i\|^2 \qquad (32)
\end{aligned}
$$

For any $B_0>0$ and $p>0$, the set $\Omega_d=\{(q^r,\dot{q}^r,\ddot{q}^r): \|q^r\|^2+\|\dot{q}^r\|^2+\|\ddot{q}^r\|^2\le B_0\}$ and the set $\Omega_1=\{\sum_{k\in\mathcal{N}_i\cup(\cup_{k\in\mathcal{N}_i}\mathcal{N}_k)}S_{k1}^2+\sum_{l\in\{i\}\cup\mathcal{N}_i}(S_{l2}^2+S_{l3}^T\bar{M}_lS_{l3}+\sum_{m=1}^{2}z_{lm}^2)\le 2p\}$, $\Omega_2=\{\sum_{k\in\mathcal{N}_i\cup(\cup_{k\in\mathcal{N}_i}\mathcal{N}_k)}S_{k1}^2+\sum_{l\in\{i\}\cup\mathcal{N}_i}(S_{l2}^2+S_{l3}^T\bar{M}_lS_{l3}+\sum_{m=1}^{3}z_{lm}^3)\le 2p\}$, $\Omega_3=\{\sum_{l\in\{i\}\cup\mathcal{N}_i}S_{l1}^2+S_{i2}^2+S_{i3}^T\bar{M}_iS_{i3}+\sum_{m=1}^{2}z_{im}^2\le 2p\}$ are compact set. In addition, by Assumption 3, $w_i,\dot{w}_i$ are all bounded. Thus, $\zeta_{ij}(j=1,2)$ has a maximum $\Pi_{ij}(j=1,2)$ on $\Omega_d\times\Omega_1$,ζ_{i3} a maximum Π_{i3} on $\Omega_d\times\Omega_2$ and ζ_{i4} has a maximum Π_{i4} on $\Omega_d\times\Omega_3$.

From above, we derive that

$$
\begin{aligned}
\dot{V}\le&-S_{*1}^T((\Lambda_1L+\Lambda_2)\otimes I_2)S_{*1}-\sum_{i=1}^{N}((k_{i2}-2)S_{i2}^2\\
&-(\lambda_{min}(K_{i3})-\frac{9}{4})\|S_{i3}\|^2-\sum_{j=1}^{2}(\frac{1}{\gamma_{ij}}-1)z_{ij}^2\\
&-(\frac{1}{\gamma_{i3}}-\frac{5}{4})z_{i3}^2-k_{W_i}\|\tilde{W}_i\|_F^2\\
&-k_{V_i}\|\tilde{V}_i\|_F^2-k_{\varrho_i}\|\tilde{\varrho}_i\|^2)+\pi
\end{aligned} \qquad (33)
$$

where $S_{*1}=[S_{11}^T,...,S_{N1}]^T\in\mathbb{R}^{2N}$,$\pi=\sum_{i=1}^{N}(\frac{1}{4}\sum_{j=1}^{4}\Pi_{ij}^2+\frac{1}{4}(\|d_i^*\|^2+\|\epsilon_i^*\|^2)+\varrho_i\delta+\frac{1}{4}k_{W_i}\|W_i\|_F^2+$

$\frac{1}{4}k_{V_i}\|V_i\|_F^2 + \frac{k_{\varrho_i}}{4}\|\varrho_i\|^2)$ and $\Lambda_1 \in \mathbb{R}^{N\times N}, \Lambda_2 \in \mathbb{R}^{N\times N}$ are diagonal matrixes with $\lambda_{min}(K_{i1}), \lambda_{min}(K_{i2}) - 1$ being the diagonal entries, respectively.

Choosing $\lambda_{min}((\Lambda_1 L + \Lambda_2) \otimes I_2) \geq \frac{\mu}{2}, k_2 \geq 2 + \frac{\mu}{2}, \frac{\lambda_{min}(K_{i3}) - \frac{9}{4}}{\lambda_{max}(M_i)} \geq \frac{\mu}{2}, \frac{1}{\gamma_{ij}} \geq 1 + \frac{\mu}{2}(j = 1, 2), \frac{1}{\gamma_{i3}} \geq \frac{5}{4} + \frac{\mu}{2}, \min\{\frac{k_{W_i}}{\lambda_{max}(\Gamma_{W_i}^{-1})}, \frac{k_{V_i}}{\lambda_{max}(\Gamma_{V_i}^{-1})}\} \geq \frac{\mu}{2}$ and substituting them into (33), it leads to

$$\dot{V} \leq -\mu V + \pi \tag{34}$$

Let $\mu > \frac{\pi}{p}$, then $\dot{V} \leq 0$ on $V = p$. Thus, $V \leq p$ is an invariant set,i.e, if $V(0) \leq p$, then $V(t) \leq p$ for all $t \geq 0$. Hence, (34) holds for all $V(0) \leq p, \forall t \geq 0$ and solving the inequality (34) gives

$$V \leq \frac{\pi}{\mu_i} + (V(0) - \frac{\pi}{\mu_i})e^{-\mu_i t} \tag{35}$$

where it shows that V is bounded by $\frac{\pi_i}{\mu_i}$, which means the signals $S_{i1}, S_{i2}, S_{i3}, \tilde{W}_i, \tilde{V}_i, z_{i1}, z_{i2}, z_{i3}, \tilde{\varrho}_i, 1 \leq i \leq N$ are all bounded. Since $S_{i1}, 1 \leq i \leq N$ is bounded, we derive that the consensus errors $q_i - \rho_i - q^r$ are bounded, which implies that $q_i - \rho_i \to q_j - \rho_j \to q^r, 1 \leq i, j \leq N$. Therefore, the cooperative control problem is solved.

Remark 6: In the foregoing theorem 1, it shows that with the proposed control and adaptive laws, all signals in the closed-loop system are bounded. Although bounded, the compact set to which the error signals converge can be made very small by choosing appropriate control parameters. For examples, by increasing the value of $K_{i1}, k_{i2}, K_{i2}, K_{i3}$, the value of μ_i can be made large, such that $\frac{\pi}{\mu_i}$ can be reduced to any prescribed value.

Remark 7: This paper considers each ASV has access to the reference trajectory q^r. However, the proposed design approach can be extended to the case where only a portion of ASVs have access to the reference trajectory q^r [11].

V. Simulation results

In this section, we simulate a scenario where five ASVs are required to maintain a desired star geometry while the formation centroid requires to follow a reference trajectory. We consider the nonlinear model of an experimental surface vehicle used in [21]. Without loss of generality, some model uncertainties and time-varying disturbance are introduced into the model, in particular, $\bar{g}_i = [0.0122uv + 0.0142v^2, 0, 0.0257ur + 0.0193r^2v]^T$ and $\tau_{iw} = [0.3 * \cos(0.5*t) + 0.1*\sin(0.3*t), 0, 0.001*\sin(t)]^T$. Let the control parameters be $\Gamma_{W_i} = 100, k_{W_i} = 0.1, \Gamma_{V_i} = 100, k_{V_i} = 0.1, K_{i1} = diag(0.1, 0.1), K_{i2} = diag(0.1, 0.1), k_{i2} = 2, K_{i3} = diag(51.6, 13.8)$ and the time constants for the filters be $\gamma_{i1} = 0.1, \gamma_{i2} = 0.1, \gamma_{i3} = 0.1$. In addition, $\hat{W}_i^T \sigma(\hat{V}_i^T \vec{\nu}_i)$ contain eight hidden neurons and the activation function is selected as $1/(1 + exp(-ax))$ with $a = 1$. Suppose the information-exchange topologies among the ASVs represented by L is given by

$$L = \begin{bmatrix} 1 & -1 & 0 & 0 & 0 \\ -1 & 2 & 0 & -1 & 0 \\ 0 & 0 & 1 & -1 & 0 \\ 0 & -1 & -1 & 3 & -1 \\ 0 & 0 & 0 & -1 & 1 \end{bmatrix} \tag{36}$$

and the reference trajectory is $q^r = [0.1*t; 5*\sin(\pi*t/50)]$

Simulation results are depicted in Fig 1-4. Fig 1 shows the entire formation geometries of the five ASVs with information-exchange given by (36). It can be observed that the star formation is well established despite the existence of the time-varying disturbances and unmodelled dynamics. The surge speed u_i and yaw angle ψ_i of the five ASVs are shown in Fig 2. By applying the proposed control algorithm, these variable are synchronized after several seconds. To verify the learning ability of NN, the uncertainties in the surge and yaw direction and outputs of NN related to ASV 1 are depicted in Fig 3. We notice that the uncertainties are efficiently compensated by NN. The control efforts τ_{iu}, τ_{ir} of the five ASVs are shown in Fig 4. It confirms that the control level is reasonable and no control saturation has occurred during the adaptation process.

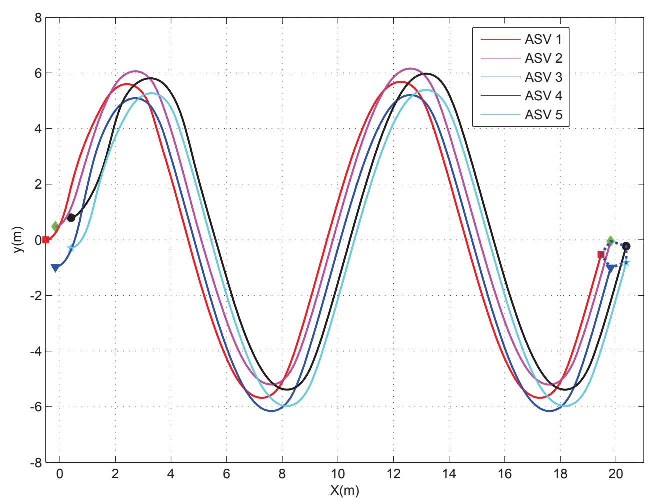

Fig. 1. Formation trajectories in 2D plane

VI. Conclusions

This paper considers the cooperative control problem for underactuated autonomous surface vehicles, in the presence of the uncertain dynamics. Compared with the existing results, the NN-based dynamic surface control approach shows some advantages to handle these uncertainties and avoids the computation of virtual control derivatives. Based on Lyapunov stability theory, the developed cooperative controller guarantees that all signals in the closed-loop system are bounded. Simulation results demonstrate the effectiveness of the cooperative controller and the learning ability of NN.

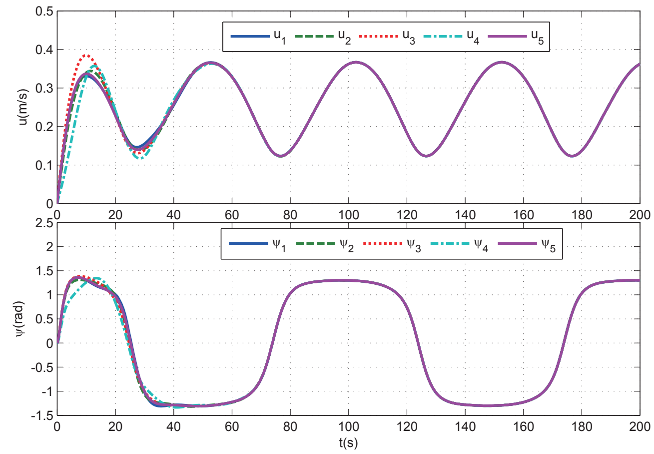

Fig. 2. Comparisions of the states of five ASVs

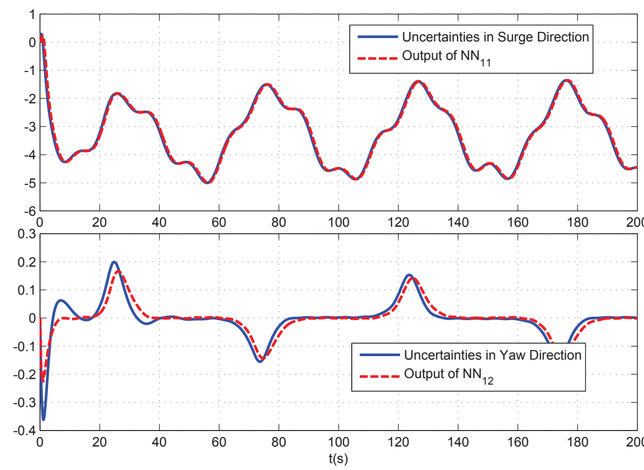

Fig. 3. NNs approximate the uncertain dynamics with respect to ASV 1

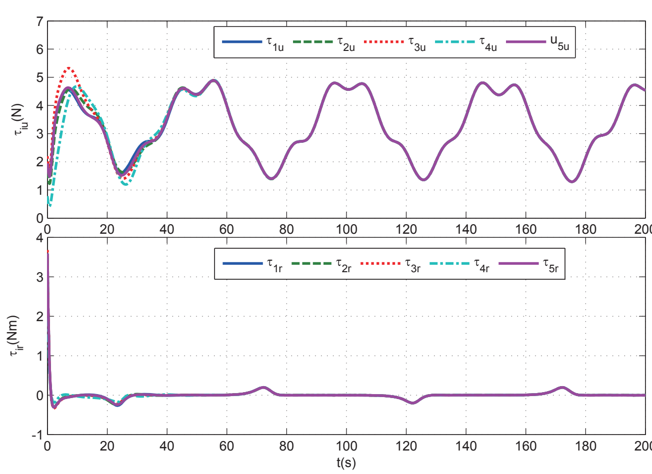

Fig. 4. Control signals

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